Application of KBc Subalgebra in String Field Theory

Syoji Zeze*)

Yokote Seiryo Gakuin High School, Yokote 013-0041, Japan

Recently, a classical solution of open cubic string field theory (CSFT) which corresponds to the closed string vacuum is found by Erler and Schnabl. In their work, a very simple subalgebra of open string star algebra — called KBc subalgebra — plays a crucial role. In this talk, we demonstrate two applications of the KBc subalgebra. One is evaluation of classical and effective tachyon potential. It turns out that the level expansion in the KBc subalgebra terminates at a certain level, so that analytic evaluation of effective potential is available. The other application is regularization of the identity based solutions. It is demonstrated that the Okawa-Erler-Schnabl type solution naturally includes gauge invariant regularization of identity based solutions.

§1. Introduction

Since its birth in '70s,[1),2)] string field theory (SFT) has been expected to explain nonperturbative phenomena which cannot be addressed easily in the traditional conformal field theory approach of string theory. Most remarkable feature of SFT is that one can 'write down' the action and equation of motion for string field at nonperturbative level. In particular, Lorentz invariant classical solutions of Witten's cubic string field theory[3)] are investigated extensively according to Sen's conjecture[4)] for tachyon condensation and unstable D-branes. First, let us focus on technical aspects of such classical solutions. Witten's action is given by

$$S[\Psi] = \frac{1}{4\pi^2}\mathrm{Tr}\left[\frac{1}{2}\Psi Q_B\Psi + \frac{1}{3}\Psi^3\right], \tag{1·1}$$

where Ψ is the open string field, Q_B is the BRST charge. The star product between string fields is omitted. The symbol Tr stands for the inner product of string fields. In general, the star product mixes each component of string fields in a nontrivial way. The mixing coefficients (called Neumann coefficients) can be calculated explicitly component by component. Thus, after suitable gauge fixing, the action (1·1) for Lorentz invariant string field reduces to

$$S(t_0, t_1, \dots) = \frac{1}{4\pi^2}\sum_{m,m,p=0}^{\infty}\left[\frac{1}{2}t_m(Q_B)_{mn}t_n + \frac{1}{3}V_{mnp}t_mt_nt_p\right], \tag{1·2}$$

where ψ_n is a component of string field, which is independent of momentum or center of mass coordinate of a string. $(Q_B)_{mn}$ and V_{mnp} are Neumann coefficients. The equation of motion for a field ψ_p is simply given by

$$\sum_{m,n=1}^{\infty}\left((Q_B)_{pn}t_n + V_{mnp}t_mt_n\right) = 0. \tag{1·3}$$

*) E-mail: ztaro21@gmail.com

These are just quadratic equations. A trouble is that components of string field and Neumann coefficients does not terminate at finite level. Finding an nontrivial solution of (1·3) has been a challenging problem since string field and Neumman coefficients does not terminate at finite n in standard expansion of string field such as level expansion in Siegel gauge. The breakthrough has been brought by Schnabl.[5] His solution is given by

$$\Psi_{Sc} = \lim_{N\to\infty}\left[\psi_N - \sum_{n=0}^{N}\partial_n\psi_n\right], \tag{1·4}$$

where ψ_n is particular excitation above so-called 'wedge state' which represents insertion of a worldsheet of width $n/2\pi$. The existence of isolated piece ψ_N, called phantom piece, forces us to take delicate limit of large N in the evaluation of physical quantities such as classical action. After some yeas later, Erler and Schnabl found the classical solution without phantom term,

$$\Psi = c(1+K)Bc\frac{1}{1+K}. \tag{1·5}$$

Here, K, B and c is central ingredients of their formalism.

1.1. *KBc subalgebra*

Now let us briefly review the notation used in 6)–9) which will be extensively used in this talk. The KBc subalgebra is a subalgebra of star algebra which is defined as

$$\{B,c\}=1,\quad [B,K]=0,\quad \{B,B\}=0,\quad \{c,c\}=0. \tag{1·6}$$

The action of the BRST charge on these elements is given by

$$Q_Bc = cKc,\quad Q_BB=K,\quad Q_BK=0. \tag{1·7}$$

In this notation, the star multiplication between elements is understood. The BPZ inner product of elements is denoted as 'trace'. The main formula in this talk is a generalization of (1·5) to arbitrary function of K as

$$\Psi_{NR} = cFBc\left(\alpha - \frac{K}{F}\right), \tag{1·8}$$

where α is a free parameter which can be absorbed into definition of F. This solution does not satisfy the reality condition of string field. However, the real form of the solution is easily obtained by 'global' symmetry of string field

$$\Psi_R = f\Psi_{NR}f^{-1} = fc\frac{K}{\alpha - f^2}Bcf, \tag{1·9}$$

where $f=\sqrt{\alpha - K/F}$ is another function of K. This form of solution was first presented by Okawa.[6] The form (1·8) is extremely useful for explicit calculations. In this talk, we would like to demonstrate two topics which can be explored by the KBc formalism — one is the tachyon potential[10] and the other is regularization of identity based solutions.[11]

§2. Tachyon potential

The classical action of open cubic string field theory (1·2) has long been studied in the context of tachyon condensation. While equation of motion is solved, the complete form of effective potential for the tachyon field t_0 is not yet known. The effective potential $\mathcal{V}(t_0)$ can be obtained by solving all equations of motion of the classical potential $V(t_0, t_1, \dots) = S(t_0, t_1, \dots)$ except for t_0, and plugging them back to the classical potential $V(t_0, t_1, \dots)$. In general, it is not so easy to get closed form of $\mathcal{V}(t_0)$. However, the simplicity of the KBc subalgebra enables us to find analytic expression of $\mathcal{V}(t_0)$.

We choose the gauge in which the Erler-Schnabl solution belongs to:

$$\frac{1}{2}\mathcal{B}_0^- \left[\Psi(1+K)\right] \frac{1}{1+K} = 0, \tag{2·1}$$

where $\mathcal{B}_0^- = \mathcal{B}_0 - \mathcal{B}_0^\dagger$ is a derivation of the star product. The general form of a string field in this gauge is given by

$$\Psi = cf(K)Bc\frac{1}{1+K}, \tag{2·2}$$

if we restrict ourselves within KBc subalgebra. Each component of the string field is given by a coefficient of taylor expansion of $f(K)$ as

$$f(K) = t_0 + t_1 K + t_2 K^2 + \cdots . \tag{2·3}$$

Note that the above expansion corresponds to level expansion with respect to eigenvalue of $\mathcal{L}$ which is obtained by anticommuting Q_B with the gauge condition (2·1). The crucial difference between the expansion in (2·3) and the traditional level expansion in Siegel gauge or $\mathcal{B}_0$ gauge is that the expansion (2·3) *terminates at finite level.* More precisely, the expansion stops at level 3 so that there are only for fields (t_0, t_1, t_2, t_3).[10] This is simply because the classical action becomes divergent for a field higher than level 4.[*)] An explicit form of the potential is

$$\begin{aligned} V(t_0,t_1,t_2,t_3) = \frac{1}{\pi^2}\Bigg(&\frac{15}{64\pi^2}t_0^3 - \frac{15t_1}{16\pi^2}t_0^2 + \frac{1}{16}t_1t_0^2 + \frac{15}{16\pi^2}t_2t_0^2 - \frac{1}{4}t_2t_0^2 - \frac{1}{12}\pi^2t_3t_0^2 \\ &+ \frac{1}{2}t_3t_0^2 - \frac{3}{32}t_0^2 + \frac{15}{16\pi^2}t_1^2t_0 - \frac{1}{16}t_1^2t_0 - \frac{15}{4\pi^2}t_2^2t_0 + t_2^2t_0 + \frac{3}{4}t_2t_0 - \frac{1}{6}\pi^2t_1t_3t_0 \\ &- \frac{15}{2\pi^2}t_1t_3t_0 + \frac{3}{2}t_1t_3t_0 + \frac{4}{3}\pi^2t_2t_3t_0 + \frac{30}{\pi^2}t_2t_3t_0 - 10t_2t_3t_0 + \pi^2t_3t_0 - 3t_3t_0 \\ &+ \frac{15}{\pi^2}t_1t_2^2 - t_1t_2^2 - \frac{15}{4\pi^2}t_1^2t_2 + \frac{1}{4}t_1^2t_2 - \frac{1}{3}\pi^2t_1^2t_3 + \frac{15}{\pi^2}t_1^2t_3 - 2t_1^2t_3\Bigg). \end{aligned} \tag{2·4}$$

2.1. *Classical solutions*

It is not so difficult to solve equations of motion $\partial_{t_1} V = 0$ for the potential (2·4). Although a solution can be obtained analytically, it is more convenient to present

*) Here we do not consider a possibility of regularization of this divergence.

Table I. Stationary points of the classical potential.

t_0	t_1	t_2	t_3	V
0.2703	−0.3928	−0.8194	−0.0358	−0.0306
0.2175	0.1770	−0.9237	0.3032	0.0078
0.1195	0.1593	−0.9076	0.3569	0.0081
1.0000	1.0000	0.0000	0.0000	−0.0507

numerical result only since it contains many square roots. Table I is a list of real valued solutions. The last solution is nothing but the Erler-Schnabl solution which corresponds to $f(K) = 1 + K$. An interesting observation can be obtained from eigenvalues of the Hessian matrix

$$H_{ij} = \frac{\partial^2 V}{\partial t_i \partial t_j}. \tag{2·5}$$

If one of eigenvalues of H_{ij} for particular solution is negative, it means that the potential is unstable around a solution. It is found that all solutions in Table I have negative eigenvalues and hence are unstable.

Physical interpretation of four solutions is also of our interest. In our case, we do not necessarily depend on the analysis in terms of power expansion in K such as (2·3) since equation of motion can be analytically solved. Using ansatz (2·2) we find an analytic solution

$$f(K) = \frac{K(1+K)}{\lambda K + (\lambda - 1)}, \tag{2·6}$$

where λ is a free parameter. One can easily see that this solution only falls into a polynomial if and only if $\lambda = 0, 1$. $\lambda = 1$ gives the Erler-Schnabl solution which appears in the last row of Table I. The other case $\lambda = 0$ does not appear as a solution of level expanded equation of motion. This solution is irregular since it does not satisfy equation of motion contracted with itself.

$$\mathrm{Tr}[\Psi(Q_B\Psi + \Psi^2)] = 3\left(-\frac{15}{\pi^4} + \frac{1}{\pi^2}\right). \tag{2·7}$$

In summary, the Erler-Schnabl solution is only physically acceptable solution in the level truncation and others should be discarded.

2.2. *Effective potential*

Since our potential depends on only finite number of fields, it is possible to get explicit form of $\mathcal{V}(t_0)$ by solving equation of motion for t_1, t_2 and t_3. This requires choice of brunch. We again have four real brunches which connect with solutions in Table I. The explicit form of the potential can be obtained analytically but too complicated to display here. A result is shown in Fig. 1. Only the brunch 2 is physical since it connects with the closed string vacuum solution. It is impressive that all brunches starts exactly from the origin. The reason is simply because the potential become complex valued in negative real axis, due to the brunch cut singularity around the origin. It is in sharp contrast with earlier results in terms of level truncation in

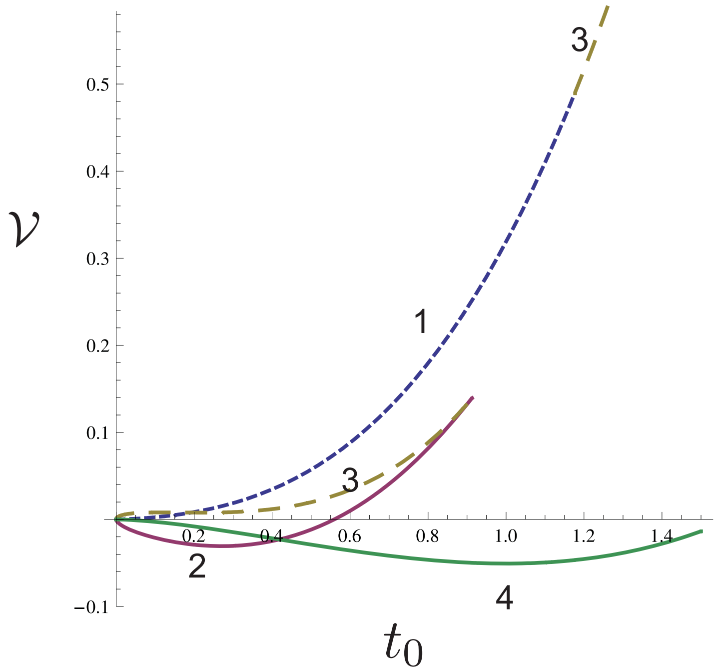

Fig. 1. Four branches of the effective potential $\mathcal{V}$. We find local minimum in branches 2 and 4. Branch 3 consists of two disconnected curves.

Siegel gauge whose potential has runaway direction towards negative axis and stops at particular point. Our result has no such inconsistency and one need not to worry about runaway direction of the tachyon potential.

§3. Regularization of identity based solutions

Identity based solutions, which are constructed upon the identity string field, have been considered since early days of string field theory.[12)–14)] More elaborated versions have been investigated[15)–26)] according to Sen's conjecture. Even after Schnabl's discovery of the analytic solution,[5)] it had been recognized that an identity based string field is useful to construct regular solutions.[7),8),27),28)] Typically a solution takes form of

$$\Psi = CI, \tag{3·1}$$

where C is certain linear combination of ghost number one operators and I is the identity string field, a surface state which represents an open string worldsheet of vanishing width. The advantage of identity based solutions to other solutions which are given by superposition of infinitely many wedge states is its simplicity. However, even though much effort has been done in the past, identity based solutions have not yet been widely accepted as a regular solution. Here we would like to demonstrate that the solution (2·2) can be naturally considered to be gauge invariant regularization of an identity based solution.

Here we focus on the identity based solution within the KBc form of (1·8). We simply require both of F and $\alpha - K/F$ are identity based, i.e. polynomials in K. This requirement leaves only two choices

1. $F(K)$ is constant. If we set $F(K) = \beta$, the solution (1·8) becomes
$$\Psi = \alpha\beta c - cK. \tag{3·2}$$
2. $F(K)$ is proportional to K. If we set $F(K) = \beta K$, we have
$$\Psi = \left(\frac{\alpha\lambda - 1}{\lambda}\right) cKBc. \tag{3·3}$$

This solution is BRST exact and considered to be trivial pure gauge solution. We consider the case 1. as nontrivial identity based solution. The solution $c - cK$, which corresponds to setting $\alpha\beta = 1$ in (3·2), is the the identity based solution found in 9) and discussed in in 29). Surprisingly, A very simple gauge transformation on this solution brings it to the form which is very convenient for regularization. The gauge transformation is given by

$$U_\lambda = 1 + \lambda cBK, \tag{3·4}$$

and the solution becomes

$$\Psi_\lambda \equiv U_\lambda Q_B U_\lambda^{-1} + U_\lambda(c - cK)U_\lambda^{-1} \tag{3·5}$$
$$= c(1+\lambda K)Bc\frac{1+(\lambda-1)K}{1+\lambda K}, \tag{3·6}$$

which is nothing but a solution of the form (1·8) with $F(K) = 1 + \lambda K$. This is no more identity based solution and evaluation of gauge invariant quantity such as D-brane tension and closed string tadpole are carried out without encountering any trouble.

3.1. *D-brane tension*

According to Sen's conjecture, the value of classical action should match with the D-brane tension. Since we are dealing with a solution of equation of motion, we only need to evaluate the kinetic term. The whole procedure is completely parallel with that of Ref. 9). The result is

$$\mathrm{Tr}[\Psi_\lambda Q_B \Psi_\lambda] = -\frac{1}{2\pi^2}\int_0^\infty due^{-u}\left\{\lambda^2u^3 + 6l(1-\lambda)u^2 + 6(1-\lambda)^2u\right\} \tag{3·7}$$
$$= -\frac{1}{2\pi^2}(6\lambda^2 + 12\lambda\lambda(1-\lambda) + 6(\lambda-1)^2) \tag{3·8}$$
$$= -\frac{3}{x}\pi^2. \tag{3·9}$$

The value $-3/\pi^2$ exactly matches with an expected answer in our convention. Note that entire process can be carried out even in $\lambda \to 0$ limit where the solution approaches to identity based configuration.

3.2. *Closed string tadpole*

Another gauge invariant quantity — the closed string tadpole — can be evaluated similarly. It is expressed as $\mathrm{Tr}[V\Psi]$ where $V = c\bar{c}V_{\mathrm{matter}}$ is on-shell closed string vertex operator at the midpoint of the string vertex. Evaluation is again quite similar to that performed in 9).

$$\mathrm{Tr}[V\Psi_\lambda] = \mathrm{Tr}[Vc\Omega] \times \int_0^\infty dt e^{-t}(1-\lambda+t\lambda) \tag{3·10}$$

$$= \mathrm{Tr}[Vc\Omega] \times 1 \tag{3·11}$$

$$= \langle \mathcal{V}(i\infty)c(0)\rangle_{C_1}, \tag{3·12}$$

where Ω is a wedge state of width $\pi/2$ and the last line is a one point function of closed string vertex operator evaluated on the cylinder of width $\pi/2$, as expected.

References

1) M. Kaku and K. Kikkawa, Phys. Rev. D **10** (1974), 1110.
2) M. Kaku and K. Kikkawa, Phys. Rev. D **10** (1974), 1823.
3) E. Witten, Nucl. Phys. B **268** (1986), 253.
4) A. Sen, J. High Energy Phys. **08** (1998), 012, hep-th/9805170.
5) M. Schnabl, Adv. Theor. Math. Phys. **10** (2006), 433, hep-th/0511286.
6) Y. Okawa, J. High Energy Phys. **04** (2006), 055, hep-th/0603159.
7) T. Erler, J. High Energy Phys. **05** (2007), 084, hep-th/0612050.
8) T. Erler, J. High Energy Phys. **05** (2007), 083, hep-th/0611200.
9) T. Erler and M. Schnabl, J. High Energy Phys. **10** (2009), 066, arXiv:0906.0979.
10) S. Zeze, Prog. Theor. Phys. **124** (2011), 567, arXiv:1004.4351.
11) S. Zeze, J. High Energy Phys. **10** (2010), 070, arXiv:1008.1104.
12) G. T. Horowitz, J. D. Lykken, R. Rohm and A. Strominger, Phys. Rev. Lett. **57** (1986), 283.
13) G. T. Horowitz, J. Morrow-Jones, S. P. Martin and R. P. Woodard, Phys. Rev. Lett. **60** (1988), 261.
14) G. T. Horowitz and A. Strominger, Phys. Lett. B **185** (1987), 45.
15) T. Takahashi and S. Tanimoto, Prog. Theor. Phys. **106** (2001), 863, hep-th/0107046.
16) I. Kishimoto and K. Ohmori, J. High Energy Phys. **05** (2002), 036, hep-th/0112169.
17) T. Takahashi and S. Tanimoto, J. High Energy Phys. **03** (2002), 033, hep-th/0202133.
18) I. Kishimoto and T. Takahashi, Prog. Theor. Phys. **108** (2002), 591, hep-th/0205275.
19) T. Takahashi and S. Zeze, Prog. Theor. Phys. **110** (2003), 159, hep-th/0304261.
20) T. Takahashi and S. Zeze, J. High Energy Phys. **08** (2003), 020, hep-th/0307173.
21) S. Zeze, Prog. Theor. Phys. **112** (2004), 863, hep-th/0405097.
22) S. Zeze, J. High Energy Phys. **10** (2005), 078, hep-th/0508196.
23) Y. Igarashi, K. Itoh, F. Katsumata, T. Takahashi and S. Zeze, Prog. Theor. Phys. **114** (2005), 695, hep-th/0502042.
24) Y. Igarashi, K. Itoh, F. Katsumata, T. Takahashi and S. Zeze, Prog. Theor. Phys. **114** (2006), 1269, hep-th/0506083.
25) I. Kishimoto and T. Takahashi, Prog. Theor. Phys. **122** (2009), 385, arXiv:0904.1095.
26) I. Kishimoto and T. Takahashi, Theor. Math. Phys. **163** (2010), 717, arXiv:0910.3026.
27) M. Schnabl, Phys. Lett. B **654** (2007), 194, hep-th/0701248.
28) I. Kishimoto and Y. Michishita, Prog. Theor. Phys. **118** (2007), 347, arXiv:0706.0409.
29) E. A. Arroyo, J. of Phys. A **43** (2010), 445403, arXiv:1004.3030.