SLAC-PUB-13631
May 2009

Application of Non-Linear Time-Domain RF Simulations to Longitudinal Emittance Studies for the LHC [*]

T. Mastorides[†], C. Rivetta, J.D. Fox, D. Van Winkle,
SLAC National Accelerator Laboratory, Stanford, CA 94309, USA

Abstract

A non-linear time-domain simulation has been developed that can determine technical limitations, effects of non-linearities and imperfections, and impact of additive noise on the interaction of the beam with the Impedance Control Radio Frequency (RF) systems [1]. We present a formalism for the extraction of parameters from the time-domain simulation to determine the sensitivity of the beam longitudinal emittance and dilution on the RF system characteristics. Previous studies [2], [3] have estimated the effect of a noise source on the beam characteristics assuming an independent perturbation source of the RF voltage and a simplified beam model with no coupling. We present the methodology for the time-domain simulation study of the dependence of the accelerating voltage noise spectrum on the various RF parameters and the technical properties (such as non-linearities, thermal noise, frequency response etc.) of the Low Level RF (LLRF) system components. Future plans to expand this formalism to coupled bunch studies of longitudinal emittance growth in the LHC at nominal and upgraded beam currents are briefly summarized.

INTRODUCTION

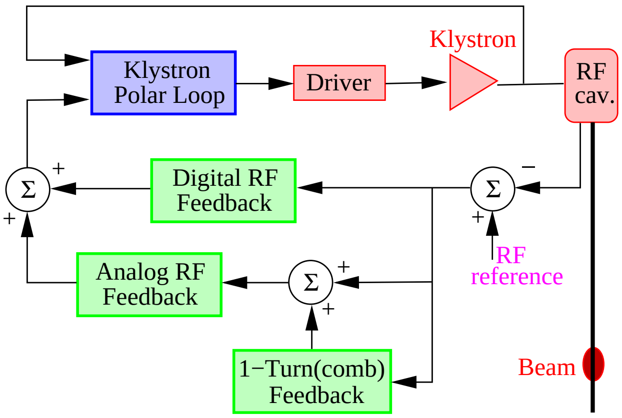

Figure 1: Simplified LHC RF block diagram.

The LHC RF system shown in Fig. 1 accelerates the beam during ramp and compensates the small energy losses during coasting. Furthermore, the RF station ensures longitudinal stability by reducing the effective impedance seen by the beam and reducing perturbations (power supply ripple and more) through the LLRF feedback loops (klystron polar and analog/digital loops). Due to this strong interaction with the beam, different operational configurations of the RF station can have a significant effect on the stability and characteristics of the beam. The close relationship between the LHC and PEP-II RF systems allow us to use our experience and tools from PEP-II operations on the LHC studies [4], [5].

In particular, a major concern for the LHC is the effect of RF station noise on the beam emittance. J. Tuckmantel at CERN has estimated the effect of a noise source on the beam characteristics assuming an independent perturbation source of the RF voltage and a simplified beam model with no coupling [2]. According to these studies, the maximum allowed phase noise from the RF system is $0.17°$ rms or 1.15 ps at 400.8 MHz. The phase noise of the RF system was estimated to 24 fs, in the absence of beam and for a particular setting of the LLRF feedback loops. A further concern involves the crossing of the synchrotron frequency f_s of the 50 Hz line during ramp. J. Tuckmantel simulated this effect, predicted non-negligible effects, and recommended an alternative ramp scheme with much smaller effects on the beam shape [3]. The noise levels for the 50 Hz line and its harmonics were also estimated for a particular setting of the LLRF.

We would like to confirm and expand J. Tuckmantel's findings with our simulation, which allows us to include the effect of:

- Changes of the phase noise floor level due to different gains in the LLRF feedback loops, which determines the sensitivity of beam emittance and diffusion on various RF parameters.
- Voltage non-linearity – since σ_z is comparable to $\lambda_{RF}/4$ for the LHC.
- Coupling between the bunches due to the RF system impedance $Z(\omega)$ (which can be estimated using our simulation).

The formalism that ties the time-domain simulation with these studies is presented in this work.

EFFECT OF RF NOISE ON BEAM EMITTANCE

In Fig. 2 from [6] we see the phase noise spectrum with the feedback loops open (faded trace), and with the analog/digital loops closed. The two noise effects of importance for the LHC beam emittance can be seen in this spec-

---

[*] Work supported by the U.S. Department of Energy under contract # DE-AC02-76SF00515 and the US-LARP program

[†] themis@slac.stanford.edu

Contributed to the 2009 Particle Accelerator Conference, 05/04/2009 −− 05/08/2009, Vancouver, Canada

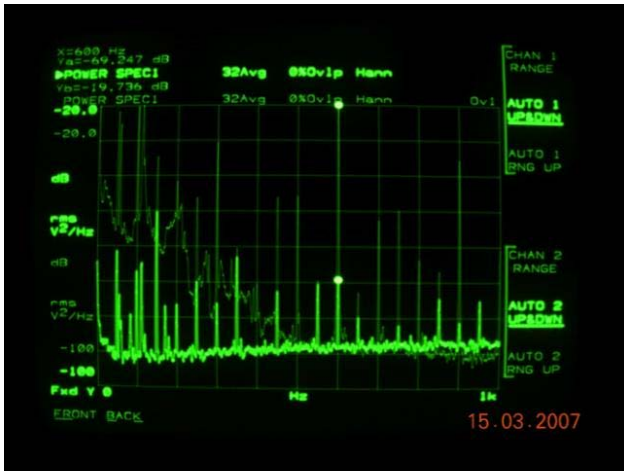

Figure 2: Phase noise spectrum in open loop (faded trace), and with analog/digital loops closed from [6].

trum. The noise floor of the spectrum defines the diffusion coefficients for the beam distribution, as will be shown below. Furthermore, the 50 Hz klystron noise line and its harmonics can interact with the beam motion. In particular, the synchrotron frequency crosses 50 Hz during the LHC ramp, which can significantly affect the beam distribution.

Noise Floor

Assuming that the beam dynamics we are studying (in the order of hours) are much slower than the synchrotron period $T_s = 16 - 47\, ms$, the radial Fokker-Planck equation in the presence of phase noise and damping can be derived from the particle equations of motion [7]:

$$\frac{\partial \Psi}{\partial t} = 2\alpha\Psi + (\alpha r + \frac{D}{r})\frac{\partial \Psi}{\partial r} + D\frac{\partial^2 \Psi}{\partial r^2} \tag{1}$$

where α is the rate of energy loss and D the diffusion coefficient, related to the power of the injected noise. Eq. 1 leads to a steady-state solution for the beam distribution:

$$\Psi(r) = \frac{1}{\sqrt{2\pi}\sigma_r} e^{-r^2/2\sigma_r^2}$$

where $\sigma_r^2 = \frac{D}{\alpha}$.

The slow nature of the beam dynamics also allows us to consider D to be proportional to the integrated noise power from DC to the revolution frequency f_{rev}. This choice – as well as the importance of the 50 Hz line – can be understood from the beam transfer function shown in Fig. 3. Since the effect of the RF noise on the beam depends on the interaction of this transfer function with the power density function, the diffusion coefficient corresponds to the integrated noise power up to f_{rev} and there is a strong resonance at 50 Hz. In [2] it was determined that white phase noise $<\Delta\phi>$ of 1.15 ps rms at 400.8 MHz would keep the LHC bunch length constant. Any higher phase noise would increase the beam longitudinal emittance. During the LLRF commissioning, the phase noise was measured to 29 fs, and it was determined that the LHC bunch would actually shrink. It was recommended that noise of 1.15 ps rms at 400.8 MHz should be injected to keep the LHC bunch length constant.

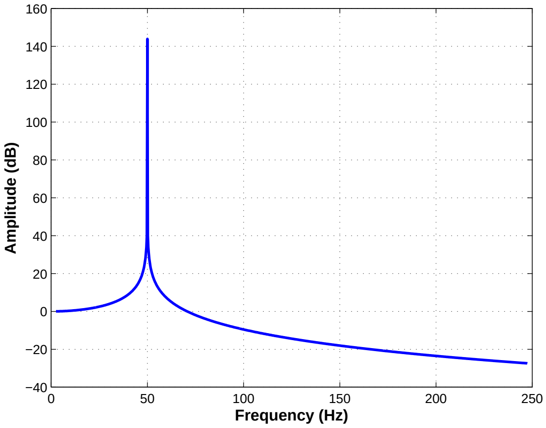

Figure 3: Beam Transfer Function

This measurement was done in the absence of beam and with the klystron polar loop off. The noise floor level though depends on the configuration of the LLRF feedback loops. Just during the LHC ramp, the open loop gain will increase by an estimated factor of nine [8], which might require adjustments in the LLRF feedback loops. If we consider the phase noise as the sum of components $A_k sin(\omega_k)$, we see that $<\Delta\phi> \propto A_k$. By linearizing around the operation point we get

$$\begin{aligned} &V_c(\omega) = \\ &H_c[I + H_k H_P + H_k H_F H_c]^{-1} N_k(\omega) \end{aligned} \tag{2}$$

where V_c is the cavity voltage, N_k the klystron output noise, and H_c, H_k, H_F, H_P are 2 by 2 matrices describing the frequency response of the cavity, klystron, analog/digital loop, and klystron polar loop respectively. Since the gains for H_F and H_P are very high, we can see that for a given N_k there is an almost inversely proportional relationship between the feedback loop gain and V_c, A_k.

This analysis shows that variations in the LLRF can have a significant effect on the cavity phase noise floor and subsequently on the beam emittance after hours of coasting. Therefore, a more systematic study of different LLRF configurations is necessary to positively determine whether the cavity phase noise will not exceed the threshold value of 1.15 ps. Using our time-domain simulation we plan to determine the cavity phase noise for each LLRF configuration, and conclude if the phase noise is below the threshold value for all possible configurations, and additionally the necessary noise level for a constant bunch length as a function of LLRF parameters.

Effect of 50 Hz line

Since the system damping is small compared to the noise excitation [9],

$$\sigma_z^2(t) = \sigma_z(0)^2 + \alpha P(f_s)t$$

where $P(f_s)$ is the power spectral density evaluated at the synchrotron frequency f_s and α is a constant. Let the 50 Hz noise be $\beta \sin(2\pi 50)$ – since the noise is slow with respect to the RF frequency an offset phase does not affect the beam distribution. Similarly to the case of the noise floor, β is a function of the LLRF parameters and β^2 is proportional to $P(f_s)$, which in turn affects σ_z^2. Therefore, the conclusions about the variation of the open loop gain with LLRF configuration will also help determine the possible amplification of the 50 Hz line effect on the beam distribution.

An added effect that we would like to study with our models and simulation is the effect of noise at ± 50 Hz around each revolution harmonic, since the beam sampling aliases that noise on top of the 50 Hz line.

SINUSOIDAL RF VOLTAGE

Since the LHC bunch length is comparable to a quarter wavelength, there was a concern about effects of the nonlinear voltage on the beam distribution. If we consider a sinusoidal RF voltage, the Fokker-Planck equation in the normalized phase-space coordinates from [7] becomes

$$\begin{aligned} \frac{\partial \Psi}{\partial t} &= 2\alpha\Psi + D_\xi \frac{\partial^2 \Psi}{\partial \omega^2} + D_\pi \frac{\partial^2 \Psi}{\partial p} - \omega_s p \frac{\partial \Psi}{\partial \omega} \qquad (3) \\ &+ \left(\frac{c\omega_s}{\omega_{RF} \cos\phi_s} \sin(\frac{\omega_{RF}\omega}{c} + \phi_s) + 2\alpha p \right) \frac{\partial \Psi}{\partial p} \end{aligned}$$

We transform Eq. 3 to radial coordinates using $<\sin\phi>= 0, \quad <\sin^2\phi>= 1/2$, since the beam dynamics are much slower than the synchrotron motion.

$$\frac{\partial \Psi}{\partial t} = 2\alpha\Psi + (\alpha r + \frac{D}{r})\frac{\partial \Psi}{\partial r} + D\frac{\partial^2 \Psi}{\partial r^2} + \mathcal{A} \qquad (4)$$

Eq. 4 is equivalent to Eq. 1 except for the term $\mathcal{A}$, which is equal to:

$$\left(\frac{c\omega_s}{\omega_{RF}\cos\phi_s} \int_0^{2\pi} \sin(\frac{\omega_{RF} r \cos\phi}{c} + \phi_s) \sin\phi d\phi \right) \frac{\partial \Psi}{\partial r}$$

This integral averages to zero over a synchrotron period. Therefore, the predicted $\Psi_{ss}(r)$ is the same for a sinusoidal voltage or its linear approximation.

FURTHER PLANS FOR LHC MODELS AND SIMULATION

Using the LHC time-domain simulations [1], the cavity phase noise level and the 50 Hz line magnitude will be determined as a function of the LLRF parameters. These studies will allow us to determine the technical implementation characteristics, the technology limits, and consider the trade-offs of RF parameters. It will also help us determine optimal control algorithms and configurations, considering the trade-off between RF stations stability, impedance reduction, and cavity noise level.

We would also like to apply the perturbation formalism [10] to include the effect of coupling on the models and calculations described in this work. Beam coupling through the RF station impedance will increase the sensitivity of the beam emittance on RF noise effects. The RF station impedance will be estimated with our simulation for each LLRF configuration and the effect on the beam emittance will be determined based on the perturbation formalism.

ACKNOWLEDGMENTS

The authors would like to thank the CERN RF group – and in particular P. Baudrenghien and J. Tuckmantel – for their interest in this work, the valuable data, their help and ideas. We would like to acknowledge the support for this LHC collaboration from the SLAC ATR department and the US LARP program. We also want to thank Alex Chao for the helpful collegial discussions.

REFERENCES

[1] T. Mastorides *et. al.*, “Modeling and Simulation of the Longitudinal Beam Dynamics - RF Station Interaction in the LHC Rings”, Proc. EPAC 2008, 23-27 June 2008, Genoa, Italy.

[2] J. Tuckmantel, “Synchrotron Radiation Damping in LHC and Longitudinal Bunch Shape”, LHC Project Report 819, June 2005.

[3] J. Tuckmantel, “Simulation of LHC Bunches under Influence of 50-Hz multiple Lines on the Cavity Field”, LHC Project Note-404, June 2007.

[4] C. Rivetta *et. al.*, “Modeling and Simulation of Longitudinal Dynamics for Low Energy Ring-High Energy Ring at the Positron-Electron Project”, Phys. Rev. ST-AB, 10, 022801 (2007) and SLAC-PUB-12374, February 2007.

[5] T. Mastorides *et. al.*, “Analysis of Longitudinal Beam Dynamics Behavior and RF System Operative Limits at High Beam Currents in Storage Rings”, Phys. Rev. ST-AB, 11, 062802 (2008) and SLAC-PUB-13287.

[6] P. Baudrenghien *et. al.*, “The LHC Low Level RF: Status and Commissioning”, LLRF 2007 Workshop, October 22-25 2007, Knoxville, TN.

[7] H. Wiedemann, “Particle Accelerator Physics II”, pages 291-302, Springer, Berlin, 1993.

[8] P. Baudrenghien *et. al.*, “The LHC Low Level RF”, Proc, EPAC 2006, 26-30 June 2006, Edinburgh, Scotland.

[9] J. Steimel *et. al.*, “Effects of RF Noise on Longitudinal Emittance Growth in the Tevatron”, Proc. PAC 2003, May 12-16 2003, Portland, OR.

[10] A. Chao, “Physics of Collective Beam Instabilities in High Energy Accelerators”, Wiley Series in Beam Physics and Accelerator Technology, 1993.