

# Application of Modal Filters for Damage Detection in the Presence of Non-Linearities

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## ABSTRACT

The aim of the paper is to study the possibility of implementing modal filtering techniques for damage detection in the presence of non-linearities in the recorded signals. Initially designed for linear damage detection the method is based on the linear combination of the sensors responses, a transformation to the frequency domain, and the computation of peak indicators which are used subsequently in an outlier analysis process. The efficiency of the method to detect both linear and non-linear damage scenarios is assessed using data recorded on the three-storey frame structure previously developed and studied at Los Alamos National Labs. Experimental data consists in four acceleration records. Besides the baseline condition, both linear (mass and stiffness changes) and non-linear (bumper device) changes have been considered. The results obtained using the modal filtering approach are compared to the ones obtained based on auto-regressive models, considering either the auto-regressive parameters or the time-domain residuals.

## INTRODUCTION

Nowadays, aging aerospace, civil and mechanical infrastructure continues to be operated under the risk of damage accumulation and possible failure. In order to detect such a degradation of performances leading to possible failure, on-line structural monitoring systems should be implemented. Over the last decades, many efforts have been made in the field of vibration based damage detection methods which are mainly focused on modal parameters changes such as modal damping, eigenfrequencies or mode shapes [1]. Recently, a fully automated approach based on modal filters and control charts has been proposed in [2,3] for damage detection and

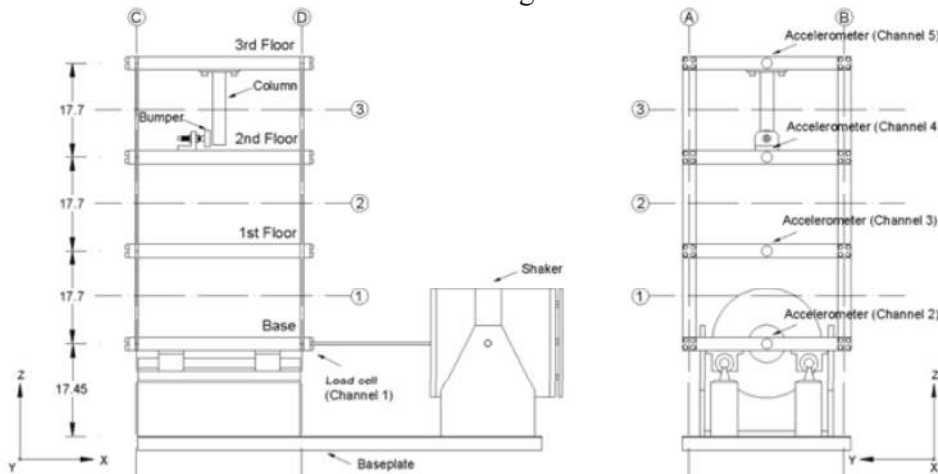
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validated in a laboratory experiment on an aircraft wing in [4]. More recently, the method has been extended to damage localization using strain measurements in [5]. In this paper, the effectiveness of this method to detect both linear and non-linear damage scenarios is assessed using data recorded on a three-storey frame structure previously developed and studied at Los Alamos National Labs. The paper is organized as follows: in the first part, the experimental setup and the available datasets are presented. In the second part, the automated method for damage detection using the modal filtering technique is briefly recalled and applied to the experimental data. Finally, in the third part, the results obtained using auto-regressive (AR) models [6] are presented and compared to the results obtained using the modal filtering technique.

## EXPERIMENTAL SET-UP AND DATA SETS DESCRIPTION

Tests were performed at Los Alamos National Labs on a three-storey frame structure shown with its basic dimensions in Figure 1.



*Figure 1. Mechanical drawing of the three-storey frame structure.*

The structure consists of aluminum columns and plates assembled using bolted joints. The structure can slide on rails allowing movement in the x-direction only. A center column is suspended from the top floor and can be used to simulate damage by inducing nonlinear behavior when it contacts a bumper mounted on the next floor. The position of the bumper can be adjusted to vary the extent of impacting that occurs at a particular excitation level. In the context of damage detection, this source of damage is intended to simulate fatigue cracks that can open and close or loose connections that can rattle under dynamic loading. An electrodynamic shaker provides a lateral excitation to the base floor along the centerline of the structure. A load cell is attached at the end of a stinger to measure the input force from the shaker to the structure. Four accelerometers are attached at the centerline of each floor on the opposite side from the excitation source to measure the system's response. A National Instruments PXI data acquisition system is used to collect and process the data. The analog sensor signals are digitized at a rate of 2560 Hz and acquired in blocks of 65536 points. The signals are subsequently downsampled into 8192 data points at 3.125 ms intervals corresponding to a sampling frequency of 320 Hz. A band-limited random excitation in the range of 20-150 Hz is used to excite the structure.

The structural state conditions can be categorized into four main groups, as shown in Table 1. Note that for each state condition, 50 tests were performed, yielding 50 time histories per channel. The first group is the baseline condition. The second group includes the states with linear changes applied to the structure (State#2-9). Initially, this set of data was designed to represent the effect of environmental and operational changes which would produce changes of eigenfrequencies of the system, but not the appearance of non-linearities in the signals. In the present study, this data is considered as a linear damage case, representing the linear change of stiffness or mass in local areas of the structure, without the appearance of non-linearities in the signals. The third group includes the state conditions with the non-linear changes; these were simulated by the introduction of nonlinearities into the structure using the bumper and the suspended column with different gaps between them (State#10-14). Finally, the fourth group includes state conditions including both linear and non-linear changes in the dynamic signals (State#15-17). More details about the test structure as well as damage scenarios can be found in [7].

**Table 1. Structural state conditions.**

Label	Samples	State condition	Description
State #1	1-50	Initial Structure	Baseline
State #2	51-100	LC	m=1.2kg at the base
State #3	101-150	LC	m=1.2kg on the 1 <sup>st</sup> floor
State #4	151-200	LC	-87.5% stiffness in column 1BD
State #5	201-250	LC	-87.5% stiffness in columns 1AD, 1BD
State #6	251-300	LC	-87.5% stiffness in column 2BD
State #7	301-350	LC	-87.5% stiffness in columns 2AD, 2BD
State #8	351-400	LC	-87.5% stiffness in column 3BD
State #9	401-450	LC	-87.5% stiffness in columns 3AD, 3BD
State #10	451-500	NLC	Gap=0.20mm
State #11	501-550	NLC	Gap=0.15mm
State #12	551-600	NLC	Gap=0.13mm
State #13	601-650	NLC	Gap=0.10mm
State #14	651-700	NLC	Gap=0.05mm
State #15	701-750	LC+NLC	Gap=0.20mm and m=1.2kg at the base
State #16	751-800	LC+NLC	Gap=0.20mm and m=1.2kg on the 1 <sup>st</sup> floor
State #17	801-850	LC+NLC	Gap=0.10mm and m=1.2kg on the 1 <sup>st</sup> floor

LC=linear change – NLC = non-linear change

## DAMAGE DETECTION USING MODAL FILTERING

### Modal filtering

Consider a vibrating structure equipped with a large network of sensors. Spatial filtering consists in condensing the data from a network of  $n$  sensors through a linear combination to form a single output response. A modal filter which isolates mode  $l$

can be constructed by properly selecting the weighting coefficients of the linear combiner in such a way that it is orthogonal to all  $N$  modes of a structure in the frequency band of interest, except the mode  $l$ . The time domain output of the modal filter can be transformed into the frequency domain by computing either the frequency response function (FRF) if the input force is known, or the power spectral density (PSD) if only ambient vibrations are measured. The typical frequency domain output of a modal filter contains a single peak as shown in Figure 2a. When damage occurs in a structure, it results in the appearance of spurious peaks in the frequency domain output of the modal filters, as demonstrated in [4,1] and shown in Figure 2b.

### Choice of features and statistical process control for damage detection

The feature used for damage detection is a peak indicator computed around each natural frequency of the initial, undamaged structure [3]. Based on the peak indicators extracted automatically from the measurements, control charts are then used in order to detect a deviation from normal condition [8]. The control chart used in this study is the multivariate Hotelling  $T^2$  control chart.

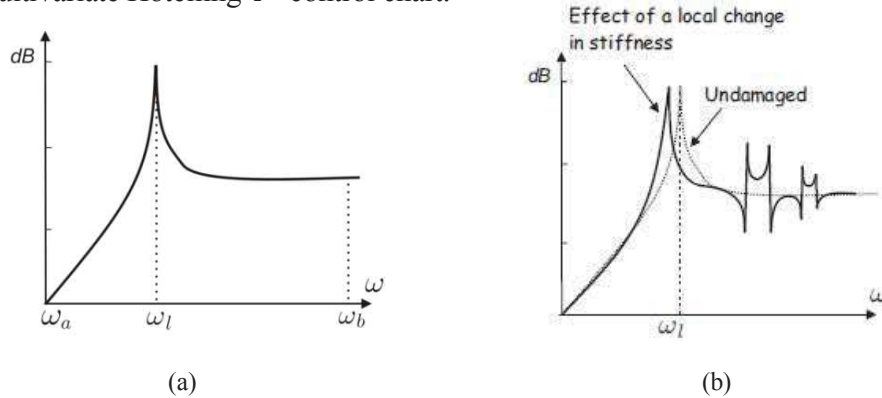
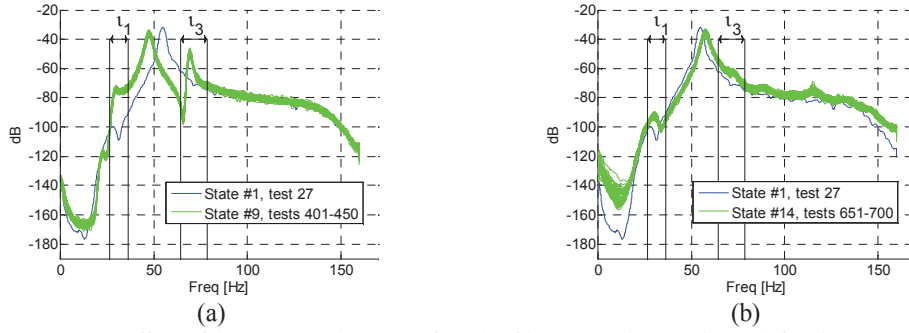


Figure 2. a) Perfect modal filter tuned to mode  $l$ , b) Effect of damage on the modal filter output.

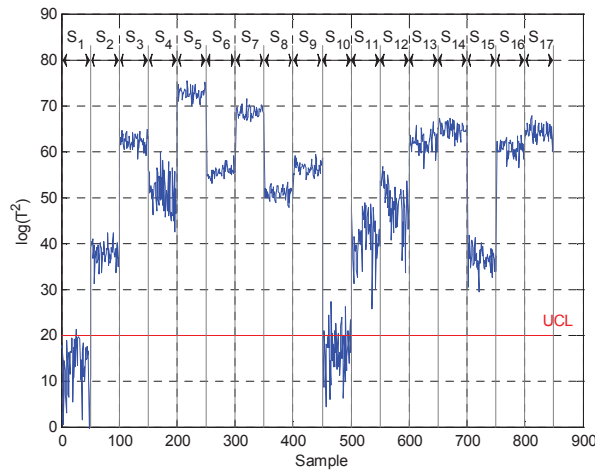
### Results

Based on the signals recorded by the accelerometers for the baseline condition, the modal parameters have been estimated using the data-driven stochastic subspace identification implemented in the MACEC toolbox under Matlab [9]. Additionally, based on the identified mode shapes, modal filter coefficients for modes 1,2 and 3 have been computed. As an example, Figure 3a shows the FRFs of the output of the modal filters tuned to mode 2 for the baseline condition (State#1) and the maximum linear damage (State#9), and Figure 3b compares the baseline condition with the maximum non-linear damage (State#14). Although the initial filter is not perfect, the figure shows clearly the appearance of the spurious peaks. For each modal filter, the peak indicator has been computed in the three following intervals:  $i_1$ [26.1880 Hz - 36.1644 Hz],  $i_2$ [36.1645 Hz - 64.2759 Hz] and  $i_3$ [64.2760 Hz - 78.5596Hz].



**Figure 3.** Effect of damage on the FRF of modal filters tuned to mode 2: a) for the state condition #9 and b) for the state condition#14.

A combined representation of the evolution of all the peak indicators for all the states is obtained using the Hotelling  $T^2$  multivariate control chart (Figure 4) in which a single control limit (Upper Control Limit – UCL) is computed using 80% of the available baseline condition tests (State #1) and a confidence level of 97.5 % ( $\alpha=0.025$ ). The figure clearly shows that all the linear changes are detected as outliers. This was expected, as the method based on modal filters is designed to react to local changes of mode shapes which are induced here by the local stiffness and mass variations induced to the structure. The figure also shows that the method is quite sensitive to the non-linear changes, except for the lowest level (State#10) for which although there is a slight increase in the statistics, it does not clearly appear as an outlier. The method is therefore sensitive to both linear and non-linear changes in the dynamics signals, but is not able to differentiate between these two types of variations.



**Figure 4.** Multivariate Hotelling  $T^2$  control chart using 6 peak indicators extracted from modal filters for all samples and state conditions.

## DAMAGE DETECTION USING AUTO-REGRESSIVE BASED METHODS

The AR models have been used in SHM to extract damage-sensitive features from time series data, either using the model parameters or the residual errors [10,11]. For a measured time series  $s_1, s_2, \dots, s_N$  the AR( $p$ ) model of order  $p$  is given by

$$s_i = \sum_{j=1}^p \alpha_j s_{i-j} + e_i \quad (1)$$

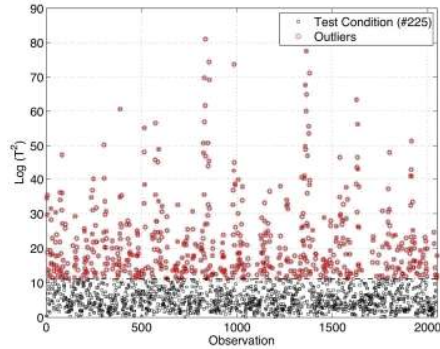
where  $s_i$  is the measured signal and  $e_i$  is an unobservable random error at discrete time index  $i$ . The unknown AR parameters,  $\alpha_j$ , can be estimated by using the least squares method. The order of the model is always an unknown integer that needs to be estimated from the data. The Akaike Information Criterion (AIC) has been reported as one of the most efficient techniques for order optimization [6]. For SHM, an AR model can be used as a damage-sensitive feature extractor based on either the (i) residual errors  $e_i$  or (ii) the parameters  $\alpha_j$ . The first approach consists of using the AR model, with parameters estimated from the baseline condition, to predict the response of data obtained from a potentially damaged structural condition. For the baseline condition, the residual errors are generally assumed to be independent and normally distributed. This approach assumes that damage will introduce either linear deviations from the baseline condition or nonlinear effects in the signal and, as a result, the linear model developed with the baseline data will no longer accurately predict the response of the damaged structure. The second approach consists of fitting AR models to signals from the undamaged and damaged structural conditions. In this approach, the AR parameters are used directly as damage-sensitive features, and some form of a multivariate classifier can be used to discriminate the damage classes. Notice that the parameters should be constant when obtained from times series of a time-invariant structural system. More details about this algorithm can be found in [7]. For this paper, the order of the model ( $p=10$ ) was established based on the AIC, as explained in previous studies [12]. The analysis present herein uses the two approaches summarized above in order to highlight the main advantages and disadvantages of each feature.

For the first case, and for each test, four individual AR(10) models, with parameters from the baseline condition, are used to fit the corresponding acceleration time series and to obtain the residuals. Then, the residual errors are reorganized into subgroups of four in order to perform dimensionality reduction, i.e. to transform residual time series with 8192 to 2048 observations. For one test from State#5, Figure 5 shows a Hotelling  $T^2$  multivariate control chart in which the UCL is computed using 80% of the available baseline condition tests (State #1) and a confidence level of 97.5%. The figure highlights 625 outliers, high above the number errors (51) suggested by the level of significance assumed a priori,  $\alpha=0.025$ , which indicates that the process is likely to operate out of control (damaged system). Thus, generalizing the same procedure to the 850 tests (samples), Figure 6 plots the number of outliers per sample. Clearly, in this case, the residual errors seem not to be appropriate to discriminate all linear and non-linear changes (51-850) from the baseline condition (1-50).

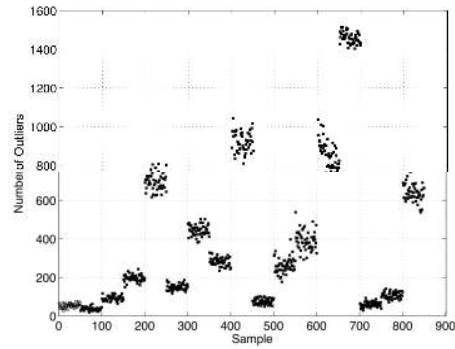
For completeness, the same procedure was carried out using residuals estimated from AR(10) models with AR parameters estimated from each test. The results plot in Figure 7 indicates that in this case, the algorithm seems to be not appropriate to discriminate the states with linear changes from the baseline condition. Indeed, it is related to fact that the linear changes do not change the dynamic dimension of the system and an AR(10) is still appropriate to fit the data. However, the nonlinearities



caused by the bumper introduce complexity into the data in such a way that a linear AR(10) model is no longer appropriate to fit the data, as demonstrated in [13].

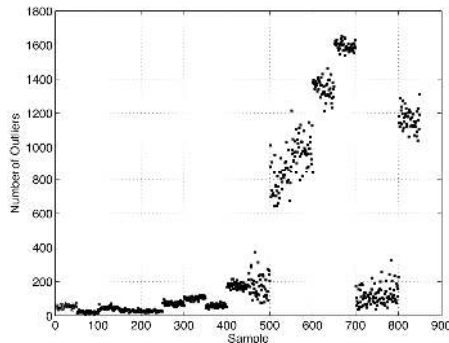


**Figure 5.** Multivariate Hotelling control chart of residuals showing the evolution of the outliers (625) for sample #225 (State#5).

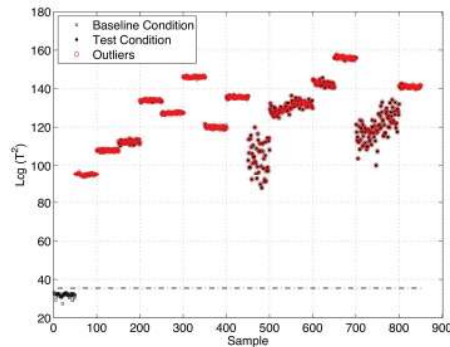


**Figure 6.** Number of outliers per sample based on multivariate control charts of residuals.

For the second case, and for each test, four individual AR(10) models are used to fit the corresponding time series from the four accelerometers and their parameters are used directly as damage-sensitive features in concatenated format, yielding 40-dimensional feature vectors. Figure 8 shows the Hotelling multivariate control chart in which a UCL is computed using 80% of the available baseline condition tests (State #1) for a confidence level of 97.5% ( $\alpha=0.025$ ). In this case, the figure clearly shows that all the conditions, with linear and non-linear changes, are detected as outliers. Indeed, it was expected because the AR parameters are related to the stiffness and mass of the system. The figure also shows more dispersion for the classification of those conditions associated with low level of nonlinearities, which is related to the randomness of the impacts.



**Figure 7.** Number of outliers per sample based on multivariate control charts of residuals and with AR parameters estimated on each test.



**Figure 8.** Multivariate Hotelling control chart using as features the AR(10) parameters from all four accelerometers in concatenated format.

## CONCLUSION

In this study, we have compared different approaches for damage detection in the case of both linear and non-linear damage using data from a three-storey building mockup previously developed at the Los Alamos National Labs. The main results of this comparison can be summarized as follows : (i) when using AR model parameters or peak indicators extracted from modal filters, it is possible to detect both linear and non-linear damages efficiently, (ii) when using AR model residuals, the detection is

not as effective when the parameters are identified on the baseline condition only and (iii) if instead, the AR parameters are identified from each state, it is possible to be almost insensitive to linear damage cases and clearly identify non-linear damages.

## REFERENCES

- [1] S.W. Doebling, C. Farrar, and M. B. Prime, A summary review of vibration based damage identification methods. *The shock and vibration digest*, 30(2), 91-105, 1998.
- [2] A. Deraemaeker and A. Preumont, Vibration based damage detection using large array sensors and spatial filters, *Mechanical systems and signal processing*, 20, 1615-1630, 2006.
- [3] A. Deraemaeker, E. Reynders, G. De Roeck and J. Kullaa. Vibration-based structural health monitoring using output-only measurements under changing environment. *Mechanical systems and signal processing*, 22, 34-56, 2008.
- [4] G. Tondreau, A. Deraemaeker, and E. Papatheou. Experimental damage detection using modal filters on an aircraft wing. In *ProcEurodyn 2011*, Leuven, Belgium, July 2011.
- [5] G. Tondreau and A. Deraemaeker. Damage localization in bridges using multi-scale filters and large strain sensor networks, *ISMA2010*, Leuven, Belgium, September 2010.
- [6] E. Figueiredo, G. Park, J.Figueiras, C. Farrar and K. Worden. Appropriate model order selection for damage detection. *Computer-aided civil and infrastructure engineering*, 26 (3), 225-238, 2011.
- [7] E. Figueiredo, G. Park, J.Figueiras, C. Farrar and K. Worden. Structural health monitoring algorithm comparisons using standard data sets. Los Alamos National Laboratory Report: LA-14393, 2009.
- [8] D.C. Montgomery. *Statistical quality control: a modern introduction*. John Wiley and Sons, New York, 2009.
- [9] E. Reynders, M. Schevenels and G. De Roeck. Macec 3.2: a Matlab toolbox for experimental and operational modal analysis. Report BWM-2011-01, Department of Civil Engineering, K.U. Leuven, 2011.
- [10] H. Sohn, C. Farrar, N. Hunter, and K. Worden. Applying the LANL Statistical Pattern Recognition Paradigm for Structural Health Monitoring to Data from a Surface-effect Fast Patrol Boat. Los Alamos National Laboratory Report: LA-13761-MS, 2001.
- [11] P. Omenzetter and J.M. Brownjohn. Application of Time Series Analysis for Bridge Monitoring. *Smart Materials and Structures*, 15, 129-138, 2006.
- [12] E. Figueiredo, G. Park, C.R. Farrar, K. Worden, and J. Figueiras. Machine Learning Algorithms for Damage Detection under Operational and Environmental Variability. *International Journal of Structural Health Monitoring*, 10(6), 559-572, 2011.
- [13] E. Figueiredo, M.D. Todd, C.R. Farrar, E. Flynn. Autoregressive Modeling with State-space Embedding Vectors for Damage Detection under Operational and Environmental Variability. *International Journal of Engineering Science*, 48, 822-834, 2010.