Journal of NUCLEAR SCIENCE and TECHNOLOGY, 8[4], p. 236~239 (April 1971).

Application of Fast Fourier Transform Algorithm to On-Line Digital Reactor Noise Analysis

Masayuki IZUMI*

Received October 26, 1970

The noise analysis technique for detecting anomalous reactor behavior through changes in the frequency characteristics of various fluctuating observables is particularly useful in reactor diagnosis for on-line monitoring in high-power-density, complex and expensive reactor systems[1]. However, the success of such a system of reactor diagnosis depends on the ability to process the raw noise signals obtained from the various sensors in almost real time.

Now, the methods that satisfy such a condition are the multichannel analog method with arrangement of band-pass filters in a row and the digital method utilizing the fast Fourier transform (FFT)[2]~[4]. For this purpose, the digital method has more advantages than the multichannel analog method, for it ease of raw noise signal processing and the possibility of computing not only the power spectral density, but also the auto- and the cross-correlation functions.

The fast Fourier transform algorithm described by Cooley & Tukey is a method for computing the finite discrete Fourier transform of a discrete periodic time series and involves less computation than other digital methods. The key elements of this algorithm are to factor N as $P_1, P_2, \cdots, P_m$, then to decompose the transform into N/P_j transforms of size P_j for $j=1,2,\cdots,m$, where N is the number of sampled data in the time series.

We consider the problem of calculating the finite discrete Fourier transform

$$G(k)=\sum_{t=0}^{N-1} X(t)W^{kt},\quad k=0,1,\cdots,N-1, \qquad (1)$$

where $X(t)$ is the given time series with period N, and W is the principal N-th root of unity. At this point, we define the notation

$$e(x)=e^{-2\pi jx},\quad (j=\sqrt{-1}). \qquad (2)$$

Using this notation, the finite discrete Fourier transform of the time series $X(t)$ is written

$$G(k)=\sum_{t=0}^{N-1} X(t)e\left(\frac{kt}{N}\right). \qquad (3)$$

If N has factors P and Q so that $N=PQ$, and further if we let $t=qP+p$ and $k=\boldsymbol{p}Q+\boldsymbol{q}$, where $p, \boldsymbol{p}=0,1,\cdots,P-1$ and $q, \boldsymbol{q}=0,1,\cdots,Q-1$, we have

$$G(\boldsymbol{p}Q+\boldsymbol{q})=\sum_{p=0}^{P-1}\sum_{q=0}^{Q-1} X(qP+p)e\left(\frac{(\boldsymbol{p}Q+\boldsymbol{q})(qP+p)}{PQ}\right)$$

$$=\sum_{p=0}^{P-1} e\left(\frac{p\boldsymbol{q}}{PQ}+\frac{p\boldsymbol{p}}{P}\right)\sum_{q=0}^{Q-1} X(qP+p)e\left(\frac{q\boldsymbol{q}}{Q}\right). \qquad (4)$$

Then the Fourier transform of Eq. (1) is performed in two stages:

a) Compute the Q-point transforms

$$\sum_{q=0}^{Q-1} X(qP+p)e\left(\frac{q\boldsymbol{q}}{Q}\right)=D(p,\boldsymbol{q}) \qquad (5)$$

of each of the P sequences.

b) Compute, then, the P-point transforms

$$G(\boldsymbol{p}Q+\boldsymbol{q})=\sum_{p=0}^{P-1} e\left(\frac{p\boldsymbol{q}}{PQ}+\frac{p\boldsymbol{p}}{P}\right)D(p,\boldsymbol{q}) \qquad (6)$$

of the Q sequences.

A straightforward calculation using Eq. (1) requires N^2 operations, which means a complex multiplication followed by a complex addition. The fast Fourier transform algorithm, however, requires only $N(P+Q)$ operations and does not involve the calculations of $e(x)$, so-called the twiddle factor. Therefore, the fast Fourier transform algorithm requires fewer number of operations than a straightforward calculation, hence the name "fast Fourier transform".

It is easy to see how successive application of the above procedure, starting with its application to Eq. (5), give an m-step algorithm requiring

$$T=N(P_1+P_2+\cdots+P_m) \qquad (7)$$

operations, where

$$N=P_1\cdot P_2\cdot\cdots\cdot P_m. \qquad (8)$$

In particular, when $P=P_1=P_2=\cdots=P_m$, the total operations require

$$T=PN\log_P N=\left(\frac{P}{\log_2 P}\right)N\log_2 N \qquad (9)$$

* Atomic Energy Research Laboratory, Hitachi Ltd., *Ozenji, Kawasaki-shi.*

operations. Therefore, the use of $P=3$ is in from the most efficient, but the gain is only about 6% over the use of 2 or 4, which have more important advantages for computers with binary arithmetic, both in addressing and in operation economy. We use a power of 2 for simplicity of programming. Thus, the ratio between straightforward calculation and the fast Fourier transform algorithm is given by

$$R=\frac{N^2}{T}=\frac{N^2}{2N\log_2 N}=\frac{N}{2\log_2 N}. \qquad (10)$$

In the analysis of the time series by digital spectrum analysis, we have the indirect method and the direct method[5]. The latter has many advantages over the former in respect of data processing and computing time. The computational sequence of the indirect method is (1) spectral estimation through computation of the correlation function and (2) Fourier transforming, while that of the direct method is (1) to obtain the periodogram and (2) smoothing. **Figure 1** shows the relation between the time domain and the frequency domain for the spectrum analysis.

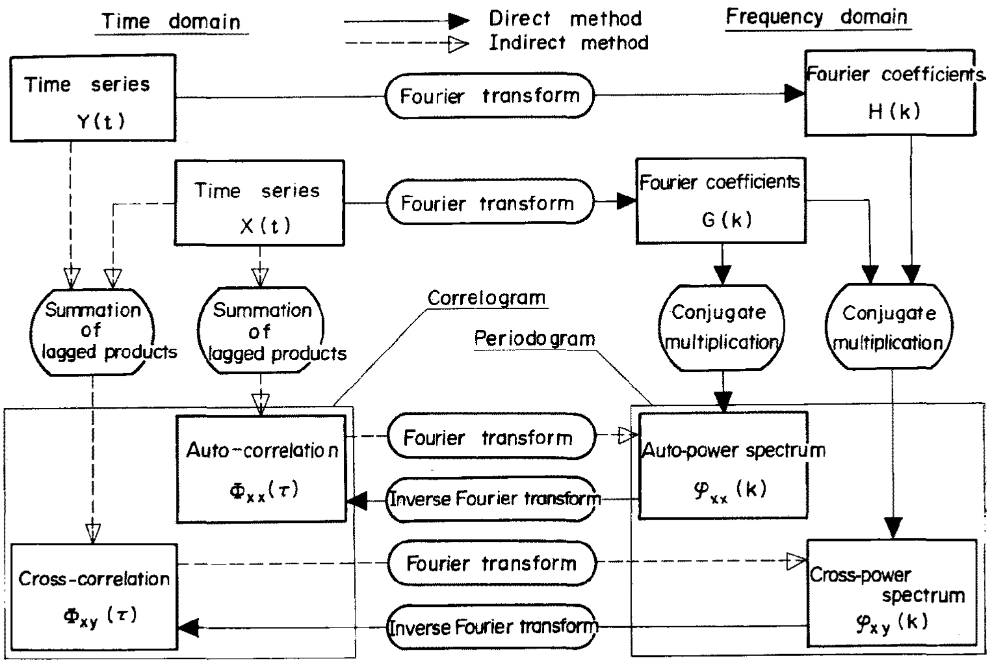

Fig. 1 Relation between time domain and frequency domain

With the direct method, the estimated power spectral density agrees with the exact power spectral density, but the experimental relative error (standard deviation/average value) is 100%. Now, if the system is ergodic, the expected value of the ensemble average is equal to one of the time average. We have here used the ensemble average method, because then the length of the time series is finite. As a result, the experimental relative error of the power spectral density is inversely proportional to the root of the ensembles.

We have analyzed the power spectra density of neutron flux and coolant flow rate in the HTR (Hitachi Training Reactor), and the results are shown in **Fig. 2.** The computing time required for the spectral analysis is only about 16 sec for the conditions of the present case (4 ensembles, N=1,024) using the HITAC-5020F (*i.e.*, about 4 sec/ensemble).

The most important uses of the fast Fourier transform algorithm have been in connection with the convolution theorem[6]. The major use of the convolusion theorem is

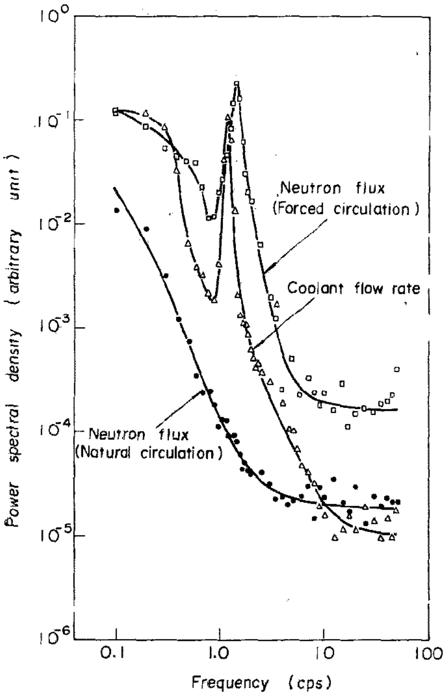

Fig. 2 Power spectral density by direct method in HTR

to compute the correlation functions for each lag time τ, *i.e.*,

the auto-correlation function

$$\Phi_{XX}(\tau)=\frac{1}{N}\Sigma X(t)X(t+\tau),\ \tau=0, \pm 1, \cdots, \pm L \tag{11}$$

and the cross-correlation function

$$\Phi_{XY}(\tau)=\frac{1}{N}\Sigma X(t)Y(t+\tau),\ \tau=0, \pm 1, \cdots, \pm L, \tag{12}$$

where the summations are overall values of t for which the products are defined. Now, we encounter two problems, which are, that the lags are performed circularly and that N may not always be convenient for use of the fast Fourier transform algorithm. Both these problems can be solved by appending zero values to one or both ends of the time series. That is,

$$\begin{cases} X'(t)=X(t), & 0\leqslant t\leqslant N-1 \\ X'(t)=0, & N\leqslant t\leqslant N'-1. \end{cases} \tag{13}$$

$X'(t)$ is periodic with period N', which is the smallest power of 2 greater than or equal to $N+L$. The Y series is analogous. The Fourier transforms of the two time series are given by

$$G'(k)=\sum_{t=0}^{N'-1}X'(t)e\left(\frac{kt}{N'}\right) \tag{14}$$

and $$H'(k)=\sum_{s=0}^{N'-1}Y'(s)e\left(\frac{ks}{N'}\right). \tag{15}$$

The cross-correlation function is the inverse Fourier transform of the sequence formed by the multiplication of the $Y'(t)$ transform and the complex conjugate of the $X'(t)$ transform. Now, if $\bar{G}'(k)$ is the complex conjugate of $G'(k)$, the cross-correlation function can be written in the form

$$\Phi_{XY}(\tau)=\frac{1}{N'}\sum_{k=0}^{N'-1}\bar{G}'(k)H'(k)e\left(-\frac{k\tau}{N'}\right)$$
$$=\sum_{t=0}^{N'-1}X'(t)Y'(t+\tau). \tag{16}$$

The auto-correlation function is precisely the cross-correlation function of the time series with itself. **Figure 3** shows the auto-correlation function of the neutron flux in the HTR with forced circulation. The time required to compute the auto-correlation function is about 9 sec using the HITAC-5020F.

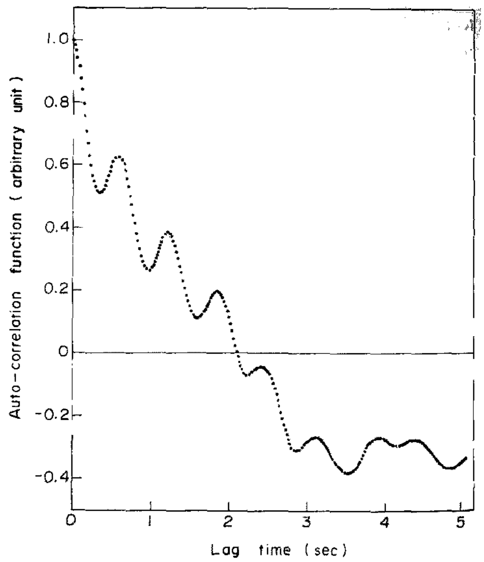

Fig. 3 Auto-correlation function of neutron flux in HTR with forced circulation

On the other hand, if both time series are only real parts, we may take advantage of the fact that the two time series can be transformed at one time. We use one series as the real part and the other series as the imaginary part, thus

$$Z(t)=X'(t)+jY'(t). \tag{17}$$

Then, we can define that

$$E(k)=\sum_{t=0}^{N'-1} Z(t)e\left(\frac{kt}{N'}\right). \tag{18}$$

From Eq. (18), we obtain the two Fourier transforms

$$G'(k)=\frac{E(k)+\bar{E}(N'-k)}{2} \tag{19}$$

and $$H'(k)=\frac{E(k)-\bar{E}(N'-k)}{2j} \tag{20}$$

using the Hermitian symmetry and its definition.

—REFERENCES—

(1) FRY, D.N., ROBINSON, J.C.: Conf. on incipient failure diagnosis, *CONF*-671011, 89~101 (1967).

(2) COOLEY, J.W., TUKEY, J.W.: *Math. Comput.*, **19**[90], 297~301 (1965).

(3) SINGLETON, R.C.: *IEEE Trans.*, Vol. AU-15, No. 2, 91~98 (1967).

(4) SINGLETON, R.C.: *Commun. ACM*, **11**[11], 773 ~776 (1968).

(5) GENTLEMAN, W.M., SANDE, G.: *Fall Joint Computer Conf., AFIPS Proc.*, Vol. 29, 563~ 578 (1966).

(6) STOCKHAM, T.G.: *Spring Joint Computer Conf., AFIPS Proc.*, Vol. 28, 229~233 (1966).