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# Application of MIMO Control Algorithm for Active Suspension System: A New Model with 5 State Variables

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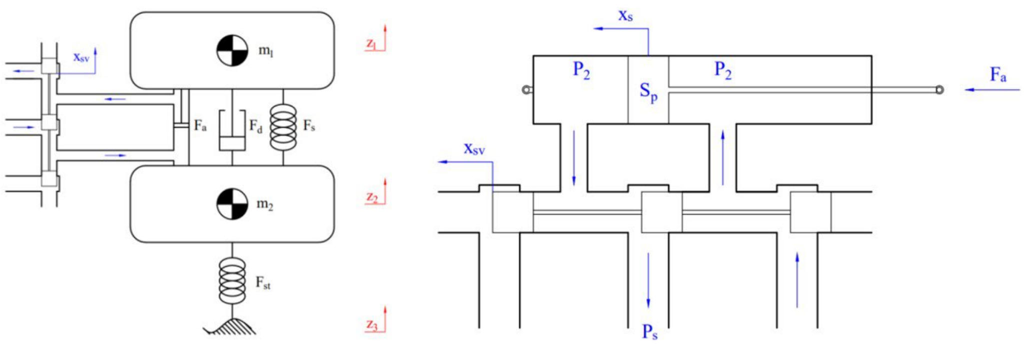
### Abstract

This paper introduces the LQR control algorithm for the active suspension system. Because the model of the vehicle dynamics used in this paper includes a hydraulic actuator, therefore, this model will include five state variables. Besides, the process of linearization of the hydraulic actuator is also shown in this paper. This is a completely novel and original method. It is possible to describe almost all the characteristics of hydraulic actuators with just one linear differential equation. Also, the parameters of the LQR controller are optimized through the in-loop optimization algorithm. The results of the research showed that the values of displacement and acceleration of the sprung mass were significantly reduced when this algorithm was used. In the investigation cases, these values usually do not exceed 2.68% and 43.00% compared to the situation of the vehicle using only a passive suspension system. Therefore, ride comfort and stability can be enhanced in many driving conditions when the active suspension system with the LQR control algorithm is used.

### Keywords

Active suspension system, Hydraulic actuator, LQR controller, MIMO system.

### Graphical Abstract

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## 1 INTRODUCTION

The vehicle's vibrations are generated by stimuli from the road surface. This oscillation can cause a loss of comfort for passengers. Besides, the life of vehicle parts can also be reduced. There are many criteria to evaluate the oscillation of the vehicle, such as the frequency of oscillation, displacement, acceleration of the vehicle body, etc. These criteria can be determined based on the maximum value or the average value. Their maximum values can be used for all cases. In contrast, average values should only be used in cases that fluctuate continuously over a certain period.

The suspension system is used to ensure the stability of the vehicle when traveling on the road. The suspension system is located between the wheel and the body, and it is seen as a "soft link" between these two components. The body of a vehicle, including passengers, cargo, etc., is called the "sprung mass". The remainder is called the "unsprung mass" (Yin, et al., 2015). The passive suspension system, which is used on most vehicles today, consists of only the spring, damper, lever arm, and stabilizer bar (Jiregna and Sirata, 2020; Tuan and Thang, 2019). According to Nguyen, the stiffness of the elements of the passive suspension system is unchanged (Nguyen, 2021a). So, the stability of the vehicle is not guaranteed in many driving conditions. Therefore, the active suspension system is proposed to replace the passive suspension system (Chen, et al., 2020; Beltran-Carbajal, et al., 2019). The active suspension system is equipped with a hydraulic actuator. Inside the hydraulic cylinder are servo valves. The process of opening and closing these valves will change the pressure inside the cylinder (Wang et al., 2018; Nguyen, 2021b). Thus, the impact force will be generated. This force will act on both the sprung and unsprung masses to reduce the vehicle's oscillations.

Many studies on the active suspension system have been published in recent years. In (Anh, 2020), Anh introduced the PID controller for the active suspension system. This controller is only suitable for the SISO (Single Input – Single Ouput) system. According to Ding et al., the PID (Proportional – Integral – Derivative) controller consists of three characteristic coefficients (Ding et al., 2021). These coefficients can be determined based on some basic principles of control theory. Besides, intelligent control algorithms such as Fuzzy Logic, GA (Genetic Algorithm), PSO (Particle Swarm Optimization), etc. are also used to optimize the coefficients of the PID controller (Mahmoodabadi and Nejadkourki, 2020; Emam, 2015; Metered, et al., 2018; Dahunsi, et al., 2020; Karam and Awad, 2020). For oscillating models, if many objects are considered, the PID controller cannot guarantee stable performance. Therefore, the LQR (Linear Quadratic Regulator) controller is proposed to apply to MIMO (Multi Input – Multi Output) systems. The LQR controller uses a linear algorithm based on the optimization of the cost function. For this method, matrices describing the oscillation state need to be established (Rodriguez-Guevara, et al., 2021). According to Brezas and Smith, the parameters of the control matrix are determined based on finding the solution to the Riccati algebraic equation. This equation can be solved by the usual numerical method (Brezas and Smith, 2014). To limit the disturbance when using the algorithm with many state variables, a Gaussian filter can be equipped (Xia, et al., 2015; Pang, et al., 2017). The biggest problem that still exists with the LQR control algorithm is the linearization of the oscillatory state matrix. When considering the hydraulic actuator of an active suspension system, it is quite complicated to linearize the equation describing its oscillation state (Nguyen, et al., 2022).

Also, some nonlinear control algorithms can bring high efficiency to the active suspension system. In (Bai and Guo, 2018), Bai and Guo introduced the SMC (Sliding Mode Control) algorithm for the suspension system with the quarter-dynamics model considered. However, this paper does not show the linearization of a hydraulic actuator. This problem is more clearly explained by Nguyen (Nguyen, 2021c). According to Nguyen, the SMC control algorithm needs to design a sliding surface (Nguyen, 2021d). The controlled object will move around this slide to reach a stable position (Azizi and Mobki, 2021). The sliding surface is designed based on the process of derivation of the signals of the state variables, which is shown by Wei and Su (Wei and Su, 2019). Therefore, the phenomenon of "chattering" can occur in many simulations and computational cases (Xiao and Zhu, 2015). This is a noise form of the signal and is characteristic of the SMC algorithm. According to Shaer et al., to limit this phenomenon, the SMC control algorithm should be combined with the Fuzzy control algorithm (Shaer, et al., 2018). According to Fuzzy theory, intermediate states of oscillation can exist (Nguyen, 2021e; Nguyen, 2021f). The "IF – THEN" condition, according to Na et al., can be used to determine Fuzzy law (Na, et al., 2020). This condition was also used in the T-S (Tagaki – Sugeno) model of Mrazgua et al. (Mrazgua et al., 2021). Besides, the Fuzzy control law is also designed based on the views and experiences of researchers (Choi and You, 2021). In addition, intelligent control algorithms such as ANN (Artificial Neural Network), RBFNN (Radial Basic Function Neural Network), ANFIS (Adaptive Neuro Fuzzy Inference System), etc. have also been introduced. In general, the efficiency of these algorithms is quite high (Sun and Zhao, 2020; Aela, et al., 2020; Xue, et al., 2021)

This paper proposes using the LQR algorithm for an active suspension system. As stated above, the most important problem that remains is the linearization of the oscillatory state matrix. This problem will be solved in a completely novel and relevant way. The control model will include five state variables with the presence of hydraulic actuators. Besides, the parameters of the control matrix are also optimized based on the in-loop algorithm. The layout of the paper consists of four parts: Introduction, Material, Simulation, and Conclusion. The main content of the paper is presented below.

## 2 MATERIAL

### 2.1 Model of the vehicle dynamics

This research uses a quarter – dynamics model to describe vehicle oscillations. This model includes two masses, $m_1$ and $m_2$. Corresponding to these two masses are two vertical displacements, $z_1$ and $z_2$.

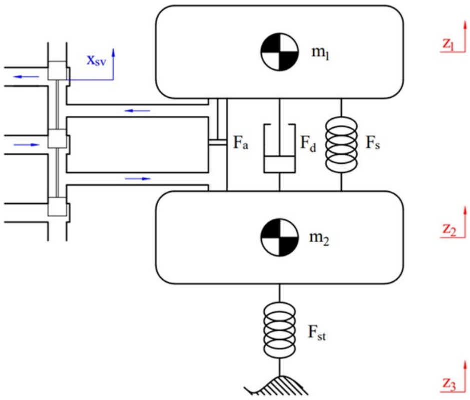


**Figure 1**: Model of the vehicle dynamics.

Unlike previous models, in this model, the hydraulic actuator is considered as a state variable (Figure 1). The equations describing the oscillations of the system are given as follows:

$$F_{i1} = F_s + F_d + F_a \tag{1}$$

$$F_{i2} = F_{st} - F_s - F_d - F_a \tag{2}$$

where:

$$F_{ii} = m_i \ddot{z}_i \tag{3}$$

$$F_s = K(z_2 - z_1) \tag{4}$$

$$F_d = C(\dot{z}_2 - \dot{z}_1) \tag{5}$$

$$F_{st} = K_t(z_3 - z_2) \tag{6}$$

The impact force that is generated by the hydraulic actuator is a nonlinear function. It depends on the control signal, the displacement of the suspension system, the dimensional parameters of the hydraulic actuator, etc. When voltage is applied, the valves inside the hydraulic actuator move (Figure 2). This displacement is shown in equation (7).

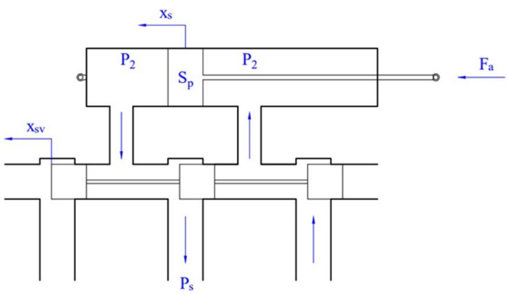


**Figure 2**: Hydraulic actuator.

$$x_{sv} = \frac{1}{t}\int (k_{sv} u(t) - x_{sv}) dt \tag{7}$$

As the valves move, a hydraulic pressure differential inside the cylinder will occur.

$$\Delta P = \sigma_3 \int \left( x_{sv} \sqrt{P_s - sgn(x_{sv}) \Delta P} \right) dt - \sigma_2 \int \Delta P dt - \sigma_1 S_p \int \dot{x}_s dt \quad (8)$$

Based on this pressure difference, an impact force can be generated. This force is proportional to the pressure difference and the cross-sectional area of the piston.

$$F_a = S_p \Delta P = S_p \left( \sigma_3 \int x_{sv} \sqrt{P_s - sgn(x_{sv}) \Delta P} dt - \sigma_2 \int \Delta P dt - \sigma_1 S_p \int \dot{x}_s dt \right) \quad (9)$$

since: $\begin{cases} |k_{sv}| \approx |\tau| \\ |\dot{x}_{sv}| \ll \tau \end{cases}$ and $sgn(x_{sv}) \Delta P \ll P_s$

Taking the derivative of both sides of (8), the linearization equation for the impact force can be simply rewritten as:

$$\dot{F}_a = \varepsilon_1 u(t) - \varepsilon_2 F_a - \varepsilon_3 (\dot{z}_2 - \dot{z}_1) \quad (10)$$

where:

$\varepsilon_1 = S_p \sigma_3 \sqrt{P_s} k_{sv}$, $\varepsilon_2 = \sigma_2$, $\varepsilon_3 = \sigma_1 S_p^2$

Substituting equations (3), (4), (5), (6), and (10), into equations (1) and (2), we get:

$$\begin{cases} m_1 \ddot{z}_1 = K(z_2 - z_1) + C(\dot{z}_2 - \dot{z}_1) + F_a \\ m_2 \ddot{z}_2 = K_t(z_3 - z_2) - K(z_2 - z_1) - C(\dot{z}_2 - \dot{z}_1) - F_a \\ \dot{F}_a = \varepsilon_1 u(t) - \varepsilon_2 F_a - \varepsilon_3(\dot{z}_2 - \dot{z}_1) \end{cases} \quad (11)$$

Let: $\begin{matrix} x_1 = z_1 \\ x_2 = \dot{z}_1 \\ x_3 = z_2 \\ x_4 = \dot{z}_2 \\ x_5 = F_a \end{matrix}$

The system of equations (11) can be rewritten as an oscillatory state matrix as follows:

$$[\dot{x}_i] = A_1 [x_i] + A_2 u(t) + A_3 z_3 \quad (12)$$

where:
*$A_1$* is the matrix of oscillation states, *$A_2$* is the matrix of the control signal, and *$A_3$* is the matrix of the disturbance signal.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{K}{m_1} & -\frac{C}{m_1} & \frac{K}{m_1} & \frac{C}{m_1} & \frac{1}{m_1} \\ 0 & 0 & 0 & 1 & 0 \\ \frac{K}{m_2} & \frac{C}{m_2} & -\frac{(K+K_t)}{m_2} & -\frac{C}{m_2} & -\frac{1}{m_2} \\ 0 & -\varepsilon_3 & 0 & \varepsilon_3 & -\varepsilon_2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & \varepsilon_1 \end{bmatrix}^T \quad A_3 = \begin{bmatrix} 0 & 0 & 0 & \frac{K_t}{m_2} & 0 \end{bmatrix}^T$$

## 2.2 Control algorithm

Consider the linear system given by the following equation:

$$\dot{x} = Ax + Bu(t) \tag{13}$$

For the system to be stable, the cost function must reach a minimum value.

$$J = \int_0^\infty \left(x^T Qx + u^T(t) Ru(t)\right) dt \to Min \tag{14}$$

This is reached when the control signal u(t) is defined through the matrix *G*.

$$u(t) = -Gx(t) = -R^{-1} B^T Sx(t) \tag{15}$$

If S does not change over time, that is:

$$\frac{dS(t)}{dt} = 0 \tag{16}$$

The Riccati algebraic equation used to determine the matrix *R* is presented in the following form:

$$A^T S + SA - SBR^{-1} B^T S + Q = 0 \tag{17}$$

where:
*Q* is the output matrix and *R* is the navigation matrix.

$$Q = \begin{bmatrix} a_1 & 0 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 & 0 \\ 0 & 0 & a_3 & 0 & 0 \\ 0 & 0 & 0 & a_4 & 0 \\ 0 & 0 & 0 & 0 & a_5 \end{bmatrix} \quad R = \left[a_6\right]$$

The determination of the coefficients for the *Q* and *R* matrices is done by the in-loop optimization algorithm. This algorithm is summarized through the following steps:

Step 1: Select the simulation and stopping conditions of the algorithm

Step 2: Determine the coefficients' upper and lower bounds. The maximum and minimum values of the coefficients are determined by the experience of the designer.

Step 3: Run the in-loop until the stopping condition is satisfied. Then the coefficients can be shown.

Step 4: Use these coefficients to run simulations in many cases.

The algorithm diagram is shown in Figure 3.

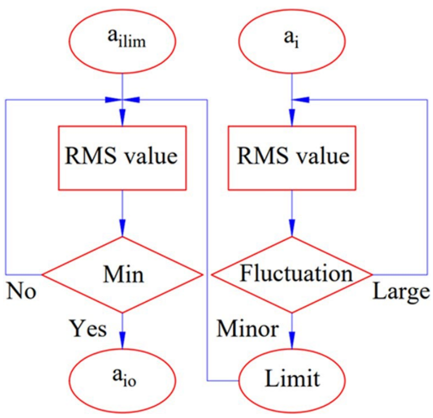


**Figure 3**: Algorithm diagram.

## 3 SIMULATIONS AND RESULTS

### 3.1 Investigation cases

The excitation from the road surface is the main cause of vehicle oscillations. To comprehensively evaluate this oscillation, the use of many different excitation cases is necessary. In this paper, four cases are considered as stimuli for numerical simulation (Figure 4). Corresponding to the four cases are the corresponding results, including displacement and acceleration of the sprung mass. Besides, three situations are given during the simulation. In the first situation, the vehicle only uses the passive suspension system. In the second and third situations, the active suspension system with a hydraulic actuator is considered. However, these two situations will use two different control methods. The simulation was performed using MATLAB-Simulink software. The parameters of the model are given in Table 1 (Nguyen, 2021g).

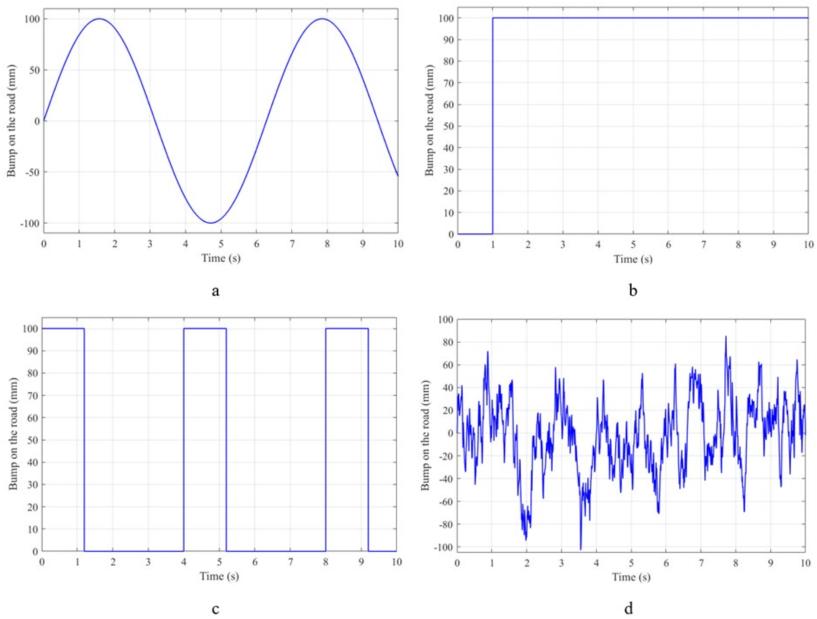


**Figure 4**: Bump on the road (a – Sine wave; b – Step; c – Pulse; d – Random).

**Table 1** Specific parameters.

| Symbol | Description | Value | Unit |
|---|---|---|---|
| $m_1$ | Sprung mass | 560 | kg |
| $m_2$ | Unsprung mass | 60 | kg |
| C | Damper coefficient | $3.5\times10^3$ | Ns/m |
| K | Spring coefficient | $42\times10^3$ | N/m |
| $K_t$ | Tire coefficient | $185\times10^3$ | N/m |
| $k_{sv}$ | Servo valve gain | $1\times10^{-3}$ | m/V |
| τ | Time constant | $2\times10^{-3}$ | s |
| $P_s$ | Supply pressure | $1\times10^6$ | N/m$^2$ |
| $S_p$ | Piston cross-sectional area | $3.6\times10^{-4}$ | m$^2$ |

### 3.2 Result and discussion

The results of the calculation and simulation process are given corresponding to the four investigation cases. The change in displacement of the sprung mass with time is given in Figure 5.

In the first case (a), the pavement excitation has a sine waveform whose amplitude is 100 (mm). This is a type of low-frequency periodic excitation commonly used in oscillation control problems. According to this graph, the value of displacement of the sprung mass is greatest when the vehicle uses only the passive suspension system. Its value can be up to 101.91 (mm). This value can be reduced if the active suspension system is fitted. For the PID control algorithm for the SISO system, the maximum value of displacement of the sprung mass is 67.53 (mm), only 66.26% of the first situation. However, this value can be greatly reduced if a linear controller for the MIMO system is applied. Its maximum value is only about 2.29 (mm), which is only approximately 2.25% compared to the situation of the vehicle without the active suspension system. Besides, the average value calculated according to RMS corresponding to three investigation situations is 70.24 (mm), 46.49 (mm), and 1.58 (mm).

In the second case (b), the pavement excitation takes the form of a step. The excitation signal begins to appear at time t = 1 (s). After that, this signal continues to be maintained throughout the investigation period with an amplitude of 100 (mm). Displacement of the sprung mass is greatest when the vehicle does not have the controller for the suspension system, reaching 153.08 (mm). It is caused by a sudden excitable from the pavement in a very short period. This results in a large inertia force appearing, and it causes the vehicle body to oscillate with a larger amplitude. When the controller for the active suspension system is used, this value drops significantly, to only 72.14 (mm) and 4.10 (mm). Besides, the oscillation of the sprung mass is also quickly extinguished if the vehicle uses the active suspension system. In contrast, this oscillation lasts for about 2.5 (s), corresponding to the first situation.

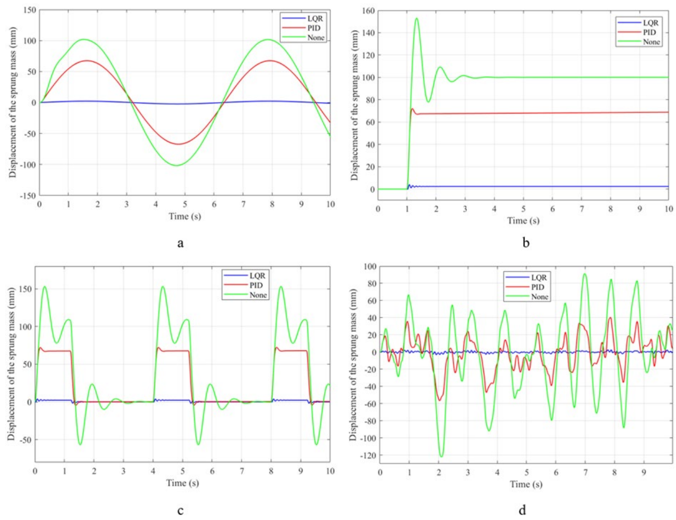


**Figure 5**: Displacement of the sprung mass (a – Sine wave; b – Step; c – Pulse; d – Random).

In the third case (c), pulsed excitation is used. These are square pulses; each pulse lasts about 1.2 (s). This stimulus also causes the body of the vehicle to oscillate strongly. After the excitation ends, the sprung mass continues to move due to the influence of inertial force. The active suspension system helps to improve this problem more effectively and thoroughly.

For the last case (d), the stimulus used is random. The power source used here is white noise, which is within acceptable standards. These stimuli change continuously over time. Therefore, the change in displacement of the sprung mass also varies continuously with time. According to this result, the maximum values of displacement are 122.23 (mm), 56.81 (mm), and 3.24 (mm) respectively. Besides, their average values are determined to be 44.82 (mm), 19.55 (mm), and 0.94 (mm) respectively. The vehicle's stability has been significantly increased when the active suspension system with the LQR control algorithm is used.

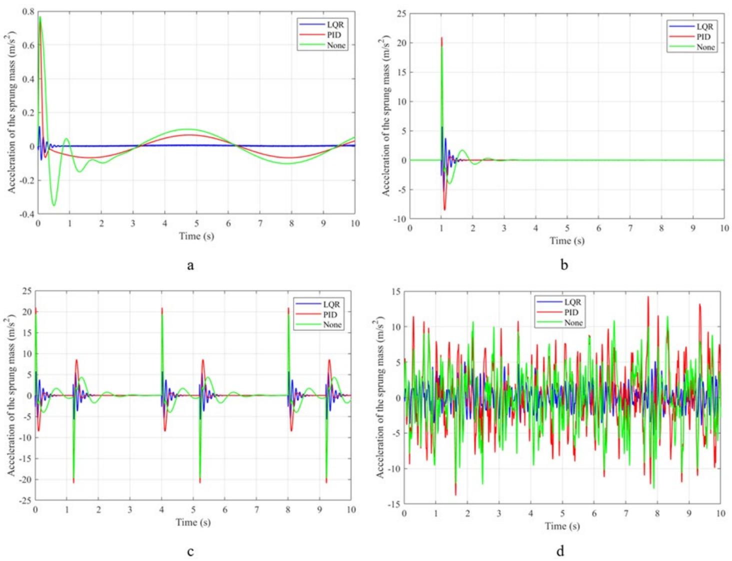


**Figure 6**: Acceleration of the sprung mass (a – Sine wave; b – Step; c – Pulse; d – Random).

The change in the acceleration of the sprung mass is shown in Figure 6 based on the four cases. According to this result, the value of acceleration was significantly reduced when the LQR algorithm for the MIMO system was used. Besides, the fluctuation of the acceleration after the excitation ends is also one of the important problems. This problem can affect the smoothness of the vehicle.

In the first case, when the vehicle used the LQR controller for the active suspension system, its maximum and average accelerations were only 0.12 (m/s$^2$) and 0.01 (m/s$^2$). If the vehicle still uses a conventional mechanical suspension system, this value can be up to 6.42 times and 13.00 times more than in the other situation. In addition, the difference in value between the vehicle using the passive suspension system and the active suspension controlled by the PID algorithm is negligible.

In the second case, the maximum value of the acceleration of the sprung mass corresponding to the three situations reaches 5.70 (m/s$^2$), 20.93 (m/s$^2$), and 19.39 (m/s$^2$), respectively. In the third case, the values are similar. Accordingly, the SISO control algorithm cannot guarantee the smoothness of the vehicle in oscillating situations with pulsed or step excitation in such a short time.

For the last case, the maximum and average values of the acceleration of the sprung mass are 5.53 (m/s$^2$), 14.31 (m/s$^2$), 12.86 (m/s$^2$), respectively; and 1.83 (m/s$^2$), 4.67 (m/s$^2$), 4.28 (m/s$^2$). In general, the efficiency of the LQR control algorithm for the MIMO system is very high. The smoothness and stability of the vehicle have been improved more in the investigation cases.

The results of the simulation are summarized in Table 2, Table 3, Table 4, and Table 5.

**Table 2** Results of the simulation process (Case 1).

| | **Situation 1** | **Situation 2** | **Situation 3** |
|---|---|---|---|
| Average of the displacement (mm) | 1.58 | 46.49 | 70.24 |
| Maximum of the displacement (mm) | 2.29 | 67.53 | 101.91 |
| Average of the acceleration (m/s$^2$) | 0.01 | 0.08 | 0.13 |
| Maximum of the acceleration (m/s$^2$) | 0.12 | 0.74 | 0.77 |

**Table 3** Results of the simulation process (Case 2).

| | **Situation 1** | **Situation 2** | **Situation 3** |
|---|---|---|---|
| Average of the displacement (mm) | 2.29 | 67.78 | 100.75 |
| Maximum of the displacement (mm) | 4.10 | 72.14 | 153.08 |
| Average of the acceleration (m/s$^2$) | 0.51 | 1.38 | 1.30 |
| Maximum of the acceleration (m/s$^2$) | 5.70 | 20.93 | 19.39 |

**Table 4** Results of the simulation process (Case 3).

| | **Situation 1** | **Situation 2** | **Situation 3** |
|---|---|---|---|
| Average of the displacement (mm) | 1.43 | 40.20 | 66.92 |
| Maximum of the displacement (mm) | 4.10 | 72.53 | 153.20 |
| Average of the acceleration (m/s$^2$) | 1.18 | 3.20 | 3.08 |
| Maximum of the acceleration (m/s$^2$) | 5.75 | 20.93 | 19.71 |

**Table 5** Results of the simulation process (Case 4).

| | **Situation 1** | **Situation 2** | **Situation 3** |
|---|---|---|---|
| Average of the displacement (mm) | 0.94 | 19.55 | 44.82 |
| Maximum of the displacement (mm) | 3.24 | 56.81 | 122.23 |
| Average of the acceleration (m/s$^2$) | 1.83 | 4.67 | 4.28 |
| Maximum of the acceleration (m/s$^2$) | 5.53 | 14.31 | 12.86 |

The change of these values is depicted in Figure 7.

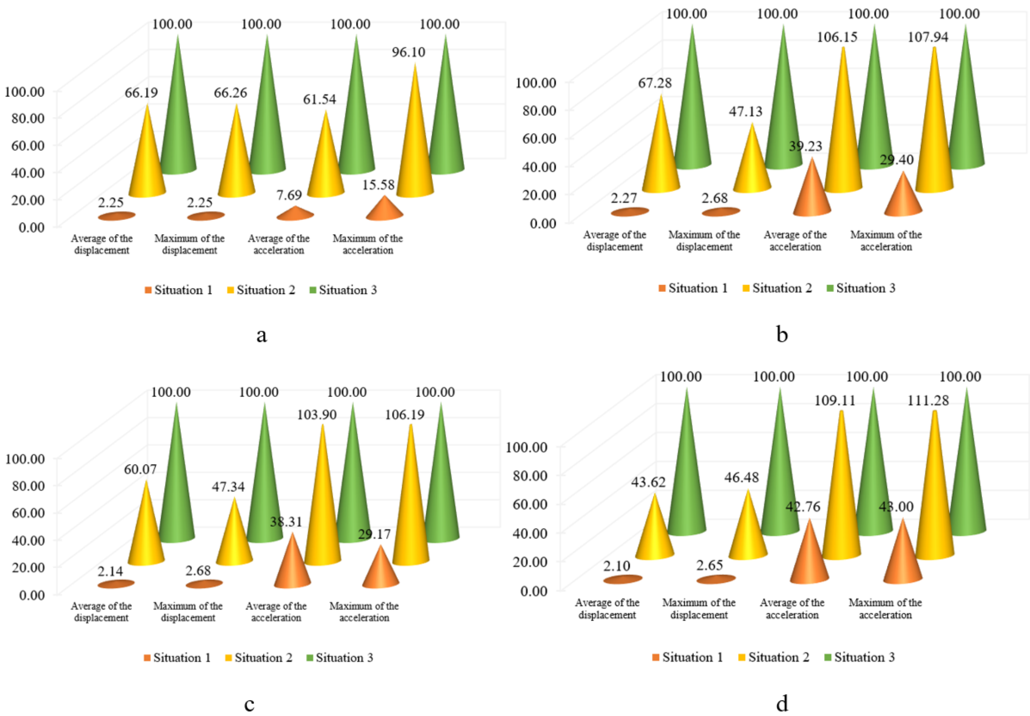


**Figure 7**: Percent of values (a – Sine wave; b – Step; c – Pulse; d – Random).

## 4 CONCLUSION

The excitement from the road surface causes oscillations in the vehicle body, which can be suppressed by the suspension system. To improve the stability of the vehicle, the active suspension system with a hydraulic actuator is used to replace the conventional passive suspension system. This research focuses on simulating the oscillation of the vehicle with the LQR control algorithm for the MIMO system.

In this paper, a quarter-dynamics model is considered. Besides, this model also includes a hydraulic actuator. The process of linearizing the hydraulic actuator is clearly described in the content of the paper. At the same time, the process of designing the controller and optimizing the parameters of the control matrix is also shown in this paper. The numerical simulation is performed in the MATLAB-Simulink environment with four specific cases. For each case, three situations are considered.

According to the results of the simulation, the values of displacement and acceleration of the sprung mass were significantly reduced when the active suspension system with the LQR control algorithm was used. In all investigation cases, the maximum value and average value of the displacement of the sprung mass when applying the LQR control algorithm do not exceed 2.68% compared to the situation where the vehicle only uses the passive suspension system. In the case of random oscillations, the values of the maximum and average accelerations of the sprung mass do not exceed 43.00%. In general, vehicle oscillations were better guaranteed when this algorithm was applied.

The LQR control algorithm is suitable for MIMO systems. However, if the system has many degrees of freedom, the design of the control matrix is quite complicated. Besides, choosing the optimal parameters is also a very important issue. It directly affects the smoothness and stability of the vehicle. The in-loop optimization method used in this research has not really given the most accurate and reasonable value for the controller. In later research, some intelligent optimization methods such as ABC, PSO, etc. will be used.

# NOMENCLATURE

$F_a$ : Force of the hydraulic actuator, N
$F_d$ : Force of the damper, N
$F_i$ : Force of the inertia, N
$F_s$ : Force of the spring, N
$F_{st}$ : Force of the tire, N
$z_1$ : Displacement of the sprung mass, m
$z_2$ : Displacement of the unsprung mass, m
$z_3$ : Bump on the road, m

# ABBREVIATION

ABC : Artificial Bee Colony
ANFIS : An
ANN : Artificial Neural Network
GA : Genetic Algorithm
LQR : Linear Quadratic Regulator
MIMO : Multi Input – Multi Output
PID : Proportional – Integral – Derivative
PSO : Particle Swarm Optimization
RBFNN : Radial Basic Function Neural Network
RMS : Root Mean Square
SISO : Single Input – Single Output
SMC : Sliding Mode Control
T – S : Tagaki – Sugeno

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## References

Aela, A. M. A., Kenne, J. P., Mintsa, H. A. (2020). Adaptive Neural Network and Nonlinear Electrohydraulic Active Suspension Control System. Journal of Vibration and Control 28: 17 pages.

Anh, N. T. (2020). Control an Active Suspension System by Using PID and LQR Controller. International Journal of Mechanical and Production Engineering Research and Development 10: 7003-12.

Azizi, A., Mobki, H. (2021). Applied Mechatronics: Designing a Sliding Mode Controller for Active Suspension System. Complexity 2021: 23 pages.

Bai, R., Guo, D. (2018). Sliding-Mode Control of the Active Suspension System with the Dynamics of a Hydraulic Actuator. Complexity 2018: 7 pages.

Beltran-Carbajal, F., et al. (2019). An Active Vehicle Suspension Control Approach with Electromagnetic and Hydraulic Actuators. Actuators 8: 18 pages.

Brezas, P., Smith, M. C. (2014). Linear Quadratic Optimal and Risk-Sensitive Control for Vehicle Active Suspensions. IEEE Transactions on Control Systems Technology 22: 543-56.

Chen, X., et al. (2020). Investigation of Sliding Mode Control for Nonlinear Suspension Systems with State Estimation. Mechanics & Industry 21: 8 pages.

Choi, H. D., You, S. H. (2021). Fuzzy Finite Memory State Estimation for Electro-Hydraulic Active Suspension Systems. IEEE Access 9: 99364-73.

Dahunsi, O. A., et al. (2020). Proportional + Integral + Derivative Control of Nonlinear Full-car Electrohydraulic Suspensions Using Global and Evolutionary Optimization Techniques. Journal of Low Frequency Noise, Vibration and Active Control 39: 393-415.

Ding, X., et al. (2021). Design of and Research into a Multiple-Fuzzy PID Suspension Control System Based on Road Recognition. Processes 9: 20 pages.

Emam, A. S. (2015). Fuzzy Self Tuning of PID Controller for Active Suspension System. Advances in Powertrains and Automotives 1: 34-41.

Jiregna, I., Sirata, G. (2020). A Review of the Vehicle Suspension System. Journal of Mechanical and Energy Engineering 4: 109-14.

Karam, Z. A., Awad, O. A. (2020). Design of Active Fractional PID Controller Based on Whale's Optimization Algorithm for Stabilizing a Quarter Vehicle Suspension System. Periodica Polytechnica Electrical Engineering and Computer Science 64: 247-63.

Mahmoodabadi, M. J., Nejadkourki, N. (2020). Optimal Fuzzy Adaptive Robust PID Control for an Active Suspension System. Australian Journal of Mechanical Engineering: 12 pages.

Metered, H., Metered, W., Emam, A. S. (2018). Optimized Proportional Integral Derivative Controller of Vehicle Active Suspension System Using Genetic Algorithm. SAE Technical Paper: 8 pages.

Mrazgua, J., et al. (2021). Static Output Feedback Stabilization of T-S Fuzzy Active Suspension Systems. Journal of Terramechanics 97: 19-27.

Na, J., et al. (2020). Adaptive Finite-Time Fuzzy Control of Nonlinear Active Suspension Systems with Input Delay. IEEE Transactions on Cybernetics 50: 2639-50.

Nguyen, D. N., Nguyen, T. A., Dang, N. D. (2022). A Novel Sliding Mode Control Algorithm for an Active Suspension System Considering with the Hydraulic Actuator. Latin American Journal of Solids and Structures 19: 16 pages.

Nguyen, T. A. (2021a). Advance the Stability of the Vehicle by Using the Pneumatic Suspension System Integrated with the Hydraulic Actuator. Latin American Journal of Solids and Structures 18: 16 pages.

Nguyen, T. A. (2021b). Control the Hydraulic Stabilizer Bar to Improve the Stability of the Vehicle When Steering. Mathematical Modelling of Engineering Problems 8: 199-206.

Nguyen, T. A. (2021c). Advance the Efficiency of an Active Suspension System by the Sliding Mode Control Algorithm with Five State Variables. IEEE Access 9: 164368-78.

Nguyen, T. A. (2021d). Study on the Sliding Mode Control Method for the Active Suspension System. International Journal of Applied Science and Engineering 18: 10 pages.

Nguyen, T. A. (2021e). Improving the Stability of the Passenger Vehicle by Using an Active Stabilizer Bar Controlled by the Fuzzy Method. Complexity 2021: 20 pages.

Nguyen, T. A. (2021f). Preventing the Rollover Phenomenon of the Vehicle by Using the Hydraulic Stabilizer Bar Controlled by a Two-Input Fuzzy Controller. IEEE Access 9: 129168-77.

Nguyen, T. A. (2021g). Improving the Comfort of the Vehicle Based on Using the Active Suspension System Controlled by the Double-Integrated Controller. Shock and Vibration 2021: 11 pages.

Pang, H., et al. (2017). Advances in Mechanical Engineering. Shock and Vibration 2017: 13 pages.

Rodriguez-Guevara, D., et al. (2021). Active Suspension Control Using an MPC-LQR-LPV Controller with Attraction Sets and Quadratic Stability Conditions. Mathematics 9: 17 pages.

Shaer, B., et al. (2018). A Chattering-free Fuzzy Hybrid Sliding Mode Control of an Electrohydraulic Active Suspension. Transactions of the Institute of Measurement and Control 40: 222-38.

Sun, J., Zhao, K. (2020). Adaptive Neural Network Sliding Mode Control for Active Suspension Systems with Electrohydraulic Actuator Dynamics. International Journal of Advanced Robotic Systems 17: 11 pages.

Tuan, A. N., Thang, B. H. (2019). Investigation of Equivalent Unsprung Mass and Nonlinear Features of Electromagnetic Actuated Active Suspension. Advances in Science, Technology and Engineering Systems Journal 4:271-75.

Wang, D., et al. (2018). Research on Robust Model Predictive Control for Electro-Hydraulic Servo Active Suspension Systems. IEEE Access 6: 3231-40.

Wei, S., Su, X. (2019). Sliding Mode Control Design for Active Suspension Systems Using Quantum Particle Swarm Optimization. International Journal of Vehicle Design 81: 93-114.

Xia, R. X., et al. (2015). Linear-quadratic-Gaussian Controller for Truck Active Suspension Based on Cargo Integrity. Advances in Mechanical Engineering 7: 9 pages.

Xiao, L., Zhu, Y. (2015). Sliding-mode Output Feedback Control for Active Suspension with Nonlinear Actuator Dynamics. Journal of Vibration and Control 21: 2721-38.

Xue, W., Jin, P., Li, K. (2021). Parameter-dependent Actuator Fault Estimation for Vehicle Active Suspension Systems Based on RBFNN. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering 235: 2540-50.

Yin, J., et al. (2015). Investigation of Equivalent Unsprung Mass and Nonlinear Features of Electromagnetic Actuated Active Suspension. Shock and Vibration 2015: 8 pages.