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Application of Automatic Differentiation for Tilt-rotor Aircraft Flight Dynamics Analysis

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Nomenclature

V = forward speed (m/s)

t = time (s)

$\boldsymbol{u}$ = control vector of the tilt-rotor aircraft (Deg)

$\boldsymbol{x}$ = state vector of the tilt-rotor aircraft

I.INTRODUCTION

THE tilt-rotor aircraft has drawn a lot of attention due to its ability to combine the advantages of the fixed-wing airplane and the helicopter, making it simultaneously possess the capability to hover and perform high speed flight.

However, the flight dynamics characteristics of this aircraft are complicated. Its characteristics are influenced by extensive aerodynamic interference effects, the possibility of redundant control strategies, and the unique regime occurring during the conversion process. The traditional approach to aircraft stability analysis often utilizes numerical differentiation methods to calculate stability derivatives and hence from a linearized representation of the aircraft model, the eigenvalues that predict the natural modes of flight [1]. The use of the numerical differentiation can reduce the efficiency (calculation speed) and accuracy of the analysis due to the presence of rounding errors. Control simulation frequently used to study the controllability in maneuvering flight can also be compromised by the use of numerical differentiation, which impedes further development of this form of flight dynamics analysis.

Automatic differentiation (AD) is a potential solution to these problems [2]. A set of techniques to numerically evaluate the derivative of a function implemented within a computer program, AD has been widely utilized for helicopter and fixed-wing aircraft CFD calculations to optimize geometry and to analyze rotor aerodynamic stability [3~5]. The results indicate that using an automatic differentiation method could accelerate the calculation speed and improve accuracy. However, little research has been conducted on the utilization of automatic differentiation methods in flight dynamics analysis, in particular for tilt-rotor aircraft. In fact, the differentiation process is widely used in both trim and stability & controllability investigations. The truncation error derived from the numerical differentiation can couple with the nonlinear aerodynamic characteristics and control features of the tilt-rotor aircraft, making the calculation results hard to interpret. Further, the time consumption of the numerical differentiation may become unacceptable when the real-time requirement needs to be met. Therefore, the application of the AD method into the flight dynamics modeling and analysis of the tilt-rotor aircraft may be helpful to further improve the accuracy and precision of flight dynamics investigation.

In light of the preceding discussion, this paper firstly details the construction of a flight dynamics model of the tilt-rotor aircraft, with the AD method embedded in the code. The accuracy of this model is then verified using trim results compared with other research, and the efficiency of the model is also assessed by comparing computing time with the conventional numerical differentiation method. Meanwhile, the stability eigenvalues derived from the proposed model are used to analyze the flight dynamics of the tilt-rotor aircraft. Then, control response is calculated using the proposed flight dynamics model, and the real-time requirement of the model is tested. Stability eigenvalues during the maneuvering flight are also obtained using the AD method to further investigate the flight dynamics characteristics in maneuvering flight.

II.METHODOLOGY

The flight dynamics model of a tilt-rotor aircraft and the application of the AD method will be introduced in this section. The model developed is generic however the configurational data used represents the XV-15 aircraft.

A. Flight Dynamics Model

The flight dynamics model of the tilt-rotor aircraft contains five parts: the rotor model, pylon-wing model, fuselage model, horizontal tail model, and the vertical tail model. Meanwhile, the aerodynamic interference among different components is considered during the modeling process.

The individual blade element method was used to construct the rotor aerodynamics characteristics [6]. Each blade is divided into several segments to calculate its aerodynamic lift and drag in various flight states. Pitt-Peters dynamic inflow model [7] is used to determine the induced velocity on the rotor disk, and the blade flapping motion is also included in the rotor aerodynamic model.

The aerodynamic characteristics of the pylon and wing are obtained using a look-up table. The aerodynamic interaction between the tilt-rotor and the wing is calculated based on the fixed wake theory and projection relationship between the rotor and wing [8]. According to this method, the wing is divided into two parts, i.e., the free-stream area and the interference area. The aerodynamic forces and moments of the free-stream area are directly obtained based on the local angle of attack and sideslip. The area and position of the interference part are decided by the relative position relation between the rotor disk, wing and the induced velocity [8]. Therefore, the velocity increment on the interference part is calculated using look-up tables and the differentiation process. Consequently, aerodynamic forces can be acquired with the wake effect included. Lastly, the resultant aerodynamic loads are obtained from the sum of forces in the free-stream area and the interference area.

The aerodynamic models of tail planes (horizontal and vertical tails) and the fuselage are constructed based on data from wind tunnel experiments [8]. The wake effect and the wing's aerodynamic interference on the tail planes are also taken into consideration using a series of look-up tables.

Therefore, the flight dynamics model of the tilt-rotor aircraft can be built based on the aerodynamics model of each part, represented as a set of nonlinear differential equations:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}, t) \tag{1}$$

where x is the state vector, including the vehicle velocities, angular velocities, attitude angles, blade flapping motions, and the induced velocities on each rotor disk; u is the control vector, which contains the collective pitch, the longitudinal cyclic pitch, the collective differential, and the differential longitudinal cyclic; t is the response time.

B. Application of Automatic Differentiation

The AD method is based on the chain rule as applied to the differentiation process. Details of the AD method can be found in references [9~12]. Therefore, only a brief overview of the method is illustrated in this article.

The execution of the flight dynamics model always boils down to a series of elementary operations such as an arithmetic operator or an intrinsic function. A simplified example is shown below, in which a and b are set to be intermediate values that depend on some independent variables x.

$$g = h(a,b) \Rightarrow \nabla_x g = \frac{\partial h}{\partial a} \bullet \nabla_x a + \frac{\partial h}{\partial b} \bullet \nabla_x b \tag{2}$$

Eq. (2) represents a simple example of the chain rule in the automatic differentiation process. By repeatedly using Eq. (2), the differentiation process can be finally simplified to a series of algebraic operations with derivative results coming from elementary operations. These derivative results are typically obtained by operator overloading or source-to-source transformation.

There are two different modes of the AD method, which are the forward mode and the reverse mode. In the flight dynamics analysis of the tilt-rotor aircraft, the forward mode is utilized. As discussed in references [3] and [12], the efficiency of the reverse mode is highly related to the adjoint method, and its potentially colossal memory requirement has been a severe impediment to its application in the flight dynamics modeling.

In the forward mode of the AD method, the derivative of the state vector $\dot{x}_i$ in Eq. (1) is selected, and the propagation of the derivate of the state vector x_j with respect to $\dot{x}_i$ is evaluated. Therefore, the operation for the forward mode can be expressed as

$$\frac{\partial \dot{x}_i}{\partial x_j} = \sum_{k=1}^{n} \frac{\partial \dot{x}_i}{\partial x_k}\frac{\partial x_k}{\partial x_j} + \sum_{l=1}^{m} \frac{\partial \dot{x}_i}{\partial u_l}\frac{\partial u_l}{\partial x_j} \tag{3}$$

where $n = \dim(x), m = \dim(u)$. By combining Eq. (3) with Eq. (2) and considering the chain rule of differentiation, the derivatives of the state vector are obtained directly. Similarly, the derivatives of the control vector can be represented as

$$\frac{\partial \dot{x}_i}{\partial u_j} = \sum_{k=1}^{n} \frac{\partial \dot{x}_i}{\partial x_k}\frac{\partial x_k}{\partial u_j} + \sum_{l=1}^{m} \frac{\partial \dot{x}_i}{\partial u_l}\frac{\partial u_l}{\partial u_j} \tag{4}$$

The benefits of the AD method on flight dynamics modeling and analysis can be concluded in Fig. 1.

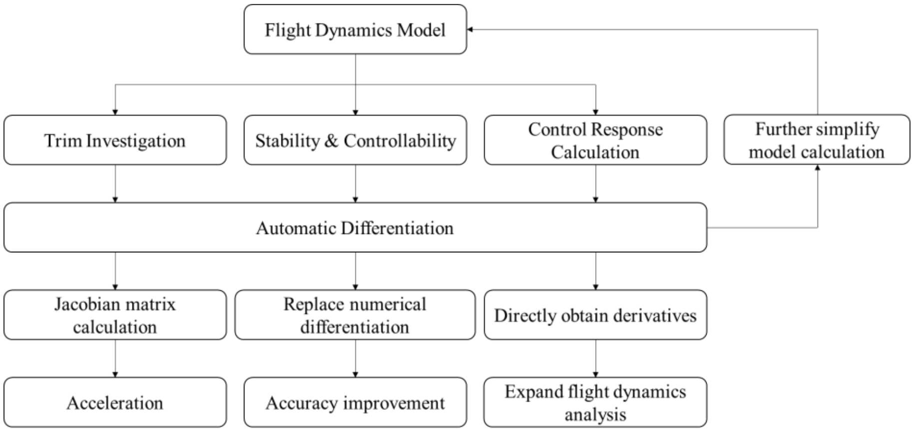


Fig. 1 Application of AD method into flight dynamics analysis

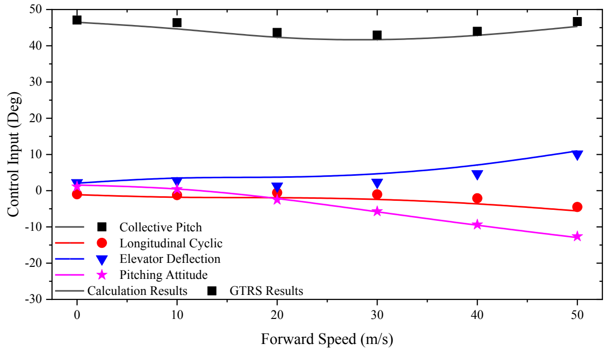
Firstly, using the derivatives directly obtained from the AD method, the Jacobi matrix is collected in a straightforward way, and the calculation process is accelerated. Furthermore, the AD method avoids the complex iteration process typical of numerical differentiation when solving the stability and controllability matrices, which prevents the inclusion of truncation errors and consequently improves the efficiency and precision of the results.

Moreover, the stability and controllability matrices can be directly output at each time step during the time response investigation. This feature could further expend the flight dynamics analysis and is beneficial to the control system design for tilt-rotor aircraft. Besides these, the aerodynamic interference calculation in the proposed model contains a series of differential equations to determine the effect of the rotor and wing's wake on the tail-planes. Therefore, the pertinent calculation process can be further simplified with the AD method.

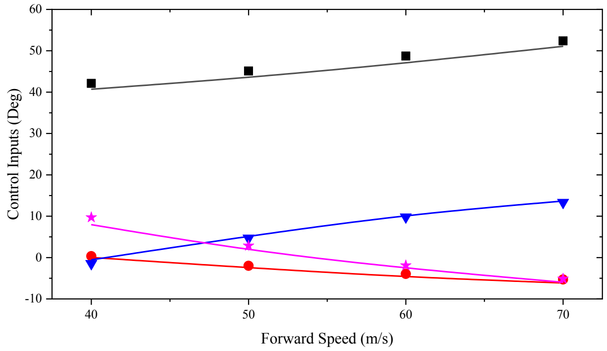
III.VERIFICATION & ANALYSES

This paper utilizes the trim results from reference 13 (GTRS report) to assess the accuracy of the AD-augmented model proposed in this article. The resulting eigenvalues are used to investigate the flight dynamics characteristics of the tilt-rotor aircraft.

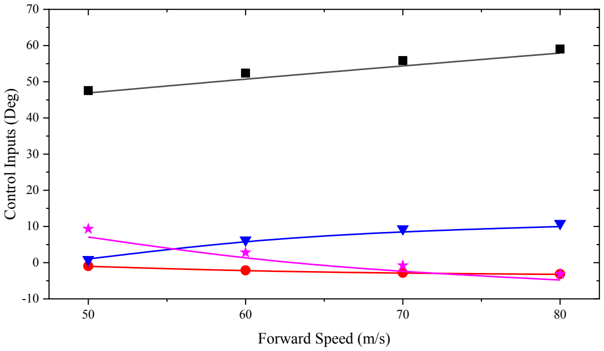
The tilt-rotor aircraft parameters used in the model are obtained from published data of the XV-15 tilt-rotor aircraft, available in references [8, 13-15]. In the trim validation, the control inputs and pitching attitude at different nacelle incidence angles are compared with simulation results from GTRS reports [13], which is shown in Fig.2.



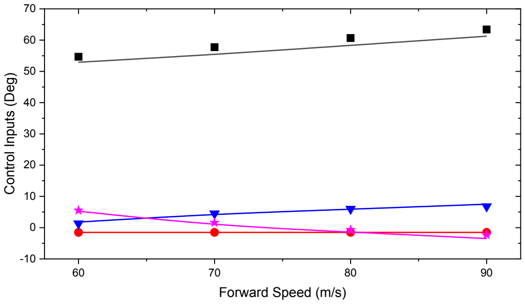
(a) Helicopter mode



(b) Conversion mode (nacelle incidence = 30 Degrees)



(c) Conversion mode (nacelle incidence = 60 Degrees)

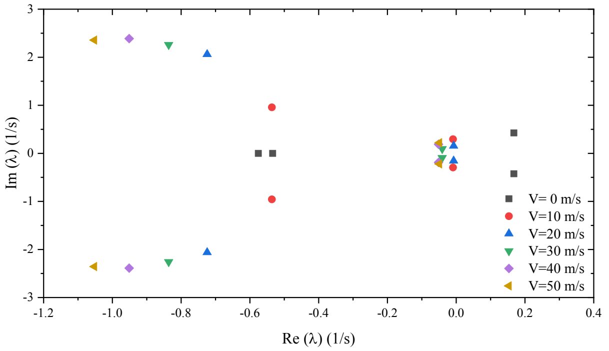


(d) Airplane mode (nacelle incidence = 90 Degrees)

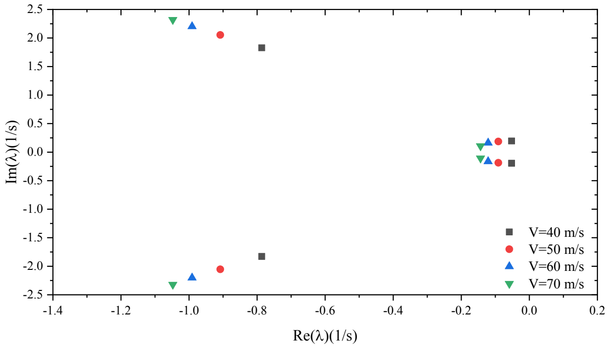
Fig.2 Trim validation

According to Fig.2, the trim characteristics obtained from the proposed model are in line with the GTRS report, demonstrating the accuracy of the proposed model. As shown in Fig. 2(a), the trim characteristics of the tilt-rotor aircraft is analogous to the conventional helicopter in helicopter mode. The collective pitch follows the saddle curve. The longitudinal cyclic pitch increases and the vehicle becomes nose-down to allow rotors to provide the longitudinal thrust needed for trimming. However, as the nacelle tilts forward, the trim characteristics of the tilt-rotor aircraft increasingly resemble the fixed-wing airplane. When the tilt-rotor aircraft is in airplane mode (Fig.2 (d)), the collective pitch increases to provide the propulsive force, and the changes of the longitudinal control inputs due to the forward speed are lower. This phenomenon arises because the effect of the forward speed increment on the pitching moment reduces as nacelle tilt angle increases.

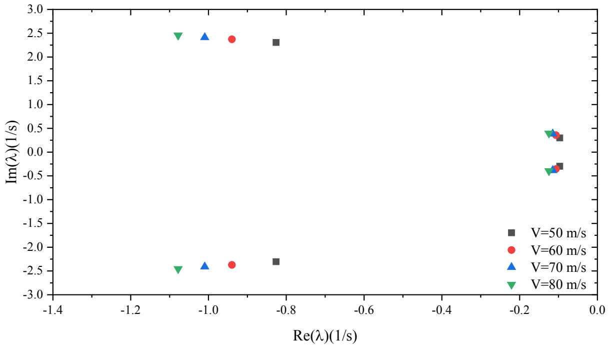
To further investigate the flight dynamics characteristics of the tilt-rotor aircraft using the proposed model, the longitudinal and lateral stability eigenvalues at different nacelle incidences are calculated, as shown in Fig. 3 and Fig.4, respectively.



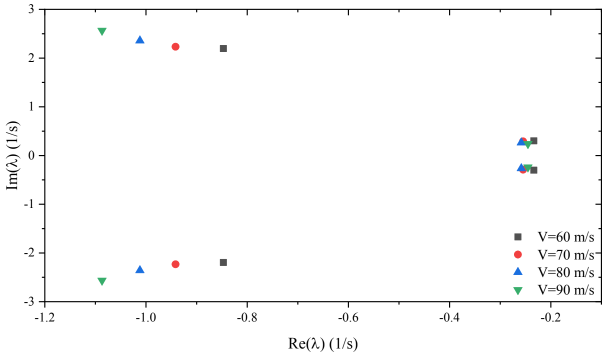
(a) Helicopter mode



(b) Conversion mode (nacelle incidence = 30 Degrees)

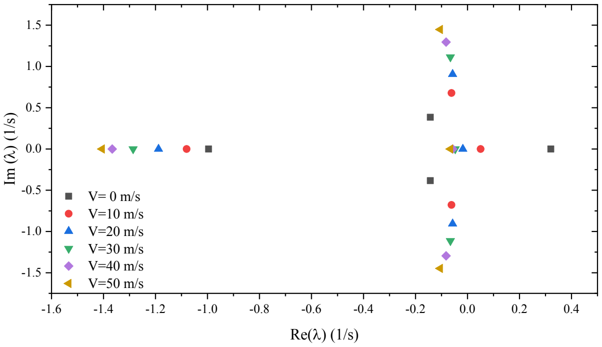


(c) Conversion mode (nacelle incidence = 60 Degrees)

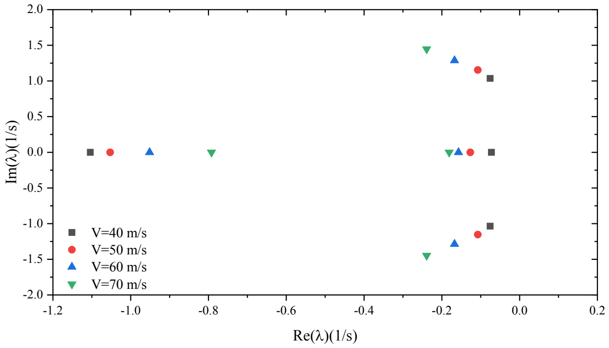


(d) Airplane mode (nacelle incidence = 90 Degrees)

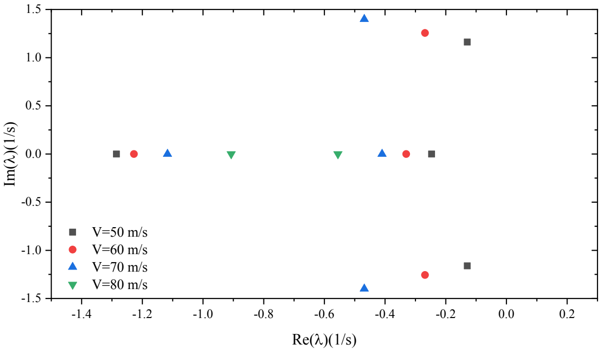
Fig.3 Longitudinal stability eigenvalues



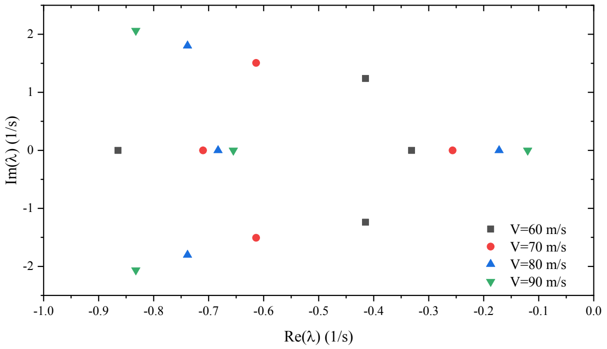
(a) Helicopter mode



(b) Conversion mode (nacelle incidence = 30 Degrees)



(c) Conversion mode (nacelle incidence = 60 Degrees)



(d) Airplane mode (nacelle incidence = 90 Degrees)

Fig.4 Lateral stability eigenvalues

In terms of the longitudinal modes of flight, in the helicopter mode we see a phugoid, pitch subsidence and heave modes. As the nacelles are tilted forward through conversion and on to airplane mode, the heave and subsidence modes form the familiar short period mode. The lateral-directional modes (spiral, roll and Dutch roll) are visible across the flight range. However, there is a large variation in both the damping and natural frequency of the Dutch Roll mode between helicopter and airplane modes. According to the longitudinal and lateral eigenvalue results, the stability of the XV-15 tilt-rotor aircraft increases with forward speed and as nacelle tilt increases. Unstable situations only occur when the aircraft is in hover or low speed flight. Based on the handling qualities specification of rotorcraft [16], eigenvalue locations indicate satisfactory stability characteristics for this configuration. Meanwhile, the eigenvalue results show similar trends to Padfield's results [14], also demonstrating the accuracy of the model we propose.

Another typical benefit of using AD is a reduction in computation time. The time required to calculate the trim and stability & controllability process using the AD method is shown in Table 1. Also listed are the corresponding execution times when using the conventional numerical differentiation method [17]. Both calculations are executed on the same platform (CPU: i7-8700K, RAM: 16G).

Table 1 Time consumption comparison

Calculation Duration (s)	Automatic Differentiation			Numerical Differentiation		
	Min	Max	Avg	Min	Max	Avg
Trim	0.23	0.31	0.27	5.42	12.8	7.14
Stability & Controllability Derivatives (Additional)	0.01	0.01	0.01	5.88	7.62	6.01

Table. 1 indicates that the AD method could reduce the duration in the trim and stability & controllability derivative calculation process. It should be mentioned that the second row in Table. 1 illustrates the additional time needed to obtain the stability & controllability matrices based on the trim states.

Based on the comparison listed above, it has been demonstrated that the application of the AD method can ensure the accuracy of the flight dynamics calculation. Furthermore, it enhances the calculation efficiency compared against the conventional numerical differentiation method.

IV.FLIGHT DYNAMICS ANALYSIS IN MANEUVRING FLIGHT

Based on the AD method, the flight dynamics analysis of the tilt-rotor aircraft can be further developed to evaluate the stability & controllability matrices in maneuvering flight.

When the tilt-rotor aircraft is flying in helicopter mode at a forward speed of 40 m/s, the response after a 1 degree increment of the collective pitch is shown in Fig. 5. It should be mentioned that the tilt-rotor aircraft is in an open-loop state. In other words, the stability control augmentation system (SCAS) is excluded from the model to clarify the analysis process.

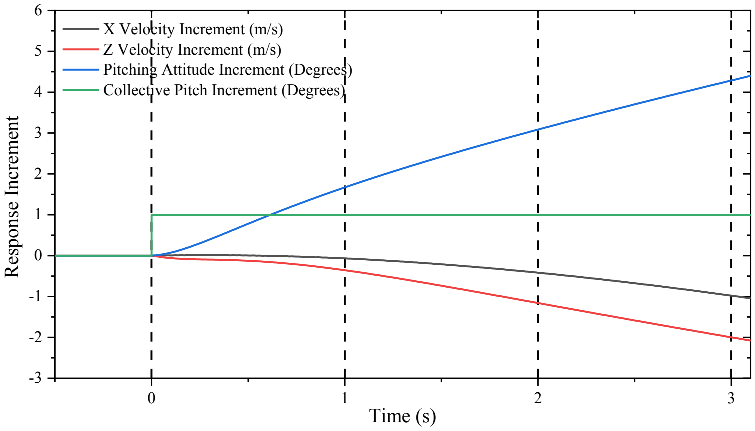


Fig.5 The time response after the increment of collective pitch

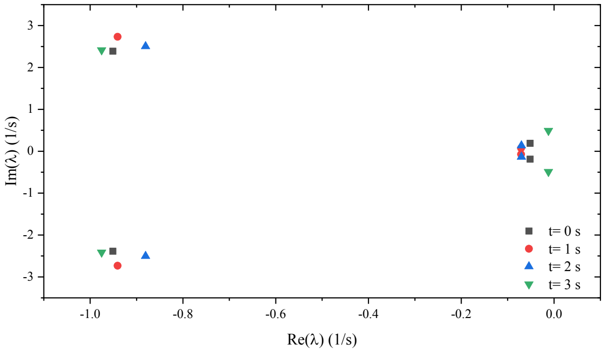
According to Fig.5, vertical speed increases with time. The forward speed is reduced due to the additional drag provided by the fuselage. Meanwhile, the vehicle is tilted backward because of the center of gravity.

The AD method accelerates the calculation speed of the time response. The derivative results can be calculated in a straightforward manner due to the additional simplification of the interference calculation. The comparison between the calculation time and the real-time period is shown in Table. 2.

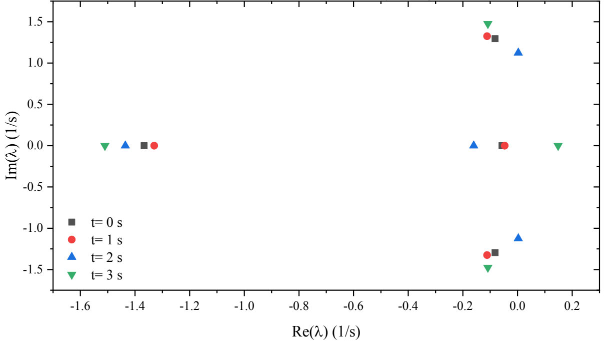
Table 2 Time consumption comparison in response calculation

Calculation Duration (s)			Time Period (s)
Min	Max	Avg	
0.64	0.71	0.66	1
1.22	1.37	1.29	2
1.79	1.94	1.88	3

According to Table. 2, the calculation time is lower than the real-time period, indicating that the flight dynamics model could achieve the real-time requirement. Furthermore, based on the AD method, the stability derivatives during response calculation can be exported at each time step directly. Therefore, the flight dynamics feature of this configuration in maneuvering flight can be analyzed in more detail. According to the control response result shown in Fig. 5, the longitudinal and lateral stability eigenvalue results at 0 s, 1 s, 2 s, and 3 s are shown in Fig. 6.



(a) Longitudinal stability eigenvalues



(b) Lateral stability eigenvalues

Fig.6 Stability eigenvalues at different time points in maneuver

According to Fig.6, the stability characteristics of the aircraft vary through the maneuver. When $t = 0$, the stability eigenvalues correspond to the results in Fig.3 (a) and Fig. 4 (a). When $t = 3s$, both the longitudinal and lateral long-term modes have become more unstable. The reasons for the stability change are proposed as follows. Firstly, accelerations of the flapping motion and induced velocity increase as time increases, which alters the stability characteristics of this aircraft. Furthermore, Z velocity increases with time due to the increment of the collective pitch, leading to additional vertical inflow during flight. This influences stability through the damping provided by the rotor. Additionally, the pitching attitude and forward velocity also change, and this will impact the response characteristics of the tilt-rotor aircraft.

Based on eigenvalue results from the AD method, the flight dynamics characteristics of the tilt-rotor aircraft can be investigated in more detail with obvious benefit to the design process of the aircraft control system. Considering how stability varies through maneuvering flight, the control system can be further developed to enhance the stability and controllability of the tilt-rotor aircraft.

V.CONCLUSION

A flight dynamics model of the tilt-rotor aircraft is built incorporating automatic differentiation methods, and its accuracy has been verified using trim comparison with an established model. Based on the automatic differentiation method, the flight dynamics analysis for the tilt-rotor aircraft can be further expanded for the maneuvering flight. Results and investigations allow the following conclusions to be drawn:

1) The trim characteristics predicted by the flight dynamics model are in line with the results from other research, indicating the accuracy of the proposed model. Additionally, the comparison of computational time also demonstrates that the model using the AD method is more efficient than its counterpart using conventional numerical differentiation methods.
2) Using an automatic differentiation approach, the flight dynamics model could meet the real-time requirement in response calculation. Moreover, the stability and controllability matrices are obtained at each time step directly during the maneuvering flight.
3) The results indicate that the tilt-rotor aircraft becomes more stable with forward speed and nacelle angle, and unstable eigenvalues only occur when the configuration is in hover or low speed forward flight in helicopter mode. Furthermore, the stability characteristics during the maneuvering vary along with time and may become unstable in some periods.

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