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Application of acoustic Bessel beams for handling of hollow porous spheres

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ABSTRACT- Acoustic manipulation of porous spherical shells, widely used as drug delivery carriers and magnetic resonance imaging contrast agents, is investigated analytically. The technique used for this purpose is based on the application of high-order Bessel beams for as a single-beam acoustic manipulation device, using which particles lying on the axis of the beam can be pulled toward the beam source. The exerted acoustic radiation force is calculated using the standard partial-wave series method and the wave propagation within the porous media is modeled using Biot's theory of poroelasticity. Numerical simulations are performed for porous aluminum and silica shells of different thicknesses and porosities. Results have shown that manipulation of low-porosity shells is possible using Bessel beams with large conical angles, over a number of broadband frequency ranges; while that for highly porous shells generally occurs over some narrowband frequency domains at much smaller conical angles.

Keywords: Acoustic manipulation, Biot' theory, porous shell, microporous

INTRODUCTION

While there has been a great deal of research directed toward acoustic handling of rigid, elastic, and porous solid particles and shells by means of sonic beams (Marston 2006, 2007, 2009; Mitri 2008, 2009a, 2009b; Azarpeyvand 2012a), almost no pertinent investigation can be found for porous shells. Acquiring knowledge about the interaction of acoustic fields with spherical and cylindrical porous shells is of great importance since they are continuously finding new applications in various engineering and medical fields. For instance, periodically arranged cylindrical porous shells can be used as sonic crystal structures to suppress sound propagation for some frequency bands (Sanchez-Dehesa et al. 2011; Umnova et al. 2005). In micro and nano scales, porous shells have found numerous applications in modern medicine, pharmacology, biotechnology and chemistry. Porous shells are now widely used as drug delivery carriers (Slowing et al. 2008; Cheng et al. 2009; Andersson et al. 2004; Lai et al. 2003; Radu et al. 2004; Mal et al. 2003; Zhao et al. 2008), magnetic resonance imaging (MRI) contrast agents (Gao et al. 2008; Campbell et al. 2011; Davis 2002), etc. In the context of drug delivery carriers, the inner cavity of such particles can store large amounts of drugs, and the encapsulating porous shells provides delivery pathway for drug molecules diffusion. The porous shelled drug carriers are also shown to be mechanically more stable than some other drug carriers, such as those made of polymers, which have revealed a naturally burst release behavior (Jing et al. 2011). Another promising application area for porous shells is in MRI contrast agents, used as a coating for high-spin, but toxic or hazardous metals. Numerous core/porous-shell combinations have been tested, among which some, such as zeolite- or clay-enclosed Gadolinium complexes, magnetite/silica core-shell (Mag@SiO2) or FePt@Fe2O3 yolk-shell

nanoparticles, have shown encouraging results (Balkus et al. 1991; Balkus and Shi 1996a; Balkus and Shi 1996b).

The ability of contact-free handling, trapping and precise transporting of small suspended objects is essential in many fields of science and technology, such as bioengineering, chemical engineering, pharmaceutical sciences, *etc.* For manipulation of a suspended particle, a force must be applied on its body. This force can be produced optically, electrokinetically, hydrodynamically, or acoustically. In the latter case, manipulation can be achieved in two different manners, using either a standing-wave field or one single focused beam (Yamakoushi and Noguchi 1998; Liu and Hu 2009; Yasuda et al. 1995, Haake and Dual 2004; Wu 1991; van West 2007). In the standing-wave method, particles are subject to the mechanical force of a standing acoustic wave, generated by one transducer (and one reflector) or more ultrasonic transducers (van West 2007; Vandaele et al. 2005). In this paper, however, we shall confine our attention to the application of the second technique. In the single-beam technique, as the name implies, only one highly focused ultrasonic transducer is required, and particle handling becomes possible by producing a negative axial force, toward the source. It was shown by Chen and Apfel (1997) and Marston (2006, 2007, 2009) that for some material properties and beam types the acoustic radiation force for a spherical or cylindrical particle can change from repulsion to attraction. This, however, occurs only at certain frequencies and beam operating conditions. Although, much research has been conducted on the viability of using single acoustic beam devices, particularly Bessel beams, for handling of particles with different mechanical properties in various media, the research in this area has remained limited to very simple cases and has not yet led to an adequate understanding of the mechanism of particle

manipulation when more complex particles are of concern (Marston 2006, 2007, 2009; Mitri 2008, 2009a, 2009b; Azarpeyvand 2012a).

As demonstrated above, despite the growing attention now being given to different aspects of the application of porous shells, their dynamical behavior when illuminated by an acoustic beam has received very little research attention. In this study, we intend to extend the previous investigations by Marston (2006, 2007, 2009), Mitri (2008, 2009a, 2009b) and Azarpeyvand (2012a) to the more realistic case of porous shells. The rest of the paper proceeds as follows: The next section is dedicated to the mathematical modeling of the problem. The formulation of a helicoidal Bessel beam is presented and the radiation force formulations are derived. Biot's theory of motion in poroelastic media is presented and the relevant parameters are defined. Numerical results and discussions are presented for hollow aluminum and silica spheres with different shell thicknesses and porosities.

MATHEMATICAL FORMULATION

Let us consider a porous spherical shell with outer radius of a and inner radius of b ($h = b/a$). The particle is positioned on the beam axis and is submerged into and filled with linearly compressible, irrotational, nonviscous fluids. The density and the sound speed in the outer medium are denoted by ρ, and c, and those in the core medium by ρ^* and c^*, respectively. The shell is illuminated by a helicoidal Bessel beam, radiating at frequency f ($= \omega/2\pi$), with a conical (or half-cone) angle of β. A schematic of the problem is shown in Figure 1. In what follows, the Roman numerals *I*, *II*, and *III* designate, respectively, the surrounding medium, the porous shell medium, and the inner inclusion medium.

The incident Bessel beam, propagating in free-space and in the positive z direction, can be expressed in cylindrical coordinates (R, z, ϕ), as (Hernández-Figueroa et al. 2008):

$$\Phi^{(inc)}(R, z, \varphi) = \Phi_0 J_\vartheta(\zeta R) \, e^{i\vartheta\varphi + i\gamma z - i\omega t}, \tag{1}$$

where Φ_0 is the incident field amplitude, $\gamma = k\cos\beta$ and $\zeta = k\sin\beta$ are the longitudinal and traverse wavenumber components of the incident field, with $k = \omega/c$, and $J_\vartheta(\cdot)$ is the Bessel function of order ϑ (Abramowitz and Stegun 1972). The plane wave field can be restored by setting $\vartheta = 0$ and $\beta = 0$, while the beam vanishes if $\vartheta = 1$ and $\beta = 0$. It is interesting to note that an axially symmetric Bessel beam is essentially the result of the superposition of plane waves whose wave vectors lay on the surface of a cone having the propagation axis as its symmetry axis and an angle equal to β (conical angle) (Hernández-Figueroa et al. 2008). General intrinsic properties of Bessel beams, such as self-healing, diffraction-free, or phase singularity, and angular momentum for higher order Bessel beams, have been explained in (Azarpeyvand 2012a, 2012b; Hernández-Figueroa et al. 2008).

To obtain a closed-form solution to the problem using partial wave expansion method, it is necessary to re-express the incident field, Eq. (1), in the coordinate system of the particle. Using a standard wave transformation technique (Stratton 1941), one can rewrite the incident sound field in the spherical coordinate system (r, θ, φ),

$$\Phi^{(inc)}(r, \theta, \varphi) = \Phi_0 e^{-i\omega t} \sum_{l=0}^{\infty} i^l \mathcal{L}_{\vartheta l} \, j_{l+\vartheta}(kr) P_{l+\vartheta}^{\vartheta}(\cos\theta) e^{i\vartheta\varphi},$$

$$\tag{2}$$

with $\mathcal{L}_{\vartheta l} = (2l + 2\vartheta + 1)\frac{l!}{(l+2\vartheta)!} P^{\vartheta}_{l+\vartheta}(\cos \beta)$. In the above equation $j_q(\cdot)$ is the spherical Bessel function of order q, and $P^p_q(\cdot)$ is the associated Legendre function (Abramowitz and Stegun 1972).

The reflected sound wave, propagating radially outwards, may be represented in terms of a series of spherical Hankel functions and Legendre polynomials, as

$$\Phi^{(I)}(r,\theta,\varphi) = \Phi_0 e^{-i\omega t} \sum_{l=0}^{\infty} i^l \mathcal{L}_{\vartheta l} x_l h_{l+\vartheta}(kr) \, P^{\vartheta}_{l+\vartheta}(\cos \theta) e^{i\vartheta \varphi},$$

(3)

where $h_q(x) = j_q(x) + i y_q(x)$ is the spherical Hankel function of the first kind of order q, and j_q and y_q are the spherical Bessel function of the first and second kind, respectively. The unknown scattering coefficients x_l have to be determined by imposing appropriate boundary conditions at particle's inner and outer surfaces. This will be dealt with later.

As the incident wave interacts with the shell, part of the incident sound energy will be transmitted into the particle. In the case of a poroelastic medium, there exist two bulk compressional waves, known as the fast and slow compressional waves and one shear wave (Bourbie et al. 1987). Thus, the wave field within the porous shell can be described as,

$$\Phi^{(II)}_{fast}(r,\theta,\varphi) = \Phi_0 e^{-i\omega t} \sum_{l=0}^{\infty} i^l \mathcal{L}_{\vartheta l} [a_l j_{l+\vartheta}(k_f r) + b_l y_{l+\vartheta}(k_f r)] \, P^{\vartheta}_{l+\vartheta}(\cos \theta) \, e^{i\vartheta \varphi},$$

(4)

$$\Phi^{(II)}_{slow}(r,\theta,\varphi) = \Phi_0 e^{-i\omega t} \sum_{l=0}^{\infty} i^l \mathcal{L}_{\vartheta l} [c_l j_{l+\vartheta}(k_s r) + d_l y_{l+\vartheta}(k_s r)] \, P^{\vartheta}_{l+\vartheta}(\cos \theta) \, e^{i\vartheta \varphi},$$

(5)

$$\psi_{shear}^{(II)}(r,\theta,\varphi) = \Phi_0 e^{-i\omega t} \sum_{l=0}^{\infty} i^l \mathcal{L}_{\vartheta l}[e_l j_{l+\vartheta}(k_t r) + f_l y_{l+\vartheta}(k_t r)] P_{l+\vartheta}^{\vartheta}(\cos\theta) e^{i\vartheta\varphi},$$

(6)

where k_f, k_s and k_t are the fast, slow and shear wavenumbers, respectively, which will be derived later. Finally, since the core medium (III) is assumed to be an inviscid compressible fluid, the transmitted sound field in this medium can be characterized by a single scalar potential, as

$$\Phi^{(III)}(r,\theta,\varphi) = \Phi_0 e^{-i\omega t} \sum_{l=0}^{\infty} i^l \mathcal{L}_{\vartheta l} g_l \, j_{l+\vartheta}(k^* r) P_{l+\vartheta}^{\vartheta}(\cos\theta) e^{i\vartheta\varphi},$$

(7)

where $k^* = \omega/c^*$.

The wave propagation in a fluid-saturated porous media can be studied using Biot's theory, which itself is constructed using the equations of linear elasticity, Navier-Stokes equations, and Darcy's law for flow of fluid through the porous matrixes. Consider a homogenous, isotropic, porous solid of density ρ_s, with porosity (pore volume fraction) ϕ_0. The solid frame is saturated with an incompressible Newtonian fluid of density ρ_{fl} and saturating fluid viscosity η. For such a two-component material two vectors may be defined to describe the displacement of the skeletal frame ($\boldsymbol{u}$), and the fluid ($\boldsymbol{U}$). In simple words, a poroelastic problem consists of four constitutive relations, stress (σ_{ij}), strain (e_{ij}), pore pressure (p_p), and increment fluid content (ξ), given by (Bourbie et al. 1987)

$$\sigma_{ij} = (\lambda_f e - \beta_K M\xi)\delta_{ij} + 2\mu e_{ij},$$

(8-1)

$$p_p = M(\xi - \beta e),$$

(8-2)

$$e = \nabla \cdot \boldsymbol{u}, \ \varepsilon = \nabla \cdot \boldsymbol{U} \tag{8-3}$$

$$\xi = \nabla \cdot \boldsymbol{w} = -\phi_0(\varepsilon - e). \tag{8-4}$$

where μ is the shear modulus of the skeletal frame in vacuum. The parameter ξ gives the quantity of fluid that enters or leaves unit volume attached to the skeletal frame, and $e = \text{div } \boldsymbol{u}, \ \varepsilon = \text{div } \boldsymbol{U}$ being the dilatations of the solid and fluid phases, respectively, and

$$\lambda_f = K_f - \frac{2\mu}{3}, \beta_K = 1 - \frac{K_0}{K_s}, M = \left(\frac{\beta_K - \phi_0}{K_s} + \frac{\phi_0}{K_{fl}}\right)^{-1}, \tag{9}$$

where K_s is the bulk modulus of the material constituting the elastic matrix, K_0 is the bulk modulus of the dry skeleton (the explicit description will be given later in section III), K_{fl} is the bulk modulus of saturating fluid, and the bulk modulus of the closed system is given by

$$K_f = \frac{\phi_0\left(\frac{1}{K_s} - \frac{1}{K_{fl}}\right) + \frac{1}{K_s} - \frac{1}{K_0}}{\frac{\phi_0}{K_0}\left(\frac{1}{K_s} - \frac{1}{K_{fl}}\right) + \frac{1}{K_s}\left(\frac{1}{K_s} - \frac{1}{K_0}\right)}, \tag{10}$$

Combining Eqs. (8-1) through (8-4) with Darcy's law for flow through a porous medium, a pair of coupled displacement equations of motion can be obtained that govern the rotational and dilatational motions in poroelastic media (Deresiewicz 1960),

$$\rho_{11}\boldsymbol{u}_{tt} + \rho_{12}\boldsymbol{U}_{tt} + b(\omega)(\rho_{11}\boldsymbol{u}_t - \rho_{12}\boldsymbol{U}_t) = \nabla(\text{P}e + \text{Q}\varepsilon) - \mu\nabla^2\boldsymbol{u}, \tag{11-1}$$

$$\rho_{12}\boldsymbol{u}_{tt} + \rho_{22}\boldsymbol{U}_{tt} - b(\omega)(\rho_{11}\boldsymbol{u}_t - \rho_{12}\boldsymbol{U}_t) = \nabla(\text{Q}e + \text{R}\varepsilon), \tag{11-2}$$

where the subscript t denotes the time derivative, and

$$\text{P} = \lambda + 2\mu, \ \text{Q} = \phi_0 M(\beta_K - \phi_0), \ \text{R} = \phi_0^2 M, \tag{12}$$

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in which λ and μ denote Lame's moduli of the material, Q is a measure of the coupling between the volume change of the solid and of the liquid, and R, a measure of the pressure which must be exerted on the fluid to force a given volume of it into the aggregate with the total volume remaining constant (Deresiewicz 1960; Allard and Atalla 2009). The dynamical mass coefficients are defined as,

$$\rho_{11} = \rho_0 + \phi_0 \rho_{fl}(\alpha_\infty - 2), \qquad (13\text{-}1)$$

$$\rho_{12} = \phi_0 \rho_{fl}(1 - \alpha_\infty), \qquad (13\text{-}2)$$

$$\rho_{22} = \alpha_\infty \phi_0 \rho_{fl}, \qquad (13\text{-}3)$$

where α_∞ is the tortuosity of the porous medium, ρ_0 is the density of the fluid-saturated material, $i.e.$ $\rho_0 = (1 - \phi_0)\rho_s + \phi_0 \rho_{fl}$. In the equations of motion, (11), the parameter b is the viscosity coupling coefficient between both phases, defined as (Allard and Atalla 2009, Hasheminejad and Badsar 2004)

$$b(\omega) = \frac{\phi_0^2 \eta}{\kappa} F(\omega). \qquad (14)$$

The quantity $\phi_0^2 \eta / \kappa$ corresponds to the ratio of the total frictional force between fluid and solid, per unit volume of bulk material, and per unit average relative velocity in the steady-state flow (Poiseuille flow, that is at zero frequency), and κ characterizes the absolute permeability of the porous medium. The frequency dependent correction $F(\omega)$ is a measure of the deviation from Poiseuille-flow, (Allard and Atalla 2009, Azarpeyvand 2012a)

$$F(\omega) = \left(1 - i\frac{4\alpha_\infty^2 \kappa^2 \rho_{fl}\omega}{\eta \Lambda^2 \phi_0^2}\right)^{1/2}, \qquad (15)$$

with the viscous characteristic length defined as $\Lambda \approx \sqrt{8\alpha_\infty \kappa / \phi_0}$.

It is more convenient to solve Eqs. (11) by representing the velocity fields, $\boldsymbol{u}$ and $\boldsymbol{U}$ in terms of scalar and vector potentials. Four potentials are used, the scalar potentials ϕ and χ for the compressional waves in the solid and fluid medium, respectively, and two vectors potentials $\boldsymbol{\psi}$ and $\boldsymbol{\Theta}$, representing the transverse wave contributions in solid and fluid media. These potentials relate to $\boldsymbol{u}$ and $\boldsymbol{U}$ through (Zimmerman and Stern 1993),

$$\boldsymbol{u} = \nabla\phi + \nabla \wedge \boldsymbol{\psi}, \tag{16-1}$$

$$\boldsymbol{U} = \nabla\chi + \nabla \wedge \boldsymbol{\Theta}. \tag{16-2}$$

Insertion of the above field decomposition equations, (16), into equations of motion (11), the following pair of Helmholtz equations can be obtained for the compressional and shear wave components, (Allard and Atalla 2009; Zimmerman and Stern 1993)

$$\left(\nabla^2 + k_j^2\right)\phi_j = 0, \quad j = \{f, s\}, \tag{17}$$

$$(\nabla^2 + k_t^2)\psi = 0, \tag{18}$$

where the fast and slow wavenumbers ($k_{f,s}$) and shear wave-number (k_t) are defined as,

$$k_{f,s}^2 = \frac{\omega^2}{2(PR-Q^2)}\left[P\tilde{\rho}_{22} + R\tilde{\rho}_{11} - 2Q\tilde{\rho}_{12} \mp \sqrt{\Delta}\right], \tag{19}$$

$$k_t^2 = \frac{\omega^2}{\mu}\left(\frac{\tilde{\rho}_{11}\tilde{\rho}_{22}-\tilde{\rho}_{12}^2}{\tilde{\rho}_{22}}\right), \tag{20}$$

in which

$$\Delta = (P\tilde{\rho}_{22} + R\tilde{\rho}_{11} - 2Q\tilde{\rho}_{12})^2 - 4(PR - Q^2)(\tilde{\rho}_{11}\tilde{\rho}_{22} - \tilde{\rho}_{12}^2), \tag{21}$$

where the modified mass density functions are defined by $\tilde{\rho}_{ij} = \rho_{ij} - ib(\omega)/\omega$.

Using the displacement representation, Eqs. (16-1) and (16-2), and the coupled displacement equations of motion, Eqs. (11-1) and (11-2), one can easily show that the total sound field due to compressional components in the fluid and solid parts are given by:

$$\phi = \phi_f + \phi_s, \tag{22}$$

$$\chi = \mu_f \phi_f + \mu_s \phi_s, \tag{23}$$

where

$$\mu_{f,s} = \frac{(\tilde{\rho}_{11} R - \tilde{\rho}_{12} Q) - k_{f,s}^2 [PR - Q^2]}{(\tilde{\rho}_{22} Q - \tilde{\rho}_{12} R)}. \tag{24}$$

It can be readily shown that the shear wave component in the fluid is related to that of the solid frame through $\boldsymbol{\Theta} = \frac{\tilde{\rho}_{12}}{\tilde{\rho}_{22}} \boldsymbol{\psi}$.

In order to determine the eight unknown scattering coefficients involved in the modeling, Eqs. (3-7), eight boundary conditions must exist (Bourbie et al. 1987). The following boundary conditions must be satisfied on both the outer ($r = a$) and the inner ($r = b$) surfaces of the shell:

(1) $\sigma_{rr} = -p$, that is to show the compatibility of the normal stress with the acoustic pressure in the surrounding medium,

(2) $\sigma_{r\theta} = 0$, *i.e.* vanishing of the tangential stress component,

(3) $w_{r,t} = \phi_0 \left(U_{r,t} - u_{r,t} \right) = s_{r,t} - u_{r,t}$, which implies the continuity of the normal component of filtration velocity (subscript t designates the time derivative). Here, s_r is the ambient fluid particle displacement.

(4) $w_{r,t} = -\kappa_s(\mathrm{p} - \mathrm{p}_p)$, indicating the consistency of the pressure drop and the normal component of filtration velocity. Here, κ_s is the interface hydraulic permeability and can vary from zero (sealed interface or no flow) to infinity (open interface or zero pressure drop). The value of the interface permeability (κ_s), for both the inner and outer surfaces, is assumed to be close to zero, *i.e.*, almost sealed interface.

The strain-displacement relations in spherical coordinate are (Kausel 2006),

$$u_r = \frac{\partial\phi}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\psi\sin\theta),$$

$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} - \frac{1}{r}\frac{\partial}{\partial r}(r\psi),$$

$$U_r = \frac{\partial\chi}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\Theta\sin\theta),$$

$$U_\theta = \frac{1}{r}\frac{\partial\chi}{\partial\theta} - \frac{1}{r}\frac{\partial}{\partial r}(r\Theta). \qquad (25)$$

The pressure and stress components in poroelastic medium are also given by (Bourbie et al. 1987)

$$p_b = Mb_f k_f^2\phi_f + Mb_s k_s^2\phi_s,$$

$$\sigma_{rr} = a_f k_f^2\phi_f + a_s k_s^2\phi_s + 2\mu\frac{\partial u_r}{\partial r},$$

$$\sigma_{r\theta} = \frac{\mu}{r}\left(\frac{\partial u_r}{\partial\theta} + r\frac{\partial u_\theta}{\partial r} - u_\theta\right), \qquad (26)$$

where $a_{f,s} = -\lambda_f + \phi_0\beta_K M(1 - \mu_{f,s})$ and $b_{f,s} = \beta_K + \phi_0(\mu_{f,s} - 1)$.

Finally, substituting Eqs. (2-7) into the above four boundary conditions for the inner and outer surfaces, and using the field equations (25), and (26), we obtain a set of eight

linear equations for each vibrational mode (l,ϑ), at a given frequency, which can then be cast into a matrix form as $[R][X] = [I]$, where $[R]_{8\times 8}$ is the matrix of coefficients, given by

$$R = \begin{bmatrix}
\mathfrak{U}_{1,h}^{\Omega_l,1} & \mathfrak{Y}_{1,j}^{\Omega_f,1} & \mathfrak{Y}_{1,y}^{\Omega_f,1} & \mathfrak{Y}_{1,j}^{\Omega_s,1} & \mathfrak{Y}_{1,y}^{\Omega_s,1} & \mathfrak{Y}_{1,j}^{\Omega_t,1} & \mathfrak{Y}_{1,y}^{\Omega_t,1} & 0 \\
\mathfrak{U}_{2,h}^{\Omega_l,1} & \mathfrak{Y}_{2,j}^{\Omega_f,1} & \mathfrak{Y}_{2,y}^{\Omega_f,1} & \mathfrak{Y}_{2,j}^{\Omega_s,1} & \mathfrak{Y}_{2,y}^{\Omega_s,1} & \mathfrak{Y}_{2,j}^{\Omega_t,1} & \mathfrak{Y}_{2,y}^{\Omega_t,1} & 0 \\
\mathfrak{U}_{3,h}^{\Omega_l,1} & \mathfrak{Y}_{3,j}^{\Omega_f,1} & \mathfrak{Y}_{3,y}^{\Omega_f,1} & \mathfrak{Y}_{3,j}^{\Omega_s,1} & \mathfrak{Y}_{3,y}^{\Omega_s,1} & \mathfrak{Y}_{3,j}^{\Omega_t,1} & \mathfrak{Y}_{3,y}^{\Omega_t,1} & 0 \\
\mathfrak{U}_{4,h}^{\Omega_l,1} & \mathfrak{Y}_{4,j}^{\Omega_f,1} & \mathfrak{Y}_{4,y}^{\Omega_f,1} & \mathfrak{Y}_{4,j}^{\Omega_s,1} & \mathfrak{Y}_{4,y}^{\Omega_s,1} & \mathfrak{Y}_{4,j}^{\Omega_t,1} & \mathfrak{Y}_{4,y}^{\Omega_t,1} & 0 \\
0 & \mathfrak{Y}_{1,j}^{h\Omega_f,2} & \mathfrak{Y}_{1,y}^{h\Omega_f,2} & \mathfrak{Y}_{1,j}^{h\Omega_s,2} & \mathfrak{Y}_{1,y}^{h\Omega_s,2} & \mathfrak{Y}_{1,j}^{h\Omega_t,2} & \mathfrak{Y}_{1,y}^{h\Omega_t,2} & \mathfrak{U}_{1,j}^{h\Omega_{II},2} \\
0 & \mathfrak{Y}_{2,j}^{h\Omega_f,2} & \mathfrak{Y}_{2,y}^{h\Omega_f,2} & \mathfrak{Y}_{2,j}^{h\Omega_s,2} & \mathfrak{Y}_{2,y}^{h\Omega_s,2} & \mathfrak{Y}_{2,j}^{h\Omega_t,2} & \mathfrak{Y}_{2,y}^{h\Omega_t,2} & \mathfrak{U}_{2,j}^{h\Omega_{II},2} \\
0 & \mathfrak{Y}_{3,j}^{h\Omega_f,2} & \mathfrak{Y}_{3,y}^{h\Omega_f,2} & \mathfrak{Y}_{3,j}^{h\Omega_s,2} & \mathfrak{Y}_{3,y}^{h\Omega_s,2} & \mathfrak{Y}_{3,j}^{h\Omega_t,2} & \mathfrak{Y}_{3,y}^{h\Omega_t,2} & \mathfrak{U}_{3,j}^{h\Omega_{II},2} \\
0 & \mathfrak{Y}_{4,j}^{h\Omega_f,2} & \mathfrak{Y}_{4,y}^{h\Omega_f,2} & \mathfrak{Y}_{4,j}^{h\Omega_s,2} & \mathfrak{Y}_{4,y}^{h\Omega_s,2} & \mathfrak{Y}_{4,j}^{h\Omega_t,2} & \mathfrak{Y}_{4,y}^{h\Omega_t,2} & \mathfrak{U}_{4,j}^{h\Omega_{II},2}
\end{bmatrix}, \tag{27}$$

and the matrix of the unknown parameters $[X]_{8\times 1}$ and the incident field contribution $[I]_{8\times 1}$ are also written as

$$X = \begin{bmatrix} x_l \\ a_l \\ b_l \\ c_l \\ d_l \\ e_l \\ f_l \\ g_l \end{bmatrix}, \quad I = \begin{bmatrix} -\mathfrak{U}_{1,j}^{\Omega_l,1} \\ -\mathfrak{U}_{2,j}^{\Omega_l,1} \\ -\mathfrak{U}_{3,j}^{\Omega_l,1} \\ -\mathfrak{U}_{4,j}^{\Omega_l,1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{28}$$

where $\Omega_l = k_l a$, with $l = \{I, II, f, s, t\}$, and the auxiliary functions used in the above matrixes $(\mathfrak{Y}, \text{ and } \mathfrak{U})$ are defined in Appendix A. The unknown scattering coefficients can then be readily calculated using $[X] = [R]^{-1}[I]$. Once the known coefficients have been determined, relevant acoustic quantities, such as pressure, particle velocity, intensity, or acoustic radiation force can be computed.

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The acoustic radiation force acting on a particle can be calculated by performing an integration of the excess of pressure over the surface of the object. The average force vector is expressed as (Hasegawa 1979; Hasegawa et al. 1981)

$$\boldsymbol{F} = \rho \left\langle \int_{\mathcal{S}} \left[-(v_r \hat{\boldsymbol{n}} + v_\theta \hat{\boldsymbol{\theta}}) v_r + \frac{1}{2} |\boldsymbol{v}|^2 \hat{\boldsymbol{n}} - \frac{1}{2c^2} \Phi_t^2 \hat{\boldsymbol{n}} \right] d\mathcal{S} \right\rangle,$$

(29)

where $\mathcal{S}$ is the boundary at its equilibrium position, $\boldsymbol{v}$ is the first-order fluid particle velocity at the surface (*i.e.* $-\nabla \Phi_t$), v_r and v_θ are, respectively, the normal and tangential components of the fluid particle velocity, and Φ_t is real part of the total velocity potential at the boundary, $\text{Re}(\Phi^{(inc)} + \Phi^{(I)})$. A detailed derivation of the force vector and the radiation force function can be found in (Hasegawa 1979; Hasegawa et al. 1981). Subsequently, the axial radiation force F_z is found to be given by (Mitri 2009b)

$$F_z(\omega, \beta) = S_c E_c Y_m(\omega, \beta),$$

(30)

where S_c is the cross-sectional area (πa^2), $E_c = \frac{1}{2} \rho k^2 |\Phi_0|^2$ is the characteristic energy density, and the radiation force function on the surface of the target sphere is given by (Mitri 2009b)

$$Y_\vartheta(\omega, \beta) = -\frac{4}{(ka)^2} \sum_{l=0}^{\infty} \frac{\Gamma(l+2)}{\Gamma(l+2\vartheta+1)} \left[a_l^\vartheta + a_{l+1}^\vartheta + 2 \left(a_l^\vartheta a_{l+1}^\vartheta + \ell_l^\vartheta \ell_{l+1}^\vartheta \right) \right] P_{l+\vartheta}^\vartheta(\cos\beta) P_{l+\vartheta+1}^\vartheta(\cos\beta),$$

(31)

where $\Gamma(\cdot)$ denotes the Gamma function (Abramowitz and Stegun 1972), and the scattering coefficient components $\left(a_l^\vartheta, b_l^\vartheta\right)$ are defined as $a_l^\vartheta = \Re(x_l)$, and $b_l^\vartheta = \Im(x_l)$, where x_l have been obtained through $[X] = [R]^{-1}[I]$.

NUMERICAL RESULTS AND DISCUSSIONS

In order to gain a better understanding of the performance of the proposed acoustic manipulation device and the dynamical behavior of a porous shell when exposed to a helicoidal Bessel beam, some numerical examples are provided. Due to the large number of parameters involved in the present model, we shall restrict our attention to the emergence of negative radiation force (NRF), due to interaction with Bessel beam of $\vartheta = 0$ (zeroth-order) and $\vartheta = 1$ (first spinning mode), acting on a spherical aluminum shells with the outer radius $a = 1\ cm$. Simulations are performed for four shell thickness ratios ($h = b/a$), $h = 0$ (no void), 0.3, 0.6, and 0.95, and three porosities, $\phi_0 = 0$ (solid), 0.3, 0.9 (almost a decoupled two-phase system). We have chosen a relatively large particle to avoid thermal and viscous losses. In addition to aluminum, discussions are also provided for silica due to its wide range of applications and extensive use in drug delivery, gene transfection, and bio-sensing. The mechanical properties of aluminum and silica, required for our model, are presented in Table 1. The surrounding ambient medium and the inclusion medium are assumed to be water at atmospheric pressure and 300 kelvin, ($\rho = \rho^* = 997\ kg/m^3$, $c = c^* = 1497\ m/s$). A Matlab code is developed for treating the boundary conditions and to calculate the unknown scattering coefficients and the radiation force at selected beam half-cone angles (β) and incident wave frequencies ($ka = \omega a/c$). The computations are

performed on a dual-core personal computer with truncation constant of $N_{max}=30$ to assure the convergence of the simulation at high frequencies.

The mechanical properties of porous materials can change with porosity. The porosity dependence of some of the parameters used in Biot's model, such as the bulk and shear moduli and tortuosity, are provided here. It is assumed that the dependence of tortuosity on porosity is given by (Berryman 1980),

$$\alpha_\infty = 1 - r\left(1 - \frac{1}{\phi_0}\right), \tag{32}$$

such that $\alpha_\infty \to \infty$ as $\phi_0 \to 0$, and $\alpha_\infty \to 1$ as $\phi_0 \to 1$. In Eq. (32), r is a variable calculated from a microscopic model of a frame moving in a fluid and is here taken as ¼. Furthermore, the porosity dependency of the skeletal frame moduli (Young's modulus, E_b, bulk modulus, K_b, and rigidity modulus, μ_b) can be expressed in terms of material volume fraction, $(1 - \phi_0)$, and Young's modulus of the solid material of the frame, E_s, as (Wagh *et al.* 1991)

$$\frac{E_b}{E_s} = (1 - \phi_0)^n, \tag{33-1}$$

$$K_b = \frac{1}{3}E_b/(1 - 2v), \tag{33-2}$$

$$\mu_b = \frac{1}{2}E_b/(1 + v), \tag{33-3}$$

where n is constant and v denotes the Poisson ratio of the frame (approximately constant regardless of porosity). The value of the empirically determined exponent n depends on the geometrical structure of the solid matrix, and lies within the range of 0.5 to about 4, where $n = 1$ holds for any straight tubular pores (honeycomb structures), $n = 2$ for homogenous and isotropic cellular open-cell foams (such as aluminium foal), and $n = 3$ for close-cell foams (Scheffler and Colombo 2005; Ashby 1983; Gibson and Ashby 1982). In this study the value of the exponent n is set to be 2.

It has been shown before by Marston (2006, 2007, 2009), Mitri (2008, 2009a, 2009b) for liquid and elastic spherical objects, and more recently by Azarpeyvand (2012a) for poroelastic spheres that Bessel beams can produce negative axial forces on spherical particles if operated over some specific $ka - \beta$ regions, referred to as the negative radiation force (NRF) *islands*. To understand the emergence of such negative forces, the radiation force (Y_ϑ), Eq. (31), has been evaluated over a wide range of radiation frequencies, $0.1 < ka < 10$, and half-cone angles, $0^\circ < \beta < 90^\circ$. The dark islands seen in figures 2 through 7 show the regions where the radiation force reverses in direction and becomes negative. Such negative forces can pull the particle toward the source. It is worth mentioning that the results presented here for a non-hollow aluminum spheres and porous aluminum spheres ($h = 0$) in Figs. 2-a through 7-a are in excellent agreement with those presented in (Azarpeyvand 2012a), which shows the overall validity of the model.

Presented in Figs. 2 through 4 are the axial radiation force caused by a Bessel beam of zeroth-order ($\vartheta = 0$). Results demonstrate how the negative radiation force regions in the $ka - \beta$ plane change by varying the shell thickness at a given porosity. Figure 2 presents results for a non-porous aluminum shell with different thicknesses. An inspection of Figure 2 reveals that the particle manipulation can be accomplished in both broadband (over a wide range of frequencies) and narrowband ways (at resonance frequencies), depending on the geometrical and mechanical properties of the particle. Similar to the statements of Marston (2006, 2007, 2009), Mitri (2008, 2009a, 2009b), and Azarpeyvand (2012a) the only NRF island for a solid aluminum particle (*i.e.* $h = 0$, $\phi_0 = 0$) appears at low frequencies, $4.8 < ka < 5.3$, and high conical angles $78^\circ < \beta < 90^\circ$, *i.e.* small longitudinal wavenumbers ($\gamma \to 0$), see Figure 2a. Results also show that increasing the inner-to-outer radius ratio (h), *i.e.* the size of the inner void, leads to the emergence of some new NRF islands with wider effective conical angle ranges. The effective conical angle for a thin solid

(non-porous) aluminum shell of $h = 0.95$ has been found to be around $56^{\circ} < \beta < 90^{\circ}$, see Figure 2-d.

Figure 3 shows the results of NRF islands for a spherical shell with low porosity of $\phi_0 = 0.3$, due to the interaction with a zeroth-order Bessel beam. A comparison of Figs. 2 and 3 shows that increasing the porosity of thick shells ($h = 0$, and 0.3) to $\phi_0 = 0.30$ results in the shift of the dominant NRF island to lower frequencies (see Figures 3-a and 3-b), and the emergence of some high-frequency narrowband NRF islands for thinner shells ($h = 0.6$ and 0.95), see Figs. 3-c and 3-d. Figure 4 presents results of the emergence of NRFs on a highly porous object ($\phi_0 = 0.9$), in a zeroth-order Bessel beam. Results for shells with $h > 0.3$, Figs. 4-b through 4-d, reveal the emergence of some harmonic, high amplitude NRF islands at low frequencies, $0 < ka < 3$, while that for a highly porous sphere, Fig. 4-a, still occurs over some broadband islands. The effective conical angle ranges for highly porous shells have been found to be around $0^{\circ} < \beta < 60^{\circ}$, *i.e.* large longitudinal wavenumbers. The most striking observation here is the possibility of the occurrence of NRFs using a plane progressive wave, *i.e.* $\vartheta = 0$ and $\beta = 0^{\circ}$. This implies that for highly porous spherical shells, even a simple acoustic plane wave can cause negative axial forces. This is a new finding and had not been mentioned before in the literature and may have practical applications.

The results for acoustics handling of elastic and porous aluminum shells using a helicoidal Bessel beam of $\vartheta = 1$ (first spinning mode), are presented in Figs. 5-7. Presented in Fig. 5 are the results for NRF islands for a non-porous aluminum shell. It can be seen that the number of NRF islands increases compared to the case of a zeroth-order Bessel beam. Inspection of the results for a low porosity shell (Fig. 6) and a solid shell (Fig. 5) has shown considerable changes to the frequencies of the dominant NRF islands for shells with a small void ($h = 0$ and 0.3) and the emergence of multiple high-amplitude narrowband NRF

islands at high frequencies for thinner shells ($h = 0.6$ and 0.95). Similar to what observed before for the case of an ordinary Bessel beam ($\vartheta = 0$), results for highly porous shells, $\phi_0 = 0.9$, have shown that multiple harmonic low-frequency tonal NRF islands can be obtained with small effective conical angles of about $5^\circ < \beta < 88^\circ$, *i.e.* large longitudinal wavenumber ($\gamma \to k$).

In addition to aluminum, a series of simulations have also been performed for stiff silica shells illuminated by zeroth-order and first-order Bessel beams. Here, we shall only outline the main conclusions of these simulations. Results have shown that zeroth-order Bessel beams cannot produce NRF on solid and low porosity ($\phi_0 = 0.3$) thick silica shells ($h < 0.7$), as per Azarpeyvand (2012a). A broadband NRF island, however, emerges at low frequencies for solid and low porosity shells with $h > 0.7$ over $60^\circ < \beta < 90^\circ$, and its frequency range increases with h, reaching $1.15 < ka < 3.65$ for very thin shells ($h = 0.95$). The multiple narrowband NRFs, observed before for aluminum cases, here for stiff silica may only occur for very thin ($h > 0.9$) and highly-porous shells $\phi_0 = 0.9$. Results have also revealed that the sensitivity of the NRF islands to the inner void and its surface condition is less for stiff materials, such as silica and that the number and the shape of the NRF islands for silica shells show very little change over $0 < h < 0.6$.

CONCLUSION

Acoustic manipulation of porous shells using an ultrasound helicoidal Bessel beam has been considered. The exerted acoustic radiation force due to a zeroth-order and first order Bessel beams are calculated in an exact manner. Results have been provided for aluminum and stiff silica shells, at different porosities. Results have shown the feasibility of exerting negative radiation forces on porous shells at some specific frequency and beam conical angles ranges. It has also been shown that under some special circumstances a plane progressive sound field can also produce negative radiation force on thin highly porous shells, which is a new result and may have many practical applications.

AKNOWLEDGMENT

Appendix A

The auxiliary functions used in the coefficient matrixes, Eqs. (27) and (28), for convenience, are defined here:

$$
\mathfrak{Y}_{1,z}^{x_i,m} = \begin{cases} \dfrac{2\mu}{q^2}\left\{\left[(l+\vartheta)(l+\vartheta-1)-x_i^2+\dfrac{a_i}{2\mu}x_i^2\right]z_{l+\vartheta}(x_i)+2x_iz_{l+\vartheta+1}(x_i)\right\}, & i=\{f,s\} \\[2ex] \dfrac{2\mu}{q^2}(l+\vartheta)(l+\vartheta+1)[(l+\vartheta-1)z_{l+\vartheta}(x_i)-x_iz_{l+\vartheta+1}(x_i)], & i=t \end{cases}
$$

$$
\mathfrak{Y}_{2,z}^{x_i,m} = \begin{cases} (1-l-\vartheta)z_{l+\vartheta}(x_i)+x_iz_{l+\vartheta+1}(x_i), & i=\{f,s\} \\[2ex] \left[1-(l+\vartheta)^2+\dfrac{x_i^2}{2}\right]z_{l+\vartheta}(x_i)-x_iz_{l+\vartheta+1}(x_i), & i=t \end{cases}
$$

$$
\mathfrak{Y}_{3,z}^{x_i,m} = \begin{cases} \dfrac{i\omega}{q}\{[\phi_0(1-\mu_i)-1][(l+\vartheta)z_{l+\vartheta}(x_i)-x_iz_{l+\vartheta+1}(x_i)]\}, & i=\{f,s\} \\[2ex] \dfrac{i\omega}{q}\{(l+\vartheta)(l+\vartheta+1)[\phi_0(1-\alpha_0)-1]z_{l+\vartheta}(x_i)\}, & i=t \end{cases}
$$

$$
\mathfrak{Y}_{4,z}^{x_i,m} = \begin{cases} \dfrac{i\omega\phi_0}{q}(\mu_i-1)[(l+\vartheta)z_{l+\vartheta}(x_i)-x_iz_{l+\vartheta+1}(x_i)]+\dfrac{\overline{\kappa}_sMb_ix_i^2}{q^2}z_{l+\vartheta}(x_i), & i=\{f,s\} \\[2ex] \dfrac{i\omega\phi_0}{q}(\alpha_0-1)(l+\vartheta)(l+\vartheta+1)\,z_{l+\vartheta}(x_i), & i=t \end{cases}
$$

$$
\mathfrak{U}_{1,z}^{x,m} = i\omega\bar{\rho}z_{l+\vartheta}(x_i), \quad i=\{I,II\}
$$

$$
\mathfrak{U}_{1,z}^{x_i,m} = 0
$$

$$
\mathfrak{U}_{3,z}^{x_i,m} = \frac{1}{q}\{x_iz_{l+\vartheta+1}(x_i)-(l+\vartheta)z_{l+\vartheta}(x_i)\}, \quad i=\{I,II\}
$$

$$
\mathfrak{U}_{4,z}^{x_i,m} = -i\omega\bar{\rho}\overline{\kappa}_sz_{l+\vartheta}(x_i), \quad i=\{I,II\}
$$

where $z=\{j,y,h\}$, and the parameter m designates the boundary condition on the outer $(m=1)$, or on the inner surface $(m=2)$. If $m=1, q=a, \bar{\rho}=\rho, \bar{c}=c, \overline{\kappa}_s=\kappa_{s,r=a}$, and $x_i=\Omega_i=k_ia$ with $i=\{I,\ II,\ f,\ s,\ t\}$, and if $m=2, q=b, \bar{\rho}=\rho^*, \bar{c}=c^*, \overline{\kappa}_s=\kappa_{s,r=b}$ and $x_i=\Omega_i=hk_ia$, with $i=\{I,\ II,\ f,\ s,\ t\}$

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CAPTIONS

Figure 1- An acoustic Bessel beam incident on a porous spherical shell.

Figure 2- Negative radiation force regions; $\vartheta = 0$, solid aluminum shell ($\phi_0 = 0$).

Figure 3- Negative radiation force regions; $\vartheta = 0$, porous aluminum shell ($\phi_0 = 0.3$).

Figure 4- Negative radiation force regions; $\vartheta = 0$, porous aluminum shell ($\phi_0 = 0.9$).

Figure 5- Negative radiation force regions; $\vartheta = 1$, solid aluminum shell ($\phi_0 = 0$).

Figure 6- Negative radiation force regions; $\vartheta = 1$, porous aluminum shell ($\phi_0 = 0.3$).

Figure 7- Negative radiation force regions; $\vartheta = 1$, porous aluminum shell ($\phi_0 = 0.6$).