Research Article

Application of Generalized Fractional Thermoelasticity Theory with Two Relaxation Times to an Electromagnetothermoelastic Thick Plate

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The fractional mathematical model of Maxwell's equations in an electromagnetic field and the fractional generalized thermoelastic theory associated with two relaxation times are applied to a 1D problem for a thick plate. Laplace transform is used. The solution in Laplace transform domain has been obtained using a direct method and its inversion is calculated numerically using a method based on Fourier series expansion technique. Finally, the effects of the two fractional parameters (thermo and magneto) on variable fields distributions are made. Numerical results are represented graphically.

1. Introduction

Fractional calculus (FC) is a very useful tool in describing the evolution of systems with memory, which typically are dissipative and complex systems such as glasses, biopolymers, biological cells, porous materials, amorphous semiconductors, and liquid crystals. Scaling laws and self-similar behavior are supposed to be fundamental features of complex systems. In recent decades the FC and in particular the fractional differential equations have attracted interest of researchers in several areas including mathematics, physics, chemistry, biology, engineering, and economics [1–4].

FC theory has been used successfully in thermoelasticity and thermoviscoelasticity, such that a quasi-static uncoupled theory of thermoelasticity based on the fractional heat-conduction equation was put forward by Povstenko [5]. The theory of thermal stresses based on the heat-conduction equation with the Caputo time-fractional derivative is used by Povstenko [6] to investigate thermal stresses in an infinite body with a circular cylindrical hole. Sherief et al. [7] introduced a fractional order theory of thermoelasticity. Raslan [8, 9] has solved 1D problems in the context of this theory and applied this theory to 2D problem of thick plate [10]. The fractional parameter effect of this theory on thermoelastic material with variable thermal variable thermal

material properties has been studied in [11, 12]. Ezzat [13] established a model of fractional heat-conduction equation by using the Taylor series expansion of time-fractional order developed by Jumarie [14].

Hamza et al. established a new mathematical model of Maxwell's equations in an electromagnetic field in [15] and derived a fractional model for thermoelasticity associated with two relaxation times in [16]. A model for unsteady thermoelectric magnetohydrodynamics (TEMHD) flow and heat transfer of two immiscible second-grade fluids with two fractional parameters was introduced in [17].

The previous work introduced by Hamza et al. [18] describes one-dimensional problems in the context of the theory [16] in which the electromagnetic field effects are ignored. The model in the previous work depends only upon one fractional parameter α which does not have electric or magnetic affects. Another application of this theory introduced in [16] depends on two fractional parameters α and β. The first fractional parameter α appears only in the heat equation and is absent from both the equation of motion and the constitutive equations. In this work, we solve a 1D problem for a thick plate that was not solved in the context of both theories [15, 16].

The new model here depends on two fractional parameters α and β. The first fractional parameter α appears in the

heat equation, the equation of motion, and the constitutive equations.

The effects of the fractional parameters corresponding to two models [15, 16] are discussed. The solution is obtained in Laplace transformed domain using a direct approach. All the studied field variables are represented graphically.

2. The Mathematical Model

The governing fractional Maxwell's equations in an electromagnetic field are given by [15]

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\tau^{\beta-1}}{\beta!} \frac{\partial^\beta \mathbf{H}}{\partial t^\beta}, \tag{2}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{3}$$
$$\nabla \cdot \mathbf{E} = 0,$$

where β is the order of the fractional derivative in Caputo sense with respect to time t such that $0 \le \beta \le 1$, μ_0 is the magnetic permeability, and ε_0 is the electric permeability. $\mathbf{H}$ and $\mathbf{E}$ are the magnetic and electric field intensities, respectively. $\mathbf{J}$ is the current density and τ is a positive constant.

Equation (2) is called "Fractional Faraday's Law of Magnetic Induction" which was derived in [15]. The proof uses Faraday's induction law and the fractional Taylor's series expansion developed by Jumarie [14].

Ohm's law for moving media states that

$$\mathbf{J} = \sigma_0 \left[\mathbf{E} + \mu_0 \frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H} \right], \tag{4a}$$

where σ_0 is the electric conductivity of the medium (assumed to be infinite) and $\mathbf{u}$ is the displacement vector.

Since $\mathbf{J}$ is bounded and σ_0 is infinite, it follows that

$$\mathbf{E} = -\mu_0 \frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H}. \tag{4b}$$

The governing equations for generalized fractional thermoelasticity associated with two relaxation times in the absence of external body forces and heat sources are given by [16] the following.

(i) The constitutive equations:

$$\sigma_{ij} = 2\mu e_{ij} + \lambda \delta_{ij} e_{kk} - \beta_1 \delta_{ij} \left(1 + \frac{\tau_1^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \theta, \tag{5}$$

where θ is the temperature of the medium, σ_{ij} are the components of the stress tensor, the constants λ and μ are Lamé's constants, and $\beta_1 = \alpha_t (3\lambda + 2\mu)$, where α_t is the coefficient of linear thermal expansion and $\alpha \in (0, 1]$ is the order of the time-fractional derivative. τ_1 is a positive constant and e_{ij} are the components of the strain tensor.

(ii) The strain-displacement relations:

$$e_{ij} = \frac{1}{2} \left(u_{i,j} + u_{j,i} \right). \tag{6}$$

(iii) The equation of motion:

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i, \tag{7}$$

where ρ is the density assumed independent of time t, u_i is the displacement vector component, σ_{ij} are the components of the stress tensor, and F_i is the component of the Lorentz force given by

$$F_i = \mu_0 \left(\mathbf{J} \times \mathbf{H} \right)_i. \tag{8}$$

(iv) The fractional heat equation:

$$k\theta_{,ii} = \beta_1 \theta_0 \dot{e}_{kk} + \rho C_E \left(\frac{\partial}{\partial t} + \frac{\tau_0^\alpha}{\alpha!} \frac{\partial^{1+\alpha}}{\partial t^{1+\alpha}} \right) \theta, \tag{9}$$

where the specific heat at constant strain is C_E and k is the thermal conductivity. τ_0 is a positive constant and θ_0 is a reference temperature assumed to be such that $|(\theta - \theta_0)/\theta_0| \ll 1$. As usual, superimposed dots denote time derivatives. The convention of summing over repeated indices is used.

3. Formulation of the Problem

We consider a magnetothermoelastic thick plate of perfect conductivity occupying the region $0 \le x \le \ell$ in an initial magnetic field $\mathbf{H}_0$ in y direction at a uniform reference temperature θ_0. This produces an induced magnetic field $\mathbf{h}$ in the direction of the y-axis and an electric field $\mathbf{E}$ in the z-axis (perpendicular to $\mathbf{H}$ and $\mathbf{u}$).

The x-axis is perpendicular to the surface of the plate. The upper surface ($x = 0$) of the plate is taken to be traction-free and is subjected to a thermal shock that is a function of time. The lower surface ($x = \ell$) of the plate is taken to be thermally isolated and laid on a rigid foundation.

It is assumed that all the state functions depend on x and t only.

Thus

$$\mathbf{u} = (u(x,t), 0, 0),$$
$$\mathbf{H} = (0, H_0 + h, 0), \tag{10}$$
$$\mathbf{E} = (0, 0, E).$$

In the context of generalized fractional thermoelasticity associated with two relaxation times, the constitutive equations, the equation of motion, and fractional heat equation can be expressed in our case as follows:

$$\sigma = \sigma_{xx} = (2\mu + \lambda) \frac{\partial u}{\partial x} - \beta_1 \left(1 + \frac{\tau_1^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \theta,$$

$$e_{xx} = \frac{\partial u}{\partial x},$$

$$\rho \frac{\partial^2 u}{\partial t^2} + F_1 = (2\mu + \lambda) \frac{\partial^2 u}{\partial x^2} - \beta_1 \left(1 + \frac{\tau_1^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \frac{\partial \theta}{\partial x}, \tag{11}$$

$$k \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial}{\partial t} \left[\beta_1 \theta_0 \left(\frac{\partial u}{\partial x} \right) + \rho C_E \left(1 + \frac{\tau_0^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \theta \right].$$

Now, (1)–(4b) yield

$$J = \frac{\partial h}{\partial x} - \varepsilon_0 \frac{\partial E}{\partial t}, \tag{12}$$

$$h = -\frac{H_0 \beta!}{\tau^{\beta-1}} \frac{\partial^{1-\beta}}{\partial t^{1-\beta}} \left(\frac{\partial u}{\partial x} \right), \tag{13}$$

$$E = -\mu_0 H_0 \frac{\partial u}{\partial t}. \tag{14}$$

From (13), (14), and (12), we obtain

$$J = \varepsilon_0 \mu_0 H_0 \left(\frac{\partial^2 u}{\partial t^2} \right) - \frac{H_0 \beta!}{\tau^{\beta-1}} \frac{\partial^{1-\beta}}{\partial t^{1-\beta}} \left(\frac{\partial^2 u}{\partial x^2} \right). \tag{15}$$

From (13), (15), and (8), we obtain

$$F_1 = \mu_0 H_0^2 \left(\frac{\beta!}{\tau^{\beta-1}} \frac{\partial^{1-\beta}}{\partial t^{1-\beta}} \left(\frac{\partial^2 u}{\partial x^2} \right) - \varepsilon_0 \mu_0 \left(\frac{\partial^2 u}{\partial t^2} \right) \right). \tag{16}$$

We shall use the following nondimensional variables

$$\left(x', u' \right) = c\eta \left(x, u \right),$$

$$\left(t', \tau'_0, \tau'_1, \tau' \right) = c^2 \eta \left(t, \tau_0, \tau_1, \tau \right),$$

$$\sigma' = \frac{\sigma}{\lambda + 2\mu},$$

$$\theta' = \frac{\beta_1 \left(\theta - \theta_0 \right)}{\lambda + 2\mu}, \tag{17}$$

$$h' = \frac{h}{H_0},$$

$$E' = \frac{E}{\mu_0 H_0 c}.$$

Now, using the above nondimensional variables, the system of equations of the problem will reduce to

$$\left(1 + \alpha_0^2 \right) \frac{\partial^2 u}{\partial t^2} = \left(1 + R_H \beta! \tau^{1-\beta} \frac{\partial^{1-\beta}}{\partial t^{1-\beta}} \right) \left(\frac{\partial^2 u}{\partial x^2} \right)$$

$$- \left(1 + \frac{\tau_1^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \theta,$$

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{\partial}{\partial t} \left[\varepsilon \left(\frac{\partial u}{\partial x} \right) + \left(1 + \frac{\tau_0^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \theta \right],$$

$$e_{xx} = \frac{\partial u}{\partial x},$$

$$h = -\beta! \tau^{1-\beta} \frac{\partial^{1-\beta}}{\partial t^{1-\beta}} \left(\frac{\partial u}{\partial x} \right),$$

$$E = -\frac{\partial u}{\partial t},$$

$$\sigma = \frac{\partial u}{\partial x} - \left(1 + \frac{\tau_1^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \theta, \tag{18}$$

where

$$c = \sqrt{\frac{\lambda + 2\mu}{\rho}},$$

$$\eta = \frac{\rho c_E}{k},$$

$$R_H = \frac{\mu_0 H_0}{\rho c^2}, \tag{19}$$

$$\alpha_0 = \frac{\mu_0 \varepsilon_0 H_0}{\rho},$$

$$\varepsilon = \frac{T_0 \beta_1^2}{(\lambda + 2\mu) k\eta}.$$

Also, we assume that the medium is initially at rest and the undisturbed state is maintained at uniform reference temperature. Then we have

$$u = u(x, 0) = \dot{u}(x, 0) = \theta(x, 0) = \dot{\theta}(x, 0) = 0. \tag{20}$$

Now, taking the Laplace transform (denoted by an overbar) with parameter s of both sides of the above system of equations, we get

$$\left(1 + \alpha_0^2 \right) s^2 \overline{u} = \left(1 + R_H \beta! \tau^{1-\beta} s^{1-\beta} \right) \left(\frac{\partial^2 \overline{u}}{\partial x^2} \right)$$

$$- \left(1 + \frac{\tau_1^\alpha s^\alpha}{\alpha!} \right) \overline{\theta}, \tag{21}$$

$$\frac{\partial^2 \theta}{\partial x^2} = s \left[\varepsilon \left(\frac{\partial \overline{u}}{\partial x} \right) + \left(1 + \frac{\tau_0^\alpha s^\alpha}{\alpha!} \right) \overline{\theta} \right], \tag{22}$$

$$\overline{e}_{xx} = \frac{\partial \overline{u}}{\partial x}, \tag{23}$$

$$\overline{h} = -\beta! \tau^{1-\beta} s^{1-\beta} \left(\frac{\partial \overline{u}}{\partial x} \right), \tag{24}$$

$$\overline{E} = -s\overline{u}, \tag{25}$$

$$\overline{\sigma} = \frac{\partial \overline{u}}{\partial x} - \left(1 + \frac{\tau_1^\alpha s^\alpha}{\alpha!} \right) \overline{\theta}. \tag{26}$$

Solving (21) and (22), we obtain

$$\left\{ \frac{d^4}{dx^4} + L_2 \frac{d^2}{dx^2} + L_1 \frac{d}{dx} + L_0 \right\} \{\overline{\theta}, \overline{u}\} = 0. \qquad (27)$$

The corresponding characteristic equation of (27) is

$$k^4 + L_2 k^2 + L_1 k + L_0 = 0, \qquad (28)$$

where

$$
\begin{aligned}
L_2 \\
&= -\frac{s\left(1 + \tau_0^\alpha s^\alpha/\alpha!\right)\left(1 + R_H\beta!\tau^{1-\beta}s^{1-\beta}\right) + \left(1 + \alpha_0^2\right)s^2}{\left(1 + R_H\beta!\tau^{1-\beta}s^{1-\beta}\right)}, \\
L_1 &= -\frac{\varepsilon s\left(1 + \tau_1^\alpha s^\alpha/\alpha!\right)}{\left(1 + R_H\beta!\tau^{1-\beta}s^{1-\beta}\right)}, \\
L_0 &= -\frac{s^3\left(1 + \tau_0^\alpha s^\alpha/\alpha!\right)\left(1 + \alpha_0^2\right)}{\left(1 + R_H\beta!\tau^{1-\beta}s^{1-\beta}\right)}.
\end{aligned}
\qquad (29)
$$

The solution of (27) compatible with (22) has the form

$$\overline{\theta} = \sum_{i=1}^{4} \varepsilon s k_i C_i e^{k_i x}, \qquad (30)$$

$$\overline{u} = \sum_{i=1}^{4} \left[k_i^2 - s\left(1 + \frac{\tau_0^\alpha s^\alpha}{\alpha!}\right) \right] C_i e^{k_i x}. \qquad (31)$$

Substituting (30) and (31) into (26), we get

$$
\begin{aligned}
\overline{\sigma} \\
&= \sum_{i=1}^{4} k_i \left[k_i^2 - s\left(1 + \frac{\tau_0^\alpha s^\alpha}{\alpha!}\right) - \varepsilon s\left(1 + \frac{\tau_1^\alpha s^\alpha}{\alpha!}\right) \right] C_i e^{k_i x}.
\end{aligned}
\qquad (32)
$$

Using (31), (23)–(25) become

$$\overline{e}_{xx} = \sum_{i=1}^{4} k_i \left[k_i^2 - s\left(1 + \frac{\tau_0^\alpha s^\alpha}{\alpha!}\right) \right] C_i e^{k_i x},$$

$$\overline{h} = -\beta!\, (\tau s)^{1-\beta} \sum_{i=1}^{4} k_i \left[k_i^2 - s\left(1 + \frac{\tau_0^\alpha s^\alpha}{\alpha!}\right) \right] C_i e^{k_i x}, \qquad (33)$$

$$\overline{E} = -s \sum_{i=1}^{4} \left[k_i^2 - s\left(1 + \frac{\tau_0^\alpha s^\alpha}{\alpha!}\right) \right] C_i e^{k_i x}.$$

We assume that the boundary conditions have the form

$$
\begin{aligned}
\theta(0, t) &= f(t), \\
\frac{\partial \theta(\ell, t)}{\partial x} &= 0, \\
\sigma(0, t) &= u(\ell, t) = 0, \\
t &> 0,
\end{aligned}
\qquad (34)
$$

where $f(t)$ is a known function of t. Equations (30)–(32) give

$$
\begin{aligned}
\sum_{i=1}^{4} \varepsilon s k_i C_i &= \overline{f}(s), \\
\sum_{i=1}^{4} k_i^2 C_i e^{k_i \ell} &= 0, \\
\sum_{i=1}^{4} \left[k_i^2 - s\left(1 + \frac{\tau_0^\alpha s^\alpha}{\alpha!}\right) \right] C_i e^{k_i \ell} &= 0, \\
\sum_{i=1}^{4} k_i \left[k_i^2 - s\left(1 + \frac{\tau_0^\alpha s^\alpha}{\alpha!}\right) - \varepsilon s\left(1 + \frac{\tau_1^\alpha s^\alpha}{\alpha!}\right) \right] C_i &= 0.
\end{aligned}
\qquad (35)
$$

4. Inversion of the Laplace Transform

We shall now outline the method used to invert the Laplace transforms in the above equations. Let $\overline{f}(s)$ be the Laplace transform of a function $f(t)$. The inversion formula for Laplace transforms can be written as [19]

$$f(t) = \frac{e^{\gamma t}}{2\pi} \int_{-\infty}^{\infty} e^{ity} \overline{f}(\gamma + iy)\, dy, \qquad (36)$$

where γ is an arbitrary real number greater than all the real parts of the singularities of $\overline{f}(s)$. Expanding the function $h(t) = \exp(-\gamma t) f(t)$ in a Fourier series in the interval $[0, 2L]$, we obtain the approximate formula [19]:

$$f(t) \approx f_N(t) = \frac{1}{2} c_0 + \sum_{k=1}^{N} c_k, \quad \text{for } 0 \le t \le 2L, \qquad (37)$$

where

$$c_k = \frac{e^{\gamma t}}{L} \operatorname{Re}\left[e^{ik\pi t/L} \overline{f}\left(\gamma + \frac{ik\pi}{L}\right) \right]. \qquad (38)$$

Two methods are used to reduce the total error. First, the "Korrektur" method is used to reduce the discretization error. Next, ε-algorithm is used to reduce the truncation error and therefore to accelerate convergence.

The Korrektur method uses the following formula to evaluate the function $f(t)$:

$$f(t) = f_{NK}(t) = f_N(t) - e^{-2\gamma L} f_{N'}(2L + t). \qquad (39)$$

We shall now describe ε-algorithm that is used to accelerate the convergence of the series in (37). Let N be an odd natural number and let $s_m = \sum_{k=1}^{m} c_k$ be the sequence of partial sums of (37). We define ε-sequence by

$$
\begin{aligned}
\varepsilon_{0,m} &= 0, \\
\varepsilon_{1,m} &= s_m, \\
& \qquad\qquad\qquad m = 1, 2, 3, \ldots, \\
\varepsilon_{n+1,m} &= \varepsilon_{n-1,m+1} + \frac{1}{\left(\varepsilon_{n,m+1} - \varepsilon_{n,m}\right)}, \\
& \qquad\qquad\qquad n, m = 1, 2, 3, \ldots.
\end{aligned}
\qquad (40)
$$

TABLE 1: Material properties.

$k = 386$ W/(m K)	$\alpha_t = 1.78\ (10)^{-5}$ K^{-1}	$c_E = 381$ J/(kg K)	$\eta = 8886.73$
$\mu = 3.86\ (10)^{10}$ kg/(m s^2)	$\lambda = 7.76\ (10)^{10}$ kg/(m s^2)	$\rho = 8954$ kg/m^3	$T_0 = 293$ K
$\varepsilon = 0.0168$	$\tau_0 = 0.025$ s	$\tau_1 = 0.025$ s	$\tau = 0.025$ s
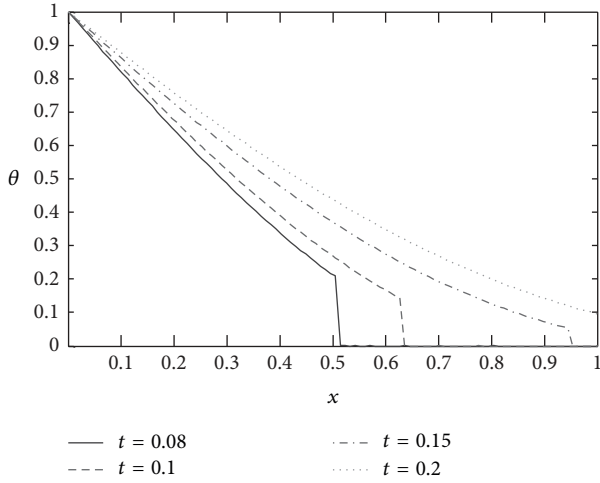
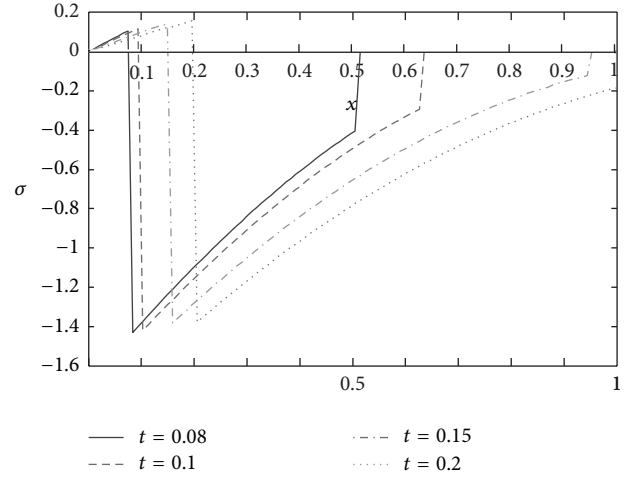FIGURE 1: Temperature distribution for different time for $\alpha = \beta = 1$.

FIGURE 3: Stress distribution for different time when $\alpha = \beta = 1$.
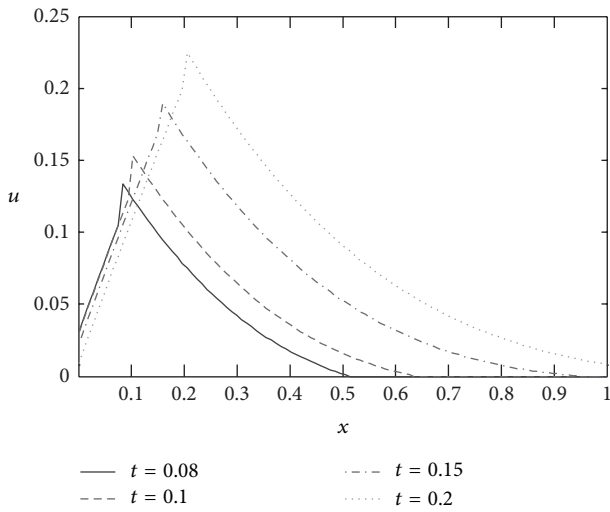
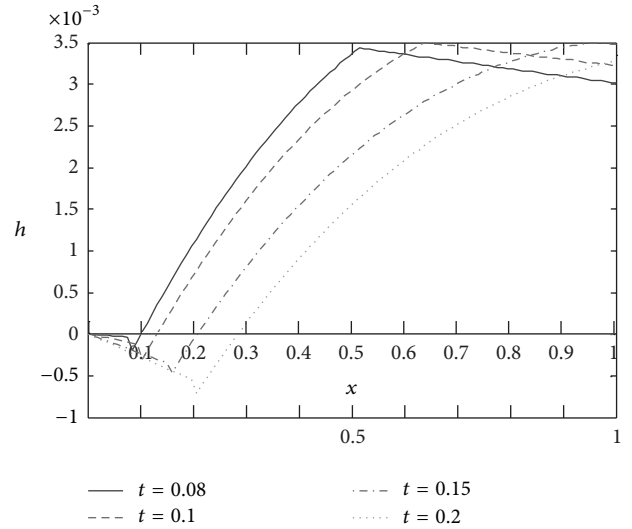FIGURE 2: Displacement distribution for different times for $\alpha = \beta = 1$.

FIGURE 4: Induced magnetic field distribution for different time for $\alpha = \beta = 1$.

It can be shown that [19] the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \ldots, \varepsilon_{N,1,\ldots}$ converges to $f(t) - c_0/2$ faster than the sequence of partial sums.

5. Numerical Results and Discussion

In order to obtain the solutions for the field functions in the physical domain, we have applied the Laplace inversion formula mentioned in the above section. FORTRAN programming language was used on a personal computer. The accuracy maintained was 7 digits for the numerical program. For computational purposes, a copper-like material has been taken into consideration. The values of the material constants are taken as in Table 1.

In order to investigate the effect of time on all field variables the computations have been carried out for $t = 0.05$, $t = 0.1$, $t = 0.15$, and $t = 0.2$. The results are displayed in Figures 1–5, for the temperature, displacement, stress, and magnetic and electric field distributions, respectively.

TABLE 2: Wave fronts and stress jump sizes.

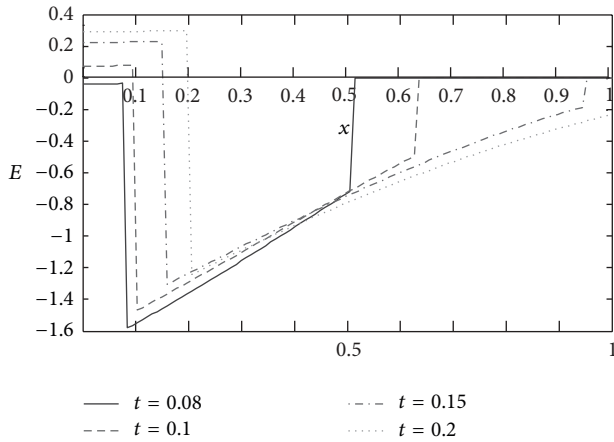| Time	First wave front of the stress		Second wave front of the stress	
	Location	Discontinuity gap size	Location	Discontinuity gap size
0.08	$x = 0.0748$	1.5341611	$x = 0.5046729$	0.4026305
0.1	$x = 0.0935$	1.5374964	$x = 0.6261683$	0.2926325
0.15	$x = 0.1495327$	1.5193814	$x = 0.9439253$	0.1216954
0.2	$x = 0.1962617$	1.5351866	$x = 0.7476634$ (reflected)	0.0026

FIGURE 5: Induced electric field distribution for different time for $\alpha = \beta = 1$.
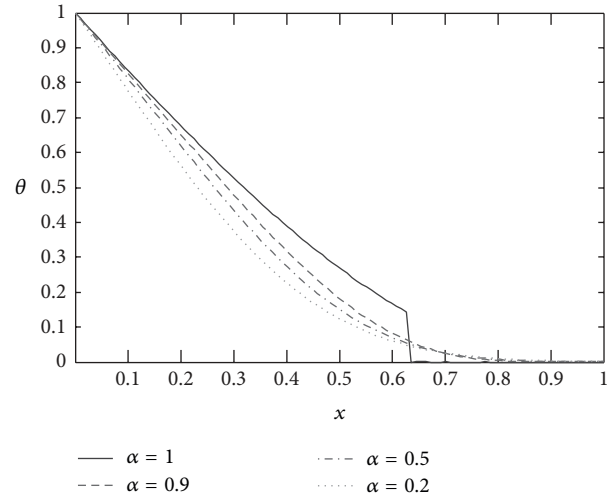

FIGURE 6: Temperature distribution for different α for $\beta = 1$ at $t = 0.1$.

FIGURE 7: Displacement distribution for different α for $\beta = 1$ at $t = 0.1$.

Next, to study the effect of the fractional parameter α, we take $\alpha = 0.01$, $\alpha = 0.2$, $\alpha = 0.5$, and $\alpha = 1$ when $t = 0.1$ and $\beta = 1$. This gives us Figures 6–10, for the temperature, displacement, stress, magnetic field, and electric field distributions, respectively.

The remaining figures illustrate the effect of the fractional parameter β on the field variables. Figures 11–14 describe the displacement, stress component, magnetic field, and electric field distributions for different values of β; namely, $\beta = 0.2$, $\beta = 0.5$, $\beta = 0.9$, and $\beta = 1$ when $t = 0.1$ and $\alpha = 1$. Here we notice that the fractional parameter β has no effect on the temperature distribution.

The conclusions from Figures 1–14 can be summarized as follows:

(1) All of the physical variables have a finite speed of wave propagation for all times when $\alpha = \beta = 1$. The speed of propagation for other values of α and β needs further theoretical investigation.

(2) As is apparent from the order of the differential equation, we have two waves. The locations of the two wave fronts are the same for all functions considered. These wave fronts appear as a jump (discontinuity) in the case of the temperature, stress, and the intensity of the electric field. On the other hand, the other two functions, namely, the displacement and the intensity of the magnetic fields, are continuous. The wave fronts, in this case, appear as a cusp signifying a discontinuous fir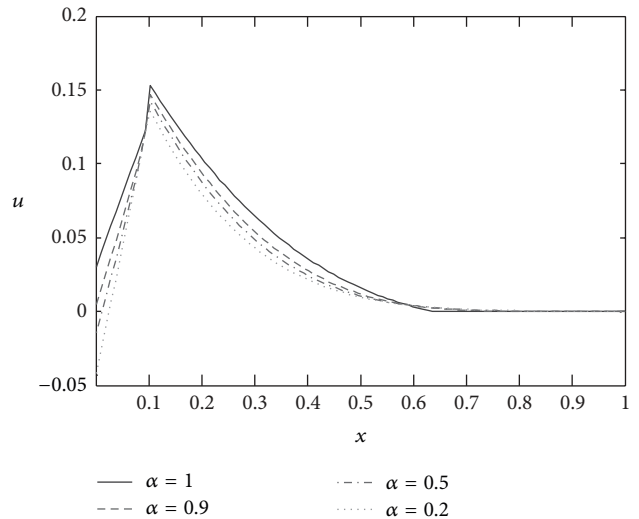st derivative. Note that the first jump in the temperature is too small to appear in the graph. For $\alpha = \beta = 1$, the waves propagate into the medium from the position $x = 0$ to fill a finite part of the region that expands with the passage of time. We note that, for $t = 0.2$, this region
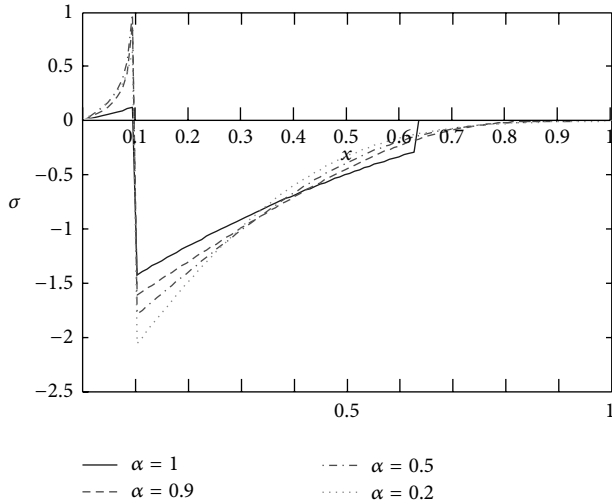
FIGURE 8: Stress distribution for different α for $\beta = 1$ at $t = 0.1$.
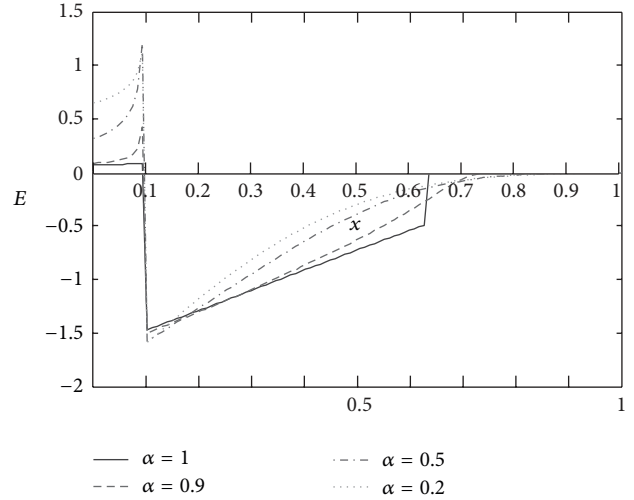

FIGURE 10: Induced electric field distribution for different α for $\beta = 1$ at $t = 0.1$.
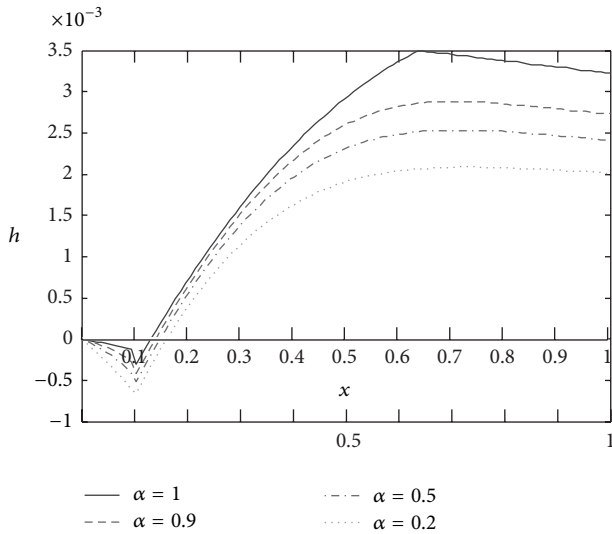

FIGURE 9: Induced magnetic field distribution for different α for $\beta = 1$ at $t = 0.1$.
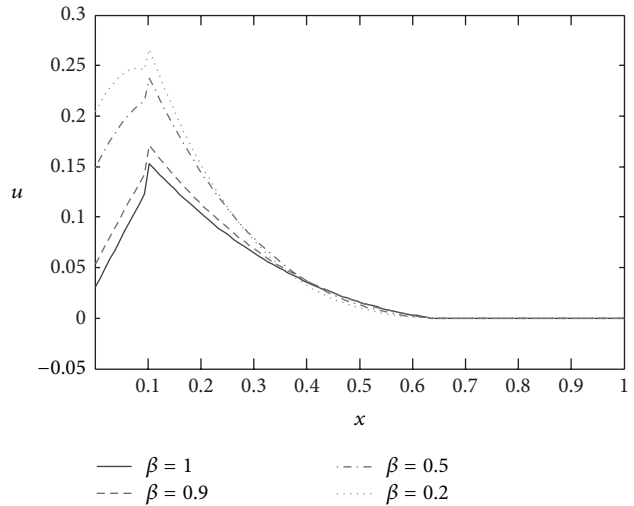

FIGURE 11: Displacement distribution for different β for $\alpha = 1$ at $t = 0.1$.

has filled the entire body of the plate. The second wave has been reflected from the other side of the plate. The positions of the wave fronts for the stress and electric field distributions as well as the size of their jump discontinuities can be found in Tables 2 and 3.

(3) From Figure 3, we notice that for the displacement, the first wave front is located at the peaks of this function. The location of these peaks for different times is introduced in Table 4.

Also, we note here that the magnitude of peaks increases with time.

(4) It is observed from Figures 6–10 that the field variables θ, u, σ, h, and E are strongly affected by the fractional parameter α. Increasing the fractional parameter α produces an increase in the peaks of the displacement and decreases the stress gap size at the location of the first wave front. Similar
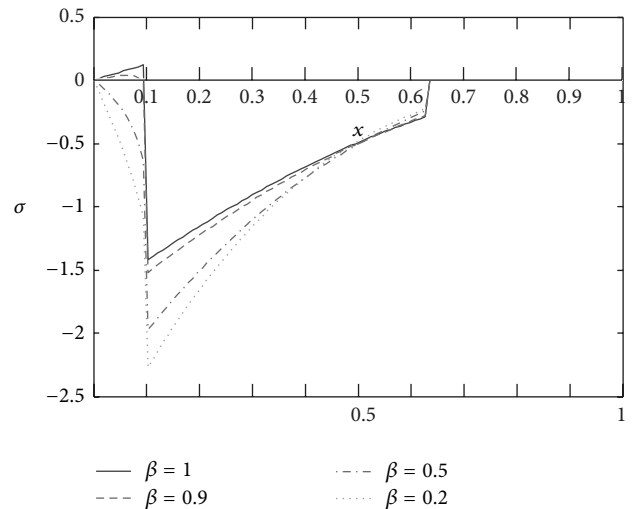

FIGURE 12: Stress distribution for different β for $\alpha = 1$ at $t = 0.1$.

TABLE 3: Wave fronts and electric field jump sizes.

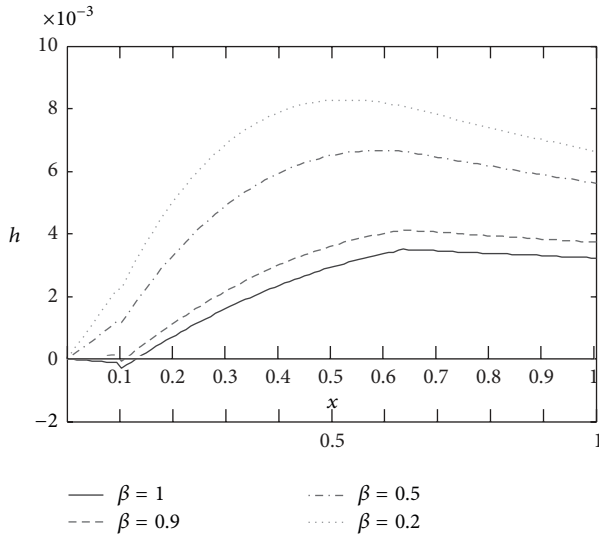| Time	First wave front of the electric field		Second wave front of the electric field	
	Location	Discontinuity gap size	Location	Discontinuity gap size
0.08	$x = 0.0748$	1.5456	$x = 0.5046729$	0.7192609
0.1	$x = 0.0935$	1.5477	$x = 0.6261683$	0.4947222
0.15	$x = 0.1495327$	1.5299	$x = 0.9439253$	0.1858727
0.2	$x = 0.1962617$	1.541777	$x = 0.7476634$ (reflected)	0.0069

FIGURE 13: Induced magnetic field distribution for different β for $\alpha = 1$ at $t = 0.1$.
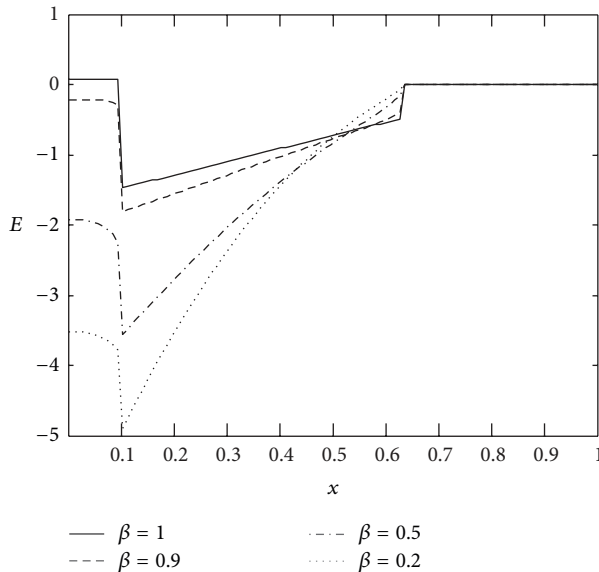

FIGURE 14: Induced electric field distribution for different β for $\alpha = 1$ at $t = 0.1$.

effects of the parameter β on u, σ, h, and E are also clear from Figures 11–14.

TABLE 4: Wave fronts and displacement peak sizes.

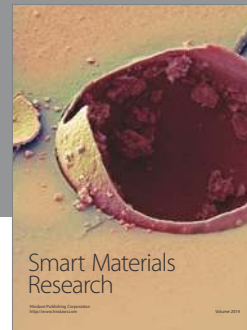| Time	First wave front location (peak)	
	Location of 1st wave front	Maximum value at peak
0.08	0.0748	0.133331
0.1	0.093457945	0.1530982
0.15	0.1495327	0.1892859
0.2	0.1962617	0.224686

Competing Interests

The author declares that there are no competing interests.

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