# Application of Evolutionary Algorithms for the Planning of Urban Distribution Networks of Medium Voltage 

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#### Abstract

Currently, an important issue in power distribution is the need to optimize medium voltage ( mv ) networks serving urban areas. This paper shows how an evolutionary algorithm can be used as the basis for the type of efficient algorithm such optimization demands. The search for optimal network solution will be restricted to a graph defined from the urban map, so each graph branch represents a trench. The solution space (networks) is assumed with "loop feeder circuits": networks with two electrical paths from the high voltage/medium voltage ( $\mathbf{h v} / \mathrm{mv}$ ) substations to the customers. In the optimization process, the investment and loss load costs are considered taking into account the constraints of conductor capacities and voltage drop. The investment costs will take into account that some cables can be lying in the same trench. The process was applied for a Spanish city of $\mathbf{2 0 0} \mathbf{0 0 0}$ inhabitants.


Index Terms-Evolutionary algorithm, medium voltage network planning, network design, urban distribution network.

## I. Introduction

URBAN medium voltage (mv) distribution networks are designed with two criteria in mind: 1) minimal cost and 2) reliability of supply. To obtain a reliable supply, the technique most employed consists of searching for configurations with loop feeders, so that all mv/lv substations have two possible paths from (high voltage/medium voltage (hv/mv) substations, in which the system is operated as radial configuration [1]-[3]. The most commonly used urban mv network configurations [2], [3] are: ring, interconnective, and clasp (see Fig. 1). The extremes of the feeders of ring and interconnective configurations are hv/mv substations. The clasp configuration has switching stations to resupply customers after a fault has been isolated and uses a reserve feeder [2], [3] from the switching station to the $\mathrm{hv} / \mathrm{mv}$ substations that is normally opened.

The search for minimal cost network configuration is done assuming certains constraints, costs, and switching needs.

The bibliography related to planning mv urban networks is scarce [2]-[10] and is based on heuristic methods. Particularly, some of the models are expansion plans based on the resolution of the multiple traveling salesman problem (m-TSP) or of multiple vehicle routing problem (m-VRP) [2]-[7]. In these heuristic optimization models the losses and voltage drop are treated only with post-optimization processes, and the

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Fig. 1. Urban distribution network configurations.
common trench problem is not considered. Only Freund [8] and Burkhardt [9], [10] consider the possibility of laying two cables in the same trench.

In this paper, a technique based on evolutionary algorithms [11] is proposed to solve mv urban networks with ring, clasp, and interconnective configurations, taking into account the investment and losses costs, the constraints of conductor capacities, and voltage drops, and trenches with more that one conductor. The position of the switch that must be open is determined to obtain the lowest losses for each open loop.

This paper is organized as follows. In Section II, a complete description of the proposed evolutionary algorithm is presented. Section III presents an example with the mv network for a city in Spain, the curves for the evolution of the optimal solution, and a table of results with different configurations. The conclusions are summed up in Section IV.

## II. Evolutionary Algorithm for Urban Distribution MV NeTworks

The design of the mv urban network can be defined with a graph $G=(N, R)$ containing all possible routes. This graph is obtained from the city map, where $R$ is the set of branches (trenches) and $N$ is the set of nodes. The nodes include the $\mathrm{hv} / \mathrm{mv}$ and $\mathrm{mv} / \mathrm{lv}$ substations, loads, switching stations, and the intersections of branches. The coordinates of the hv/mv and $\mathrm{mv} / \mathrm{lv}$ substations, mv customers and switching stations, and the load of the mv/lv substations and mv customers are known.

Evolutionary algorithms work with a population of individuals (codified solutions), which is able to evolve in a given environment by application of the selection, crossover, and mutation operators. The "elite" or best individuals (solutions) survive during the optimization process.
Each individual or "chromosome," represents a complete solution of the mv network. The chromosomes are integer strings that codify the connection between the nodes of the mv network.


Fig. 2. Example of an mv network and the codification.

The first step of an evolutionary algorithm is to generate the initial population. The initial population consists of $\mu$ different chromosomes generated randomly, that represent $\mu$ different solutions for the configurations in Fig. 1.

A random sampling of this initial population is selected to create an intermediate population, and the crossover and mutation operators are applied to them in order to obtain a new population with $\lambda$ chromosomes. This process is called a generation and it is repeated until a stop function is decided.

The population of the next generation must have $\mu$ chromosomes. These $\mu$ chromosomes will be the best (minimal cost function) of the set composed by the $\mu$ and $\lambda$ chromosomes of the previous generation.

The crossover and mutation operators must obtain new chromosomes that represent urban distribution networks in Fig. 1 (with loop feeders).

## A. Codification

The codification used to represent a network is a variablelength integer string $\vec{x}$. The chromosome $k$ on generation $t$, is defined by

$$
\begin{equation*}
\vec{x}_{k}^{t}=\left(x_{1}, x_{2}, \ldots, x_{r}\right), \quad 1 \leq k \leq \mu \tag{1}
\end{equation*}
$$

where $x_{i}$ are the genes of the chromosome and contain node numbers of the graph. The string represents the sequence of nodes that shape the network loops and the values can be positive (cross) or negative (connection). When the network has more than one loop, the string represents the consecutive sequence of the nodes of all loops (see Fig. 2). When there are some loops going through a node, the node number will be repeated on the chromosome (e.g., nodes 4, 7, 8, and 12 in Fig. 2).

The negative value of a gene means that the $\mathrm{hv} / \mathrm{mv}$ or $\mathrm{mv} / \mathrm{lv}$ substation or the switching station located at this loop node is connected. These nodes will be called principal nodes. The positive values represent the sequence of nodes of the path between two consecutive principal nodes across the graph.

In the example of Fig. 2, the sets of principal nodes of the three loops are $\{4,1,3,7\},\{4,5,13,7\}$, and $\{4,10,8,7\}$, respectively. Segments $(8,12)$ and $(7,9)$ belong to loops 2 and 3. The mv/lv substation of node 8 is connected on loop 3 and, therefore, node 8 is a principal node of loop 3 and a normal node of loop 2.


Fig. 3. Reconnection of two segments of the same loop.

## B. Initial Population

The initial population, randomly generated, consists of $\mu$ chromosomes. The network codified by each chromosome must have, for all the mv/hv substations, two possible paths from $\mathrm{hv} / \mathrm{mv}$ substations or switching stations. These paths are selected by applying the heuristic algorithm proposed in Section II-F.

## C. Mutation Operator

The mutation operator is applied individually to each selected chromosome of the population, and the result is a new individual of the intermediate population. The mutation is the most important operator of the algorithm, and the modification implies changes of connections between the customers and the number of loops. This operator is based on a topological transformation of the codified network. The method consists of selecting two segments of the network associated to the chromosome, defined by:

1) both extremes of the segments are principal nodes (normal nodes can be between the principal nodes); or
2) fictitious segments between $\mathrm{hv} / \mathrm{mv}$ substations and/or switching stations.
The probability of selecting two segments is a function of the relation between the current and the new length after the change. The pairs of segments with greater length reduction have more probability of being selected. The two selected segments will be eliminated and replaced by two new segments reconnecting the principal nodes with two paths crossed with the previous. In the next examples the different possibilities when the branches are selected are represented. To simplify the examples, they have only principal nodes.
3) The segments can belong to the same loop, and the result is a new loop (see Fig. 3).
4) When the segments belong to different loops, there are two possible solutions [see Fig. 4(a) and (b)]. When the loops have a switching station on an extreme, only the solution with a hv/mv substation on both loops can be selected.
5) When the two selected segments belong to different loops and both are incident with $\mathrm{hv} / \mathrm{mv}$ substations or switching stations, the result is one loop and a fictitious branch (see Fig. 5).
6) Likewise, a fictitious segment and a segment of a loop can be selected and the result is two loops (see Fig. 6).
7) When the extreme principal nodes of a new segment are not adjacent in the graph, it is necessary to determine a



(a)



(b)

Fig. 4. (a) Reconnection type 1 of segments of different loops. (b) Reconnection type 2 of segments of different loops.


Fig. 5. Reconnection resulting in a fictitious segment.


Fig. 6. Connection with eliminated fictitious segment.
path that connects these nodes. The employed method is developed in Section II-F.

## D. Crossover Operator

The crossover operator belongs to the designated group "bisexual," because it is applied to two chromosomes (parents), obtaining two new chromosomes (children), with mixed characteristics of both parents. The steps are as follows (see Fig. 7):

Step 1) select two chromosomes of the population (parents);
Step 2) obtain the set of pairs of principal nodes that are eliminated (each pair separately), make disconnected the graph corresponding to the union of the networks associated to both individuals (taking into


Fig. 7. Example of crossover operation with two changes.
account that the hv/mv substations and switching stations are connected with fictitious branches);
Step 3) for each pair of nodes, the decision is taken to exchange the existing isolated subnetworks between both individuals, with a probability of $50 \%$.
The example Fig. 7 has pairs of nodes $(3,6)$ and $(11,17)$ that, if one of them is eliminated, the graph is disconnected. In the example all the nodes of the graph are principal nodes.

## E. Selection Operator

The selection operator belongs to the type called "elitist," because it is used for the selection of the $\mu$ best chromosomes. The elements of the population will be selected from among the chromosomes with minor costs from the set formed by the $\mu$ chromosomes of the previous population, plus the $\lambda$ individuals obtained with the mutation and crossover operators.

## F. Search for the Path Between Two Principal Nodes

When two principal nodes must be connected (crossover operator and initial population), it is necessary to obtain a path on the graph, and this does not have to be the shortest.

The process begins from the two principal nodes at the same time, until both paths are connected. Let node $i$, be the extreme of a calculated section of the path from node $n_{1}$, and $A D(i)$ the set of adjacent nodes to $i$; the probability of the following node selected being node $k$ is

$$
\begin{equation*}
\operatorname{Pr} \%(k)=\frac{p(k)}{\sum_{r \in A D(i)} p(r)} \cdot 100 \tag{2}
\end{equation*}
$$



Fig. 8. Weight ellipses defined by the points $i$ and $e$.


Fig. 9. Resultant path from $n_{1}$ to $n_{2}$.
where $p(k)$ is the weight of the node $k$ :

$$
\begin{equation*}
p(k)=\frac{d(i, e)}{d(i, k)+d(k, e)} \tag{3}
\end{equation*}
$$

To obtain the weights for the different adjacent nodes, the ellipses with the focus on points $i$ and $e$ will be considered (see Figs. 8 and 9)

$$
\begin{aligned}
\left(x_{e}, y_{e}\right) & =\left(\frac{x_{i}+\delta \cdot x_{n_{2}}}{1+\delta}, \frac{y_{i}+\delta . y_{n_{2}}}{1+\delta}\right) \\
d(i, e) & =\min \left\{d\left(i, n_{2}\right), \max \{d(i, k) / \forall k \in A D(i)\}\right\} \\
\delta & =\frac{d(i, e)}{d\left(i, n_{2}\right)-d(i, e)}
\end{aligned}
$$

## G. Objective Function

The objective function to minimize is the sum of the total cost for the loops, trenches, and $\mathrm{hv} / \mathrm{mv}$ substations plus a penalty function for unfeasible networks. It is important to allow unfeasible networks into the population, because good solutions can be the result of operating with unfeasible networks and because the operators do not ensure feasible descendants when the parents are feasible.

The optimal solution of the mv network must fulfill the constraints of conductor capacities and voltage drops. Each one of the loops of the network must be able to supply all their loads from both extremes (emergency operation). The cost of the trenches is calculated separately, because it cannot be considered a term of the cost of the loops.

When one of the extremes of the loop is a switching station the backup line is open, and only the conductor type of the loop must be determined. If the loop has two hv/mv substations on the extremes, it is necessary to obtain the open branch of the loop to minimize the losses.


Fig. 10. The mv network of Vigo with three hv/mv substations and six switching stations.

Given a loop whose extremes are hv/mv substations (both different or the same), the obtaining of the minimal cost of the loop, implies determining: the optimal conductor of each loop and the location of the open branch. The cost of a loop for a type $t$ conductor will be

$$
\begin{equation*}
C_{l}^{t}=\sum_{(i, j) \in \mathrm{loop}}\left(K_{i n v}^{t} \cdot L_{i, j}+K_{\mathrm{loss}}^{t} \cdot I_{i, j}^{2} L_{i, j}\right) \tag{4}
\end{equation*}
$$

where $K_{i n v}^{t}$ and $K_{\text {loss }}^{t}$ are the investment and losses unitary costs, respectively, for the type $t$ conductor.

In the proposed algorithm, different types of conductor are admitted, but each loop will be formed by only one type. The most unfavorable situations are given when all the power of the loop is supplied from one extreme (minimal section).

When the cost of a loop with branch $(i, i+1)$ opened is known, the cost with the branch $(j, j+1)$ opened can be obtained as

$$
\begin{equation*}
C_{l}^{t}(j, j+1)=C_{l}^{t}(i, i+1)+\Delta S_{i, j}^{t} \tag{5}
\end{equation*}
$$

where $\Delta S_{i, j}^{t}$ is the cost variation of the losses, and its value is

$$
\begin{gather*}
\Delta S_{i, j}^{t}=K_{\mathrm{loss}}^{t} \cdot\left[I_{j, j+1}^{2} \cdot \Gamma_{i}-2 \cdot I_{j, j+1} \cdot \Lambda_{i}+2 \cdot I_{j, j+1} \cdot \Lambda_{i+1}\right. \\
\left.+I_{j, j+1}^{2} \cdot \Gamma_{i+1}+I_{j, j+1}^{2} \cdot L_{i, i+1}\right] \tag{6}
\end{gather*}
$$

with

$$
\begin{aligned}
\Gamma_{k} & =\sum_{r \in U(k)} L_{r} ; \quad \Lambda_{k}=\sum_{r \in U(k)}\left(I_{r} . L_{r}\right) \\
\Psi_{k} & =\sum_{r \in U(k)}\left(I_{r}^{2} \cdot L_{r}\right)
\end{aligned}
$$

where $U(k)$ is the set of branches located between node $k$ and the $\mathrm{hv} / \mathrm{mv}$ substation that is supplying it power.

Only when $\Delta S_{i, j}$ is negative is it possible to reduce the cost. To prove when an open branch reduces the cost, (7) must be verified

$$
\begin{equation*}
\Delta S_{i, j}^{t} \leq 0 \Leftrightarrow I_{j, j+1} \leq 2 \cdot \frac{\Lambda_{i}-\Lambda_{i+1}}{L_{l o o p}} \tag{7}
\end{equation*}
$$



Fig. 11. (a) Cost evolution curve. (b) Cables and trenches lengths evolution curve. (c) Number of loops evolution curve.

Through the recursive checking of the adjacent branches, the branch that must be opened can be determined, taking into account that the cost curve of the losses is convex with respect to the optimal branch.

Once the optimal open branch is obtained, it is necessary to determine the optimal conductor type. When the cost of a loop is known, with branch $(i, i+1)$ opened and the conductor type $t$, the cost with the branch $(j, j+1)$ opened and the conductor type $u$ can be obtained as

$$
\begin{equation*}
C_{l}^{u}(j, j+1)=C_{l}^{t}(i, i+1)+\Delta S_{i, j}^{t}+\Delta Q^{t, u}(j, j+1) \tag{8}
\end{equation*}
$$

The cost variation $\Delta Q^{t, u}(j, j+1)$, when the conductor type is changed from $t$ to $u$ and the open branch is $(j, j+1)$ is

$$
\begin{gather*}
\Delta Q^{t, u}(j, j+1)=\left(K_{i n v}^{u}-K_{i n v}^{t}\right) \cdot L_{l o o p}+\left(K_{\mathrm{loss}}^{u}-K_{\mathrm{loss}}^{t}\right) \\
\cdot\left[\Psi_{i}+\Psi_{i+1}+I_{j, j+1}^{2} \cdot L_{l}+2 . I_{j . j+1} \cdot\left(\Lambda_{i+1}-\Lambda_{i}\right)\right] \cdot \tag{9}
\end{gather*}
$$

Equations (6) and (9) are functions only of the data of the two implicated branches in the change and their adjacent nodes.
The values of $\Gamma_{k}, \Lambda_{k}$ and $\Psi_{k}$ of each principal node and $L_{\text {loop }}$ and the minimal cost of each loop can be associated to the chromosomes and must only be recalculated by the loops modified with the mutation and crossover operators.

## H. Stop Function

The number of generations (iterations) of the algorithm is variable. The process is stopped when the population is homogeneous and there are not variations of the population during enough generations.III.

## III. EXAMPLE

The example of Fig. 10 represents the mv network of Vigo city (Spain) with 200000 inhabitants, $242 \mathrm{mv} / \mathrm{lv}$ substations

TABLE I
Results for Vigo City

| $\begin{aligned} & \text { helmv } \\ & \text { substat:* } \end{aligned}$ | switch stations | Total cost (ME) | $\begin{array}{\|l\|} \hline \mathrm{n}^{0} \text { of } \\ \text { loops } \\ \hline \end{array}$ | $\begin{aligned} & \text { length } \\ & \text { cable }(\mathrm{km}) \end{aligned}$ | length trench (km) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 6 | 7.84 | 19 | 71.8 | 56.9 |
| 3 | 0 | 9.87 | 18 | 88.4 | 63.1 |
| 2 | 6 | 11.60 | 21 | 106.5 | 74.2 |
| 2 | 0 | 13.87 | 18 | 130.6 | 74.8 |

(195 MW installed), three hv/mv substations, and six switching stations. The graph has 1852 nodes and 2996 branches.

Some of the initial networks are unfeasible. The evolution of the population toward to feasible solutions is very fast, due to the mutation operator. Later evolution is slower, and it corresponds with a fine fitting, in which the crossover operation has more effect. The total cost, after 60000 iterations of a $(6,10)$-ES, is $7.84 \mathrm{M} €(1 € \approx 0.9 \$)$. The network is composed of 19 loops, with 56.9 km of trenches and 71.8 km of cables. The curves of Fig. 11(a)-(c) represent the evolution of the cost, the cable, and trench lengths, and the number of loops for the best chromosome of the population of each generation. Table I presents the optimal results with different numbers of $\mathrm{hv} / \mathrm{mv}$ substations (2 or 3), with or without switching stations.

## IV. Conclusions

The proposed method makes it possible to obtain the mv network from a large city, knowing the position of the mv/lv substations, hv/mv substations, and switching stations, by considering investment and losses costs and constraints of conductor capacities and voltage drop. The investment costs will take into account that some cables can be lying in the same trench. The cost of the losses are calculated considering the switch which must be opened in each loop, for optimal radial operation. The


Fig. 12. Costs for typical Spanish commercial conductors.

TABLE II
Costs for Commercial Substation Arrangements

| level | Voltage (kV) | Type | Cost (c) |
| :---: | :---: | :---: | :---: |
| hv | 132 | aerial | 420,000 |
|  | 132 | SF6 | 660,000 |
|  | 66 | aerial | 180,000 |
|  | 66 | SF6 | 270,000 |
| mv | 20 | SF6 | 30,000 |

TABLE III
Costs for Commercial hv/mv Transformers

| Power (MVA) | cost invest <br> (c) | costofiron losses ( $\mathbf{C}$ ) | cost of joule losses (emvai) |
| :---: | :---: | :---: | :---: |
| 10 | 192,000 | 54,500 | 631 |
| 15 | 270,000 | 65,450 | 352 |
| 25 | 300,000 | 109,000 | 180 |
| 40 | 360,000 | 190,000 | 110 |
| 50 | 403,000 | 196,000 | 88 |

resolution for large cities is feasible in reasonable times, without making simplifications in the objective function or in the graph. The proposed method can be modified for application to an expansion planning model. In this case, the objective function must consider the existing lines.

## ApPENDIX

The conductor costs are represented by Fig. 12.
The trench costs are $48000 € / \mathrm{km}$ under the sidewalks and $84000 € / \mathrm{km}$ under the road.

The hv/mv substation costs are shown in Tables II and III.
The economic parameters are: $0.04 € / \mathrm{kW}$-losses, 25-year planning, $25 \%$ overload factor, $1 \%$ annual inflation and $5 \%$ annual interest.

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