Application of fuzzy inferencing principles in reservoir operation analysis

I. Goulter, Suharyanto
Central Queensland University, Rockhampton, Queensland 4702, Australia

Abstract

Successful application of fuzzy control to an optimum control problem relies on the ability to make appropriate inferences from fuzzy information. In the reservoir operation problem, the operational rule adopted for simulation of the performance of a reservoir under historical or generated inflows, demands, etc. usually relates to the concept of an optimum release for the 'current' period. The main source of uncertainty in this process arises from the prediction of the value of the inflow during the current period. The value of this inflow is generally known in terms of its distribution. Since the storage volume at the end of each period is highly dependent on this inflow, it also is influenced by this uncertainty. Most stochastic simulation techniques for reservoir operation, however, operate on the basis of strict compliance to, or interpolation of, the operating policies and use as input stochastically generated inflows to account for the inflow uncertainty. Little attention, if any, is given to accounting for uncertainty in the decision itself. Since the optimum release decision obtained from a 3-state variable (storage volume at the beginning of the current period, the inflow in the previous time period, and the reservoir release during the current period) stochastic dynamic program is based on evaluation of the *expected* value of the return to the system, such a release decision should only be considered as a 'guide' such that, in certain circumstances, deviation of the release decision from the operating rule might be necessary. In this paper, a rational approach for selecting a release decision different from that envisaged in the operation rule is derived from application of the principles of fuzzy inference. The approach is demonstrated by application to the Wadaslintang Reservoir in Prembun, Central Java, Indonesia.

1 Introduction

The advantages and limitations of a wide range of simulation and optimisation techniques for reservoir operation have been addressed in many studies (e.g., Yeh[17]). In attempting to exploit the advantages of simulation and optimisation techniques, the system analyst generally applies optimisation to obtain a broad view of optimal operation policies and then uses simulation to evaluate the efficiency and effectiveness of the policies in more detail. In studies of the operation of reservoirs, some attempts have also been made to embed a one-time-step re-optimisation technique within a simulation process to determine how much water to be released during the current-time period (Tejada-Guibert et al.[14] and Johnson et al.[7]).

The optimisation technique most popular for reservoir operation is stochastic dynamic programming technique (SDP). This popularity is due in part to the ability of the technique to incorporate easily the stochastic nature of streamflow, to handle non-linearities in the objective function and constraints, and to address almost any configuration of reservoir system. Successful applications of SDP can be found, for example, in Terry et al.[15] and Bras et al.[2]

An important by-product of SDP optimisation is the 'cost-to-go' values for the whole discretisation grid for the reservoir storage and inflow values. Some researchers have reported that implementation of the 'cost-to-go' function within the simulation process can be used as guidance for a more optimum operation (Tejada-Guibert et al.[14] and Braga et al.[1]). Braga et al.[1] used the 'cost-to-go' function obtained from multi-reservoir SDP model for re-optimisation of one reservoir at a time within the simulation process. Tejada-Guibert et al.[14] solved the spline SDP for the two reservoir Shasta-Trinity system and then used the 'cost-to-go' function by-product in the re-optimisation of the implemented policy within a simulation process.

Both approaches are based on the fact that the 'cost-to-go' function is calculated at the grid points, normally the mid points, of the intervals of the discretised variables. In the simulation, and also, in real-world operation, however, the state variable value of either inflow or reservoir storage or both may fall anywhere in the interval associated with a particular grid point, and rarely, if ever, falls at the actual grid point. To address this situation, and also to avoid possible constraint violations arising from the actual condition being some distance from the grid point value, the operating policies obtained from the optimisation technique need to be adjusted either by an interpolation and adjustment approach, or by one-step re-optimisation.

It should be noted that these two approaches emphasize avoiding violation of the constraints. The approaches, therefore, tend to have the greatest impact if the reservoir state occurring as a result of application of the 'optimal' operating policy is close to the constraint boundaries. The inconsistency between the use of grid points of the state variable interval to derive operating policies during the optimisation process and the use of actual values of the reservoir state variables in the one-step re-optimisation within the simulation process should also be acknowledged as it may result in the release volume which is actually selected being different from that defined by the derived operating policies.

In this paper, explicit consideration of the likelihood of the value of the state variable not being exactly at the corresponding grid point is proposed through application of the fuzzy membership function concept in both the SDP and the simulation processes. The underlying issue addressed in the paper is whether the influence of conditions in adjacent storage intervals should be considered if, during the optimisation process, the storage volume does not fall exactly at the mid-point of the interval, or more correctly lies close to the boundary with a neighbouring interval. This issue is examined in the context of, if, during the simulation process, the actual storage volume does not fall close to the grid-point of a storage interval, the process of identifying the optimal operating policy necessity to move from crisp operating policies to inferenced policies. The degree of the membership function of a particular value of actual storage volume in the circumscribing classes of fuzzy storage intervals is employed in this assessment. The approach has particular relevance to answering the question of which storage interval which actually posseses or contains a particular storage volume, and therefore, which storage interval determines the associated 'cost-to-go' function value to be considered in the optimisation process and the resulting operating policy.

The framework of this paper is generation of operating policies through application of 3-state-variable stochastic fuzzy dynamic programming (Suharyanto and Goulter[13]) followed by evaluation of the performance of the system by implementing the derived operation policies through a simulation process. The contribution of the paper lies in the application of the membership function concept from fuzzy set theory to i) assess the influence of adjacent storage intervals during the optimisation process and ii) provide a mechanism for assessing the necessity of complying with or deviating from the derived operating policies during the simulation process.

2 Mathematical Formulation

2.1 Stochastic Fuzzy Dynamic Programming (SFDP)

Let the storage volume be divided into NS intervals, with sharp or crisp boundaries between each interval, and where $S_{k,t}$ represents the mid-point of the k^{th} storage class interval at the beginning of the current time period t. The possible reservoir releases are also discretised into NR classes, with $R_{r,t}$ representing the mid-point of the release class r during month t. (The reservoir storage and the reservoir release are discretised according to the Savarenskiy's scheme.) The monthly inflow ranges are similarly discretised into NI intervals. The term $Q_{j,t}$ is the representative inflow of class or interval j during month t.

The representative inflow of each class interval is determined, if there is adequate historical data falling in the interval, by taking the mean of inflows falling within that interval, or else, by taking the mid-point of the interval.

Consider now a reservoir operating within an environment in which it has to meet specified target demands, e.g., irrigation demands. Let the demand in time period t be denoted by T_t. The continuity equation for this simple situation is:

$$S_{t+1} = S_{k,t} + Q_{j,t} - R_{r,t} - L_t \qquad (1)$$

where L_t is the loss through evaporation, seepage, etc. from the reservoir during time period t and S_{t+1} is the resulting storage volume at the end of period t, i.e., at the beginning of period $t+1$. S_{t+1}, which will fall into one of the storage intervals defined at the end of period t, i.e., at the beginning of period $t+1$, is constrained to lie in the feasible storage volume range, i.e.,

$$S_{\min,t+1} \le S_{t+1} \le S_{\max,t+1} \quad \forall t \qquad (2)$$

where $S_{min,t+1}$ and $S_{max,t+1}$ are the minimum and maximum storage volumes, respectively, during time period $t+1$.

A typical simple objective function for this type of system is minimisation of the sum of the square of deviations of the actual releases from target releases if the release value is outside the acceptable range of the target value. In this case, this short-term return may be expressed in an exponential functional form as used by Karamouz and Houck[9] and shown in Equation (3).

$$B_{r,t} = \begin{cases} 1.580K_1\left(e^{-\left(\frac{R_{r,t}}{T_t}\right)/0.80} - e^{-1.0}\right) & \text{if } R_{r,t} < 0.80T_t \\ 0.0 & \text{if } 0.80T_t \le R_{r,t} \le 1.20T_t \\ 0.388K_2\left(e^{\left(\frac{R_{r,t}}{T_t}\right)/1.20} - e^{1.0}\right) & \text{if } R_{r,t} > 1.20T_t \end{cases} \qquad (3)$$

where K_1 and K_2 are weighting constants on under and over achievement of the target release respectively, the value of release between 0.80 T_t and 1.20 T_t represents the safe range of releases within which no penalty is incurred, and $B_{r,t}$ is short-term return as a consequence of releasing $R_{r,t}$ during period t. Note that the short-term return in Equation (3) is calculated on the basis of the deviation of the release from the respective target demands. An additional penalty function on the deviation of the storage from a pre-determined target storage volume can also be incorporated in Equation (3).

Proceeding backwards in time, the general recursive equation of the stochastic crisp dynamic programming is given in Equation (4) below :

$$f_t^n(k,i) = \min_{R_{r,t} \in \overline{R}_t} \left\{ B_{r,t} + \sum_{j=1}^{NI} P_{i,j}^{t-1} \ f_{t+1}^{n-1}(l,j) \right\} \tag{4}$$

where $f_t^n(k,i)$ is the optimum objective function associated with being in storage interval or class k at the beginning of period t with n periods of the optimisation remaining and having observed an inflow in interval i during the previous time period. $\overline{R}_t$ is the set of possible discretised releases in time period t. $P^{t-1}_{i,j}$ is the probability, given that the inflow observed during the previous time period $t-1$ falls into the flow interval $Q_{i,t-1}$, that the inflow during the period t will be in class j, resulting, when the release is $R_{r,t}$, in a storage level in class l at the end of time period t.

The value of the term $f_{t+1}^{n-1}(l,j)$ in this process is the long-term return associated with the storage class interval l which encloses the value of S_{t+1} when the inflow in the current time period is in class j. Note that in this process, the determination of which storage class is actually enclosing the value of S_{t+1} is decided on the basis of crisp interval. As long as the value of S_{t+1} is within the interval, it does not matter whether it is actually at the grid-point or close to the interval boundary or anywhere in between. Similarly, the process does not consider the influence of adjacent storage intervals, as only the value of $f_{t+1}^{n-1}(l,j)$ of the enclosing storage interval is used in the determination of the optimal release decision. This type of formulation is, therefore, referred to as Stochastic Crisp Dynamic Programming (SCDP) wherein crisp storage and inflow class intervals are adopted.

For each initial current period storage state k, the optimum objective function value is obtained by minimising the right hand side of Equation (4) with respect to all of the possible discretised release decision values. This SCDP formulation is essentially equivalent to that of Butcher[3].

The following additional chance constraints may also be imposed in this formulation to exercise control on the probability of violation of the minimum and maximum storage volumes.

$$p\left[S_{t+1} \geq S_{\min,t+1}\right] \leq P_{\min} \tag{5}$$

$$p\left[S_{t+1} \leq S_{\max,t+1}\right] \leq P_{\max} \tag{6}$$

where P_{min} and P_{max} are the acceptable limits of the probability of violation of the minimum storage volume and the maximum storage volume, respectively.

A fuzzy modification of the above general recursive equation is based upon the observation discussed above that the value of S_{t+1} can lie anywhere within an enclosing storage interval. This situation raises the issue of the potential influence of adjacent intervals in the determination of which $f_{t+1}^{n-1}(l,j)$ values are most appropriate to use. In this study, the storage intervals are considered as fuzzy intervals with some degree of overlap of the membership functions of adjacent storage intervals for particular storage values. The membership functions for the fuzzy storage intervals considered in this study are either triangular or trapezoidal in form as shown in Figure 1.

This strategy is directed at facilitating a mechanism for assessment of non-specificity of a particular storage volume within storage volume intervals, i.e., at addressing explicitly the fact that the actual storage in a storage class interval can lie anywhere within that class range, and for evaluation of the influence of the adjacent storage intervals and their associated long-range returns in the determination of the optimal release decision.

The modification of the general recursive formulation required to incorporate these fuzzy concepts is shown in Equation (7) where the previous long-term return is determined by selecting the storage interval, among the 'triggered' storage intervals, which gives the minimal weighted long-term return. 'Triggered' storage intervals in this context refer to those storage intervals for which the actual storage volume has a non-zero membership function value. The use of this minimal weighted long-term return was shown previously by Suharyanto and Goulter[13] to be more appropriate than other inferencing approaches.

$$f_t^n(k,i) = \min_{R_{r,t} \in \bar{R}_t} \left\{ B_{r,t} + \sum_{j=1}^{NI} P_{i,j}^{t-1} \cdot f_{t+1}^{n-1}(u^*, j) \right\} \qquad (7)$$

where u^* is the class of the storage interval which results in:

$$\min_u \left(\frac{f_{t+1}^{n-1}(u,j)}{\mu_{S_{t+1}}(u)} \right) \quad \forall u \,\middle|\, \mu_{S_{t+1}}(u) > 0.0 \qquad (8)$$

where $\mu_{S_{t+1}}(u)$ is the membership level of the actual storage volume at the end of time period t, i.e., at the beginning of time period $t+1$, in storage class u. The 'triggered' storage intervals are defined mathematically as those intervals with $\mu_{S_{t+1}}(u) > 0.0$.

In order to account for the economic value of the long-term return, i.e., the economic values of the long-term returns of deficits and surpluses, Equation (7) becomes :

$$f_t^n(k,i) = \min_{R_{r,t} \in \bar{R}_t} \left\{ B_{r,t} + \sum_{j=1}^{NI} P_{i,j}^{t-1} \frac{1}{(1+d)} f_{t+1}^{n-1}(u^*, j) \right\} \qquad (9)$$

Transactions on Ecology and the Environment vol 12, © 1996 WIT Press, www.witpress.com, ISSN 1743-3541

where d is the seasonal discount rate.

2.2 Implementation of Operating Policies

During the simulation process, the implemented operating policy is chosen such that it results in the most minimum weighted current time period 'cost-to-go' value. The selection of the minimum weighted 'cost-to-go' value is shown mathematically in Equation (10).

$$\min_k \left\{ \frac{CTG_t(R_t(k,i))}{\mu_{S_t}(k)} \right\} \quad \forall k \mid \mu_{S_t}(k) > 0.0 \tag{10}$$

where $CTG_t(R_t(k,i))$ is the current time period 'cost-to-go' function as a consequence of adopting release policy $R_t(k,i)$. $\mu_{S_t}(k)$ is the 'triggered' level or the membership level of storage interval k for the actual storage volume S_t. Note that this minimal selection process is equivalent to that used in the optimisation procedure in which the 'cost-to-go' value is weighted by the reciprocal of its membership level.

The value of $CTG_t(R_t(k,i))$ is calculated through a one-time-step re-optimisation procedure as shown in Equation (11).

$$CTG_t(R_t(k,i)) = \left\{ B_{R_t(k,i)} + \sum_{j=1}^{NI} P_{i,j}^{t-1} \ \mathrm{CTG}_{t+1}(u^*, j) \right\} \tag{11}$$

where $\mathrm{CTG}_{t+1}(u^*, j)$ is the long-term 'cost-to-go' value obtained as a by-product of the optimisation process. The storage interval which gives the minimum weighted cost-to-go' value, $u*$, is the class of storage interval which results in:

$$\min_u \left(\frac{CTG_{t+1}(u,j)}{\mu_{S_{t+1}}(u)} \right) \quad \forall u \mid \mu_{S_{t+1}}(u) > 0.0 \tag{12}$$

A discount rate factor can be incorporated in Equation (11) through :

$$CTG_t(R_t(k,i)) = \left\{ B_{R_t(k,i)} + \sum_{j=1}^{NI} P_{i,j}^{t-1} \ \frac{1}{(1+d)} \ \mathrm{CTG}_{t+1}(u^*, j) \right\} \tag{13}$$

3 Application and Discussion

The methodology described above was applied to the Wadaslintang reservoir which is located on the Bedegolan river in Central Java, Indonesia. The reservoir is used primarily to supply water to the surrounding irrigation areas, for hydro power generation of 2x8 MW capacity, low flow augmentation, and to meet other domestic, municipal, and industrial demands. The reservoir has minimum and maximum capacities of 30.0 million cubic metres (mcm) and 432.0 mcm, respectively.

In this study, the reservoir is operated to meet the irrigation demands only which are assumed to have a far higher priority than other demands. The formulation presented in this paper, however, is easily modified to incorporate other types of objectives. K_1 and K_2 are both assigned values of 10^6 in this study. The irrigation target demands incorporated in the optimisation and the simulation processes are those demands which must be met at least 90.0 % of the time. Discount rate values of 0.0% and 20.0% were used in the study. Note that 20.0% is in fact a realistic value for the economic situation in Indonesia where the case study is located. The values of *NI*, *NS*, and *NR* used in the study were 5, 20, and 25, respectively. The starting storage volume for the simulation process was 30.0 mcm.

Evaluation of the operating policies generated by the SCDP and SFDP formulations was performed through simulation of the reservoir for 25 years of operation. A sequence of 25 years of stochastically generated inflow was used in these simulations. This inflow sequence was generated by 3-parameter log-normal transformation (McMahon and Mein[12]). The overall performance of the reservoir was evaluated on the basis of the Expected Annual Deviation (EAD) which indicates the extent to which the actual releases from the system deviate from the target releases. A comparison of the EAD values resulting from the SCDP and SFDP approaches is shown in Table 1. Additional system performance measures used in the evaluation were reliability and resiliency as defined by Hashimoto et al.[6] and vulnerability which is defined as the average of maximum deficit ratio to its respective target demand as used in, for example, Tickle and Goulter[16]. The values of those additional performance measures for the two approaches are shown in Tables 2(a) and 2(b).

It can be seen from Tables 1 and 2 that for a discount rate equal to 0.0%, the SFDP approach does not produce significant improvements in values of EAD over those obtained from SCDP approach. The improvements which did occur were only of the order of 4.0% for the triangular membership function and about 6.0% for the trapezoidal membership function. With the discount rate equal to 20.0%, however, it can be seen that the EAD values resulting from the SFDP were significantly better, in some cases over 90.0%, than those obtained from the corresponding SCDP model. This result gives an indication of the important influence of the true present time economic value of the long-term returns of deficit and surplus and is, in fact, a more realistic approach.

It was also observed that the EAD values obtained by incorporating trapezoidal storage intervals were consistently lower, i.e., more optimal, than those obtained using triangular storage intervals. This phenomenom may be partly a result of the fact that, during the process of minimal inferencing for trapezoidal storage intervals, it is only the influence (return) of the adjacent 'triggered' storage interval which is actually weighted by the reciprocal of its 'triggered' level. The return in the storage interval which would 'crisply' circumscribe the actual storage volume is essentially not weighted (increased) because its 'triggered' level is always unity. This mechanism, therefore, results in a more strict selection criteria through which the model will adopt the consequence or the 'cost-to-go' value associated with any adjacent 'triggered' storage interval only if the influence of the adjacent 'triggered' storage interval is very strong, i.e., the adjacent storage is part of a significantly more optimum path. In another words, a change to the optimum path which occurs as a result of considering the condition in adjacent storage intervals indicates a very strong pull to that new 'optimal' path.

For the triangular membership function storage interval, on the other hand, a weighting is applicable to all the 'triggered' storage intervals, including that interval which would 'crisply' circumscribe the actual storage volume. As a consequence, this approach is less 'strict' than that of the trapezoidal membership function storage interval in the sense that even though the optimal policy may change due to consideration of the impacts of adjacent intervals, such changes do not necessarily indicate that there is very strong pull to that new 'optimal' path.

4 Conclusion

It has been demonstrated that Stochastic Fuzzy Dynamic Programming (SFDP) approaches, using either a triangular or trapezoidal fuzzy membership function for the storage intervals, are able to identify a more 'optimum' path and, therefore, improved operating policies compared to those obtained from traditional Stochastic Crisp Dynamic Programming (SCDP). Results from the optimisation process show that the Expected Annual Deviation (EAD) values resulting from SFDP using triangular membership functions for the storage intervals are always bigger, i.e., less optimal, than those observed for SFDP using trapezoidal membership function. The simulated system performance measures resulting from different discount rates also show that the operating policies resulting from the application of the SFDP approaches are clearly superior to those obtained from SCDP for higher values of discount rate factors.

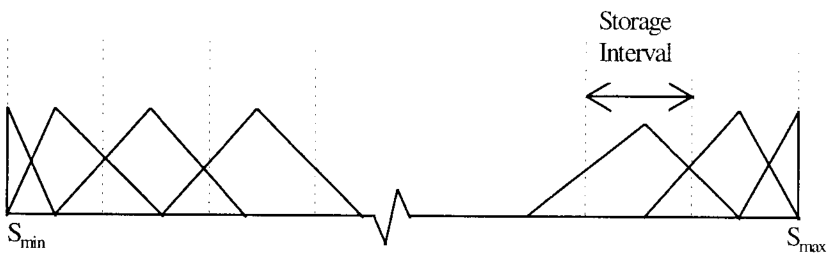
These results demonstrate the value of using fuzzy theory concepts in addressing issues arising from the discretisation of storage required for dynamic programming and from the associated inability, when using traditional crisp stochastic dynamic programming formulation, to address the potential impact of actual storage values being some distance from the representative grid point for the storage interval in which that actual storage volume falls and perhaps being close to an adjacent storage interval which has a significantly different release policy and associated return. Ongoing work on this topic by the authors is expected to expand and validate further the concept of using fuzzy set theory in optimisation of reservoir operation.

References

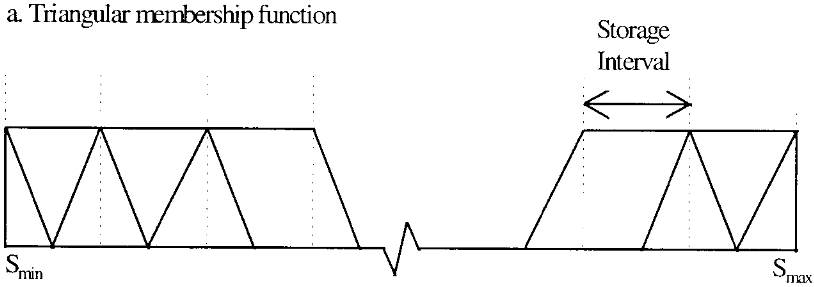
1. Braga, B. P. F., Yeh, W. W-G., Becker. L., and Barros, M. T. L. Stochastic Optimisation of Multiple-Reservoir-System Operation, *Journal of Water Resources Planning and Management*, ASCE, 1991, 117(4), 471-481.
2. Bras, R. L., Buchanan, R., and Curry, K. C. Real Time Adaptive Closed Loop Control of Reservoir with the High Aswan Dam as a Case Study, *Water Resources Research*, 1983, 19(1), 33-52.
3. Butcher, W. S., Stochastic Dynamic Programming for Optimum Reservoir Operation, *Water Resources Bulletin*, 1971, 7(1), 115-123.
4. ECI, *Feasibility Report of the Karangsambung/ Wadaslintang Multipurpose Project*, Volume I - III, ECI, 1978, Gombong, Indonesia.
5. ECI, *The Karangsambung/ Wadaslintang Multipurpose Project, Appendixes*, 1978, ECI, Denver, Colorado.
6. Hashimoto, T., Stedinger, J. R., and Loucks, D. P. Reliability, Resiliency, and Vulnerability Criteria for Water Resource System Performance Evaluation, *Water Resources Research*, 1982, 18(1), 14-20.
7. Johnson, S. A., Stedinger, J. R., and Staschus, K. Heuristic Operating Policies for Reservoir System Simulation, *Water Resources Research*, 1991, 27(5), 673-685.
8. Johnson, S. A., Stedinger, J. R., Shoemaker, C. A., Li, Y., and Tejada-Guibert, J. A. Spline Stochastic Dynamic Programming, in Water Resources Planning and Management (ed S. C. Harris), pp. 137-140, *Proceedings of the 16th Annual Conference on Water Resources Planning and Management*, Sacramento, California, May 12-25, 1989.
9. Karamouz, M., and Houck, M. H. Annual and Monthly Reservoir Operating Rules Generated by Deterministic Optimisation, *Water Resources Research*, 1982, 18(5), 1337-1344.
10. Kaufmann, A. and Gupta, M. M. *Introduction to Fuzzy Arithmetic : Theory and Applications*, Van Nostrand Reinhold, New York, 1991, xvii+361p.
11. Loucks, D. P., Stedinger, J. R., and Haith, D. A. *Water Resources Systems Planning and Analysis,* Prentice-Hall, Inc., New Jersey, 1981, xv+559p.
12. McMahon, T. A., and Mein, R. G. *River and Reservoir Yield* Water Resources Publications, Littleton, Colorado, USA, 1986, ix+368p.
13. Suharyanto and Goulter, I. C. Use of Fuzzy Sets Theory for Consideration of Storage Non-specificity in Stochastic Dynamic Programming for

Reservoir Operation, 1996, (accepted for presentation in *7th IAHR International Symposium on Stochastic Hydraulics*), Mackay, Australia.

14. Tejada-Guibert, J. A., Johnson, S. A., and Stedinger, J. R. Comparison of Two Approaches for Implementing Multireservoir Operating Policies Derived Using Stochastic Dynamic Programming, *Water Resources Research*, 1993, 29(12), 3969-3980.
15. Terry, L. A., Pereira, M. V. F., Araripe Neto, T. A., Silva, L. F. C. A., and Sales, P. R. H., Coordinating the Energy Generation of the Brazilian National Hydrothermal Electrical Generating System, *Interfaces*, 1986, 16(1), 16-38.
16. Tickle, K. and Goulter, I. C. Assessment of Performance Metrics for a Reservoir Under Stochastic Conditions, in Stochastic Hydraulics (ed Kuo, J. T. and Lin, G. F.), pp. 583-590, *Proceedings of the Sixth IAHR International Symposium on Stochastic Hydraulics*, Taipei, May 18-20, 1992.
17. Yeh, W. W. G. Reservoir Management and Operations Models: A State-of-the-Art Review, *Water Resources Research*, 1985, 21(12), 1797-1818.



a. Triangular membership function



b. Trapezoidal membership function

Figure 1: Membership Functions of the Storage Intervals.

Table 1. Expected Annual Deviation (EAD) Values Obtained from SCDP and Two Approaches of SFDP.

		EAD	
Program	Discount Rate (%)	0.0	20.0
SCDP		3.00×10^5	2.08×10^5
SFDP	Triangular Storage Interval	2.87×10^5 (4.63)	1.36×10^4 (93.44)
	Trapezoid Storage Interval	2.81×10^5 (6.39)	1.22×10^4 (94.13)

Note : Values in parentheses show percentage of improvement compared to the SCDP results

Table 2.a Simulation Results for Discount Rate = 0.0 %.

Performance Measures	Simulation results by Implementing the Operating Policies Obtained From :		
	SFDP with Triangular Storage Intervals	SFDP with Trapezoidal storage Intervals	SCDP
p[fail]	28.667	28.333	29.333
p[full]	0.000	0.000	0.000
p[empty]	0.000	0.000	0.000
p[R_in_safe_range]	58.000	58.000	57.000
p[R<safe_range]	28.667	28.333	29.333
p[R>safe_range]	13.333	13.667	13.667
Reliability	71.333	71.667	70.667
Resiliency	45.349	47.059	46.591
Total Deviation	1.447×10^7	1.471×10^7	1.684×10^7
Vulnerability	35.000	35.600	34.700
Max_deficit_ratio	0.700	0.700	0.800
Mean_storage	177.300	178.200	178.600
CV_storage	0.441	0.439	0.423

Table 2.b Simulation Results for Discount rate =20.0%.

Performance Measures	Simulation results by Implementing the Operating Policies Obtained From :		
	SFDP with Triangular Storage Intervals	SFDP with Trapezoidal storage Intervals	SCDP
p[fail]	18.000	19.333	28.000
p[full]	0.000	0.000	0.000
p[empty]	1.000	1.333	0.000
p[R_in_safe_range]	71.333	71.333	59.667
p[R<safe_range]	17.000	18.667	28.000
p[R>safe_range]	11.667	10.000	12.333
Reliability	82.000	80.667	72.000
Resiliency	66.667	62.069	46.429
Total Deviation	1.451×10^7	1.478×10^7	1.515×10^7
Vulnerability	40.500	39.000	35.500
Max_deficit_ratio	0.900	1.000	0.800
Mean_storage	167.700	166.300	177.300
CV_storage	0.478	0.484	0.434

Table 3. Influence of Discount Rate to EAD Values.

Program		EAD				
	Discount Rate (%)	0.0	5.0	10.0	15.0	20.0
SCDP		3.00×10^5	2.74×10^5	2.26×10^5	1.96×10^5	2.08×10^5
SFDP	Triangular Storage Interval	2.87×10^5 (4.63)	1.10×10^5 (59.73)	7.47×10^4 (66.87)	3.84×10^4 (80.37)	1.36×10^4 (93.44)
	Trapezoid Storage Interval	2.81×10^5 (6.40)	9.91×10^4 (63.82)	3.53×10^4 (84.37)	2.61×10^4 (86.64)	1.22×10^4 (94.13)

Note : Values in parentheses show the percentage of improvement compared to the SCDP results