# Application of Network DEA Model to Vertically Integrated Electric Utilities

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## Abstract

Liberalization of the electric power industry has been underway in various countries. Under this situation, it is necessary for electric power companies to improve their management efficiency in order to survive in a competitive market. This study examines management efficiency of vertically integrated electric companies, which consist of several divisions, such as generation, transmission, distribution and so forth. Previous studies mainly focused on efficiency of only a specific division independently. However, we should evaluate firm-level management efficiency based on divisional efficiencies, because these divisions are closely linked each other. For measuring firm-level efficiency, we apply a network DEA model that considers a streamlined relationship among divisions under vertical integration. Furthermore we newly proposed a constrained network DEA model to obtain more practical efficiency index.

Keywords: Network DEA, electric utilities, vertical integration

## 1 Introduction

Deregulation of the electric power industry commenced in the early 1990s in several countries in Europe, e.g. the UK and Norway, and subsequently many countries and regions worldwide followed suit. In order to survive in the new competitive market, efficient management is essential for incumbent electric power companies that have been regulated for a long period of time by the authorities. At the same time, it is also important to adopt appropriate methods for measuring and evaluating management efficiency.

There are several functions (activities) in electric industry in order to produce and provide electricity to customers, like generation, transmission, distribution, retail sales and so forth. Since the technology used in these activities are different each other, there exist function-specific companies, e.g. generation companies and transmission companies. On the other hand, a vertical integrated electric power company, which consists of all of these activities as divisions within one company, is also typical supplier in many countries, such as Japan, the U.S., and Germany. This study focuses on these vertically integrated companies.

The overall efficiency of such companies depends on their divisional efficiencies. Furthermore, it should be noted that these divisions are interdependent, and the efficiency of a certain division exerts an influence on those of the other divisions. Actually, it may happen that the excellent performance of a certain division can be achieved at the sacrifice of other divisions' performance. For instance, even if the generation division only performs very well, the other divisions, e.g. transmission and distribution divisions might perform worse than those of the other companies. From the firm-level viewpoint, such a vertically integrated company cannot be qualified as an efficient company. Thus, in order to evaluate companies of this type, we should focus on the overall firm-level efficiency resulting from well-balanced divisional efficiencies.

Since electric power has taken the role of a major source of energy, considerable numbers of authors have addressed efficiencies in this industry (see Jamasb and Pollitt

[2001]). These previous studies mainly focused on efficiency of only a specific division of vertically integrated companies or function-specific companies. On the other hand, studies that measured the firm-level management efficiency of vertically integrated electric companies were very limited, and most of them focused on a simple input-output correspondence, e.g. capital, labor and material as inputs, and generated power as an output. This implies these studies regarded the inner structure of this industry as a “black box.” Thus, for measuring firm-level efficiency, we employ a network DEA model to take into account the streamlined structure among divisions under vertical integration. Furthermore, in applying network DEA, this study develops the new constrained network DEA (CNDEA) model that explicitly restricts the intensity of connectivity among activities within a company in order to obtain more practical efficiency indices.

This study is organized as follows. In section 2, we review previous studies on management efficiency of electric utilities and the network DEA model, and then explain several network DEA models introduced in the previous studies and develop them to CNDEA model in the 3rd section. We apply this model to vertically integrated electric power companies in the U.S. as a numerical example in section 4, and compared the result with the traditional network DEA model. Section 5 concludes this study and mentions future extensions.

## 2 Literature review

Studies on management efficiency of electric power companies emerged primarily after the 1990s (Jamasb and Pollitt [2001] and Qassim *et al.* [2005]). These studies evaluated utilities in various countries and applied several alternative methods for the efficiency measurement such as Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA).

Most of the previous studies have focused on the efficiency of the specific functions, particularly the network function (transmission and distribution), e.g. Hjalmarsson and Veiderpass [1992], Førsund and Kittelsen [1998], and Hattori [2002]. On the other hand, only a few studies dealt with the firm-level efficiency of vertically integrated electric power companies.

Goto and Tsutsui [1998] examined the firm-level efficiency using DEA for major vertically integrated power companies in Japan and the U.S. from 1984 to 1993. The employed data were four inputs; nameplate generation capacity (MW) as proxy for total assets of firms, fuel used (kilo calories), number of employees, and purchased power (GWh: Giga Watt hour), and two outputs; sales to residential and sales to non-residential customers (GWh). This study did not consider efficiency of each division and assumed that transmission and distribution assets were roughly proportional to the asset of generation division. However, this assumption is not always valid.

Delmas and Tokat [2005] also examined the efficiency of vertically integrated companies in the U.S. and uncovered the relationship between efficiency scores and proportion of supplied electricity that was generated by their own power plants. This study considered divisional inputs such as generation, transmission, distribution, sales and administrative expenses. However, these inputs were treated as independent and the streamlined structure of vertical integration was not considered.

In our study, we employ a network DEA model that incorporates the streamlined structure into the traditional DEA model and measures the firm-level efficiency based on the closely linked divisional efficiencies.

The basic DEA model such as CCR reports an efficiency score based on input and output data. Basically, DMUs using small inputs and producing large outputs are judged as efficient in the DEA model. However, it is generally presumed that the mechanism between input and output is hidden in a “black box”. Färe and Grosskopf (F&G) [1996, 2000] are the first to introduce a network structure into DEA model to look into the technology structure hidden in the black box of the standard DEA model. The network DEA model assumes *k* nodes (activities) inside the black box. At each node, intermediate (and/or exogenous) inputs are used in order to produce intermediate (and/or final) outputs. These nodes are regarded as subsections in which different activities are performed.

The network DEA model is suitable to evaluate DMUs with complicated and hierarchical structures. In the F&G model, activities at nodes are linked by intermediate inputs/outputs; however, the intensity of connectivity among activities is not restricted. This implies that efficiency of subsections, i.e. divisional efficiency, is independent of each other. It can be pointed out that F&G approach is inadequate to take into account the closely linked and streamlined structure of vertically integrated utilities. Furthermore, this model may turn out to give a good score to DMUs with unbalanced efficiency performance, e.g. efficiency of only one division is extremely high and others are low as we explain later. For the sake of practical efficiency evaluation, we should give a good score only to well-balanced DMUs.

Another network DEA model proposed by Lewis and Sexton (L&S) [2004] has a multi-stage structure as an extension of the two-stage DEA model proposed in Sexton and Lewis [2003]. This study solves a DEA model for each node independently. For an output-oriented model, firstly a general DEA model is solved for the upstream node at the 1st stage to obtain the optimal solution of outputs. At the next stage, a part of (or all of) optimal outputs obtained at the upstream node are applied as intermediate inputs to the next node. After solving DEA models for all nodes in turn, a final optimal output is obtained at the last node. The firm-level efficiency score is measured as the final optimal output divided by an observed output. Similar to F&G, the intensity of connectivity among activities is not restricted in this model and divisional efficiencies are assumed to be independent of each other.

In our study, we modify the F&G network DEA model and propose a constrained network DEA (CNDEA), which explicitly restricts the intensity of connectivity among activities. This constraint has not been considered in previous studies. In our model, only DMUs that perform well in all divisions will be evaluated as more efficient, while companies with unbalanced divisional efficiency score worse.

# 3 A mathematical formulation

## 3.1 The traditional network DEA

A general DEA model such as output oriented CCR is formulated as (1).

[CCR-O]

$$\begin{aligned} & \eta^* = \max\ \eta \\ \text{s.t.}\quad & x_o = X\lambda + s_x^-, \\ & \eta y_o = Y\lambda - s_y^+, \\ & \lambda \geq 0, s_x^- \geq 0, s_y^+ \geq 0, \end{aligned} \tag{1}$$

where $X$ and $Y$ are $m\times n$ input and $r\times n$ output matrices for $n$ Decision Making Units (DMUs), respectively, and $x_o$ and $y_o$ are the $m\times 1$ input and $r\times 1$ output vectors of $DMU_o$ ($o = 1,\Lambda\ ,n$), respectively. $s_x^-$ and $s_y^+$ are $m\times 1$ and $r\times 1$ slack vectors for inputs and outputs. $\eta$ is a scalar variable indicating an efficiency score, and $\lambda$ is a $n\times 1$ vector to represent the intensity of reference DMUs. In this model, inputs are transformed into outputs within a hidden "black box."

The network DEA model introduced by F&G assumes $k$ nodes (activities) inside the black box. At each node, intermediate (and/or exogenous) inputs are used in order to produce intermediate (and/or final) outputs. In this model, the intensity vector $\lambda$ is assumed to be different among $k$ nodes ($\lambda_k$). It means there are $k$ frontiers in this model and the efficient DMUs on the frontiers are different from node to node.

Figure 1 explains a simple example of the network DEA model. In the general DEA framework, two exogenous inputs $x^1$ and $x^2$ are used to produce final outputs $y^2$, whereas the network DEA model accounts for intermediate outputs/inputs. This example supposes two subsections and an output of activity 1 ($y^1$) at node 1 is used as an intermediate input for activity 2 at node 2.

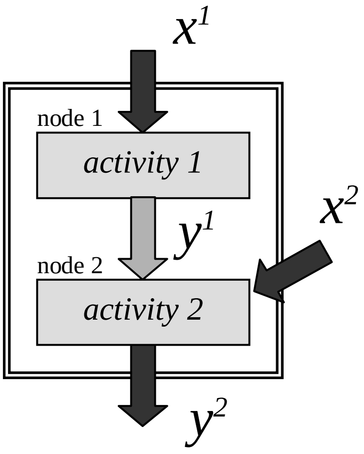


Figure 1: Simple example of a network DEA model

A DEA formulation for $DMU_o$ to measure output oriented technical efficiency of this example is shown in (2). In this model, $\lambda$s are different between the nodes 1 and 2.

[CCR-O-Network]

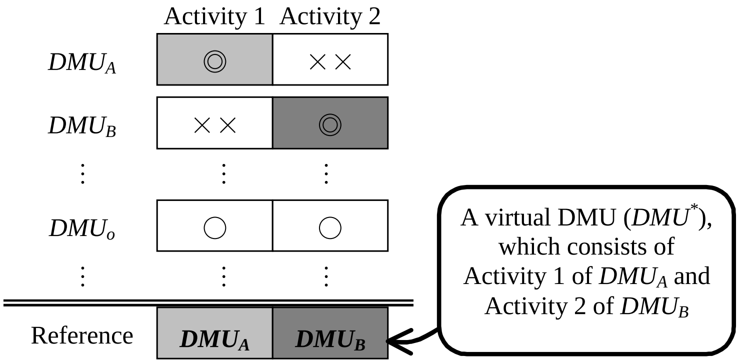
$$
\begin{array}{ll}
 & \eta^* = \max\ \eta \\
s.t. & x_o^1 = X^1\lambda^1 + s_x^{1-}, \quad \textit{(Exogenous Input at Node 1)} \\
 & \eta y^1 = Y^1\lambda^1 - s_y^{1+}, \quad \textit{(Output at Node 1)} \\
 & y^1 = Y^1\lambda^2 + s_y^{1-}, \quad \textit{(Intermediate Input at Node 2)} \\
 & x_o^2 = X^2\lambda^2 + s_x^{2-}, \quad \textit{(Exogenous Input at Node 2)} \\
 & \eta y^2 = Y^2\lambda^2 - s_y^{2+}, \quad \textit{(Output at Node 2)} \\
\multicolumn{2}{l}{\lambda^1,\ \lambda^2, s_x^{1-}, s_y^{1+}, s_y^{1-}, s_x^{2-}, s_y^{2+} \geq 0,}
\end{array}
\tag{2}
$$

where the number of superscript indicates the node number.

Similar to F&G, the multi-stage network DEA proposed by L&S assumes that the intensity vector $\lambda^k$ s are different among $k$ nodes, i.e. the reference group for $DMU_o$ is different by node.

## 3.2 Constrained network DEA model

In the models with different $\lambda$s as (2), the efficient DMUs could be very different among nodes. This means that the efficient DMUs at the overall firm level might be virtual ones. For instance, suppose that the performance of $DMU_A$ is superior to the other DMUs in the activity 1 at the expense of activity 2, while $DMU_B$ is superior in the activity 2 at the expense of the activity 1. They are, so to speak, unbalanced efficient DMUs. If we solve the different $\lambda$s network DEA model with them, $DMU_o$ will refer to $DMU_A$ for the activity 1, and $DMU_B$ for the activity 2. It implies that the firm-level efficient DMU in this model will be the virtual DMU ($DMU^*$) that consists of $DMU_A$ and $DMU_B$ for activities 1 and 2, respectively.



A virtual DMU ($DMU^*$), which consists of Activity 1 of $DMU_A$ and Activity 2 of $DMU_B$

Figure 2: A virtual DMU referred by $DMU_o$

If activities are completely independent between activities, it might be possible for $DMU_o$ to achieve the efficiency level of $DMU^*$. However, in the case that activities are interdependent, it must be impossible or unreasonable for $DMU_o$ to aim for the efficiency level of $DMU^*$, which nibbles here and there and stands at the sacrifice of the other activity. Indeed, this model gives $DMU_o$ relatively low scores, and sometimes no DMU is scored unity. This means there are no efficient DMU and all DMUs refer to virtual one. It is doubtful whether this virtual $DMU^*$ is a reasonable target for $DMU_o$. For instance, in the case $DMU_o$ is scored 0.7, our question is whether it is really possible for it to improve its efficiency by the remaining 0.3 point in order to achieve the virtual efficiency level.

This unreasonable situation is attributed to different $\lambda$s, i.e. different frontiers among nodes. If $\lambda$ is common among nodes, every DMU in every node will refer to the same frontier. This case might be more reasonable because $DMU_o$ refers to existing DMUs on the frontier. It would present a realistic target to improve efficiency. However, this situation is nearly the same as the model without intermediate inputs, and hence more DMUs will be scored unity than those in the different $\lambda$s model. Thus, the discriminatory power of the common $\lambda$ model will drop.

To cope with this inconvenience, we propose a Constrained Network DEA (CNDEA) model, which is a midpoint between the different $\lambda$s and the common $\lambda$ models. We add constraints on $\lambda^k$ to avoid $DMU_o$ referring to completely different DMUs node by node.

$$\left|\lambda_j^k - \lambda_j^l\right| \le \Lambda^{kl}, \quad (j = 1, \ldots, n) \quad k \ne l, \quad k, l \in K, \quad K \text{ is a set of nodes.} \tag{3}$$

In the case $\Lambda^{kl}=\infty$, the model is the same as (2). That is, the intensity vectors are independent node by node. This model would be suitable when the activities of subsections are mutually independent. On the other hand, the case of $\Lambda^{kl} = 0$ means that all $\lambda$s are same and the efficient frontier is common to all nodes. The model is the nearly same as (1). Thus, we can appropriately choose the value of $\Lambda^{kl}$ depending on the connectivity among the subsections.

Compared with previous studies that assume no connectivity among activities ($\Lambda^{kl}=\infty$), our CNDEA model produces a more reasonable and practical efficiency index because it reflects connectivity among activities. This model is potentially applicable to a broad range of other industries with network structures, such as telecommunications, railways, education and so forth.

# 4 Numerical Example

## 4.1 The network model structure and dataset

We used investor owned vertically integrated electric power companies in the U.S. from 1990 to 2001 as a numerical example. After eliminating missing values and outliers by box plots, we obtained 314 unbalanced panel dataset of 56 companies. In this study the vertical structure of electric power companies is defined as described in Figure 3 There are four nodes ($k$ = 4) that imply generation, transmission, distribution and sales divisions, respectively.

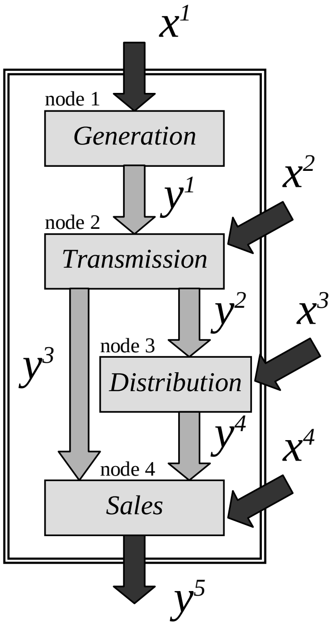


Figure 3: Structure of our network DEA model

In the generation division (node 1), companies utilize capital, labor and fuel inputs

($x^1$). The capital input is total nameplate capacity of electricity power plants measured in Mega Watts (MW), the labor input is number of employees of this division, and fuel input is consumed fuel at power plants. Since fuel consumption units differ amongst gas, coal, and petroleum, they were converted to British Thermal Units (BTU) in order to sum up the fossil fuel data. In contrast, the heat quantity from consumed nuclear fuel is difficult to measure. We thus performed backward calculations with the amount of nuclear power generation, assuming the thermal efficiency to be 0.32.

Using these three inputs, the generation division produces electric power ($y^1$), which is measured in Mega Watt hours (MWh). Then it becomes an intermediate input for the transmission division (node 2).

In the transmission division, we assumed three exogenous inputs ($x^2$) and one intermediate input ($y^1$). The capital input is transmission line length (kilo meter: km) and the labor input is the number of employees in this division. Furthermore, we employed purchased power from outside measured in MWh as an exogenous input. Electric power companies have two alternative power sources for supplying energy to customers; their own electric power plants and purchased power from other companies. The intermediate input ($y^1$) corresponds to the former source, and the third exogenous input corresponds to the latter one.

Electricity through transmission lines is sent to distribution lines. However distribution lines are used by small customers such as residential. This study assumes that large customers such as industrial do not use distribution lines and are supplied electricity directly from transmission lines, while residential customers are supplied via distribution lines. Therefore, outputs of the transmission division are divided into two parts, i.e. electricity sent to small customers ($y^2$) and large customers ($y^3$).

The distribution division (node 3) uses capital and labor inputs ($x^3$) and the intermediate input from the transmission division ($y^2$). The capital input is the total capacity of transformers measured in Mega Volt Ampere (MVA), and the labor input is the number of employees in this division. The output of this division is also electricity to small customers ($y^4$) after eliminating the estimated distribution losses.

The sales division (node 4) provides electricity supply services to large and small customers. In our structure, this division uses a labor input as an exogenous input ($x^4$) and two intermediate inputs ($y^3$ and $y^4$), and produces the final output ($y^5$), which is the sum of $y^3$ and $y^4$.

Dataset was constructed from the “FORM No.1” and “FORM No.423” published by the Federal Energy Regulatory Commission (FERC) and “Form EIA-860” published by Energy Information Administration (EIA). Table 1 shows input and output for all four divisions.

Table 1: Dataset of all divisions

| | | | Input and output factors | |
|---|---|---|---|---|
| Generation | $x^1$ | G1 | Capital Input | Nameplate Capacity (MW) |
| | | G2 | Labor Input | Number of Employess (#) |
| | | G3 | Fuel Input | Fuel Consumption (BTU) |
| | $y^1$ | | Output | Electric Power Generated (MWh) |
| Transmission | | | ⇒ Intermediate Input | |
| | $x^2$ | T1 | Capital Input | Transmission Line Length (km) |
| | | T2 | Labor Input | Number of Employess (#) |
| | | T3 | Purchased Power | Purchased power (MWh) |
| | $y^3$ | | Output | Electric Power Transmitted to large customers(MWh) |
| | $y^2$ | | Output | Electric Power Transmitted to small customers(MWh) |
| Distribution | | | ⇒ Intermediate Input | |
| | $x^3$ | D1 | Capital Input | Transformer Capacity (MVA) |
| | | D2 | Labor Input | Number of Employess (#) |
| | $y^4$ | | Output | Electric Power Distributed to small customers(MWh) |
| Sales | | | ⇒ Intermediate Input | |
| | $y^3$ | | ⇒ Intermediate Input | Electric Power Transmitted to large customers(MWh) |
| | $x^4$ | S1 | Labor Input | Number of Employess (#) |
| | $y^5$ | | Final Output | Total Electric Power Sales (MWh) |

## 4.2 Results

Figure 4 compares the results of the network DEA model with various $\Lambda$s in the production possibility set $P$ under CRS. $\Lambda=\infty$ and $\Lambda=0$ correspond to the different $\lambda$s and common $\lambda$ models, respectively.

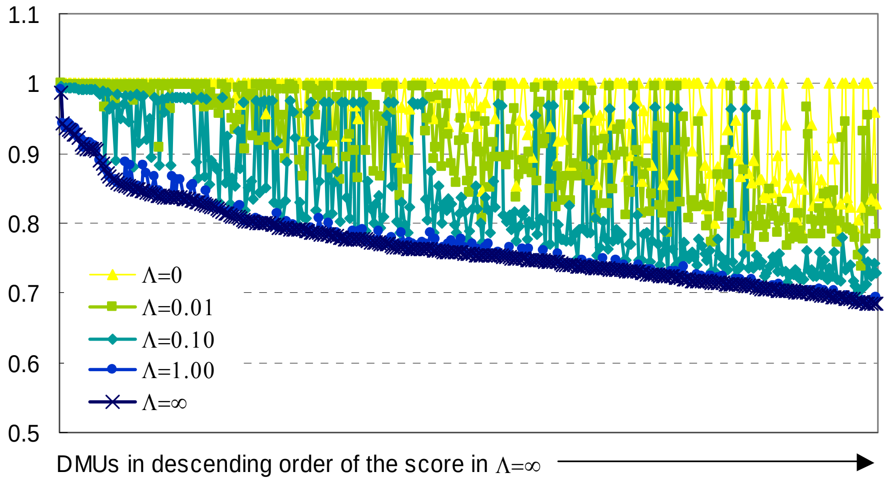


Figure 4: Results of the network DEA with various Λs under CRS

As this figure shows, the results of Λ=∞ and 1.00 are nearly the same, and there is no efficient DMU scoring unity[1]. This implies all DMUs refer to a virtual DMU. On the other hand, 54% of all DMUs score unity in the other extreme case where Λ=0, thus the discriminatory power of this model is weaker than the other models. In the models in which connectivity restrictions are imposed, the efficiency scores appear between those of Λ=0 and 1, and the scores become generally higher as Λ gets closer to 0.

Figure 5 describes the results under VRS. On this figure, we do not show the result of Λ=∞ because it is completely the same as that of Λ=1.

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[1] The maximum efficiency score is 0.988.

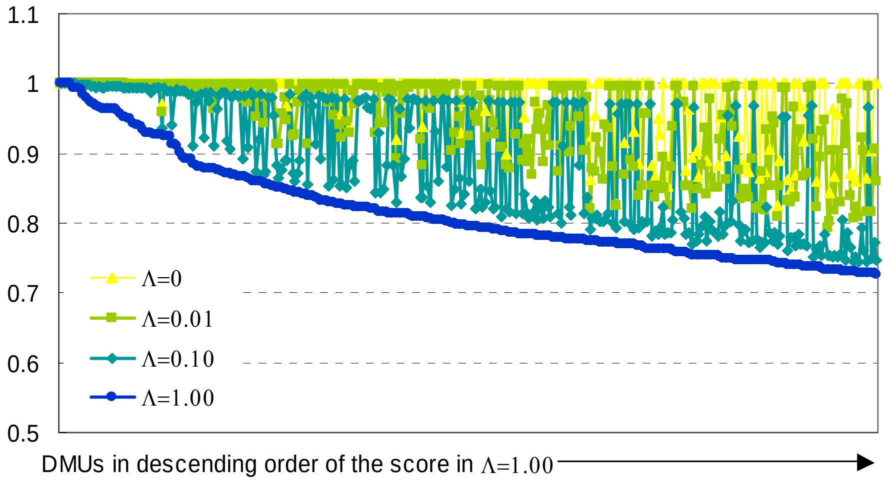

Figure 5: Results of the network DEA with various Λ under VRS

Compared with Figure 4, we can find DMUs scoring unity in the case of Λ=1; however, they are only 2% of the sample. The share of DMUs scoring unity in Λ=0 is 68%, which is larger than that under CRS and implies weak discriminatory power.

These results demonstrate that we can obtain more reasonable DEA scores if we appropriately choose the value of Λ depending on the connectivity among the subsections. Unfortunately, we have not discovered a practical way how to define Λ, and we leave this as a future project.

## 5 Conclusion

This study newly proposed constrained network DEA (CNDEA) to evaluate management efficiency of vertically integrated electric companies, which have the streamlined structure with closely linked divisions such as generation, transmission, distribution, and so forth. Through this model with an appropriate value of Λ, reasonable and practical efficiency scores can be obtained. It has potential use in a broad range of applications, including other industries with network structure.

In terms of future research, we should investigate and clarify how to find the appropriate $\Lambda$ that is depending on the connectivity among the subsections. And then, we will apply this model to the vertically integrated electric power companies to evaluate their management efficiency and provide useful and practical information to these companies in order to survive in the competitive market, and also to regulatory authority to evaluate those companies appropriately.

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