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Review Article

Application of AdS/CFT in Quark-Gluon Plasma

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We review some important applications of AdS/CFT correspondence to gain insight into properties of the quark-gluon plasma. We study some important quantities such as drag force, screening length, and jet-quenching parameter of an external probe quark and also quark-antiquark configuration. In particular, we focus on the STU background and compare our results with other important backgrounds.

1. Introduction

The relation between gauge theories and string theory has been the subject of many important studies in the last three decades. First, Maldacena [1] proposed the AdS/CFT correspondence therefore the AdS/CFT correspondence, sometimes called Maldacena duality. According to this conjecture there is a relation between a conformal field theory (CFT) in d-dimensional space and a supergravity theory in $(d + 1)$-dimensional anti-de Sitter (AdS) space. Maldacena suggests that a quantum string in $(d + 1)$-dimensional AdS space, mathematically, is equivalent to the ordinary quantum field theory with conformal invariance in d-dimensional spacetime which lives on the boundary of AdS_{d+1} space. The preliminary formulation of Maldacena is developed and completed by independent works of Witten [2] and Gubser et al. [3]. The famous example of AdS/CFT correspondence is the relation between type IIB string theory in $AdS_5 \times S^5$ space and $\mathcal{N} = 4$ super Yang-Mills gauge theory on the 4-dimensional boundary of AdS_5 space. For more studying about the AdS/CFT correspondence and its applications see [4–9]. One of the most interesting applications of the AdS/CFT correspondence is to study the quark-gluon plasma (QGP). The study of the QGP is a testing ground for finite temperature field theory. Such studies are important to understand the early evolution of our universe. Already, there are many attempts to study QCD by using gauge/gravity duality, where the $\mathcal{N} = 4$ super Yang-Mills (SYM) plasma is considered. The most important quantities of QGP are the shear viscosity, drag force, and jet-quenching parameter. The shear viscosity is one of the important hydrodynamical quantities of QGP which relates to the important thermodynamical quantity so-called entropy; in particular it is found that the ratio of shear viscosity η to the entropy density s had a universal value: $\eta/s = 1/4\pi$ [10–39]. However, for the several cases, this value may be modified [28–39].

The calculation of energy loss of moving heavy charged particle through a thermal medium known as the drag force [40–45]. The moving heavy quark in context of QCD has dual picture in the string theory where an open string was attached to the D-brane and stretched to the horizon of the black hole. Already the issue of the drag force was considered in the $\mathcal{N} = 4$ super Yang-Mills thermal plasma with several interesting backgrounds [40–50].

Another important property of the QGP is called the jet-quenching parameter ($\hat{q}$). The knowledge about this parameter increases our understanding about the QGP. In that case the jet-quenching parameter was obtained by calculating the expectation value of a closed light-like Wilson loop and using the dipole approximation. In order to calculate this parameter in QCD one needs to use perturbation theory. But, by using AdS/CFT correspondence, the jet-quenching parameter was calculated in nonperturbative quantum field theory. This calculations were already performed in the several interesting backgrounds [51–58]. In this paper we shall investigate some important properties of the QGP, specially in the STU background. The STU model is a special example of $D = 5$, $\mathcal{N} = 2$ gauged supergravity theory which is dual to the $\mathcal{N} = 4$ SYM theory with finite chemical potential. Also the solutions of $\mathcal{N} = 2$ supergravity may be solutions of supergravity theory with more supersymmetry. Already the duality between gravity and $\mathcal{N} = 2$ gauged theory was investigated and found that $\mathcal{N} = 2$ supergravity is an ideal laboratory [59–62]. Therefore, we may consider the STU model as a gravity dual of a strongly coupled plasma. Indeed, we review some of the previous results and also add some new things and collect all of them in this review article.

2. General Solutions

One can consider the following general line element [45]:

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{xx}\delta_{ij}dx^i dx^j, \tag{1}$$

where $i = 1, 2, \ldots, d-1$. The geometry of this general space-time is asymptotically AdS_{d+1}. Also, it is assumed to have a horizon at $r = r_h$, and g_{xx} is finite at horizon. Another important assumption is that the metric components depend only on the radial coordinate. The Hawking temperature of this black hole space-time, which is dual to the temperature of the field theory, is given by

$$T = \left.\frac{\sqrt{-(g_{tt})'(g^{rr})'}}{4\pi}\right|_{r=r_h}, \tag{2}$$

where prime denotes derivative with respect to the radius r. The entropy density of the field theory is given by

$$s = \left.\frac{(g_{xx})^{(d-1)/2}}{4G_N}\right|_{r=r_h}, \tag{3}$$

where G_N is gravitational Newton constant.

The open string is described by the following Nambu-Goto action:

$$S = -T_0 \int d\tau\, d\sigma \sqrt{-G}, \tag{4}$$

where T_0 is the string tension. The coordinates τ and σ are corresponding to the string world-sheet. Also, G is determinant of the world-sheet metric G_{ab}, which is obtained as the following:

$$\begin{aligned} -G &= \left(\dot{X}\cdot X'\right)^2 - \left(X'\right)^2\left(\dot{X}\right)^2 \\ &= -g_{rr}g_{tt} - g_{xx}g_{tt}\left(x'\right)^2 - g_{rr}g_{xx}(\dot{x})^2, \end{aligned} \tag{5}$$

where it is assumed that the string only extends in one direction $x(\tau,\sigma)$ and therefore string profile described by the $X(t,r,x)$. Hence, the equation of motion reads

$$\partial_r \frac{-g_{xx}g_{tt}x'}{\sqrt{-G}} - g_{rr}g_{xx}\partial_t \frac{\dot{x}}{\sqrt{-G}} = 0, \tag{6}$$

which yields to the following canonical momentum densities associated to the string

$$\begin{aligned} \pi_\mu^0 &= -T_0 g_{\mu\nu} \frac{\left(\dot{X}X'\right)\left(X^\nu\right)' - \left(X'\right)^2\left(\dot{X}^\nu\right)}{\sqrt{-G}}, \\ \pi_\mu^1 &= -T_0 g_{\mu\nu} \frac{\left(\dot{X}X'\right)\left(\dot{X}^\nu\right) - \left(\dot{X}\right)^2\left(X^\nu\right)'}{\sqrt{-G}}. \end{aligned} \tag{7}$$

These components are used to obtain the total energy and momentum of string by using the following integrals:

$$\begin{aligned} E &= -\int_{r_h}^{r_m} \pi_t^0 dr, \\ P &= \int_{r_h}^{r_m} \pi_x^0 dr, \end{aligned} \tag{8}$$

where r_m is location of $D7$-brane.

We will use above relations to obtain the drag force and the jet-quenching parameter in various backgrounds. Specially we focus on STU background introduced in the next section.

3. STU Background

In this paper we are interested in the special form of the $\mathcal{N} = 2$ supergravity in five dimensions. This model, which is called STU model, has generally 8-charged (4 electric and 4 magnetic) nonextremal black holes. However, there are many situations with less charges such as four-charged and three-charged black holes. In 5 dimensions the situation is different and actually much simpler; there is no distinction between BPS and non-BPS branches. So, in 5 dimensions, the three-charged configurations are the most interesting ones. The STU model admits a chemical potential for the $U(1)^3$ symmetry, and this makes it more interesting. For instance, presence of a baryon number chemical potential for heavy quark in the context of AdS/CFT correspondence yields to introducing a macroscopic density of heavy quark baryons. Indeed the STU model, which contains a nonextremal charged black hole, was obtained from five-dimensional gauged supergravity theory. Therefore, we may to consider the STU model as a gravity dual of a strongly coupled plasma. So, we begin with the three-charged nonextremal black hole solution in $\mathcal{N} = 2$ gauged supergravity, described by the following solution [63]:

$$ds^2 = -\frac{f_k}{\mathcal{H}^{2/3}}dt^2 + \mathcal{H}^{1/3}\left(\frac{dr^2}{f_k} + \frac{r^2}{R^2}d\Omega_{3,k}^2\right), \tag{9}$$

where

$$f_k = k - \frac{\mu}{r^2} + \frac{r^2}{R^2}\mathscr{H},$$
$$\mathscr{H} = \prod_{i=1}^{3} H_i,$$
$$H_i = 1 + \frac{q_i}{r^2}, \quad i = 1, 2, 3,$$
$$A_t^i = \sqrt{\frac{kq_i + \mu}{q_i}}\left(1 - H_i^{-1}\right), \tag{10}$$

where R is the constant AdS radius and relates to the coupling constant via $R = 1/g$ (also, coupling constant relates to the cosmological constant via $\Lambda = -6g^2$) and r is the radial coordinate along the black hole, so the boundary of AdS space was located at $r \to \infty$ (or $r = r_m$ on the $D7$-brane). The black hole horizon was specified by $r = r_h$ which is obtained from $f_k = 0$. In this background there are three real scalar fields, which is also solution of the metric (1), as $X^i = \mathscr{H}^{1/3}/H_i$, which satisfy the following condition: $\prod_{i=1}^{3} X^i = 1$. In another word, if we set $X^1 = S$, $X^2 = T$, and $X^3 = U$, then there is the STU $= 1$ condition. For the three R-charges q_i, in (2), there is an overall factor such as $q_i = \mu\sinh^2\beta_i$, where μ is called nonextremality parameter and β_i are related to the three independent electrical charges of the black hole. Finally, the factor of k indicates the space curvature, so the metric (9) includes a S^3 (three-dimensional sphere) for $k = 1$, a pseudosphere for $k = -1$, and a flat space for $k = 0$. So, for $k = 1$, $k = 0$, and $k = -1$, one can write, respectively,

$$d\Omega_{3,k}^2 \equiv \begin{cases} R^2\left(d\rho^2 + \sin^2\rho\, d\theta^2 + \sin^2\rho \sin^2\theta\, d\phi^2\right) \\ dx^2 + dy^2 + dz^2 \\ R^2\left(d\rho^2 + \sinh^2\rho\, d\theta^2 + \sinh^2\rho \sin^2\theta\, d\phi^2\right). \end{cases} \tag{11}$$

These are general properties of our interesting background which are used to study QGP by using AdS/CFT correspondence.

Here, we study thermodynamics of STU black hole and extract some important thermodynamical quantities such as temperature and entropy. Also we discuss dual picture of the STU model which is $\mathcal{N} = 4$ SYM with finite chemical potential.

First of all we compute some thermodynamical quantities in the STU model with three different black hole charges for the arbitrary spaces. Some of these quantities such as temperature and entropy will be useful to study QGP in the next sections. According to the previous works, the Hawking temperature of the black hole solution (9) will be as follows [63]:

$$T = \frac{r_h}{2\pi R^2}\frac{2 + \left(1/r_h^2\right)\sum_{i=1}^{3} q_i - \left(1/r_h^6\right)\prod_{i=1}^{3} q_i}{\sqrt{\prod_{i=1}^{3}\left(1 + q_i/r_h^2\right)}}. \tag{12}$$

There is also a chemical potential which is given by the following relation:

$$\phi_i^2 = q_i\left(r_h^2 + q_i\right)\left(\frac{1}{R^2 r_h^2}\prod_{j\neq i}\left(r_h^2 + q_j\right) + k\right). \tag{13}$$

Also, the entropy density in $d = 4$ dimension is given by the following expression, which is valid for $k = \pm 1$ and $k = 0$:

$$s = \frac{1}{4GR^3}\left(r_h^3\sqrt{\mathscr{H}\left(r_h\right)}\right), \tag{14}$$

where G is Newton's constant and relates to the AdS curvature as $G = \pi R^3/2N^2$, where N is the number of colors.

As we mentioned already, the STU solution (9) is dual to the $\mathcal{N} = 4$ SYM with finite chemical potential in Minkowski space. It can be shown by the following rescaling [63]:

$$r \longrightarrow \lambda^{1/4} r, \qquad t \longrightarrow \frac{t}{\lambda^{1/4}}, \qquad \mu \longrightarrow \lambda\mu,$$
$$q_i \longrightarrow \lambda^{1/2} q_i, \tag{15}$$

and taking $\lambda \to \infty$ limit while

$$d\Omega_{3,k}^2 \longrightarrow \frac{1}{R^2\lambda^{1/2}}\left(dx^2 + dy^2 + dz^2\right), \tag{16}$$

and also we set $r_0^4 \equiv \mu R^2$. Then, solution (9) reduces to the following:

$$ds^2 = e^{2A(r)}\left[-\frac{f}{\mathscr{H}^{2/3}}dt^2 + \mathscr{H}^{1/3}d\vec{X}^2 + \frac{\mathscr{H}^{1/3}}{f}dr^2\right],$$
$$f = \mathscr{H} - \frac{r_0^4}{r^4},$$
$$\mathscr{H} = \prod_i\left(1 + \frac{q_i}{r^2}\right), \tag{17}$$

where the geometric function $A(r)$ is defined as $A(r) \equiv \ln(r/L)$ and r_0 is the horizon radius in the $\mathcal{N} = 4$ SYM theory. In that case the chemical potential conjugate to the physical charge for the $U(1)$ R-charges is given by

$$\phi_i = \frac{r_h^2}{R^2}\frac{2q_i}{r_h^2 + q_i}\sqrt{\prod_j\left(1 + \frac{q_j}{r_h^2}\right)}. \tag{18}$$

This is dual expression of the chemical potential which is given by the relation (13). For the special case of $q_1 = q_2 = q_3 = q$ the Hawking temperature reads as

$$T_H = \frac{q + 2r_h^2}{2\pi R^2\sqrt{q + r_h^2}}, \tag{19}$$

where the radius of the horizon (root of $f = 0$) is given by

$$r_h^2 = \frac{1}{2}\left(\sqrt{4r_0^4 + q^2} - q\right). \tag{20}$$

4. Other Backgrounds

In this section we review some important backgrounds where the drag force and jet-quenching parameter are calculated by previous papers.

4.1. AdS Black Brane. It is possible to study QGP by using the metric of the AdS black brane solution in $d + 1$ dimensions which is given by [44]

$$ds^2 = \frac{dr^2}{R^2 f(r)} - R^2 f(r)\, dt^2 + \frac{r^2}{R^2}\delta_{ij} dx^i dx^j, \tag{21}$$

where

$$f(r) = \frac{r^2}{R^4}\left[1 - \left(\frac{r_h}{r}\right)^d\right]. \tag{22}$$

We can see that the black hole horizon is located at $r = r_h$, where $f(r)$ vanishes. It is clear that the metric (22) in $d = 4$ is indeed similar to the metric (17) with $q = 0$. This solution is corresponding to $\mathcal{N} = 4$ super Yang-Mills theory [46]. The Hawking temperature is given by

$$T = \frac{d}{4\pi R^2} r_h. \tag{23}$$

Also the entropy is obtained as the following:

$$s = \frac{1}{4G}\left(\frac{r}{R}\right)^{d-1}. \tag{24}$$

4.2. NR NC Yang-Mills. Another interesting background is the nonrelativistic, noncommutative Yang-Mills theory which is described by the following metric [64]:

$$\begin{aligned} ds^2 = \frac{r^2}{KR^2} \Big[& \left(1 - \beta^2 r^2 f\right) \left(dx^-\right)^2 - \left(1 + \beta^2 r^2\right) \\ & \times f\left(dx^+\right)^2 + 2\beta^2 r^2 f\, dx^- dx^+ \Big] \\ & + \frac{hr^2}{R^2}\left(\left(dx^2\right)^2 + \left(dx^3\right)^2\right) + \frac{R^2}{fr^2} dr^2, \end{aligned} \tag{25}$$

where the 5-sphere (S^5) part of the metric is neglected. Indeed, the metric (25) represents the AdS_5 space, and r denotes vertical direction to D-branes. In the above solution $K \equiv 1 + \beta^2 (r_h^4/r^2)$, and $R^2 = r_h^2 \sinh\varphi$; also r_h denotes the horizon radius, and φ is called the boost parameter; also β is a physical parameter related to the chemical potential of the Yang-Mills theory on the boundary. Moreover,

$$\begin{aligned} f &= 1 - \frac{r_h^4}{r^4}, \\ h &= \frac{1}{1 + a^4 r^4}, \end{aligned} \tag{26}$$

where

$$a^4 = \frac{1}{r_h^4 \sinh^2\varphi \cos^2\theta}. \tag{27}$$

$D3$-branes are lying along x^1, x^2, and x^3, and $D1$-branes are lying along x^1. The angle θ in the relation (26) measures the relative numbers of D-branes. So, for N $D3$-branes, and M $D1$-branes one can write $\cos\theta = N/\sqrt{N^2 + M^2}$.

In this configuration there is a large magnetic field in the $x^2 - x^3$ directions, so these directions satisfy the noncommutativity relation $[x^2, x^3] = i\Theta$, where Θ is called the noncommutativity parameter [65]. It has shown that $a^4 r_h^4 \sim \Theta^2$ [64], so the parameter a measures the noncommutativity. Also $\beta \to 0$ limit recovered the relativistic cases, so the parameter β specifies the nonrelativistic feature. The temperature of the nonrelativistic, non-commutative Yang-Mills theory is given by

$$T = \frac{r_h}{\pi R^2} = \frac{r_h}{\sqrt{2\hat{\lambda}}\pi\alpha'}, \tag{28}$$

where α' is the slop parameter ($\alpha' = 1/2\pi T_0$) and $\hat{\lambda}$ is the 't Hooft coupling of the nonrelativistic, non-commutative theory, which is related to the 't Hooft coupling of the ordinary Yang-Mills theory by the relation $\lambda = (\alpha'/\Theta)\hat{\lambda}$.

There are also other backgrounds which we can not represent here; we just try to recall results of them in the next sections.

5. Drag Force

Study of drag force on a moving heavy quark through a thermal plasma is interesting point to understand physics of charm and bottom quark at RHIC [66–68].

5.1. Single Quark. It is known that a moving quark in the $\mathcal{N} = 2$ thermal plasma corresponds to the stretched string from $r = r_m$ on the D-brane to the black hole horizon. So, calculating the energy loss of a heavy quark is reduced to find components of momentum density along the string. Detailed calculations of drag force for special case of STU model moved to Appendix A. These calculations are similar for other backgrounds. Therefore, here we just represent results of previous studies.

Drag force is obtained by using momentum components (see Appendix A) which in STU model is obtained as the following:

$$\begin{aligned} \frac{dP}{dt} &= \pi_x^1\Big|_{r=r_m} = -T_0 C v, \\ \frac{dE}{dt} &= \pi_t^1\Big|_{r=r_m} = T_0 C v^2. \end{aligned} \tag{29}$$

In order to find C we use reality condition for x'^2 and $\sqrt{-g}$; in that case one can fix the constant C as the following:

$$C = \left[\prod_{i=1}^{3}\left(1 + \frac{q_i}{r_c^2}\right)\right]^{1/3} \frac{r_c^2}{R^2}, \tag{30}$$

where

$$r_c = r_h + \left(\left(r^2 v^2 \mathscr{H} \right) \times \left(2R^2 \left[\frac{\mu}{r^3} + \frac{r\mathscr{H}}{R^2} - \left(\left(q_1 H_2 H_3 + q_2 H_1 H_3 + q_3 H_1 H_2 \right) \times \left(rR^2 \right)^{-1} \right) \right] \right)^{-1} \right)_{r=r_h} + \mathscr{O}\left(v^4\right). \quad (31)$$

It is important to note that this result is independent of curvature parameter, k; however, we should set $k = 1$ in the relation (10) to have $AdS_5 \times S^5$ space. Therefore the drag force may be written as

$$\frac{dP}{dt} = -T_0 v \left[\prod_{i=1}^{3} \left(1 + \frac{q_i}{r_h^2} \right) \right]^{1/3} \times \frac{r_h^2}{R^2} \left(1 + \mathscr{O}\left(v^2\right) \right). \quad (32)$$

It is clear that the black hole charge increases the value of drag force.

The drag force corresponding to the solution (21) was calculated by [44–46] and found that

$$\frac{dP}{dt} = -T_0 \frac{r_h^2}{R^2} \frac{v}{\left(1 - v^2\right)^{2/d}}. \quad (33)$$

It is clear that both expressions (32) and (33) are the same at $q = 0$ limit.

The drag force corresponding to the solution (25) was calculated by [64] and found that

$$\frac{dP}{dt} = -T_0 \frac{r_h^2}{R^2} \frac{v}{1 + a^4 r_h^4} = -T_0 \frac{r_h^2}{r_c^2} \frac{v}{\sinh\varphi \left(1 + a^4 r_h^4\right)}, \quad (34)$$

where the critical radius r_c is the root of the following:

$$\left(r^4 - r_h^4\right)\left(1 + a^4 r^4\right)\left(1 + \beta^2 r^2\right) - v^2 \left(r^4 + \beta^2 r_h^4 r^2\right) = 0. \quad (35)$$

It is easy to check that the special case of $\beta = a = 0$ with $\sinh\varphi = -\sqrt{1 - v^2}$ was reduced to (33).

5.2. Quark-Antiquark Configuration. Now, we consider a moving quark-antiquark pair which may be interpreted as a meson. Indeed there is a moving meson with the constant speed v in the $\mathscr{N} = 2$ supergravity thermal plasma. Already the energy of a moving quark-antiquark pair in $\mathscr{N} = 4$ SYM plasma was calculated [69]. Now, we would like to repeat same calculations in the STU background (see Appendix A). The quark-antiquark pair in the thermal QGP corresponds to an open string in AdS_5 space with two endpoints on the D-brane in the (X, Y) plan. Two end points of string on the D-brane represent quark and antiquark which are separated from each other by a constant l. We assume that at the $t = 0$ string is straight and two endpoints of string move with the constant velocity v along the X direction. The dynamics of such configuration is discussed in detail in [69] for the $\mathscr{N} = 4$ SYM plasma.

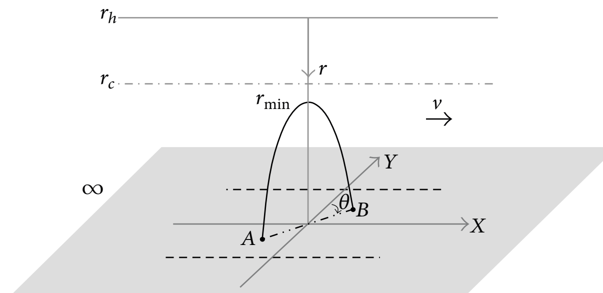

Figure 1: A rotating ∩-shape string dual to a $q\bar{q}$ pair which can be interpreted as a meson.

Figure 1 shows the general configuration of string. The points A and B in Figure 1 represent quark and antiquark with separating length l. The radial coordinate r varies from r_h (black hole horizon radius) to $r = r_m$ on D-brane. r_c is a critical radius, obtained for single quark solution, which the string cannot penetrate beyond it, and $r_{min} \geq r_c$. $r_{min} = r_c$ is satisfied if points A and B are located at origin ($l = 0$); in that case there is the straight string which is dual picture of the single static quark. θ is assumed to be the angle with Y-axis and the string center of mass moves along X-axis with velocity v. Solutions of this configuration satisfy boundary conditions $x(\infty, t) = vt \pm (l/2)\sin\theta$ and $y(\infty, t) = \pm (l/2)\cos\theta$, which reduced to the boundary condition of the case without rotational motion for $\theta = 0$.

By using calculations given by Appendix A one can obtain

$$\left(\pi_x^1\right)^2 = \frac{1}{2a}\left[\pm\sqrt{b^2 - 4ac} - b\right], \qquad \left(\pi_y^1\right)^2 = \frac{1}{2a'}\left[\pm\sqrt{b'^2 - 4a'c'} - b'\right], \quad (36)$$

where

$$a = \prod_i \left(1 + \frac{q_i}{r_{min}^2}\right)^{1/3} \cos^2\theta \left(\mathscr{R}_{min}^2 \varsigma + \xi\chi\right),$$

$$b = T_0^2 \prod_i \left(1 + \frac{q_i}{r_{min}^2}\right)^{2/3} \chi \times \left(2\mathscr{R}_{min}^4 \varsigma + \cos^2\theta \left(\mathscr{R}_{min}^2 \xi\chi - \varsigma\right)\right),$$

$$c = T_0^4 \prod_i \left(1 + \frac{q_i}{r_{\min}^2}\right) \mathscr{R}_{\min}^6 \sin^2\theta \varsigma \chi^2,$$

$$a' = \prod_i \left(1 + \frac{q_i}{r_{\min}^2}\right)^{2/3} \left(\mathscr{R}_{\min}^2 \varsigma + \xi\chi\right),$$

$$b' = T_0^2 \mathscr{R}_{\min}^2 \xi \left(\mathscr{R}_{\min}^4 \varsigma \prod_i \left(1 + \frac{q_i}{r_{\min}^2}\right)^{2/3} - 2\mathscr{R}_{\min}^2 \varsigma - \prod_i \left(1 + \frac{q_i}{r_{\min}^2}\right)^{2/3} \xi\chi\right),$$

$$c' = T_0^2 \prod_i \left(1 + \frac{q_i}{r_{\min}^2}\right)^{2/3} \mathscr{R}_{\min}^6 \varsigma \xi^2 \times \left(\prod_i \left(1 + \frac{q_i}{r_{\min}^2}\right)^{4/3} - T_0^2\right), \quad (37)$$

with

$$\varsigma = \prod_i \left(1 + \frac{q_i}{r_{\min}^2}\right) y^2 \dot{\theta}^2 \mathscr{R}_{\min}^2 \sin^2\theta \left(v + x\dot{\theta}\cos\theta\right)^2,$$

$$\xi = \frac{f_k(r_{\min})}{\prod_i (1 + q_i/r_{\min}^2)^{1/3}} - \prod_i \left(1 + \frac{q_i}{r_{\min}^2}\right)^{2/3} \times \left(v + x\dot{\theta}\cos\theta\right)^2 \mathscr{R}_{\min}^2, \quad (38)$$

$$\chi = \prod_i \left(1 + \frac{q_i}{r_{\min}^2}\right)^{1/3} y^2 \dot{\theta}^2 R_{\min}^2 \sin^2\theta - \frac{f_k(r_{\min})}{\prod_i (1 + q_i/r_{\min}^2)^{2/3}},$$

where $\mathscr{R}_{\min} = r_{\min}/R$ and $r_{\min}$ is the turning point. The direct consequence of rotational motion is that drag force is no longer constant. But, this result is not appropriate description of a meson. According to previous works [69] the $q\bar{q}$ pair should be close enough together and not moving too quickly. The presence of functions $x(r)$ and $y(r)$ is consequence of relativistic motion, which is not acceptable. On the other hand, because of nonvanishing drag forces, it is expected that the velocity of a $q\bar{q}$ pair decreases. So, we consider a moving heavy $q\bar{q}$ pair with non-relativistic speed, which rotates by angel $\theta = \omega t$ around the center of mass. Indeed this situation is corresponding to the motion of the heavy meson with large spin. Actually, in the very large angular momentum limit, a classical approximation is reliable. In this case, the angular velocity of the string is very small. Therefore, we are going to discuss the case of non-relativistic motion ($\dot{\theta}^2 \to 0$ and $\dot{\theta}v \to 0$). In that case $\varsigma = c = c' = 0$ and we have

$$\left(\pi_x^1\right)^2 = \frac{r_{\min}^2}{R^2} T_0^2 f_k(r_{\min}) \mathscr{H}^{-1/3}(r_{\min}),$$

$$\left(\pi_y^1\right)^2 = \frac{r_{\min}^2}{R^2} T_0^2 \left(f_k(r_{\min}) - \mathscr{H}(r_{\min}) \frac{r_{\min}^2 v^2}{R^2}\right) \mathscr{H}^{-1/3}(r_{\min}). \quad (39)$$

Now, we assume that $v^2 \to 0$ and angular velocity is infinitesimal constant ($\dot{\theta} = \omega \ll 1$), and the quark-antiquark pair rotates around origin. In that case we neglect ω^4 terms and obtain values of momentum densities as the following:

$$\pi_x^1 = \pi_y^1 = T_0 \frac{r_{\min}}{R} \left(\left[k - \frac{\mu}{r_{\min}^2} + \frac{r_{\min}^2}{R^2} \times \prod_i \left(1 + \frac{q_i}{r_{\min}^2}\right)\right]^{1/2} \times \left(\prod_i \left(1 + \frac{q_i}{r_{\min}^2}\right)^{1/6}\right)^{-1}\right). \quad (40)$$

6. Screening Length

Also it is possible to calculate screening length [70–74] for a quark-antiquark pair by using the following relations:

$$L_x = 2\int_{r_{\min}}^{\infty} dr x', \qquad L_y = 2\int_{r_{\min}}^{\infty} dr y'. \quad (41)$$

In the case of $q = 0$ limit one can obtain

$$L_x = 2\int_1^{\infty} dz \frac{\alpha\rho\left(\left(1 - v^2\right) z^4 - \rho^4\right)}{\pi T r_{\min} \sin\theta z^4 \left(z^4 - \rho^4\right)} f(z),$$

$$L_y = 2\int_1^{\infty} dz \frac{\beta\rho\left(\left(1 - v^2\right) z^4 - \rho^4\right)}{\pi T r_{\min} \cos\theta z^4} f(z), \quad (42)$$

where

$$f(z) = z^4 \left(T_0^2 \left(z^4 - \rho^4\right)\left(\left(1 - v^2\right) z^4 - \rho^4\right) + \rho^4 \left(\alpha^2 + \beta^2 - z^4\left(\alpha^2 + \beta^2 - \alpha^2 v^2\right)\right)\right)^{-1/2}, \quad (43)$$

and we defined the following variables:

$$\alpha = \frac{\pi_x^1 l^2}{r_{\min}^2}, \quad \beta = \frac{\pi_y^1 l^2}{r_{\min}^2}, \quad z = \frac{r}{r_{\min}}, \quad \rho = \frac{r_h}{r_{\min}}. \tag{44}$$

In [73] the screening length of a heavy quark-antiquark pair in strongly coupled gauge theory plasmas has been studied by using the AdS/CFT correspondence and found that the screening length is proportional to inverse of boosted energy density with power $1/d$, and some examples in $(d+1)$-dimensions were studied. We find that our results agree with the previous works.

7. Jet-Quenching Parameter

One of the interesting properties of the stronglycoupled plasma at RHIC is the jet quenching of partons, produced with high transverse momentum [75]. This parameter controls the description of relativistic partons and it is possible to employ the gauge/gravity duality and determine this quantity in the finite temperature gauge theories. In order to obtain the jet-quenching parameter one needs to rewrite the metric background in the light-cone coordinates. In Appendix B we give detailed calculation of the jet-quenching parameter in STU background. So, we can specify the jet-quenching parameter as the following [51]:

$$\widehat{q} = \frac{(I(q))^{-1}}{\pi\alpha'}, \tag{45}$$

where

$$I(q) = R^2 \int_{r_h}^{\infty} \frac{dr}{\sqrt{\left(\mathscr{H}^{2/3} r^2/R^2 - f_k/\mathscr{H}^{1/3}\right) f_k r^4}}. \tag{46}$$

In order to obtain the explicit expression of the jet-quenching parameter we set $k = 1$ and consider three special cases of one-, two-, and three-charged black hole.

In the case of one-charged black hole we set $q_1 = q$, $q_2 = q_3 = 0$ and obtain

$$I(q_1) = R^4 \int_{r_h}^{\infty} \sqrt{\frac{\left(1 + q/r^2\right)^{1/3}}{\left(r^2 - \mu\right)\left(r^4 + \left(q + R^2\right) r^2 - \mu R^2\right)}} dr. \tag{47}$$

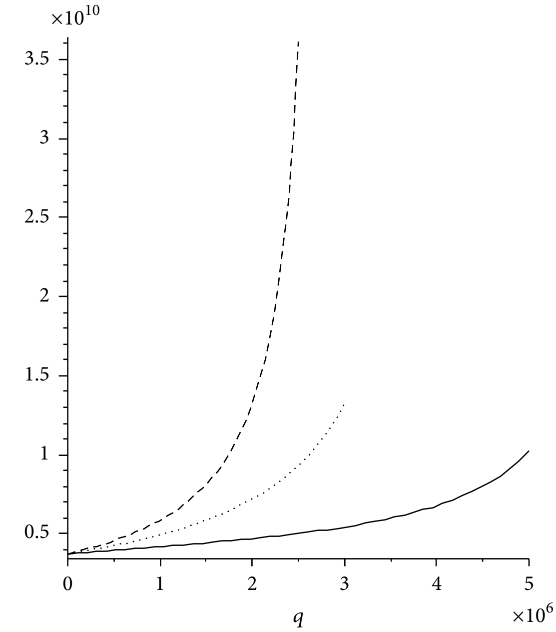

Figure 2: Plot of the jet-quenching parameter in terms of the black hole charge. We fixed our parameters as $\alpha' = 0.5$, $\lambda = 6\pi$, and $T = 300$ MeV. The solid line represents the case of $q_1 = q$, $q_2 = q_3 = 0$. The dotted line represents the case of $q_1 = q_2 = q$, $q_3 = 0$. The dashed line represents the case of $q_1 = q_2 = q_3 = q$. It shows that increasing the number of black hole charges increases the value of the jet-quenching parameter.

In the similar work the jet-quenching parameter in medium with chemical potential has been studied [54].

In the case of metric background (21) it is found that [58]

$$\widehat{q} = \frac{\pi^{3/2}\Gamma(3/4)}{\Gamma(5/4)} \sqrt{\lambda} T^3. \tag{48}$$

From (48), one finds $\widehat{q} = 4.5$, 10.6, and 20.7 GeV^2/fm for $T = 300$, 400, and 500 MeV. Comparing (45) and (48) tells that the jet-quenching parameter was enhanced for STU background. For example, by choosing $R^2 = \alpha'\sqrt{\lambda}$, $\alpha' = 0.5$, $\lambda = 6\pi$, $q = 10^6$, and $T = 300$ MeV one can obtain $\widehat{q} = 42$ GeV^2/fm in STU model. It means that the black hole charge increases the jet-quenching parameter.

Numerically, we give plots of the jet-quenching parameter in terms of the black hole charges and the temperature in Figures 2 and 3, respectively. These plots show that the jet-quenching parameter of the $\mathcal{N} = 2$ theory is larger than the jet-quenching parameter of the $\mathcal{N} = 4$ theory. Also we find that the jet-quenching parameter of the three-charged black hole is larger than the jet-quenching parameter of the one-charged and two-charged black holes.

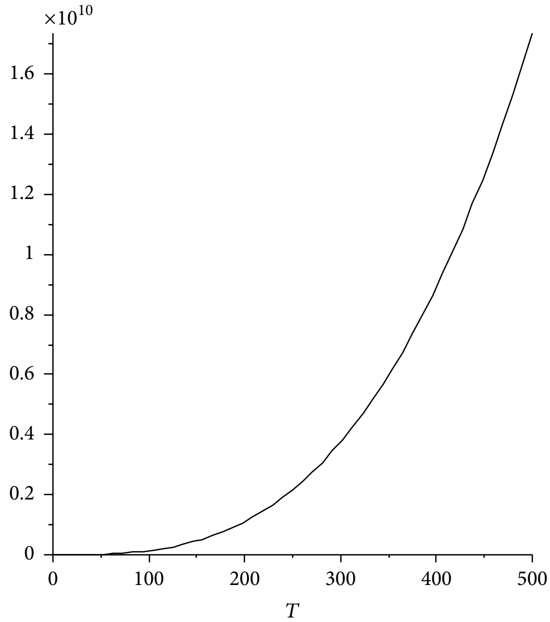

FIGURE 3: Plot of the jet-quenching parameter in terms of the temperature for small black hole charge. We fixed the parameters as $\alpha' = 0.5$, $\lambda = 6\pi$. In that case three different cases of one-, two-, and three-charged black holes have similar behaviors.

Also the jet-quenching parameter corresponding to metric background (17) is given by

$$\widehat{q} = \frac{r_0^2}{\pi\alpha' R^4}\left[\int_{r_h}^{\infty}\frac{dr}{r^2\sqrt{f/H}}\right]^{-1}, \tag{49}$$

where

$$f = H^3 - \frac{r_0^4}{r^4}, \quad H = 1 + \frac{q}{r^2}, \tag{50}$$

which agree with the results of the previous works, where the jet-quenching parameter is calculated with the chemical potential [56]. The horizon radius r_0 is obtained for the case of zero-charged black hole. For the black hole with nonvanishing charges, it is clear that the horizon radius decreases ($r_h < r_0$). We know that the $q = 0$ limit is equal to $\phi = 0$ limit, and one can say that the jet-quenching parameter from the $\mathcal{N} = 2$ supergravity theory with zero chemical potential is equal to the jet-quenching parameter from the $\mathcal{N} = 4$ SYM theory.

On the other hand by using the metric background (25) we find the following expression of the jet-quenching parameter [76]:

$$\widehat{q} = 2\sqrt{2}T_0\left[\int_{r_h}^{\infty} dr\frac{R^4}{r^4}\sqrt{\frac{K}{fh^2\left(r_h^4/2r^4 - 2r^2\beta^2 f\right)}}\right]^{-1}. \tag{51}$$

It is found that for the large chemical potential (for both infinitesimal and large non-commutativity parameters) the jet-quenching parameter is obtained as (48), but for the case of infinitesimal non-commutativity parameter and large β the jet-quenching parameter is proportional to temperature. We have found that the presence of non-commutativity is necessary to obtain the jet-quenching parameter in the experimental range.

8. Conclusion

In this paper we used AdS/CFT correspondence and reviewed some important quantities to understand the nature of QGP more exactly. Indeed, we considered thermal QGP includes a chemical potential. This chemical potential comes from $\mathcal{N} = 2$ supergravity in 5 dimensions. This theory contains a nonextremal black hole with three electrical charges and is well known as STU model. First of all we introduced STU model and wrote their important properties. We studied thermodynamics of STU background and extracted the Hawking temperature and entropy density. In order to compare our results with the $\mathcal{N} = 4$ SYM plasma we used special rescaling that bring the spherical-horizon metric to the flat-horizon metric.

We considered problem of the drag force and found energy loss of single quark and quark-antiquark pair. We showed that the value of the drag force enhanced due to the black hole charges. Also we calculated screening length for the special case of $q = 0$.

Finally we studied the jet-quenching parameter and found that the jet-quenching parameter enhanced due to the black hole charges. It means that the energy of the string in $\mathcal{N} = 2$ thermal plasma is larger than the string in $\mathcal{N} = 4$ thermal plasma hence the string in $\mathcal{N} = 2$ thermal plasma loses more energy than the string in $\mathcal{N} = 4$ thermal plasma. We examine our solution for three special cases of one-, two-, and three-charged black holes. All cases yield to the same value of the jet-quenching parameter for the small black hole charge. However, thermodynamical stability allows choosing the black hole charge of order 10^6. In that case we found $\widehat{q} = 42, 49$, and $58\ \mathrm{GeV}^2/\mathrm{fm}$ for one-, two-, and three-charged black hole, respectively. These values of the jet-quenching parameter are far from experiments of RHIC; experimental data tell us that ($5 < \widehat{q} < 25$). There is no worry for this statement because the temperature of the $\mathcal{N} = 2$ supergravity theory should be smaller than that of the $\mathcal{N} = 4$ SYM theory. In that case with the temperature about 155 MeV we obtained the jet-quenching parameter in the experimental range.

Appendices

A. Drag Force

It is assumed that the string moves along x direction and uses static gauge, where $\tau = t$ and $\sigma = r$. Therefore, the string world-sheet is described by $x(r, t)$.

By using Euler-Lagrange equation one can obtain the string equation of motion as the following expression:

$$\frac{\partial}{\partial r}\left(\frac{f_k r^2}{\mathscr{H}^{1/3}\sqrt{-g}}x'\right) = \frac{\mathscr{H}^{2/3}r^2}{f_k}\frac{\partial}{\partial t}\left(\frac{\dot{x}}{\sqrt{-g}}\right). \tag{A.1}$$

In order to obtain the total energy and momentum, drag force, or energy loss of particle in the thermal plasma, we have to calculate the canonical momentum densities. In that case one can obtain the following expressions:

$$\begin{pmatrix} \pi_x^0 & \pi_x^1 \\ \pi_r^0 & \pi_r^1 \\ \pi_t^0 & \pi_t^1 \end{pmatrix} = -\frac{T_0}{\mathscr{H}^{1/3}\sqrt{-g}} \times \begin{pmatrix} -\frac{\mathscr{H}r^2}{f_k R^2}\dot{x} & \frac{f_k r^2}{R^2}x' \\ \frac{\mathscr{H}r^2}{f_k R^2}\dot{x}x' & 1-\frac{\mathscr{H}r^2}{f_k R^2}\dot{x}^2 \\ 1+f_k\frac{r^2}{R^2}x'^2 & -\frac{f_k r^2}{R^2}\dot{x}x' \end{pmatrix}. \tag{A.2}$$

Corresponding to the single quark, in CFT side, we have an open string in AdS space which stretched from $r = r_m$ on $D7$-brane to $r = r_h$ at the horizon. An ansatz made in order to describe the system may be

$$x(r,t) = x(r) + vt, \tag{A.3}$$

where v is the constant velocity of the single quark. In that case by using equation of motion (15) one can find

$$\frac{f_k r^2}{R^2 v \mathscr{H}^{1/3}\sqrt{-g}}x' = C, \tag{A.4}$$

where C is an integration constant and $\sqrt{-g}$ is obtained by using the following:

$$-g = \frac{1}{\mathscr{H}^{1/3}}\left[1-\frac{\mathscr{H}r^2}{f_k R^2}v^2 + \frac{f_k r^2}{R^2}x'^2\right]. \tag{A.5}$$

By using these results in the canonical momentum densities (15) we find

$$\begin{aligned} \pi_x^1 &= -T_0 C v, \\ \pi_t^1 &= T_0 C v^2. \end{aligned} \tag{A.6}$$

In the case of quark-antiquark, one can obtain the following equations, respectively,

$$\begin{aligned} &\frac{\partial}{\partial r}\left[\frac{1}{\sqrt{-g}}\left(\frac{r^4}{R^2}\mathscr{H}^{2/3}\left(\dot{y}^2 x' - \dot{x}\dot{y}y'\right) + \frac{f_k r^2 x'}{\mathscr{H}^{1/3}}\right)\right] \\ &\quad + r^2\mathscr{H}^{2/3}\frac{\partial}{\partial t}\left[\frac{1}{\sqrt{-g}}\left(\frac{\dot{x}}{f_k} + \frac{r^2}{R^2}\left(y'^2\dot{x} - x'\dot{y}y'\right)\right)\right] = 0, \\ &\frac{\partial}{\partial r}\left[\frac{1}{\sqrt{-g}}\left(\frac{r^4}{R^2}\mathscr{H}^{2/3}\left(\dot{x}^2 y' - \dot{x}\dot{y}x'\right) - \frac{f_k r^2 y'}{\mathscr{H}^{1/3}}\right)\right] \\ &\quad + r^2\mathscr{H}^{2/3}\frac{\partial}{\partial t}\left[\frac{1}{\sqrt{-g}}\left(\frac{\dot{y}R^2}{f_k} + r^2\left(x'^2\dot{y} - x'\dot{x}y'\right)\right)\right] = 0, \end{aligned} \tag{A.7}$$

and the momentum densities are obtained by the following:

$$\begin{pmatrix} \pi_x^0 & \pi_x^1 \\ \pi_y^0 & \pi_y^1 \\ \pi_r^0 & \pi_r^1 \\ \pi_t^0 & \pi_t^1 \end{pmatrix} = -T_0\frac{r^2\mathscr{H}^{1/3}}{R^2\sqrt{-g}} \times \begin{pmatrix} \frac{r^2}{R^2}\mathscr{H}^{1/3}x'\dot{y}y' - \left(\frac{\mathscr{H}^{1/3}}{f_k} + \frac{r^2}{R^2}\mathscr{H}^{1/3}y'^2\right)\dot{x} & \frac{r^2}{R^2}\mathscr{H}^{1/3}y'\dot{y}\dot{x} - \left(\frac{\mathscr{H}^{1/3}r^2}{R^2}\dot{y}^2 - \frac{f_k}{\mathscr{H}^{2/3}}\right)x' \\ \frac{r^2}{R^2}\mathscr{H}^{1/3}y'\dot{x}x' - \left(\frac{\mathscr{H}^{1/3}}{f_k} + \frac{r^2}{R^2}\mathscr{H}^{1/3}x'^2\right)\dot{y} & \frac{r^2}{R^2}\mathscr{H}^{1/3}x'\dot{y}\dot{x} - \left(\frac{\mathscr{H}^{1/3}r^2}{R^2}\dot{x}^2 - \frac{f_k}{\mathscr{H}^{2/3}}\right)y' \\ \frac{\mathscr{H}^{1/3}}{f_k}\left(\dot{x}x' + \dot{y}y'\right) & \frac{R^2}{\mathscr{H}^{2/3}r^2} - \frac{\mathscr{H}^{1/3}}{f_k}\left(\dot{x}^2 + \dot{y}^2\right) \\ \frac{R^2}{\mathscr{H}^{1/3}r^2} + \frac{f_k}{\mathscr{H}^{1/3}}\left(x'^2 + y'^2\right) & -\frac{f_k}{\mathscr{H}^{2/3}}\left(\dot{x}x' + \dot{y}y'\right) \end{pmatrix}. \tag{A.8}$$

There are two interesting motions for the meson. The first one is the moving quark-antiquark pair with constant speed v. The second case is the rotational motion of the quark-antiquark pair.

The first system may be described by the following ansatz:

$$\begin{aligned} x(r,t) &= vt + x(r), \\ y(r,t) &= y(r). \end{aligned} \tag{A.9}$$

These solutions satisfy boundary conditions as $x(\infty,t)=vt$ and $y(\infty)=\pm l/2$. In order to obtain drag force, we calculate π_x^1 and π_y^1 components, solve them for x' and y', respectively, and obtain

$$\begin{aligned} x'(r) &= \pi_x^1 \frac{R}{r}\left(1-\frac{\mathscr{H} r^2 v^2}{f_k R^2}\right) \\ &\times \left[\left(\frac{f_k}{\mathscr{H}}-\frac{r^2v^2}{R^2}\right)\left(T_0^2\frac{r^2}{R^2} f_k \mathscr{H}^{2/3} -\mathscr{H}\left(\pi_x^1\right)^2\right)-f_k\left(\pi_y^1\right)^2\right]^{-1/2}, \\ y'(r) &= \pi_y^1 \frac{R}{r}\left[\left(\frac{f_k}{\mathscr{H}}-\frac{r^2v^2}{R^2}\right) \times\left(T_0^2\frac{r^2}{R^2} f_k\mathscr{H}^{2/3}-\mathscr{H}\left(\pi_x^1\right)^2\right)-f_k\left(\pi_y^1\right)^2\right]^{-1/2}. \end{aligned} \tag{A.10}$$

As before, by using reality condition one can obtain

$$\left(\pi_y^1\right)^2 = \left[\left(\frac{f_k}{\mathscr{H}}-\frac{r^2v^2}{R^2}\right)\times\left(T_0^2\frac{r^2}{R^2}\mathscr{H}^{2/3}-\frac{\mathscr{H}}{f_k}\left(\pi_x^1\right)^2\right)\right]_{r=r_{\min}}, \tag{A.11}$$

where $r_{\min}$ is turning point of string. If $\pi_y^1=0$, then $r_{\min}=r_c$ and above solutions are similar to single quark solution ($l=0$). Here, as the string have a chance of turning around smoothly, it requires that $\partial y/\partial x = y'/x' = \infty$ at $r_{\min}$. So, it is necessary to have $\pi_x^1=0$. Therefore, one can find the momentum component as

$$\pi_y^1 = \frac{T_0}{R} r_{\min}\mathscr{H}^{1/3}\left(r_{\min}\right) \times \sqrt{\frac{f_k\left(r_{\min}\right)}{\mathscr{H}\left(r_{\min}\right)}-\frac{r_{\min}^2 v^2}{R^2}}. \tag{A.12}$$

In the second case we add a rotational motion with angular velocity $\dot{\theta}$ to the motion of meson. Therefore, the string may be described by the $x(r,t)=vt+x(r)\sin\theta$ and $y(r,t)=y(r)\cos\theta$ profiles.

Also there is another condition due to our conjecture, $y'/x'=\cot\theta$, which reduces to $y'/x'\to\infty$ at the $\theta\to 0$ limit, which agrees with the first case.

These boundary conditions can also satisfy with two separated string which move at velocity v along X-axis and simultaneously swing a circle with radius $l/2$. Specifying these boundary conditions does not lead to a unique solution for equation of motion, so we should specify additional conditions for this motion.

Here, we assume that the string is initially upright, moves at velocity v, and rotates around its center of mass.

Now, one can obtain the following:

$$\begin{aligned} A x'^2 + B y'^2 + C x' y' + D &= 0, \\ A' x'^2 + B' y'^2 + C' x' y' + D' &= 0, \end{aligned} \tag{A.13}$$

where

$$\begin{aligned} A &= \mathscr{R}^2\sin^2\theta\left[\pi_x^{1\,2}\left(\frac{f_k}{\mathscr{H}^{1/3}}-\mathscr{H}^{2/3}y^2\dot{\theta}^2\mathscr{R}^2\sin^2\theta\right) - T_0^2\mathscr{H}^{2/3}\mathscr{R}^2\left(\mathscr{H}^{1/3}y^2\dot{\theta}^2\mathscr{R}^2\sin^2\theta -\frac{f_k}{\mathscr{H}^{2/3}}\right)^2\right], \\ B &= \mathscr{R}^2\cos^2\theta\left[\pi_x^{1\,2}\left(\frac{f_k}{\mathscr{H}^{1/3}}-\mathscr{H}^{2/3}\left(v+x\dot{\theta}\cos\theta\right)^2\mathscr{R}^2\right) - T_0^2\mathscr{H}^{4/3}y^2\dot{\theta}^2\mathscr{R}^6\sin^2\theta\left(v+x\dot{\theta}\cos\theta\right)^2\right], \\ C &= -2y\dot{\theta}\mathscr{R}^4\sin^2\theta\left[\pi_x^{1\,2}\mathscr{H}^{2/3}\cos\theta + T_0^2\mathscr{H}\mathscr{R}^2 \times\left(\mathscr{H}^{1/3}y^2\dot{\theta}^2\mathscr{R}^2\sin^2\theta-\frac{f_k}{\mathscr{H}^{2/3}}\right)\right] \times\left(v+x\dot{\theta}\cos\theta\right), \\ D &= \mathscr{R}^2\pi_x^{1\,2}\frac{\mathscr{H}^{2/3}}{f_k}\left[\frac{f_k}{\mathscr{R}^2\mathscr{H}}-y^2\dot{\theta}^2\sin^2\theta -\left(v+x\dot{\theta}\cos\theta\right)^2\right], \\ A' &= \mathscr{R}^2\sin^2\theta\left[\pi_y^{1\,2}\left(\frac{f_k}{\mathscr{H}^{1/3}}-\mathscr{H}^{2/3}y^2\dot{\theta}^2\mathscr{R}^2\sin^2\theta\right) - T_0^2\mathscr{H}^{4/3}y^2\dot{\theta}^2\mathscr{R}^6\sin^2\theta\left(v+x\dot{\theta}\cos\theta\right)^2\right], \\ B' &= \mathscr{R}^2\cos^2\theta\left[\pi_y^{1\,2}\left(\frac{f_k}{\mathscr{H}^{1/3}}-\mathscr{H}^{2/3}\left(v+x\dot{\theta}\cos\theta\right)^2\mathscr{R}^2\right) - T_0^2\mathscr{H}^{2/3}\mathscr{R}^2\left(\mathscr{R}^2\mathscr{H}^{1/3}\left(v+x\dot{\theta}\cos\theta\right)^2 -\frac{f_k}{\mathscr{H}^{2/3}}\right)^2\right], \\ C' &= -2y\dot{\theta}\mathscr{R}^4\sin^2\theta\cos\theta \times\left[\pi_y^{1\,2}\mathscr{H}^{2/3}+T_0^2\mathscr{H}\mathscr{R}^2\left(\mathscr{R}^2\mathscr{H}^{1/3}\left(v+x\dot{\theta}\cos\theta\right)^2 -\frac{f(r)}{\mathscr{H}^{2/3}}\right)\right]\left(v+x\dot{\theta}\cos\theta\right), \\ D' &= \mathscr{R}^2\pi_y^{1\,2}\frac{\mathscr{H}^{2/3}}{f_k}\left[\frac{f_k}{\mathscr{R}^2\mathscr{H}}-y^2\dot{\theta}^2\sin^2\theta -\left(v+x\dot{\theta}\cos\theta\right)^2\right], \end{aligned} \tag{A.14}$$

and we set $r/R \equiv \mathscr{R}$. Therefore, from (A.13), one can obtain

$$x'(r) = 2\left[\frac{D\left(B - \left({\pi_y^1}^2/{\pi_x^1}^2\right)B'\right)}{C^2 - C'^2 - 4\left(BA - B'A'\right)}\right]^{1/2},$$
$$y'(r) = 2\left[\frac{D\left(A - \left({\pi_y^1}^2/{\pi_x^1}^2\right)A'\right)}{C^2 - C'^2 - 4\left(BA - B'A'\right)}\right]^{1/2}. \qquad \text{(A.15)}$$

Here, if the rotational motion vanishes ($\dot{\theta} = 0$), one can see that coefficients of $x'y'$ vanish ($C = C' = 0$) and our solutions recover the motion of quark-antiquark pair without rotation. In order to obtain drag force we use reality condition and find

$$\frac{A}{A'} = \frac{B}{B'} = \frac{C}{C'} = \frac{D}{D'} = \left(\frac{\pi_y^1}{\pi_x^1}\right)^2. \qquad \text{(A.16)}$$

Then one can find two equations as $C^2 - 4AB = 0$ and $C'^2 - 4A'B' = 0$. These equations specify π_x^1 and π_y^1, respectively.

B. Jet-Quenching

First of all we introduce light-cone coordinates $x^{\pm} = (t \pm x^1)/\sqrt{2}$ and rewrite the metric (1) in the following form:

$$ds^2 = \frac{1}{2}\left(\frac{\mathscr{H}^{1/3}r^2}{R^2} - \frac{f_k}{\mathscr{H}^{2/3}}\right)\left(\left(dx^+\right)^2 + \left(dx^-\right)^2\right) - \left(\frac{\mathscr{H}^{1/3}r^2}{R^2} + \frac{f_k}{\mathscr{H}^{2/3}}\right)dx^+dx^- + \mathscr{H}^{1/3}\left(\frac{r^2}{R^2}\left(dx_2^2 + dx_3^2\right) + \frac{dr^2}{f_k}\right). \qquad \text{(B.1)}$$

We begin with the general relation for the jet-quenching parameter,

$$\widehat{q} \equiv 8\sqrt{2}\frac{S_I}{L^- L^2}, \qquad \text{(B.2)}$$

where $S_I = S - S_0$ (S denotes $q\overline{q}$ pair action and S_0 denotes the action of isolated q and $\overline{q}$). It means that the jet-quenching parameter is proportional to energy of the string, so we expect that this quantity will be opposite to the drag force which is indeed energy loss of the string. Therefore, calculation of the jet-quenching parameter reduces to obtain actions S and S_0.

One can image the situation with an open string whose endpoints lie on the brane. In the light-cone coordinates, the string may be described by $r(\tau, \sigma)$. We use the static gauge where $\tau = x^-$ and $\sigma = x^2 \equiv y$, and all other coordinates are considered as constants. In that case $-L/2 \le y \le L/2$ and $L^- \le x^- \le 0$, and because of $L^- \gg L$ one can assume that the world-sheet is invariant along the x^- direction. Therefore, the string may be described by the function $r(y)$, so the boundary condition is $r(\pm L/2) = \infty$. Then the Nambu-Goto action is given by

$$S = \frac{\sqrt{2}L^-}{2\pi\alpha'}\int_0^{L/2} dy\sqrt{\left(\frac{\mathscr{H}^{2/3}r^2}{R^2} - \frac{f_k}{\mathscr{H}^{1/3}}\right)\left(\frac{r^2}{R^2} + \frac{1}{f_k}r'^2\right)}. \qquad \text{(B.3)}$$

One can remove the r' by using the equation of motion. For the low energy limit ($E \to 0$) one can obtain

$$S = \frac{L^-}{2\pi\alpha'}\int_{r_h}^{\infty} dr\sqrt{\frac{2\mathscr{H}^{1/3}}{f_k}\left(\frac{\mathscr{H}^{1/3}r^2}{R^2} - \frac{f_k}{\mathscr{H}^{2/3}}\right)} \times\left[1 + \frac{R^2}{\left(\mathscr{H}^{2/3}r^2/R^2 - f_k/\mathscr{H}^{1/3}\right)Br^2}\right]. \qquad \text{(B.4)}$$

Now, one can extract action S_0 which can be interpreted as the self-energy of the isolated quark and the isolated antiquark. In that case one can obtain

$$S_0 = \frac{L^-}{2\pi\alpha'}\int_{r_h}^{\infty} dr\sqrt{\frac{2\mathscr{H}^{1/3}}{f_k}\left(\frac{\mathscr{H}^{1/3}r^2}{R^2} - \frac{f_k}{\mathscr{H}^{2/3}}\right)}. \qquad \text{(B.5)}$$

Therefore, we can extract S_I as the following:

$$S_I = \frac{1}{\sqrt{B}}\frac{L^-}{2\pi\alpha'}\int_{r_h}^{\infty} dr\sqrt{\frac{2R^4}{\left(\mathscr{H}^{2/3}r^2/R^2 - f_k/\mathscr{H}^{1/3}\right)Bf_k r^4}}. \qquad \text{(B.6)}$$

These relations give us the jet-quenching parameter.

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