APPLICATION OF DTM ON MHD JEFFERY HAMEL PROBLEM WITH NANOPARTICLE

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In this paper, the MHD Jeffery Hamel problem with nanoparticles for various values of Hartmann number has been investigated. The present study discusses about the velocity profile in the steady 2-dimensional flow of a MHD fluid with nanoparticles between two nonparallel walls. At first a similarity transformation is used to reduce the partial differential equations modeling the flow, to a single third-order nonlinear differential equation containing the semi angle between the plates, Reynolds number, the magnetic field strength and nanoparticle volume fraction as parameters. Differential Transformation method (DTM) has been used in order to study the problem and finally the obtained analytical results have been compared with numerical solutions and results achieved from pervious works in some numerical cases.

Keywords: Magneto Hydro Dynamic flow, Jeffery Hamel problem, Differential
transformation method, Nanoparticle.

1. Introduction

The incompressible viscous fluid flow through convergent and divergent channels is one of the most applicable cases in fluid mechanics, electrical and bio mechanical engineering. Jeffery [1] and Hamel [2] were the first persons who discussed about this problem and so, it is known as Jeffery–Hamel problem, too.

One of the most significant examples of Jeffery Hamel problems are those subjected to an applied magnetic field. The MHD systems are used effectively in many applications including power generators, pumps, accelerators, electrostatic filters, droplet filters, the design of heat exchangers, the cooling of reactors etc. The investigation on MHD fluid flow was the main purpose of many pervious researches [3-5].

The term nanofluid was envisioned to describe a fluid in which nanometer-sized particles were suspended in conventional heat transfer basic fluids. Nanotechnology aims to manipulate the structure of the matter at the molecular level with the goal for innovation in virtually every industry and public

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endeavor including biological sciences, physical sciences, electronics cooling, transportation, the environment and national security [6, 7].

In recent years, much attention has been devoted to the newly developed methods for constructing an analytic solution of a nonlinear equation. Such methods include the Homotopy Perturbation Method (HPM) [8], Variational Iteration Method (VIM) [9], Differential Transformation Method (DTM) [10], Homotopy Analysis Method (HAM) [11] and Adomian Decomposition Method (ADM) [12].

Ordinary differential equations are generally solved using integral transformation methods such as the Laplace or Fourier transform. These methods transform the ordinary differential equation into a corresponding algebraic equation. The differential transformation method (DTM) is an analytical method for solving problems of this type [13].

The aim of this study is to investigate the velocity profile in MHD Jeffery Hamel flow with nanoparticles by using differential transformation method. The obtained approximate results will be compared to numerical solutions in some numerical cases.

2. Mathematical modeling

We consider the boundary layer flow of an electrically conducting viscous fluid with nanoparticle. A magnetic field $B(x)$ acts transversely to the flow. As it can be seen in Fig.1 the steady 2 dimensional flow of an incompressible conducting viscous fluid from a source or sink at the intersection between two non parallel plane walls is considered. We assume that the velocity is purely radial and depends on r and θ only. The Navier-Stokes equations in polar coordinates are:

$$\frac{\rho_{nf}}{r}\frac{\partial(ru(r,\theta))}{\partial r}(ru(r,\theta)) = 0 \tag{1}$$

$$u(r,\theta)\frac{\partial u(r,\theta)}{\partial r} = -\frac{1}{\rho_{nf}}\frac{\partial P}{\partial r} + v_{nf}\left(\frac{\partial^2 u(r,\theta)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u(r,\theta)}{\partial \theta^2} - \frac{u(r,\theta)}{r^2}\right)$$

$$-\frac{\sigma B_0^2}{\rho_{nf} r^2}u(r,\theta) = 0 \tag{2}$$

$$\frac{1}{r\rho_{nf}}\frac{\partial P}{\partial \theta} - \frac{2v_{nf}}{r^2}\frac{\partial u(r,\theta)}{\partial \theta} = 0 \tag{3}$$

Here B_0 is the electromagnetic induction, $u(r)$ is the velocity along radial direction, P is the fluid pressure, σ is the conductivity of the fluid, ρ_{nf} is the density of fluid and v_{nf} is the coefficient of kinematic viscosity. By introducing ϕ

as a solid volume fraction, fluid density, dynamic viscosity and the kinematic viscosity of nanofluid can be written as follows:

$$\begin{cases} \rho_{nf} = \rho_f (1-\phi) + \rho_s \phi_s \\ \mu_{nf} = \dfrac{\mu_f}{(1-\phi)^{2.5}} \\ v_{nf} = \dfrac{\mu_f}{\rho_{nf}} \end{cases} \tag{4}$$
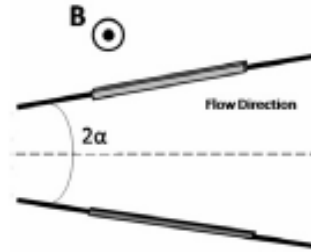
Fig. 1. Geometry of problem.

Using $\eta = \dfrac{\theta}{\alpha}$ as the dimensionless degree, the dimensionless form of the velocity parameter can be yield by dividing that to its maximum values as:

$$f(\eta) = \frac{f(\theta)}{f_{max}} \tag{5}$$

Substituting dimensionless parameters into equations.1-3 and eliminating the pressure term implies the following nonlinear third order boundary value problem:

$$f''' + 2\alpha \operatorname{Re} A^* (1-\phi)^{2.5} f f' + \left(4 - (1-\phi)^{1.25} H\right)\alpha^2 f' = 0 \tag{6}$$

where $A^* = (1-\phi) + \dfrac{\rho_s}{\rho_f} \phi$ is the particle parameter, $\operatorname{Re} = \dfrac{\alpha U_{max}}{v}$ is a Reynolds number and $H = \sqrt{\dfrac{\sigma B_0^2}{\rho v}}$ is the Hartmann number.

The following boundary conditions are considered:

$$f(0) = 1, \quad f(1) = 0, \quad f'(0) = 0 \tag{7}$$

3. Basic concept of Differential Transformation Method

Let $x(t)$ be analytic in a domain D and let $t = t_i$ represent any point in D. The function $x(t)$ is then represented by one power series whose center is located at t_i. The Taylor series expansion function of $x(t)$ is in the form of:

$$x(t) = \sum_{k=0}^{m} \frac{(t - t_i)}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=t_i} \qquad \forall t \in D \tag{8}$$

The particular case of Equation.7 when $t_i = 0$ is referred to as the Maclaurin series of $x(t)$ and is expressed as:

$$X(k) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=0} \qquad \forall t \in D \tag{9}$$

As explained in [13] the differential transformation of the function $x(t)$ is defined as follows:

$$X(k) = \sum_{k=0}^{\infty} \frac{H^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=0} \tag{10}$$

where $x(t)$ is the original is function and $X(k)$ is the transformed function. The differential spectrum of $X(k)$ is confined within the interval $t \in [0, H]$, where H is constant. The differential inverse transform of $X(k)$ is defined as follows:

$$x(t) = \sum_{k=0}^{\infty} \left(\frac{t}{H!} \right)^k X(k) \tag{11}$$

It is clear that the concept of the differential transformation is based upon the Taylor series expansion. The values of function $X(k)$ at values of argument k are referred to as discrete, i.e. $X(0)$ is known as the zero discrete, $X(1)$ as the first discrete, etc. the function is expressed by a finite series and equation.11 can be written as:

$$x(t) = \sum_{k=0}^{n} \left(\frac{t}{H!} \right)^k X(k) \tag{12}$$

Mathematical operations performed by DTM are listed in Table.1.

Some of the basic operations of Differential Transformation Method.

Original Function	Transformed Function
$x(t) = \alpha f(t) \pm \beta g(t)$	$X(k) = \alpha F(k) \pm \beta G(k)$
$x(t) = f(t)g(t)$	$X(k) = \sum_{l=0}^{k} F(l)G(k-l)$
$x(t) = \dfrac{df(t)}{dt}$	$X(k) = (k+1)F(k+1)$
$x(t) = \dfrac{d^2 f(t)}{dt^2}$	$X(k) = (k+1)(k+2)F(k+2)$

4. Application procedure

Equation.6 can be rewritten as following form:

$$f''' + bff' + cf' = 0 \tag{13}$$

where $b = 2\alpha \, \mathrm{Re} \, A^*(1-\phi)^{2.5}$ and $c = \left(4 - (1-\phi)^{1.25} H\right)\alpha^2$. In order to attack equation.13 by DTM, we consider the following boundary conditions:

$$f(0) = 1, \quad f'(0) = 0, \quad f''(0) = a \tag{14}$$

where a should be determined furthermore.

Taking the differential transform of equation.13 with respect to t, and considering $H = 1$ gives:

$$(k+1)(k+2)(k+3)F(k+3) + b\sum_{m=0}^{k} F(m)(k-m+1)F(k-m+1)$$
$$+ c(k+1)F(k+1) = 0 \tag{15}$$

From equation.4 we can write:

$$\begin{aligned} F(0) &= 1 \\ F(1) &= 0 \\ F(2) &= a \end{aligned} \tag{16}$$

We will have:

$$F(3) = 0$$

$$F(4) = -\frac{1}{12}ba - \frac{1}{12}ca$$

$$F(5) = 0$$

$$F(6) = \frac{1}{360}b^2a + \frac{1}{180}cba - \frac{1}{60}ba^2 + \frac{1}{360}c^2a$$

$$F(7) = 0$$

$$F(8) = -\frac{1}{20160}b^3a - \frac{1}{6720}cb^2a + \frac{1}{560}b^2a^2$$

$$-\frac{1}{6720}bc^2a + \frac{1}{560}bca^2 - \frac{1}{20160}c^3a$$

$$F(9) = 0$$

(17)

We avoid listing the other components. However it can be yielded that the closed form of the solutions is:

$$F(t) = F(0) \times t^0 + F(1) \times t^1 + F(2) \times t^2 + F(3) \times t^3 + F(4) \times t^4 + \cdots \quad (18)$$

According to Equation.7, inserting $t = 1$ into Equation.18 a can be yield as follows:

$$a = \frac{\left(\begin{array}{l} -20160 + 1680b + 1680c - 56b^2 - 112bc - 56c^2 + b^3 + 3b^2c \\ \\ +c^3 + \left(\begin{array}{l} -40642560b - 67737600c + 2177280b^2 + 5080320c^2 \\ -228480c^3 - 228480b^3 - 685440b^2c - 685440bc^2 \\ 7257600bc + 6496b^4 - 112b^5 + 9496c^4 - 112c^5 \\ +25984b^3c + 38976b^2c^2 + 25984bc^3 - 560b^4c \\ -1120b^3c^2 - 1120b^2c^3 - 560bc^4 + 6b^5c + 15b^4c^2 \\ 20b^3c^3 + 15b^2c^4 + 6bc^5 + b^6 + c^6 + 406425600 \end{array} \right)^{1/2} \end{array} \right)}{24b(-28 + 3b + 3c)} \quad (9)$$

5. Results and Discussions

In this section we want to investigate the accuracy and validity of this approximate solution for a special case. We also want to compare the obtained result with Runge Kutta forth order solution.

In Fig. 2, the velocity profile has been presented for a numerical case in which $Re = 75$, $\alpha = -5$, $H = 250$ and $\phi = 0.05$. As it can be seen in Fig. 2 the DTM have a good agreement with numerical solution.
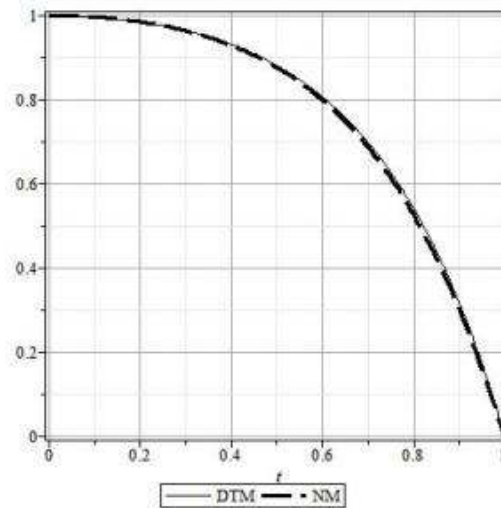

Fig.2. Comparison between Numerical solution and DTM result for $Re = 75$, $\alpha = -5$, $H = 250$ and $\phi = 0.05$.

A comparison between DTM solution and ADM solution [14] for velocity when $H = 250$, $Re = 25$, $\alpha = 5$ $\phi = 0$. Is listed in Table 2.

Table 2

Comparison between DTM solution and ADM solution [14] when $H = 250$, $Re = 25$, $\alpha = 5$, $\phi = 0$

η	DTM	ADM [14]	Error
0	1.000000	1.000000	0.000
0.2	0.954700	0.960841	0.007
0.4	0.821345	0.811225	0.012
0.6	0.614891	0.604866	0.009
0.8	0.339890	0.325834	0.014
1	0.000000	0.000000	0.000

6. Conclusions

In this study, the differential transformation method (DTM) has been successfully applied to a nonlinear MHD Jeffery Hamel problem with nanoparticle. We used a similarity transformation in order to reduce the partial differential equations modeling the flow, to a single third-order nonlinear

differential equation. A comparison between DTM and ADM has been presented. The obtained approximate result has been compared with Runge Kutta solution in a numerical case. The good agreement between numerical solution and Analytical Results proves the accuracy and validity of method.

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