Application of modified Manning formula in the determination of vertical profile velocity in natural rivers

S. Song, B. Schmalz, J. X. Zhang, G. Li and N. Fohrer

ABSTRACT

Seldom studied before, the vertical profile velocity is indicative of the flood process and nutrient transportation process. In this paper, a substitution of cross section hydraulic radius with vertical depth was made to the Manning formula, which was then applied in the vertical profile velocity determination. Simultaneously, the determination accuracy and its relationship with hydraulic conditions were discussed, based on the 1050 vertical profiles sampled from 140 cross sections in flood and moderate level seasons. The observations show the following. (1) The modified Manning formula provides a simplified approach for vertical profile velocity determination with acceptable accuracy. (2) The fitting quality of the profile velocity from the middle region of the cross section and the flood season were higher than that from near the bank or the moderate level season. The coefficient of determination (R^2) of the regression for the moderate level season and the flood season were 0.55 and 0.58, while the Nash–Sutcliffe coefficients were 0.64 and 0.82, respectively. (3) Analysis of the determination error and the coefficient of variation showed a positive correlation with the river aspect ratio. This seems to suggest that the modified Manning formula tends to be more applicable in narrow and deep rivers. More measurements from rivers or channels with a high aspect ratio would be meaningful for future research.

Key words | horizontal averaged profile velocity determination, mean vertical profile depth, modified Manning empirical formula, Northern German lowland catchment

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INTRODUCTION

Velocity determination and distribution have been a recurring problem in recent hydraulic science. The Manning empirical formula is widely used in hydraulic engineering and surface hydrology to derive mean flow velocity of natural gravity-driven uniform rivers (Hauser 1995; Sedghi-Asl & Rahimi 2011; Seo *et al.* 2016). The Manning formula was first presented in 1775 and re-developed by other scientists in the following century (Gauckler 1867; Newbury 1995). The equation concisely expresses the relationship between channel mean velocity, bed roughness coefficient and channel geometry (Chow 1959; Gioia & Bombardelli 2001):

$$V = \frac{k}{n} * \mathrm{R}^{2/3} * \mathrm{S}^{1/2} \quad (1)$$

where V (m/s) is the averaged river velocity, n is the Manning's roughness coefficient, R (m) is the hydraulic radius of the cross section and S is the channel slope; k is a conversion coefficient, internationally accepted as 1 (SI units). The hydraulic radius is the ratio between the cross-sectional area and the wet perimeter. In U-shaped cross sections, it is commonly approximated by the width (W) along with the depth (D) (W*D/(W + 2D)). The accuracy and efficiency of the Manning formula has been demonstrated by empirical research in various water conditions from catchments worldwide (Grimaldi *et al.* 2010; Retsinis *et al.* 2013). Theoretical studies related to improving the Manning formula mainly focus on the precision of k for different research areas, calibration of the roughness coefficient,

doi: 10.2166/nh.2016.131

channel slope and modification of the formation for certain water environments, etc. (Ruf 1988; Yu & Lim 2003; Yu *et al.* 2009).

The velocity distribution attracts more attention these days, especially in hydrological process research (Chiu 1989; Marini *et al.* 2011; Fontana *et al.* 2013). Empirical models have been developed by earlier investigators for predicting the point velocity distribution within the whole cross section based on experimental and field data. Entropy theory has been widely used in velocity distribution, and its limitations analyzed (Marini *et al.* 2011; Bonakdari & Moazamnia 2014; Corato *et al.* 2014). Because the specific assumptions and hydraulic conditions during the experiment or field sampling vary widely, the equation derived from one cross section is not suitable for another cross section or under different water levels. The vertical distribution of the velocity has been systematically studied and distribution functions have been proposed under various hydraulic conditions (Bergstrom *et al.* 2001; Huai *et al.* 2009; Bowers *et al.* 2012). However, widely accepted lateral distribution formula for the flow vertical profile velocity have only rarely been studied (Wiberg & Smith 1991; Chen *et al.* 1999; Cheng & Gartner 2003; Hu *et al.* 2008).

Manning's equation reveals that the flow velocity is positively proportional to the river cross section hydraulic radius and channel slope, and inversely proportional to channel roughness. It is reasonable to expect a similar linkage in vertical profile velocity according to the gravity effect. The absence of hydraulic radius for each profile is undoubtedly the first problem in the application of the Manning formula in profile velocity determination. One possible solution suggested by literature is the substitution of R with profile depth (Deng *et al.* 2001; Wilkerson & McGahan 2005). Then the Manning formula was modified as follows,

$$V_P = \frac{1}{n} * \mathrm{D}^{2/3} * \mathrm{S}^{1/2} \tag{2}$$

where V_P (m/s) is the vertical profile velocity in the stream wise direction. The mentioned substitution has already been adopted for the reason of simplification even in cross section velocity determination (Kirby *et al.* 2005; Gill & Pugh 2009). The error caused by the substitution has been analyzed over the last decades (Yang & Wen 2007; Fu & Zhang 2008). The mean error caused by substitution was believed to be within ±2% at all tested cross sections.

In this paper, we present the application of the modified Manning empirical formula in river vertical profile velocity determination, with hydraulic data collected from a Northern Germany lowland catchment. The main aim was to test the practicality of determining the vertical profile velocity based on Manning roughness, vertical depth and river bed slope; to evaluate the influence of the water level and aspect ratio on the velocity determination; and finally to provide a new and simple approach to vertical velocity determination, especially for a network area without gauge data.

STUDY AREA AND DATA PROCESS

Study area

The Upper Stör catchment is located in the middle of Schleswig-Holstein, Northern Germany, with a river length of 25 km and a drainage area of 468 km^2 (Figure 1(a)). As part of the Northern Germany lowland area, the altitude of this area falls from 90 and 60 m in the western and eastern parts, respectively, to 2 m a.s.l. at the outlet. In most of the catchment the gradients are smaller than 1°, except the south western part, which has gradients of more than 3° (LAV S-H 1995). Due to the high latitude and low lying terrain, shallow groundwater tables and older glacial and glaciofluvial sediments cover the basin, and there are criss-cross ditches, drainage pipes and canalization. In addition, this area is controlled by a temperate maritime climate, with an annual precipitation of 831 mm/year and a mean annual temperature of 8.3 °C (Müller-Wohlfeil *et al.* 2000; Schmalz *et al.* 2008; Schmalz & Fohrer 2009). The combination of terrain and climate has promoted agricultural activity, and has resulted in a land use pattern dominated by arable land (48.1%), pasture (29.5%) and forest (9.1%). Ten river sections were selected according to the following criteria: a smooth and regular inner river wall; no flood regulation; no turbulence caused by large obstacles, dams or waterfalls. The attributes of the ten river sections are shown in Table 1.

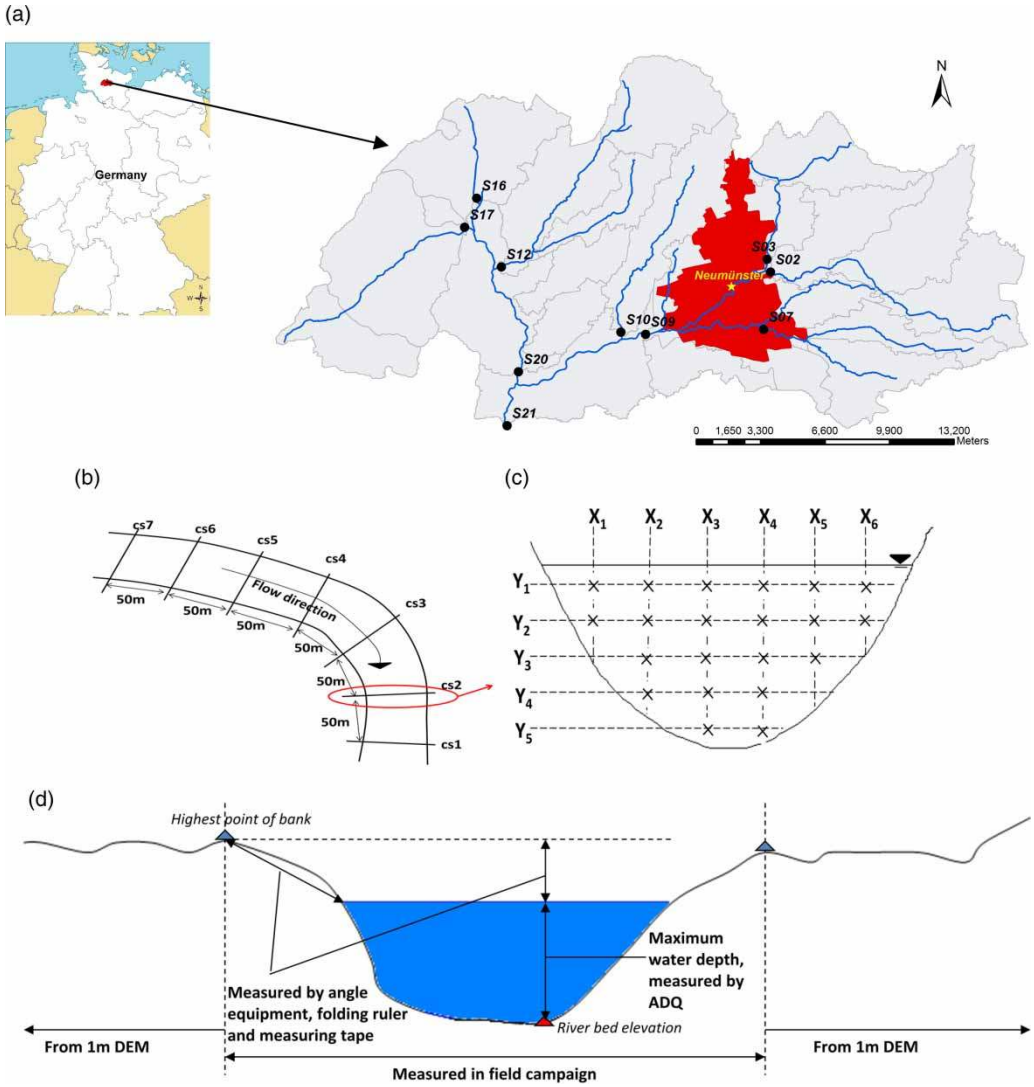

Figure 1 | Locations of the upper Stör catchment in Northern Germany (a), measurement of cross sections along the river section (b), measurement point distribution throughout the cross-sectional area (c), and (d) measurement of the river bed elevation.

Data collection and processing

Field campaign

Ten river sections were surveyed under a moderate water level period (September 2011), and surveys were then repeated at the same cross sections in the flood season (January 2012). This provided cross-sectional profiles of water surface width, depth, velocity, discharge and river bed elevation for seven cross sections evenly distributed along the 300 m river reach. At every cross section, five to ten vertical profiles evenly distributed along the river width were sampled. Each vertical profile was positioned 0.5–2 m apart depending on the river top width (Figure 1(b) and 1(c)). In addition, the roughness coefficient of each cross section was estimated during the field campaign. In total, 140 cross sections and 1050 vertical profiles were sampled.

River slope calculation

River gradient is an essential parameter for the application of the modified Manning formula in a river vertical profile.

Table 1 | The hydraulic information of the 10 selected river reaches (January 2012)

			Gradient (%)[a]		Width[b]		Depth[b]		Velocity[b]		Discharge[b]	
Site no.	River name	Catchment size (km^2)	M	H	(m)	CV	(m)	CV	(m/s)	CV	(m^3/s)	CV
S02	Schwale	70.09	0.21	0.15	7.29	0.85	0.88	0.41	0.44	0.24	2.85	0.53
S03	Dosenbek	30.98	0.20	0.28	4.84	0.79	0.51	0.53	0.43	0.19	1.05	0.49
S07	Stör-Luxemburg	33.29	0.09	0.06	7.27	0.46	0.53	0.34	0.33	0.15	1.23	0.38
S09	Stör-Padenstedt	196.05	0.43	0.59	9.14	0.44	1.12	0.42	0.79	0.26	7.90	0.29
S10	Aalbek	32.34	0.10	0.11	5.42	0.78	0.70	0.63	0.58	0.22	2.13	0.47
S12	Mitbek	60.63	0.25	0.32	5.49	0.92	0.81	0.51	0.47	0.28	2.03	0.51
S16	Fuhlenau	32.33	0.09	0.09	13.21	0.50	0.87	0.42	0.32	0.15	3.45	0.41
S17	Buckener Au	56.92	0.05	0.02	5.46	0.68	0.89	0.55	0.39	0.19	1.86	0.44
S20	Bücener	203.02	0.19	0.21	12.14	0.52	1.63	0.43	0.62	0.15	12.33	0.32
S21	Stör-Willenscharen	461.74	0.10	0.15	15.96	0.37	1.85	0.24	0.59	0.13	17.46	0.30

[a]The M column is the real measured river gradients, and the H column is the HEC-RAS calibrated gradients.
[b]The first column under width, depth, velocity and discharge were averaged values of seven cross sections surveyed in the field from the flood season. The second column is the CV of all the cross sections sampled from the same river in the flood season and moderate water level season.

Nowadays, a total station is normally used to collect large-scale land surface elevation data, but high-precision river gradient data is still unavailable (Huang *et al.* 2002). Dhondia & Stelling (2002) pointed out that the hydraulic model is a relatively reliable data source for river gradients under current technological constraints. In our study, the river bed elevation was derived from the vertical distance between the highest bank point and the lowest river bed point. The elevation of the highest bank point was extracted from the 1 m Digital Elevation Model (DEM), while the maximum water depth and vertical distance between the highest bank point and water surface were measured from the field (Figure 1(d)). The bed slope between cross sections was then calculated and calibrated in the hydraulic model.

Acoustic Doppler Qliner

The river cross-sectional and vertical-profile hydraulic information was gathered with an emerging device called the Acoustic Doppler Qliner (ADQ, OTT Company, Kempten/Germany). The ADQ is the latest acoustic device to determine water velocity, water depth and discharge in medium-size rivers with high accuracy (Song *et al.* 2012). It communicates via Bluetooth with a field computer (Personal Digital Assistant), where all the data are collected and stored (Figure 2(a)).

The measurement is carried out in classical vertical profiles across the river width. At each position, the ADQ automatically records the water depth and flow velocity of the cells from top to bottom. For velocity measurement, three ultrasonic transducers emit sound pulses (beams 1, 2, 3), which are then reflected by moving particles in the water column (plankton, branches, leaves, air bubbles etc.). According to the frequency shift between the transmitted and received pulses, the relative velocity between the instrument and the suspended material can be calculated (Figure 2(b) and 2(c)). River depth is calculated from the time difference between the emitted and reflected sound pulse 4, which is directed downward to the riverbed. This methodology provided a reliable database for our study.

Model calibration and validation

Hydrologic Engineering Centers River Analysis System (HEC-RAS, US Army Corps of Engineers) is a one-dimensional (1D) flow routing approach based on the St Venant/Shallow Water Equations (Horritt & Bates 2002; Pappenberger *et al.* 2005). It has been around in hydraulic design and engineering research for a few decades and is adopted in this study because of its well-performing theoretical basis (Horritt & Bates 2002; Drake *et al.* 2010; Morche *et al.* 2010). Based on the data collected in September 2011

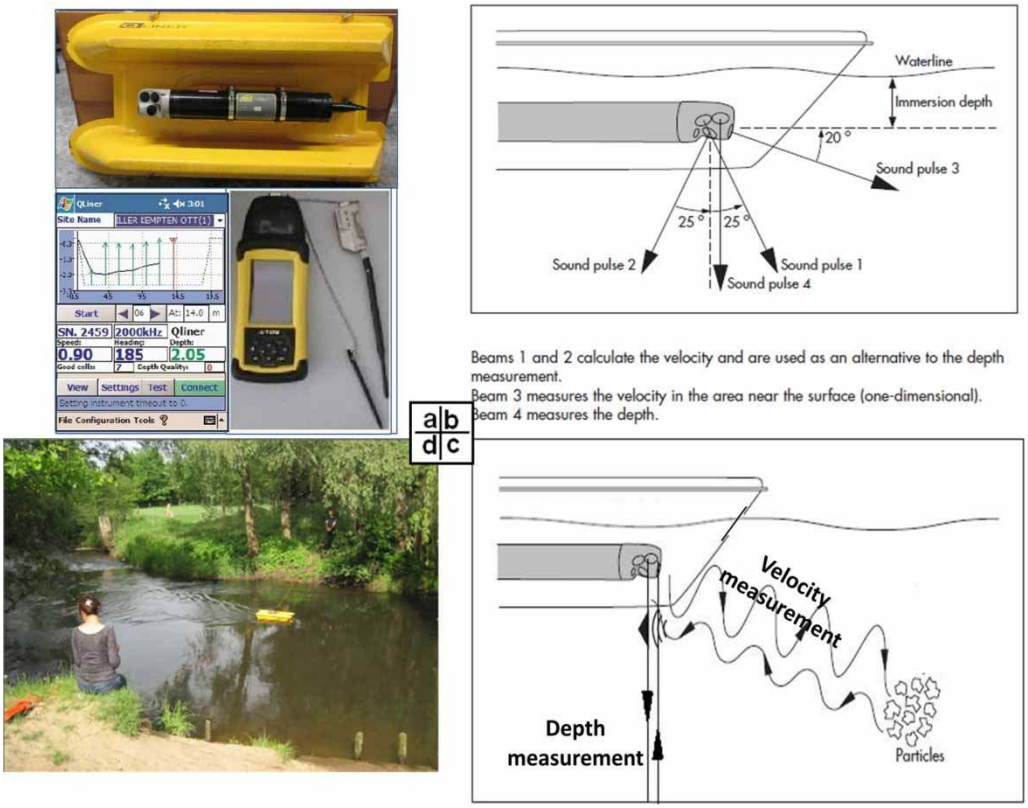

Figure 2 | Equipment and measurement principle of ADQ.

and 1 m DEM data, HEC-RAS models for ten selected river sections in the Stör catchment were set up and calibrated, and during this process a combination of roughness and bed gradient calibration was involved. Due to the difficulty in determining a representative Manning's n value, our main attention was focused on roughness calibration to achieve the minimum error between real measured and modeled water surface elevations, maximum depth, hydraulic depth and mean cross-sectional velocity. Finally, the averaged errors between measured and modeled data of all the models were within ±5% (Song *et al.* 2014). The dataset from January 2012 was used for model validation, and the validation quality is shown in Table 2. The HEC-RAS output and field surveyed data were positively correlated in the validation model. All the correlation coefficients were higher than 0.9. This indicated reliable performance of the steady model, and provided the basis for the application of the modified Manning formula.

With the calibrated and validated river slope and roughness, the modified Manning formula was applied to determine the vertical profile velocity. The synthetic vertical velocity was compared with the real measured data to assess the determination accuracy and uncertainty. The flow diagram of this study is shown in Figure 3.

RESULTS ANALYSIS

Regression of synthetic data against measured velocities of all the data

Based on the ADQ-sampled water depth, the empirical-calibrated roughness coefficient and the measured-calibrated river gradient for the 1050 vertical profiles from 140 cross sections, the synthetic mean vertical profile velocity (V_S) was calculated according to the improved Manning formula (2). Then linear regression analysis was made between V_S and the vertical profile velocity measured by ADQ (V_M), and the results are shown in Figure 4(a). Linear fit analysis for all the data showed regression coefficients of 1. The coefficient of determination (R^2) of the

Table 2 | Relative error of steady models

	Calibration				Validation			
Relative error (%)	**WSE**	**MD**	**HD**	**MCV**	**WSE**	**MD**	**HD**	**MCV**
cs7	0.31	3.37	1.01	3.75	4.43	8.10	5.44	10.20
cs6	6.78	5.14	1.18	1.66	7.24	4.27	1.38	8.90
cs5	1.71	1.50	1.15	1.65	2.31	5.81	4.48	1.86
cs4	2.04	2.51	0.27	1.58	4.24	3.04	2.90	2.02
cs3	8.59	0.70	1.72	0.55	5.98	8.39	7.58	1.67
cs2	1.70	0.37	0.44	0.32	3.47	0.82	2.70	2.33
cs1	0.87	1.02	1.35	0.57	3.10	1.54	5.37	1.64
Mean	3.14	2.09	1.02	1.44	4.39	4.57	4.26	4.09
Error	1.92				4.33			

WSE: water surface elevation; MD: maximum depth; HD: hydraulic depth; MCV: mean cross-sectional velocity.

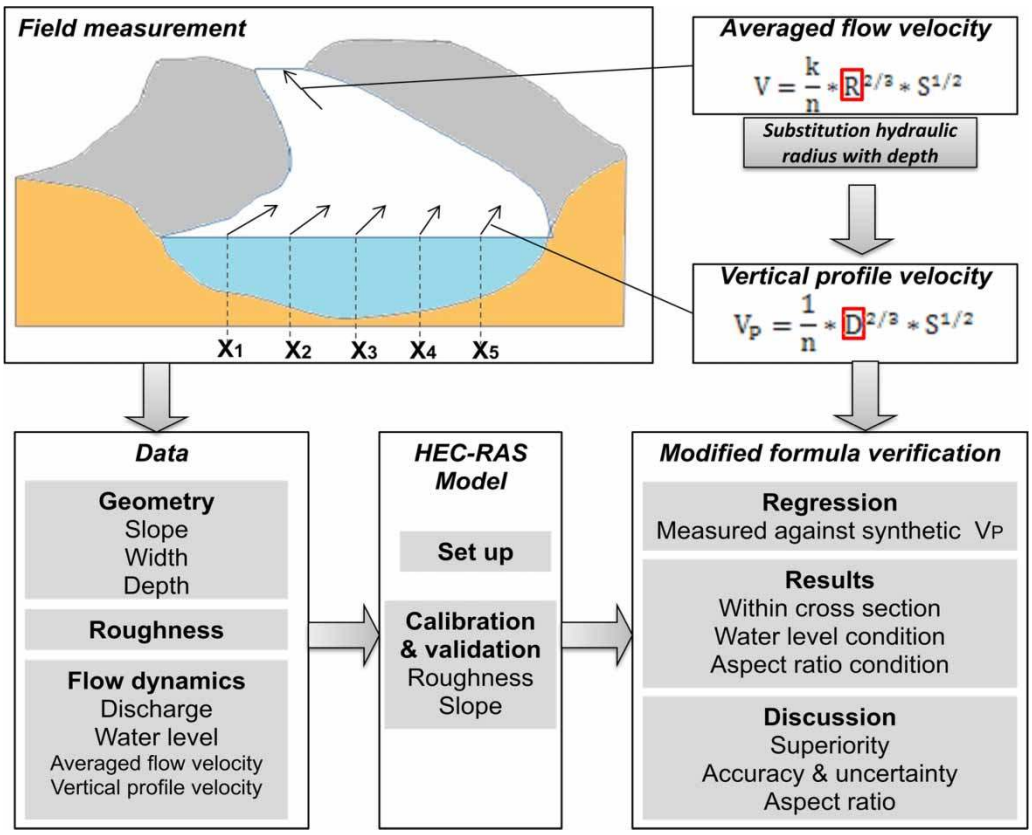

Figure 3 | The flow diagram of this study.

fitting was 0.58, while the Nash–Sutcliffe coefficient was 0.38. This indicated the directly proportional relationship between V_S and V_M, and showed the possibility of determining the vertical profile velocity with the improved Manning formula. Figure 4(b) and 4(c) display the regression of data from a moderate level season and flood season individually. The fitting quality with data from the flood season was higher than that from the moderate

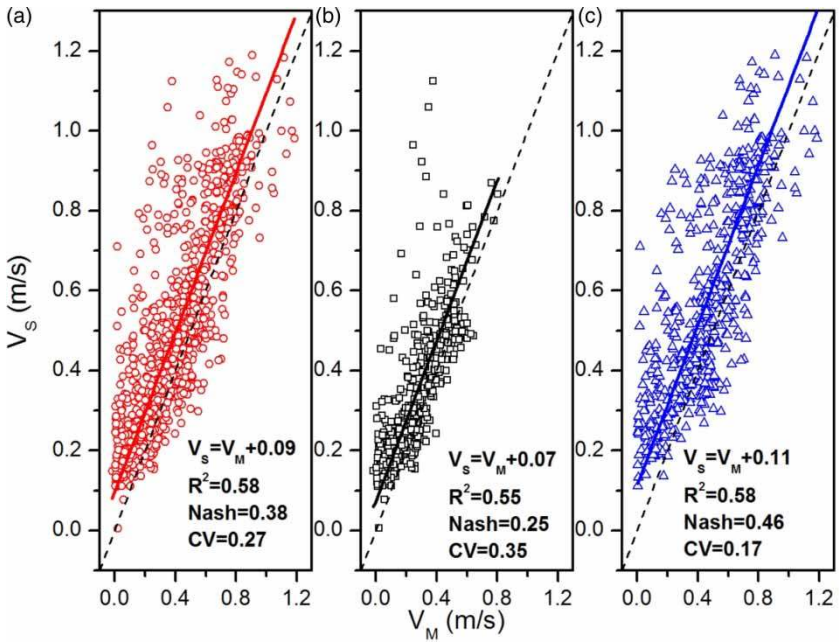

Figure 4 | Comparison of V_M (measured mean vertical velocity) and V_S (synthetic mean vertical velocity): (a) all the profiles; (b) profiles from moderate level season; (c) profiles from flood season. Solid lines are the linear fits of V_S and V_M, dashed lines are the ideal lines; Nash means the Nash–Sutcliffe coefficient; CV refers to the coefficient of the variation of the estimation error.

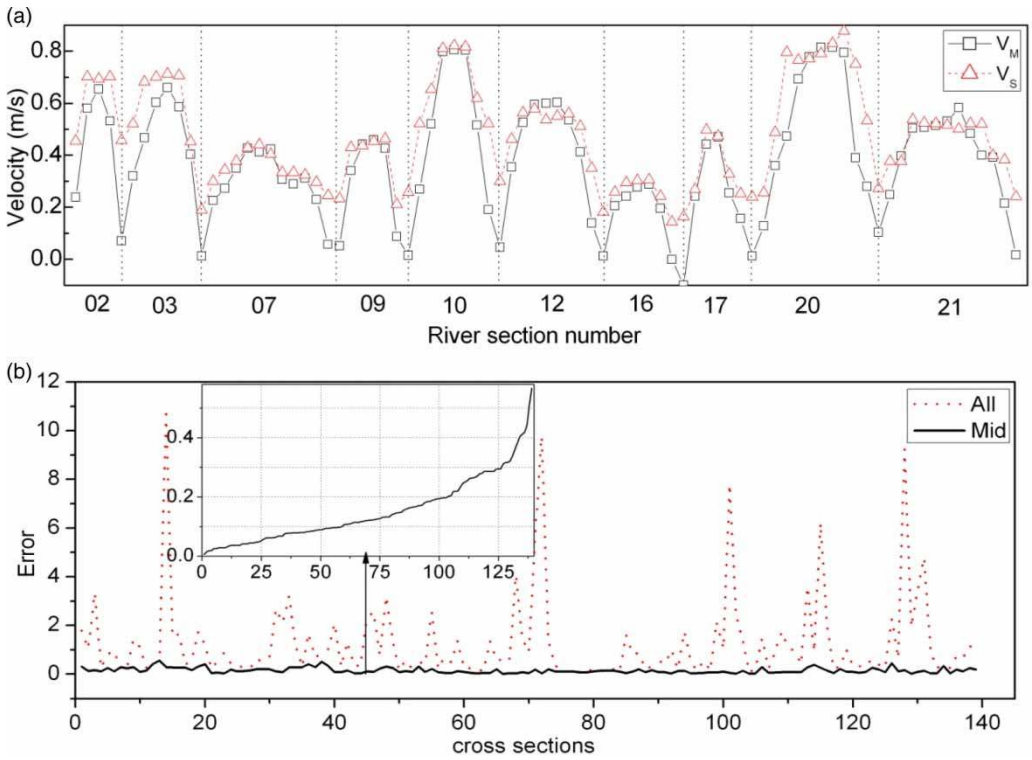

Figure 5 | Comparison of V_S and V_M of the whole cross section and the middle part of the cross section. (a) VS and VM of CS4 from each river section in flood season. Vertical dotted lines are the separation of river section. (b) The mean error of V_S to V_M. All: vertical profiles from the whole cross section, Mid: the middle three profiles of each cross section. Smaller plot: statistic plot of error of Mid.

level season. This seems to suggest better fit quality under higher water levels.

Analysis within the same cross section

Further analysis of V_S within the same cross section found that the V_S and V_M of the middle vertical profiles fitted each other better than those from the bank region. The V_S and V_M of cross section 4 (CS4) from all the river sections are displayed in Figure 5(a). There is a disparity between the measured and synthetic results from the bank region, with the synthetic results generally higher. The mean errors of V_S to V_M of all the vertical profiles from the same cross section (All) and that from the middle three profiles of each cross section (Mid) are shown in Figure 5(b). On average, the middle three profiles covered 38% of the middle part of the river width. After removal of the profiles near the bank, the averaged error dropped distinctly. Nearly 50% of Mid is below 0.1, while 20% of the Mid value falls into the 0.1–0.2 band, and only 10% of them are higher than 0.3 but lower than 0.6. This means the determination for the mean vertical profile velocity worked much better when we excluded the influence of the bank.

Regression of vertical profiles in the middle part of cross section

The middle three vertical profiles of every cross section were picked up and processed for the new linear regression. In total, 420 profiles from 140 cross sections were taken into account in this part. The results are shown in Figure 6. Regression coefficients of all three datasets are equal to 1, with R^2 of 0.82, 0.64 and 0.82 individually. The Nash–Sutcliffe coefficients were 0.48 for the data from the moderate level season, 0.62 for all the data, and reached 0.76 for the data from the flood season. Both the Nash–Sutcliffe coefficient and R^2 improved distinctly compared with those obtained with profiles from the bank region. This shows the good performance of the improved Manning formula (2) in the determination of vertical profile velocity in the middle part of the cross section.

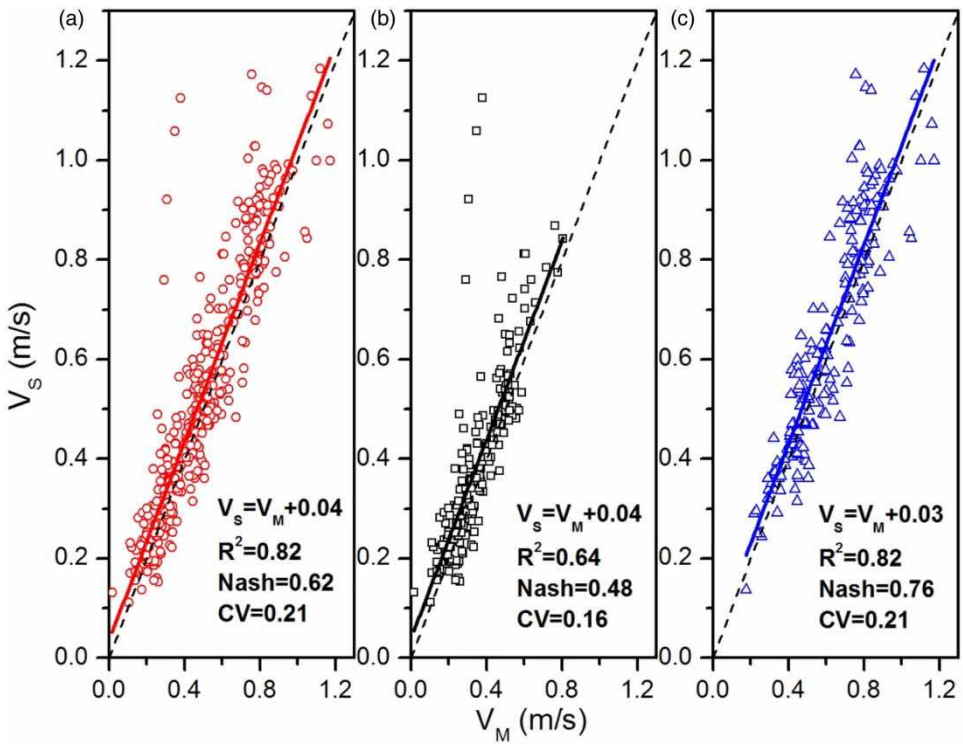

Figure 6 | Comparison of V_M (measured mean vertical velocity) and V_S (synthetic mean vertical velocity) of the middle vertical profiles: (a) all the profiles; (b) profiles from the moderate level season; (c) profiles from the flood season.

Table 3 | The correlation among river hydraulic parameters

	Catchment size (km^2/s)	Discharge (m^3/s)	Width (m)	Depth (m)	Aspect ratio (w/D)
Catchment size (km^2/s)	1				
Discharge (m^3/s)	0.986931	1			
Width (m)	0.851444	0.850256	1		
Depth (m)	0.90108	0.918353	0.771241	1	
Aspect ratio (w/D)	–0.26513	–0.29486	0.172885	–0.48114	1

Figure 6(b) and 6(c) reveal a similar phenomenon to Figure 3(b) and 3(c). The regression R^2 and Nash–Sutcliffe coefficient were higher for the data from deeper profiles. This strengthens the conclusion mentioned before – the improved Manning formula (2) works better under higher water depth.

Determination error, water depth and aspect ratio (w/d)

The velocity determination was affected by different hydraulic conditions. According to Table 3, the catchment size, discharge, water depth and width are correlated with each other, while the aspect ratio is independent from other parameters. Besides, the previous results showed higher determination quality in the flood season and in the middle part of the cross section. Therefore, we mainly focus on the effects of water depth and aspect ratio on the determination quality of the modified Manning formula.

Water depth

The error and water depth of each vertical profile was analyzed and the results are shown in Figure 7. The error of the synthetic velocity was inversely proportional to the increase in depth. It is identical to the results of accuracy analysis, which suggested that the accuracy of the velocity determination from the flood season and middle cross section is superior to that of the moderate level season or bank region. According to the statistic plot in Figure 7, about 60% of the simulated absolute relative errors were within 0.1, and 90% of them were below 0.35.

The cross-sectional absolute relative error and depth of the middle region from 140 cross sections were calculated and are shown in Figure 8. According to Figure 8(a), the mean relative cross-sectional error was inversely

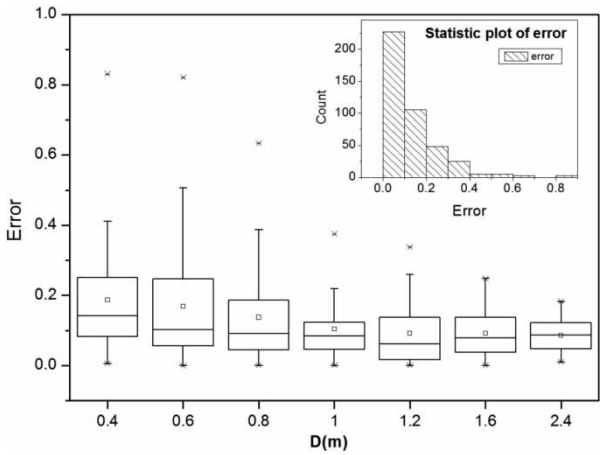

Figure 7 | The changing trend of relative error of profile velocity determination to the water depth.

proportional to the increase in water depth. As is shown in the distribution histogram of error, about 45% of absolute relative errors were below 0.1, and 35% of them fall within the range of 0.1–0.2. Compared to the profile scale, the accuracy of determination decreased slightly.

A similar trend existed between the CV of relative error and water depth, and the inverse proportional trend was even more regular and distinct (Figure 8(b)). This seems to suggest that when the water depth was greater, the determination errors of the profile velocities from the middle part of the cross sections were closer to each other and the determination quality was relatively higher. The distribution histogram exposed that nearly 50% of the CV were less than 0.1.

Aspect ratio

The averaged cross-sectional relative error and its CV under similar aspect ratio bands were calculated, and the results are shown in Figure 8(c) and 8(d). The plot revealed the

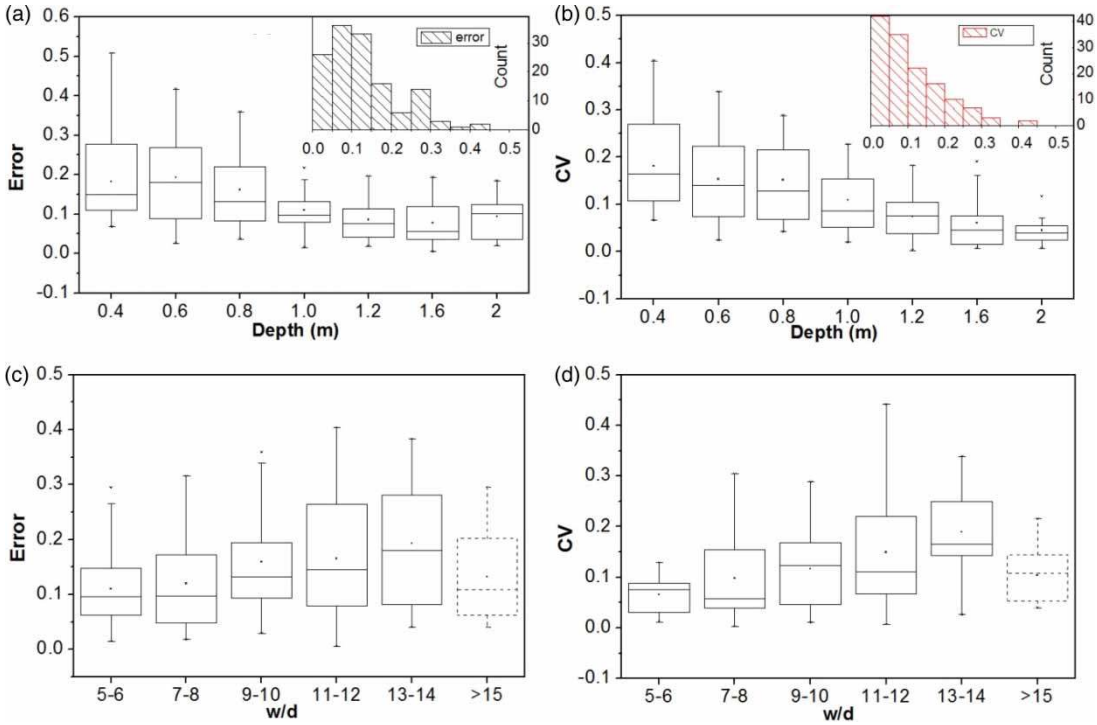

Figure 8 | The changing trend of mean relative cross-sectional error and CV of the water depth and aspect ratio (w/d). The small windows are the histogram statistic of error and CV individually.

positive proportion between aspect ratio and the other two parameters when the aspect ratio was under 15. When the aspect ratio was higher than 15, the trend becomes uncertain. There were around ten cross sections with an aspect ratio higher than 15 and valued from 15 to 30 in our study. Due to the insufficient number and wide range of the samples, the analysis of the cross-sectional parameters with an aspect ratio higher than 15 failed to represent the real situation.

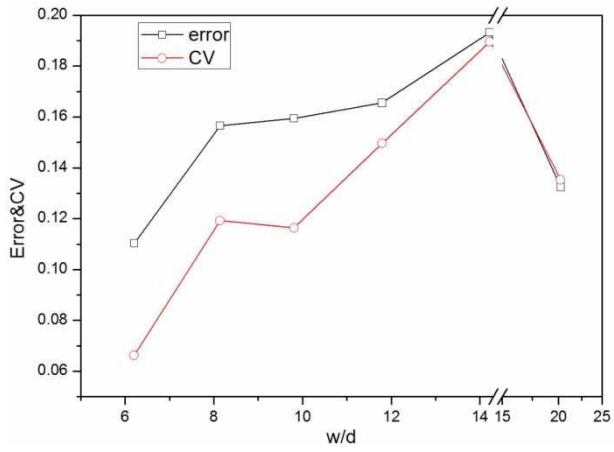

Figure 9 | Mean relative error and CV of different aspect ratio (w/d) bands.

Figure 9 reveals the reach averaged determination error and its CV under different aspect ratio conditions. When the aspect ratio was under 15, the error and its CV increased with the aspect ratio. Although the determination error and its CV from river reaches with higher aspect ratios decreased, the insufficient amount of measurements leads to a need for further experiment in the future.

DISCUSSION

The advantage of the modified formula

The method proposed by Shiono and Knight (SKM) provides an analytical solution, and has been most widely adopted for the vertical profile velocity and its lateral distribution determination (Tang & Knight 2008; Liu *et al.* 2013; Choi & Lee 2015). Along slightly different lines to the SKM, many vertical profile

velocity models have been developed. Ervine *et al.* (2000) improved geometry and roughness boundaries (Ervine *et al.* 2000). Castanedo *et al.* (2005) introduced the λ-method, including three different forms of expression for the lateral turbulent shear stress (Castanedo *et al.* 2005). All these models involve parameters such as water density, gravitational acceleration, flow depth, bed slope in the stream wise direction, bed slope in the lateral direction, lateral eddy viscosity, bed shear stress, geometric factors and secondary flow effects, etc. The accuracy of these methods is considered to be high, but the applicability was limited due to the inaccessibility of most parameters, especially the bed slope in the lateral direction, lateral eddy viscosity, bed shear stress and the secondary flow.

The modified formula proposed in this paper adopted three parameters: the Manning coefficient, water depth and river bed slope. All three parameters are easier to measure compared to the parameters in SKM, which simplified the calculation process significantly. Therefore, the main advantage of this formula is the distinct reduction of data collection and computation effort with acceptable accuracy. Although the synthetic results near the bank showed higher error, this method is still quite significant for basic hydrology research due to the low velocity and discharge portion near the bank.

Accuracy and uncertainty

Results analysis indicated that the modified Manning formula is applicable for the determination of vertical profile velocity, especially in the middle part of the river. Data from the flood season show a better regression quality. Further analysis for vertical data from the middle region of each cross section revealed that the removal of data near the river bank will largely improve the conformity of V_S and V_M. Five factors may be responsible for this phenomenon:

1. The relatively low accuracy of measured velocity near the river bank sampled by ADQ. Former measurements proved that the distances from the bank are negatively correlated with the mean vertical profile velocity error, and the repeatability of measurement is lower near the river bank (Song *et al.* 2012). The relative error of the ADQ velocity measurement near the river bank was around 0.2, while the relative error of the synthetic river bank velocity was 0.5 or even higher. Due to the uncertainty of the measurement for the verticals near the bank, the determination error of the modified formula were with higher uncertainty.
2. The difficulty of roughness coefficient determination. Roughness was estimated during the field campaign according to the empirical table and then calibrated with the HEC-RAS model. Due to the vegetation and debris on the intertidal area and its inconsistency during the moderate level season and flood season, it is a challenge to find the proper Manning coefficient for each cross section in different seasons. In addition, the Manning roughness varied for the vertical profiles near the river bank and the vertical profiles in the middle part of the cross section. One constant roughness for the whole river section and the calibration of the roughness value together with the calibration of the river gradient could cause more uncertainty in the results.
3. The inaccuracy of using the unique gradient through the whole cross section. The gradient changed from profile to profile even at the same cross section, especially near the river bank, because of the irregular river geometry. However, the 1D HEC-RAS model treats the river bed along river sections as a single curve and gives an overall gradient for every cross section.
4. Model error. The output of calibrated models for the ten river sections under research had errors within ±10% for all parameters, including velocity, maximum depth and hydraulic depth. These are all factors that can affect gradients and Manning roughness calibration.
5. The influence of the secondary flow. One hypothesis made in this study was the laminar flow of the research flow section. However, this was only a simplified ideal model for the field conditions. The longitudinal secondary flow developed due to the anisotropic turbulence in corner regions or near the water surface (Knight *et al.* 2007). This indicated the higher disturbance from secondary flow near the river bank region and in the dry season, and resulted in better applicability of the modified Manning formula in the middle part of the cross section and in the flood season. The number, strength and position of the secondary flow cells was considered to be one of the

main factors disturbing the lateral vertical profile velocity distribution (Tang & Knight 2015). Leaving out the effects of the secondary flow would lead to some uncertainty.

The effect of aspect ratio

The hydraulic radius is defined as the ratio of the cross section area to the wet perimeter (R = W*D/P). In the rectangular or trapezoid-shaped cross sections where the width is much larger than the depth, the wet perimeter is closer to the river width, while the hydraulic radius is closer to the depth. This suggests better interchangeability between the hydraulic radius and water depth in rivers with a large aspect ratio. When researchers estimated mean river velocity with river depth instead of hydraulic radius with the Manning formula in the laboratory, the error of determination was inversely proportional to the aspect ratio (Yang & Wen 2007; Jia *et al.* 2010). Further research on the rectangular river cross section clearly demonstrated that when the aspect ratio is higher than 100, the error caused by substitution of the hydraulic radius with depth was no more than 2%, but when the aspect ratio is around 20, the error was as high as 7%, and it was even higher under a lower aspect ratio. In a triangular or U-shaped cross section, the determination accuracy is slightly higher than the rectangular cross section under the same aspect ratio conditions (Yang & Wen 2007).

However, in our study, a positively proportional relationship between determination uncertainty and the aspect ratio was found. One possible reason might be the lateral shear stress, which is related to the aspect ratio. According to research involving 14 rivers with an aspect ratio ranging from 5 to 25, the influence of lateral shear stress decreased with the increase of aspect ratio (McGahey *et al.* 2006). In wide, shallow channels, the side walls influenced the channel centre less and the flow was mainly dominated by bed-generated turbulence, while in a narrow deep river the side walls had a stronger influence with greater lateral shearing. The higher disturbance from the bed-generated turbulence in wide shallow rivers would require more detailed hydraulic information near the measured vertical, in addition to depth and Manning roughness. This might corroborate the better applicability of the modified Manning formula in narrow, deep rivers.

CONCLUSIONS

Based on data sampled from field campaigns during the moderate water level season and flood season in ten subcatchments, the application of the modified Manning formula (2) in determining the river vertical profile velocity was analyzed in this paper. Determination accuracy was studied using the linear regression method. The influence of the water depth and aspect ratio on the relative error and coefficient of variation (CV) of the error were explored. The observations in this paper led to the following conclusions:

1. Regression analysis between V_S and V_M showed that the modified Manning formula provides a valid and effective method for vertical profile velocity determination, especially in the middle part of the cross section or during the flood season.
2. Determination error and CV were negatively proportional to the river depth, and were positively proportional to the aspect ratio.
3. The modified formula tends to be more applicable in relatively narrow and deep rivers. More measurements need to be collected to verify the accuracy in rivers with an aspect ratio higher than 15.

The applicability and uncertainty study of the replacement of the hydraulic radius with real water depth in determining the vertical profile velocity with a modified Manning formula is worth further research to reveal the effect of hydraulic conditions on lateral flow dynamics. More application of the modified Manning formula in catchments with various hydrological conditions, especially under high aspect ratio conditions, would be a good resource for future research and would provide more knowledge related to concrete situations.

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First received 21 June 2015; accepted in revised form 7 February 2016. Available online 3 March 2016