Transactions on Engineering Sciences vol 13, © 1996 WIT Press, www.witpress.com, ISSN 1743-3533

Application of a discontinuous strip yield model to multiple site damage in stiffened sheets

A. Yohannes,[a] D.J. Cartwright,[a] R.A. Collins[b]
[a]*Department of Mechanical Engineering, Bucknell University, Lewisburg, PA 17837, USA*
[b]*British Aerospace Airbus Ltd, Filton, Bristol, UK*

ABSTRACT

Stiffened panels are widely used in aircraft wing and fuselage structures. As a result of the cyclic nature of the loads associated with each flight, and material and manufacturing defects it is known that wide spread fatigue cracks may develop particularly around high stress regions such as fastener holes. This simultaneous development of relatively small fatigue cracks at multiple sites in the same structural element can give rise to their joining to form one large crack. This main crack, resulting from the multiple site damage (MSD) may cause immediate and complete failure of the structure.The effect of multiple site damage on the fracture strength of a stiffened sheet is examined using the Strip Yield Model of a crack. This model is relatively simple to apply and has contributed significantly to the fracture analysis of unstiffened structures and, to a more limited extent, to the analysis of stiffened structures. Following previous developments the MSD is represented by a discontinuous strip yield zone ahead of the main crack. The solution is extended in the present work to a crack in a stiffened sheet and results are determined for the crack tip opening displacement and the strip yield zone length for a crack in a typical MSD distribution. The model is used to compare the effect of MSD on the main crack for these structural variables.

INTRODUCTION

The Displacement Compatibility Method (DCM), has been used extensively (see citations in ref 1) for the analysis of cracked, reinforced sheets typical of those used in the aircraft industry. Applications of the DCM have proved both efficient and versatile for the determination of attachment forces and stress intensity factors in stiffened sheet structures. For high strength materials it is possible to use the principles of Linear Elastic Fracture Mechanics (LEFM) and to base the analysis on the Stress Intensity Factor of the crack. For medium and low strength materials where yielding at the crack tip may exceed the limits required for LEFM to apply it is necessary to use the principles of Non-linear Fracture Mechanics (NLFM). One method of doing this is to use a Strip Yield Zone (SYZ) model of the crack for which the fracture analysis is based on the Crack

Tip Opening Displacement (CTOD) or Energy Release Rate of the crack. The SYZ model [2] has been used in the analysis of reinforced sheets structures [3-5]. This previous work was restricted the a single lead crack in the stiffened sheet. The model has also been extended to the analysis of multiple cracks [6] in which the interaction between the multiple cracks, such as occurs in MSD, was accounted for by using the displacement and stress field for the single crack together with the crack line Greens Function for a single crack in a Schwarz alternating process. In the present work the MSD is represented by introducing regions of zero traction into the strip yield zone between which are regions of uniform traction having a magnitude equal to the yield strength of the material. This representation of the MSD enables the single SYZ model to be directly entended to take account of the effect of multiple site damage by having the strip yield zone distributed discontinuously. Each gap in the yield zone tractions corresponds to the region of damage and each junction corresponds to the yielded ligament between the damage sites. A similar approach, based on a dislocation model of the MSD, has been used [7]. This model was used to analyze the effect of MSD on a lead crack in an unstiffened sheet and then create a modified single SYZ model for the analysis of stiffened sheets. In the present work the MSD is represented as a discontinuous distribution of tractions for both the unstiffened and stiffened sheet.

THEORETICAL FORMULATION AND SOLUTION OF EQUATIONS

The analysis is based on the complex variable technique due to Muskhelishvili [8] which states that the stress/displacement state within a multiply connected, two dimensional body subjected to in-plane loading may be completely specified in terms of two complex stress functions which can be written in series form. The system equations are formed by satisfying equilibrium of forces at, and compatibility of displacements between the attachment points, and by satisfying the strip yield crack condition that the stresses be bounded at the tip of the yield zone (Dugdale Condition). The system equations are solved for the unknown distribution of attachment forces and the unknown ratio of the sheet stress to the yield stress of the sheet material.The CTOD of the crack is determined from the attachment forces and the applied stress ratio for a specified distribution of MSD in the yield zone.

CONFIGURATION STUDIED

The configuration to be studied in the present work is shown in Fig. 1. It consists of an infinite sheet containing a lead crack of length $2a$ centered at the origin of the coordinates ($x = 0, y = 0$). The lead crack has a series of of collinear cracks symmetrically located on either side of each tip which are joined by ligaments at the yield stress of the material the outer ones of which have strip yield zones at their tip. The region of MSD and the outer tip yield zones extend a distance s from the tip of the lead crack. The collinear cracks, which represent the MSD, are located under doubly riveted stiffeners with a stiffener across the centre of the lead crack. The sheet is subjected to a uniform stress σ perpendicular to the crack line.

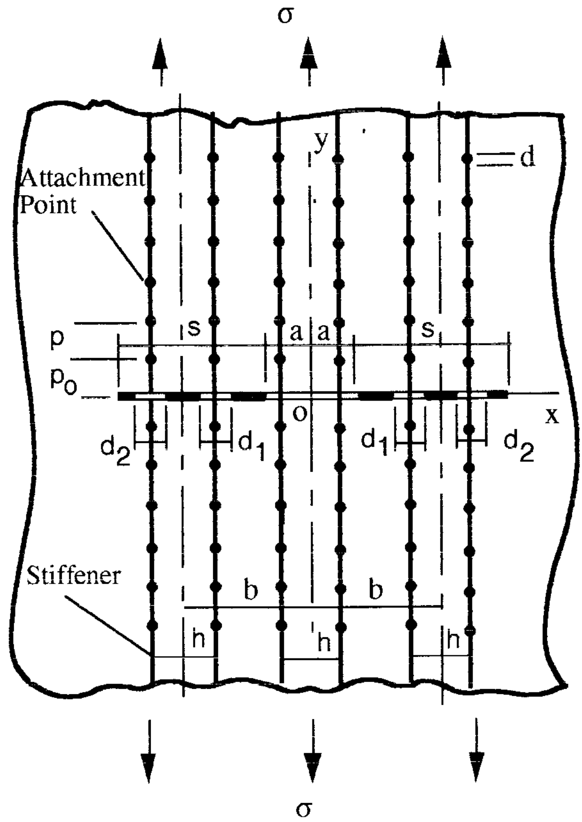

Fig. 1 Lead crack and Multiple Site Damage in a Uniformly Stressed Stiffened Sheet

In general the sheet is reinforced by arbitrarily spaced stiffeners parallel to the y-axis. The stiffeners are attached to the sheet at discrete points symmetrically either side of the x-axis. The first attachment point is p_0 from the x axis and all the other attachment points are a distance p apart . The attachment points are assumed to be represented by localised forces at the center of a rigid insert of diameter d. The sheet has a modulus of elasticity E, Poisson's ratio ν and thickness t. The Young's modulus and area of each stiffener is E_s and A_s respectively. The effect of the in-plane and out of plane bending stiffness of each stiffener is assumed negligible compared to its axial stiffness. In the present work only a symmetrical distribution of two equal length MSD cracks, each side of the lead crack, is considered although the formulation allows for an arbitrary length and position of the MSD cracks It is also assumed that the center of the MSD cracks are symmetrically located across the center line of the attachments.

The parameters used for modelling the stiffened sheet are given in Table 1 and represent a typical aircraft stiffened structure [3]. The stiffeners were attached at 15 points either side of the crackline along each rivet line.

Parameter	Magnitude
Sheet Modulus E	$73.8\ GNm^{-2}$
Sheet Yield Strength σ_Y	$386\ MNm^{-2}$
Stiffener Modulus E_s	$74.5\ GNm^{-2}$
Sheet Poissions Ratio υ	0.3
First Rivet Pitch p_o	38.1 mm
Rivet Pitch p	38.1 mm
Sheet Thickness t	18.1 mm
Stiffener Area A	$1710\ mm^2$
Rivet Diameter d	8 mm
Attachment Line Spacing h	58.4 mm
Stiffener pitch b	185.4 mm
Lead Crack Length a	148.2 mm
MSD Crack Size d_1 and d_2	10 mm

Table 1. Dimensions and Material properties for the stiffened sheet containing MSD

COMPARISION WITH KNOWN SOLUTIONS

Table 2 shows the crack tip opening displacement for the SYZ model normalised with that for a SYZ crack in an unstiffened sheet. The SYZ crack is modeled as a single crack ($d_1 = d_2 = 0$) and by simulating this single crack by modelling the SYZ with a longer strip yield zone and eliminating part of the zone with contiguous MSD regions to give the same strip yield length as the single crack model. It can be seen that these two models give identical results as is to be expected.

$\frac{a}{a+s}$	Unstiffened Sheet		Stiffened Sheet	
	Single Crack	Simulated Single Crack	Single Crack	Simulated Single Crack
0.1	1.000	1.000	0.086	0.086
0.5	1.000	1.000	0.191	0.191
0.9	1.000	1.000	0.291	0.291

Table 2 Comparision of the normalised crack tip opening displacement for a single SYZ crack in a sheet with that for a single crack simulated using the MSD model.

Transactions on Engineering Sciences vol 13, © 1996 WIT Press, www.witpress.com, ISSN 1743-3533

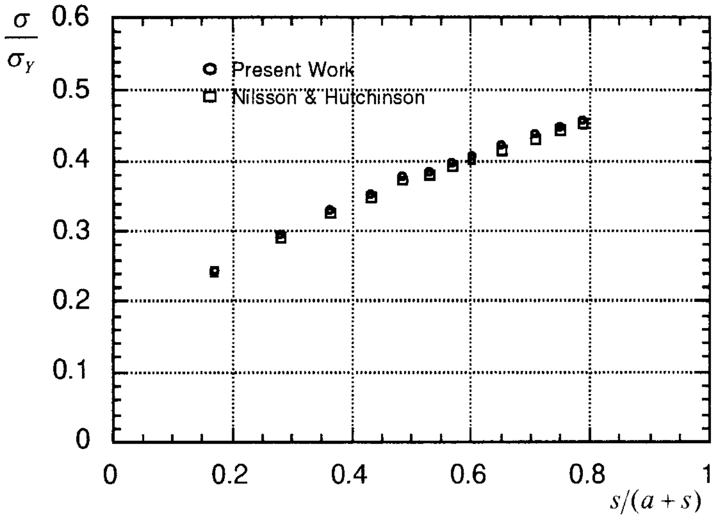

Fig 2 Comparision strip yield lengths

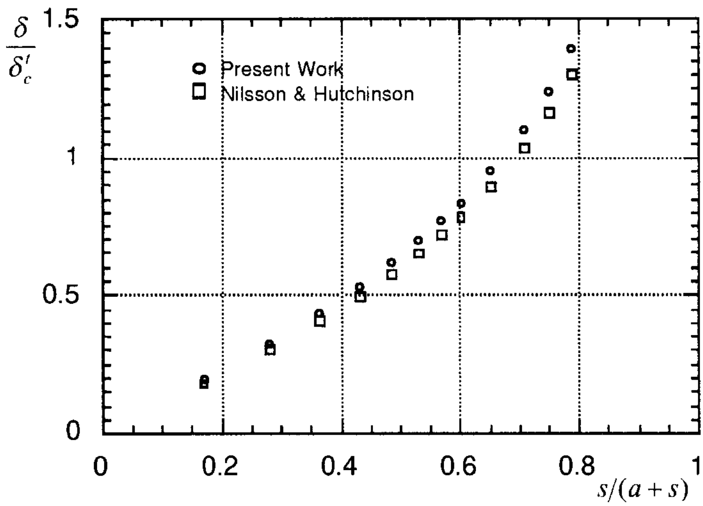

Fig 3. Comparision of normalised crack tip opening displacements

Transactions on Engineering Sciences vol 13, © 1996 WIT Press, www.witpress.com, ISSN 1743-3533

Figures 2 and 3 show a comparision of the present work with the results of the model developed [7], in which an approximate dislocation representation was used to model the MSD ahead of the outer tips of the plastic zone. This effect was estimated to be small [7] and was neglected in the present work. The value of the cricital crack tip opening displacement δ_c^t was taken to be 1.13 *mm* which is the same as that used for normalising the results given in [7].

STIFFENED SHEET WITH MULTIPLE SITE DAMAGE

Figures 3 and 4 show a comparision of the results for the configuration studied [2] and shown in fig.1. For both the stiffened and the unstiffened sheet the effect of the MSD is to increase the extent s of the damage region as the number of MSD sites increases. The results for two MSD sites are plotted for $s/(a+s) > 0.39$ since the yield zone tip must reach the outer MSD crack before it can for part of the discontinuous yield zone.

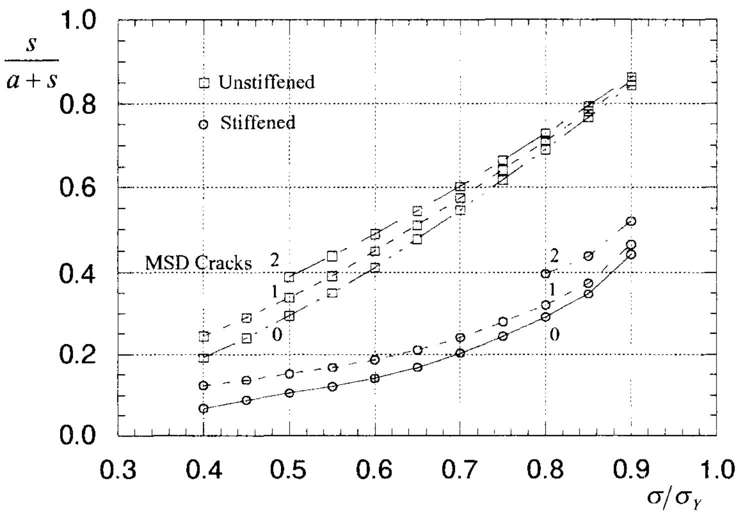

Fig 4. Normalised lengthof MSD region and Strip Yield Zone for an unsiffened and a stiffened Sheet

The normalised crack tip opening displacement for an unsiffened and a stiffened sheet containing MSD is shown in fig. 5. The crack opening displacement δ is normalised by δ_{ref} which is the crack tip opening displacement in the absence of MSD. For the unstiffened sheet results with MSD δ_{ref} is for the unstiffened sheet with no MSD and for the stiffened sheet with MSD δ_{ref} is for the stiffened sheet with MSD. It can be seen that the effect of the MSD, for the example presented, is to significantly increase the crack tip opening displacement of the unstiffened sheet. The effect of the MSD is less for the stiffened sheet but in

Transactions on Engineering Sciences vol 13, © 1996 WIT Press, www.witpress.com, ISSN 1743-3533

practice there will be yielding at the attachment points and the stiffened sheet will behave more like an unstiffened sheet at failure[3]. In the present model the rivets were assumed rigid for the purposes of developing the model but the theory and associated computer software was developed so that both stiffener and rivet yield can be introduced.

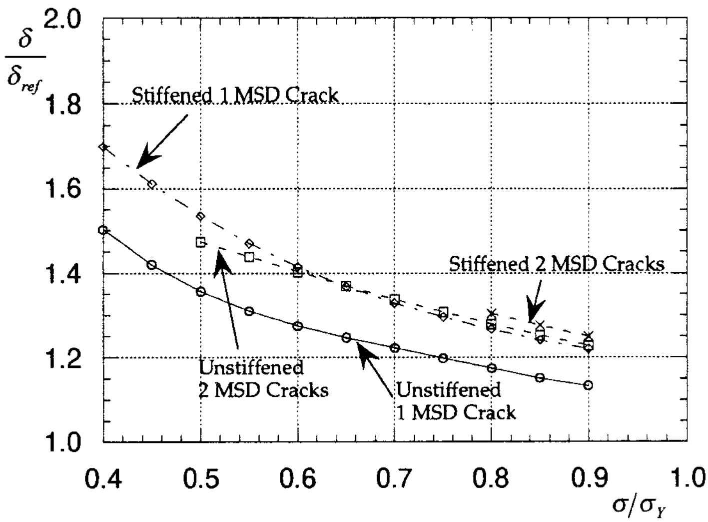

Fig 5. Normalised crack tip opening displacement for an unstiffened and a stiffened sheet containing MSD

CONCLUSIONS

The SYZ model has been extended to the analysis of stiffened sheet structures containing multiple site damage in which the ligaments between the MSD sites are modelled as entirely yielded or broken.

The model has been shown to give results that are consistent with a single strip yield crack for both a stiffened and an unstiffened sheet.

The model has been compared with one based on the representing the MSD with a dislocation model and it has been shown that the crack tip opening dislplacement and the size of the MSD region are in close agreement for the two models for an unstiffened sheet.

A typical aircraft configuration has been analysed with and without MSD present and the damage zone length and the crack tip opening displacement determined.

The model has been developed to include the effects of stiffener and rivet yielding and can also allow for ligament failure.

Transactions on Engineering Sciences vol 13, © 1996 WIT Press, www.witpress.com, ISSN 1743-3533

REFERENCES

1 Dowrick G, Cartwright, D.J. "Biaxial Stress Effects in a Reinforced Cracked Sheet", *Jl. Strain Analysis*, Vol 19, pp. 61-69, 1984.

2. Dugdale D. S., " Yielding of Steel Sheets Containing Slits" Journ. Mech. Phys. Solids, Vol 8, pp100-104, 1960.

3. Low M.C., Cartwright D.J., "Fracture Diagrams for Stiffened Aircraft Structures: Effect of Material Non-linearity,"*Proceedings of the 13th Symposium on Aeronautical Fatigue*, Pisa, Italy, (Eds A Salvetti Giorgio Cavallini), EMAS Ltd UK, pp. 551-583, 1985.

4. Cartwright D. J., T.P. Rich T. P., "A Strip Yield Crack in a Stiffened Sheet," *Proceedings of the 1st International Conference on Numerical Methods in Fracture Mechanics,* (Eds. D.R.J. Owen and A.R. Luxmoore), Pineridge Press, Swansea, pp. 550-561, 1978.

5. Rich T. P., Cartwright D. J., "Effect of Attachment and Stiffener Damage on Plastic Strip Yielding for a Crack in a Stiffened Sheet," *Proceedings of the 2nd International Conference on Numerical Methods in Fracture Mechanics*, (Eds. D.R.J. Owen and A.R. Luxmoore), Pineridge Press, pp. 385-400, 1980.

6. Nishimura T., "Strip Yield Analysis for Multiple Cracked Sheet with Riveted Stiffeners" Trans ASME, Journ Engng Mater and Technol, Vol 115, pp 398 - 403, 1993.

7. Nilsson K.F., Hutchinson J. W., "Interaction Between a Major Crack and a Small Crack Damage in Aircaft Sheet material" Division of Applied Sciences Report, Harvard University, August 1993.

8. Muskhelishvili N.I., *Some Basic Problems of Mathematical Theory of Elasticity*, Noordhoff,1953