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Application of modified Laplace decomposition method for solving boundary layer equation

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Abstract In this paper, we apply the modified Laplace decomposition method (MLDM) to obtain series solutions of the boundary layer equation. The technique is based on the application of Laplace transform to boundary layers in fluid mechanics. The nonlinear terms can be easily handled by the use of Adomian polynomials. The obtained series solution is combined with the diagonal Padé approximants to handle the boundary condition at infinity. Comparison of the present solution is made with the existing solution and excellent agreement is noted.

1. Introduction

The study of laminar boundary layer flow of an incompressible fluid has several important engineering applications such as the aerodynamic extrusion of plastic sheets, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along liquid film condensation process, glass and polymer industries. The pioneering work in this area was done by Sakiadis (1961a,b). Since then, the characteristics of momentum and energy transfer over a flat plate have been extensively studied, both numerically and analytically, and similarity solutions for a large variety of boundary conditions were proposed, like for stretching walls (Crane, 1970; Banks, 1983; Magyari and Keller, 2000; Kuiken, 1981), porous media (see Ingham and Pop, 2002; Nield and Bejan, 1999), permeable surfaces (Magyari and Keller, 2000; Magyari et al., 2003), etc. Although the numerical approach allows studying more complex boundary conditions, the importance of analytical solutions is undeniable and it is witnessed by the large amount of work performed, particularly in recent years, on this subject. Various kinds of solutions methods (Wazwaz, 1997, 2006; Xu, 2007; Noor and Mohyud-Din, 2009; He, 2006, 2003; Abbasbandy, 2007; Rashidi, 2009; Ganji et al., 2009; Fathizadeh and Rashidi, 2009) were used to handle the boundary layer problem. One of those solution methods is the Laplace decomposition method was proposed by Khuri (2001) and developed by Yusufoglu (2006). A reliable modification of the Laplace decomposition algorithm has been done by Khan (2009). Furthermore, the Laplace transformation method was also combined with the well-known homotopy perturbation method (Madani and Fathizadeh, 2010; Mohyud-Din et al., 2010; Yildirim, 2010), the variational iteration method (Hesameddini and Latifzadeh, 2009; He et al., 2010) and the Adomian

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decomposition method (Khan and Faraz, 2010; Islam et al., 2010; Wazwaz, 2010) to produce a highly effective technique for handling many nonlinear problems.

This technique basically illustrates how the Laplace transform can be used to approximate the solutions of the nonlinear differential equations by manipulating the decomposition method which was first introduced by Adomian (1994). The method is very well suited to physical problems since it does not require unnecessary linearization, perturbation and other restrictive methods and assumptions which may change the problem being solved, sometimes seriously. The basic motivation of the present study is to extend our previous approach proposed in Khan (2009) to solve boundary layer problem on semi-infinite domain. The modified Laplace decomposition method is much easier to implement as compared with the Adomian decomposition method where huge complexities are involved. To the best of authors knowledge no attempt has been made to exploit this method to solve boundary layer equation on semi-infinite domain. Also our aim in this article is to compare the results with solutions to the existing ones (Wazwaz, 2006; Xu, 2007; Noor and Mohyud-Din, 2009).

2. Modified Laplace decomposition method

In order to elucidate the solution procedure of the modified Laplace decomposition method, we consider the following general form of 3rd order non-homogeneous nonlinear ordinary differential equation with initial conditions is given by

$$f''' + b_1(x)f'' + b_2(x)f' + b_3(x)f = g(y), \tag{1}$$

$$f(0) = \alpha,\ f'(0) = \beta,\ f''(0) = \gamma, \tag{2}$$

According to Laplace decomposition method (Khuri, 2001; Yusufoglu, 2006), we apply Laplace transform (denoted throughout this paper by L) on both sides of Eq. (1):

$$\begin{aligned} s^3L[f] - s^2\alpha - s\beta - \gamma + L[b_1(x)f''] + L[b_2(x)f'] + L[b_3(x)f] \\ = L[g(y)], \end{aligned} \tag{3}$$

Using the differentiation property of Laplace transform, we have

$$L[f] = \frac{\alpha}{s} + \frac{\beta}{s^2} + \frac{\gamma}{s^3} + \frac{1}{s^3}L[g(y)] - \frac{1}{s^3}L[b_1(x)f'' + b_2(x)f' + b_3(x)f]. \tag{4}$$

The Laplace decomposition method (Khuri, 2001; Yusufoglu, 2006) admits a solution in the form

$$f = \sum_{m=0}^{\infty} f_m. \tag{5}$$

The nonlinear term is decomposed as

$$g(y) = \sum_{m=0}^{\infty} A_m, \tag{6}$$

where A_m are Adomian polynomials of $f_0, f_1, f_2, f_3, \ldots, f_n$ and it can be calculated by the following formula

$$A_m = \frac{1}{n!}\frac{d^n}{d\lambda^m}\left[N\left(\sum_{i=0}^{\infty}\lambda^i f_i\right)\right]_{\lambda=0}, \quad m = 0, 1, 2, 3 \ldots. \tag{7}$$

Using Eqs. (5) and (6) in Eq. (4) we get

$$\begin{aligned} L\left[\sum_{m=0}^{\infty} f_m\right] &= \frac{\alpha}{s} + \frac{\beta}{s^2} + \frac{\gamma}{s^3} + \frac{1}{s^3}L\left[\sum_{m=0}^{\infty} A_m\right] \\ &\quad - \frac{1}{s^3}L\left[b_1(x)\sum_{m=0}^{\infty} f''_m + b_2(x)\sum_{m=0}^{\infty} f'_m + b_3(x)\sum_{m=0}^{\infty} f_m\right], \end{aligned} \tag{8}$$

Matching both sides of Eq. (8), we have the following relation;

$$L[f_0] = \frac{\alpha}{s} + \frac{\beta}{s^2} + \frac{\gamma}{s^3}, \tag{9}$$

$$\begin{aligned} L[f_1] &= \frac{1}{s^3}L[A_0] - \frac{1}{s^3}L[b_1(x)f''_0 + b_2(x)f'_0 + b_3(x)f_0], \\ L[f_2] &= \frac{1}{s^3}L[A_1] - \frac{1}{s^3}L[b_1(x)f''_1 + b_2(x)f'_1 + b_3(x)f_1]. \end{aligned} \tag{10}$$

In general the recursive relation is given by

$$L[f_{m+1}] = \frac{1}{s^3}L[A_m] - \frac{1}{s^3}L[b_1(x)f''_m + b_2(x)f'_m + b_3(x)f_m], \quad m \geqslant 0. \tag{11}$$

Taking the inverse Laplace transform from both sides of Eqs. (9)–(11), one obtains

$$f_0(x) = H(x), \tag{12}$$

$$\begin{aligned} f_{m+1}(x) = L^{-1}\left[\frac{1}{s^3}L[A_m] - \frac{1}{s^3}L\left[b_1(x)f''_m + b_2(x)f'_m\right.\right. \\ \left.\left. + b_3(x)f_m\right]\right], \quad m \geqslant 0. \end{aligned} \tag{13}$$

where $H(x)$ represents the term arising from source term and prescribe initial condition. The modified Laplace decomposition method (Khan, 2009) suggests that the function $H(x)$ defined above in (12) be decomposed into two parts, namely $H_0(x)$ and $H_1(x)$. Such that

$$H(x) = H_0(x) + H_1(x). \tag{14}$$

The initial solution is important, and the choice of Eq. (12) as the initial solution always leads to noise oscillation during the iteration procedure. Instead of the iteration procedure, Eqs. (12) and (13), we suggest the following modification

$$\begin{aligned} f_0(x) &= H_0(x), \\ f_1(x) &= H_1(x) + L^{-1}\left[\tfrac{1}{s^3}L[A_0] - \tfrac{1}{s^3}L[b_1(x)f''_0\right. \\ &\qquad \left. + b_2(x)f'_0 + b_3(x)f_0]\right], \\ f_{m+1}(x) &= L^{-1}\left[\tfrac{1}{s^3}L[A_m] - \tfrac{1}{s^3}L[b_1(x)f''_m + b_2(x)f'_m\right. \\ &\qquad \left. + b_3(x)f_m]\right], \quad m \geqslant 1. \end{aligned} \tag{15}$$

The solution through the modified Laplace decomposition method highly depends upon the choice of $H_0(x)$ and $H_1(x)$. We will show how to suitably choose $H_0(x)$ and $H_1(x)$ by example.

3. Series solution for the boundary layer equation

In this section, we apply the modified Laplace decomposition method (MLDM) for solving boundary layer problem in an infinite domain. Let us consider the following nonlinear third order boundary layer problem which appears mostly in the mathematical modeling of physical phenomena in fluid mechanics (Wazwaz, 2006; Xu, 2007; Noor and Mohyud-Din, 2009)

$$f''' + (n-1)ff'' - 2n(f')^2 = 0, \quad n > 0,$$
$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) = 0, \tag{16}$$

where $f''(0) = \alpha < 0$, will be examined in this work. By applying the aforesaid method subject to the initial conditions, we have

$$L[f] = \frac{s+\alpha}{s^3} + \frac{1}{s^3}L[2n(f')^2 - (n-1)ff''], \tag{17}$$

The inverse of Laplace transform implies that

$$f(\eta) = \eta + \frac{\alpha\eta^2}{2} + L^{-1}\left[\frac{1}{s^3}L[2n(f')^2 - (n-1)ff'']\right], \tag{18}$$

Following the technique, if we assume an infinite series solution of the form (5) we obtain

$$\sum_{m=0}^{\infty} f_m(\eta) = \eta + \frac{\alpha\eta^2}{2} + L^{-1}\left[\frac{1}{s^3}L\left[2n\left(\sum_{m=0}^{\infty} A_m(\eta)\right) - (n-1)\left(\sum_{m=0}^{\infty} B_m(\eta)\right)\right]\right], \tag{19}$$

In the above equation $A_m(\eta)$ and $B_m(\eta)$ are the Adomian polynomials (Adomian, 1994) that represent the nonlinear terms. So Adomian polynomials are given below

$$\sum_{m=0}^{\infty} A_m(\eta) = (f')^2(\eta), \tag{20}$$

The few components of the Adomian polynomials are given as follow

$$\begin{aligned}
A_0(\eta) &= (f_0')^2(\eta),\\
A_1(\eta) &= 2f_0'(\eta)f_1'(\eta),\\
A_2(\eta) &= (f_1')^2(\eta) + 2f_0'(\eta)f_2'(\eta),\\
A_3(\eta) &= 2f_1'(\eta)f_2'(\eta) + 2f_0'(\eta)f_3'(\eta),\\
A_4(\eta) &= (f_2')^2(\eta) + 2f_1'(\eta)f_3'(\eta) + 2f_0'(\eta)f_4'(\eta),\\
A_5(\eta) &= 2f_0'(\eta)f_5'(\eta) + 2f_1'(\eta)f_4'(\eta) + 2f_2'(\eta)f_3'(\eta),\\
&\vdots\\
A_m(\eta) &= \sum_{i=0}^{m} f_i'(\eta)f_{m-i}'(\eta).
\end{aligned} \tag{21}$$

And for $B_m(\eta)$ we find

$$\sum_{m=0}^{\infty} B_m(\eta) = f(\eta)f''(\eta), \tag{22}$$

$$\begin{aligned}
B_0(\eta) &= f_0(\eta)f_0''(\eta),\\
B_1(\eta) &= f_0(\eta)f_1''(\eta) + f_1(\eta)f_0''(\eta),\\
B_2(\eta) &= f_0(\eta)f_2''(\eta) + f_1(\eta)f_1''(\eta) + f_2(\eta)f_0''(\eta),\\
B_3(\eta) &= f_0(\eta)f_3''(\eta) + f_1(\eta)f_2''(\eta) + f_2(\eta)f_1''(\eta) + f_3(\eta)f_0''(\eta),\\
B_4(\eta) &= f_0(\eta)f_4''(\eta) + f_1(\eta)f_3''(\eta) + f_2(\eta)f_2''(\eta) + f_3(\eta)f_1''(\eta)\\
&\quad + f_4(\eta)f_0''(\eta),\\
B_5(\eta) &= f_0(\eta)f_5''(\eta) + f_1(\eta)f_4''(\eta) + f_2(\eta)f_3''(\eta) + f_3(\eta)f_2''(\eta)\\
&\quad + f_4(\eta)f_1''(\eta) + f_4(\eta)f_1''(\eta),\\
&\vdots\\
B_m(\eta) &= \sum_{i=0}^{m} f_i(\eta)f_{m-i}''(\eta).
\end{aligned} \tag{23}$$

Through the modified Laplace decomposition method (Khan, 2009) the function $H(\eta)$ can be written as

$$H(\eta) = \eta + \frac{\alpha\eta^2}{2} = f_0(\eta) + f_1(\eta), \tag{24}$$

By this consideration, we first set modified recursive relations in the form

$$\begin{aligned}
f_0(\eta) &= \eta,\\
f_1(\eta) &= \tfrac{\alpha\eta^2}{2} + L^{-1}\left[\tfrac{1}{s^3}L[2n(A_0) - (n-1)(B_0)]\right],\\
f_{m+1}(\eta) &= L^{-1}\left[\tfrac{1}{s^3}L[2nA_m(\eta) - (n-1)B_m(\eta)]\right], \quad m \geqslant 1
\end{aligned} \tag{25}$$

Writing $f_0(\eta) = \eta$ in Eq. (25), the other components are

$$\begin{aligned}
f_1(\eta) &= \frac{\alpha\eta^2}{2} + \frac{n\eta^3}{3},\\
f_2(\eta) &= \frac{\alpha(3n+1)\eta^4}{24} + \frac{n(n+1)\eta^5}{30},\\
f_3(\eta) &= \frac{\alpha^2(3n+1)\eta^5}{120} + \frac{\alpha(19n^2+18n+3)\eta^6}{720} + \frac{n(2n^2+2n+1)\eta^7}{315},\\
f_4(\eta) &= \frac{\alpha^2(27n^2+42n+11)\eta^7}{5040} + \frac{\alpha(167n^3+297n^2+161n+15)\eta^8}{40320}\\
&\quad + \frac{n(13n^3+38n^2+23n+6)\eta^9}{22680},
\end{aligned} \tag{26}$$

The series solution is given as

$$\begin{aligned}
f(\eta) &= \eta + \frac{\alpha\eta^2}{2} + \frac{n\eta^3}{3} + \frac{\alpha(3n+1)\eta^4}{24}\\
&\quad + \left(\frac{1}{30}n^2 + \frac{1}{40}n\alpha^2 + \frac{1}{120}\alpha^2 + \frac{1}{30}n\right)\eta^5\\
&\quad + \left(\frac{19}{720}n^2\alpha + \frac{1}{240}\alpha + \frac{1}{40}n\alpha\right)\eta^6\\
&\quad + \left(\frac{1}{120}n^2\alpha + \frac{1}{315}n + \frac{2}{315}n^3 + \frac{11}{5040}\alpha^2 + \frac{3}{560}n^2\alpha^2 + \frac{2}{315}n^2\right)\eta^7\\
&\quad + \left(\frac{11}{40320}\alpha^3 + \frac{33}{44880}n^2\alpha + \frac{3}{4480}\alpha^3 n^2 + \frac{23}{5760}n\alpha + \frac{1}{2688}\alpha\right.\\
&\quad \left. + \frac{167}{40320}n^3\alpha + \frac{1}{960}\alpha^3 n\right)\eta^8\\
&\quad + \left(\frac{1}{3780}n + \frac{527}{362880}n^3\alpha^2 + \frac{19}{11340}n^3 + \frac{709}{362880}n\alpha^2 + \frac{23}{8064}n^2\alpha^2\right.\\
&\quad \left. + \frac{23}{22680}n^2 + \frac{13}{22680}n^4 + \frac{43}{120960}\alpha^2\right)\eta^9 + \cdots
\end{aligned} \tag{27}$$

4. The Padé approximants

Our aim in this section is mainly concerned with the mathematical behaviour of the solution $f(\eta)$ in order to determine the value of free parameter $\alpha = f(0)$. It was formally shown by Wazwaz (2007), Boyd (1997) and Mohyud-Din et al. (2010) this goal can easily be achieved by forming the Padé approximants (Baker, 1975) which have the advantage of manipulating the polynomial approximation into a rational function to obtain the more information about $f(\eta)$. It is well-known fact that Padé approximants will converges on the entire real axis if $f(\eta)$ is free of singularities on the entire real axis. More importantly, the diagonal approximants are most accurate approximants, therefore we will construct on diagonal approximants. Using the boundary condition $f'(\infty) = 0$, the diagonal approximants $[M/M]$ vanish if the coefficients of numerator vanish with the highest power in the η. Choosing the coefficients of the highest power of η equal

Table 1 Comparison of the numerical value of $\alpha = f''(0)$ obtained by MLDM with MADM and MVIM.

n	Padé approximants	Present method	MADM (Wazwaz, 2006)	MVIM (Noor and Mohyud-Din, 2009)
0.2	[2/2]	−0.3872983347	−0.3872983347	−0.3872983347
	[3/3]	−0.3821533832	−0.3821533832	−0.3821533832
	[4/4]	−0.3819153845	−0.3819153845	−0.3819153845
	[5/5]	−0.3819148088	−0.3819148088	−0.3819148088
	[6/6]	−0.3819121854	−0.3819121854	−0.3819121854
0.3	[2/2]	−0.5773502692	−0.5773502692	−0.5773502692
	[3/3]	−0.5615999244	−0.5615999244	−0.5615999244
	[4/4]	−0.5614066588	−0.5614066588	−0.5614066588
	[5/5]	−0.5614481405	−0.5614481405	−0.5614481405
	[6/6]	−0.561441934	−0.561441934	−0.561441934
0.4	[2/2]	−0.6451506398	−0.6451506398	−0.6451506398
	[3/3]	−0.6397000575	−0.6397000575	−0.6397000575
	[4/4]	−0.6389732578	−0.6389732578	−0.6389732578
	[5/5]	−0.6389892681	−0.6389892681	−0.6389892681
	[6/6]	−0.6389734794	−0.6389734794	−0.6389734794
0.6	[2/2]	−0.8407967591	−0.8407967591	−0.8407967591
	[3/3]	−0.8393603021	−0.8393603021	−0.8393603021
	[4/4]	−0.8396060478	−0.8396060478	−0.8396060478
	[5/5]	−0.8395875381	−0.8395875381	−0.8395875381
	[6/6]	−0.8396056769	−0.8396056769	−0.8396056769
0.8	[2/2]	−1.007983207	−1.007983207	−1.007983207
	[3/3]	−1.007796981	−1.007796981	−1.007796981
	[4/4]	−1.007646828	−1.007646828	−1.007646828
	[5/5]	−1.007646828	−1.007646828	−1.007646828
	[6/6]	−1.007792100	−1.007792100	−1.007792100

Table 2 Comparison of the numerical value of $\alpha = f''(0)$ obtained by MLDM with MADM, MVIM and HPM.

n	Present method	MADM (Wazwaz, 2006)	HPM (Xu, 2007)	MVIM (Noor and Mohyud-Din, 2009)
4	−2.483954032	−2.483954032	−2.5568	−2.483954032
10	−4.026385103	−4.026385103	−4.0476	−4.026385103
100	−12.84334315	−12.84334315	−12.8501	−12.84334315
1000	−40.65538218	−40.65538218	−40.6556	−40.65538218
5000	−104.8420672	−104.8420672	−90.9127	−104.8420672

The above tables clearly reveal that present solution method namely MLDM shows excellent agreement with the existing solutions in the literature (Wazwaz, 2006; Xu, 2007; Noor and Mohyud-Din, 2009). This analysis shows that MLDM suits for boundary layer flow problems.

to zero, we get a polynomial equations in α which can be solved very easily by using the built in utilities in the most manipulation languages such as Maple and Mathematica.

5. Conclusion

The main aim of this work is to provide the series solution of the Boundary layer equation by using the modified Laplace decomposition method (MLDM). The new modification of Laplace decomposition method (LDM) is a powerful tool to search for solutions of various nonlinear problems. The method overcomes the difficulty in other methods because it is efficient. We derived the same results by combining the series, obtained by the modified Laplace decomposition method, with the diagonal Padé approximants. The convergence of MLDM is also shown in Tables 1 and 2. Comparison of the present solution is made with the existing solution (Wazwaz, 2006; Xu, 2007; Noor and Mohyud-Din, 2009). An excellent agreement between the present and existing solutions is achieved. The analysis given here further shows confidence on MLDM.

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