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Application of permutations to lossless compression of multispectral thematic mapper images

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Abstract. The goal of data compression is to find shorter representations for any given data. In a data storage application, this is done in order to save storage space on an auxiliary device or, in the case of a communication scenario, to increase channel throughput. Because remotely sensed data require tremendous amounts of transmission and storage space, it is essential to find good algorithms that utilize the spatial and spectral characteristics of these data to compress them. A new technique is presented that uses a spectral and spatial correlation to create orderly data for the compression of multispectral remote sensing data, such as those acquired by the Landsat Thematic Mapper (TM) sensor system. The method described simply compresses one of the bands using the standard Joint Photographic Expert Group (JPEG) compression, and then orders the next band's data with respect to the previous sorting permutation. Then, the move-to-front coding technique is used to lower the source entropy before actually encoding the data. Owing to the correlation between visible bands of TM images, it was observed that this method yields tremendous gain on these bands (on an average 0.3 to 0.5 bits/pixel compared with lossless JPEG) and can be successfully used for multispectral images where the spectral distances between bands are close.

Subject terms: permutations; sorting; lossless compression; multispectral images; spectral distances.

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1 Introduction

With the development of remote sensing systems that can transmit large volumes of data, the transmission and storage requirements of these data have become a significant concern. For example, it is estimated that the Earth Observation System Data and Information System (EOSDIS) will generate more than one terabyte (10^{12} bytes) of data per day.[1] To handle these quantities, there is a critical need for data compression.

The goal of data compression is to represent the given image data with the minimum number of bits. Compression schemes can be broadly classified into two categories—lossy and lossless. For many applications, the loss of information that is not perceptually significant can be easily tolerated if a good visual approximation to the original is preserved. Conversely, some applications, such as medical imaging and remote sensing, cannot tolerate any loss of information. Lossless compression, as the name implies, provides compression without losing any information (i.e., when the data are uncompressed, the original information is obtained).

A lossless image compression technique consists of two main components—modeling and encoding.[2] A model captures the structure inherent in the raw data and extracts it. The residual, also called *error*, is then encoded using an entropy encoding technique. Shannon[3] showed that the minimum average rate at which the output of a source can be coded is its entropy. If an image is modeled as a source of independent pixels, then the first-order entropy defines the average information (bits/pixel) by which an image can be encoded. Well-known encoding techniques, such as arithmetic and Huffman encoding, can encode a source optimally (closer to its first-order entropy). Hence, the critical task in data compression is modeling.

For a typical image, since the values of adjacent pixels are often highly correlated, a significant amount of information about a pixel value may be obtained by examining the neighboring pixels. One of the most popular techniques in image compression is the linear predictive technique, which exploits the correlations among the neighboring pixels. This technique scans the image in a fixed order (usually raster order) and predicts the current pixel by taking a linear combination of neighboring pixels that have been previously transmitted. The Joint Photographic Expert Group (JPEG) still picture compression standard[4] uses linear predictive techniques in its lossless mode, and has eight different schemes, which are listed in Table 1. The first scheme makes no prediction. The next three are one-dimensional predictors, which scan the image in raster, vertical, and diagonal order, while the last four schemes are two-dimensional predictors. In this work, we use first-order entropy as a measure to compare our work with the well-known techniques, such as JPEG.

Table 1 Lossless JPEG predictors.

Mode	Prediction for $x_{i,j}$
0	No Prediction
1	$x_{i-1,j}$
2	$x_{i,j-1}$
3	$x_{i-1,j-1}$
4	$x_{i,j-1}+x_{i-1,j}-x_{i-1,j-1}$
5	$x_{i,j-1}+(x_{i-1,j}-x_{i-1,j-1})/2$
6	$x_{i-1,j}+(x_{i,j-1}-x_{i-1,j-1})/2$
7	$(x_{i,j-1}+x_{i-1,j})/2$

The correlation of adjacent pixels implies spatial correlation, and well-known image compression techniques, such as JPEG, utilize this spatial correlation to decorrelate image data before actually encoding it. However, unlike single-band images, multispectral images also have spectral correlation between adjacent spectral bands. In the case of Landsat Thematic Mapper (TM) data, it is well recognized that bands 1, 2, and 3 are highly correlated with each other as also are bands 5 and 7. Although much work has been done toward developing algorithms for compressing single-band image data,[5,6] the multidimensional (spatial and spectral) nature of multispectral remotely sensed data has not been thoroughly explored.[7,8] A review of the literature indicates that some recent studies have examined the spectral relationships in multiband data and have developed algorithms that take advantage of such relationships.[7–10]

Tate[9] used spanning trees to order the bands before using linear prediction among the bands in the decorrelation step. Wang, Zhang, and Tang[10] calculated the standard correlation (r) between the bands to classify the bands of a TM image prior to using multivariate regression on spatial and spectral neighbors to determine the optimal coefficients for prediction in the decorrelation step. Memon et al.[8] took a different approach. Instead of using linear or regression-based predictive techniques, they stated that since the bands are imaging the same physical structure, they should have similar patterns that could be captured by some structure. Hence, if a structure can optimally decorrelate one of the bands, such as prediction trees, then that structure could be used for other bands. Their observations proved to be correct based on the results presented.

This research investigated the spectral and spatial correlation for the Landsat TM multispectral image data using permutations. A new spectral scheme is proposed that utilizes permutations for the visible bands of Landsat TM data. The results indicate that on an average, an additional 0.3 bit per pixel or more may be saved over the standard lossless JPEG technique.

2 Permutations for Image Compression

In the study of sorting algorithms, permutations are of special importance since they represent unsorted input data.[11] Knuth[11] explains this relationship with respect to different sorting algorithms. Given a set S, of size $n=|S|$, there are $n!$ unique possible orderings of the set S. Let P be an $n\times n$ digital image, that is, $P_{i,j}$ is a pixel representing the i'th row and j'th column, for $i,j=1,\ldots,n$, and suppose that $P_{i,j}$ is a byte representing any of 256 brightness (gray) values (BVs). Similarly, a multispectral image may be represented by a three-dimensional array where $P_{i,j,b}$ represents the pixel value along row i, column j, in band b. We convert a two-dimensional square array into a linear array P' in the usual way where $P'_k = P_{i,j}$ for $i,j=1,\ldots,n$ and $k=(i-1)n+j$. Thus, k ranges over the set $\{1,2,\ldots,n^2\}$. In this way, each pixel position has an integer index, and we can regard the image as a sequence of gray values, $[P'_1,P'_2,\ldots,P'_j,\ldots,P'_{n^2}]$. If π is a permutaion of $\{1,2,\ldots,n^2\}$ that sorts the sequence $[P'_1,P'_2,\ldots,P'_j,\ldots,P'_{n^2}]$ in ascending order, then the sorted sequence is

$$P'_{\pi_1}\leqslant P'_{\pi_2}\leqslant \ldots \leqslant P'_{\pi_{n^2}}.$$

Indices	$\pi_1,\ldots,\pi_{k_1}$	$\pi_{k_1+1},\ldots,\pi_{k_1+k_2}$	$\ldots$	$\pi_{k_1+k_2+\ldots+k_g+1},\ldots,\pi_{n^2}$
Data	$0,\ldots,0$	$1,\ldots,1$	$\ldots$	$g-1,\ldots,g-1$

Fig. 1 View of data after sorting.

Since n^2 is in fact much larger than the number g of gray values, the sorted sequence gives rise to a partition of $\{1,2,\ldots,n^2\}$ into blocks, $B_1,B_2,\ldots,B_g$ where $P'_b = i$ for all $b\in B_i$. In other words,

$$P'_{\pi_1}=P'_{\pi_2}=\ldots=P'_{\pi_{k_1}}<P'_{\pi_{k_1+1}}=\ldots=P'_{\pi_{k_1+k_2}}$$
$$<\ldots<P'_{\pi_{k_1+k_2+\ldots+k_g+1}}=\ldots=P'_{\pi_{n^2}},$$

where $k_i=|B_i|$ for $1\leqslant i\leqslant g$.

Here there are $g-1$ jumps, where g is the number of gray values attained in the image. This generates a partition of indices, as in Fig. 1. Note that the indices within each block can be arranged in any order. Hence, they can be arranged in ascending order. This sorting permutation based on P' is called the *canonical sorting permutation* and is represented with π^s.

Depending on $|P'|$, and the number of gray values (intensities), there may be more than one sorting permutation. Indeed, if all the 256 possible gray values occur with some frequency f_i, then the number of sorting permutations is:

$$\prod_{i=0}^{255} f_i!.$$

Since this number is very large for a typical image, we chose the canonical sorting permutation in order to create a unique correspondence between the transmitter and receiver.

On single images, one can compress the data by sorting it and then taking the differences of consecutive elements. If we assume that all possible 256 intensities occur in an image, then upon sorting the image, blocks of f_0 elements with value 0, f_1 elements with a value 1, and so on, result. In that case, if the differences of consecutive elements are taken, only 0s and 1s will remain, and owing to the high frequency of 0s, the data are likely to be coded at a very low cost rate. In fact, the simulation results show that such data can be coded at a rate of 0.015 to 0.035 bits/pixel in the case of 256×256 images. For bigger images, the cost range is even smaller.

Although such tremendous minimization of data is possible, the additional cost that results from transmitting a sorting permutation reverses the process to an overall loss. However, in the case of multispectral images, we do not encounter the transmission cost of a sorting permutation, and as we will demonstrate in later sections, we even have gains.

3 Permutations for Multispectral Image Compression

In the case of multispectral images, the pixel values in neighboring bands may be different; however, the relationship between a pixel and its spatial neighbors may be very similar in adjoining spectral bands. If spectrally such a relationship occurs between the adjoining spectrally different pixels and their respective neighbors, then the relationship can be captured by permutations. From the correlation matrices of the images (Tables 2 to 7), it is clear that the visible bands (bands 1 to 3) have high correlation, as do the middle-infrared bands (bands 5 and 7) of a TM image. The correlation between the visible and mid-infrared bands is well recognized by the remote sensing community. In fact, the classification algorithm used by Wang, Zhang, and Tang[10] creates four classes: bands {1, 2, 3}, {4}, {6} and {5, 7}. It is generally accepted that if the correlation among two vectors is high (e.g., >0.8), then the sorting permutations are closer. Therefore, the sorting permutations of visible and mid-infrared bands may create more orderly data (which have locality of reference) when applied to the next band in the order.

When data have locality of reference, then the move-to-front coding may be a good choice[12] for transforming the original source data into another source that may have lower first-order entropy with respect to the original source. Consequently, if the spectral correlation can be captured by permutations, we may transform the source data in some bands to a different ordering using the canonical sorting permutations of some other band, so that the move-to-front coding can generate a lower source entropy.

In this section, we present an algorithm based on permutations and move-to-front coding to compress a multiband image.

The encoder uses the following steps:

1. Obtain the canonical sorting permutation, π_1^s of band b_1 (where b_x represents the image $P_{i,j,x}$).
2. Compress b_1 with JPEG. Encode the data and transmit them to the decoder along with 3 bits to indicate which JPEG operator was used.
3. Starting from $x=2$, obtain in the canonical sorting permutation, π_x^s, of band b_x. Then, apply π_{x-1}^s to b_x, to get b_x'.
4. Apply move-to-front coding to b_x'; encode the resulting data; then transmit them to the decoder.

The decoder:

1. Decode b_1 to undo the effect of the JPEG operator and obtain the π_1^s.
2. Starting from $x=2$, for each band received, decode the data, undo the effect of the move-to-front coding, and using π_{x-1}^s, reorder the data and construct band b_x.

From the above algorithms it is clear that permutations do not require an additional overhead for encoding or decoding purposes for the multispectral images.

4 Simulation Results

For our simulations, we used the TM images ''Omaha,'' ''Chernobyl,'' ''Butler,'' and ''Crescent Lake,'' which were obtained from the Center for Advanced Land Management Information Technologies (CALMIT) at the University of Nebraska, Lincoln. The ''Washington D.C.'' image is well known among the data compression community and was supplied by K. Zhang.[10] The test images, shown in Fig. 2, differ in terms of size and are as follows: ''Omaha,'' 1323×1325; ''Butler,'' ''Chernobyl,'' and ''Washington D.C.,'' 512×512; ''Crescent Lake,'' 376×331; and ''Wyoming,'' 780×664.

In Table 8, results of the original source entropy of bands 2 to 7, and the results obtained by applying our algorithm defined in the last section, are presented for some of the test images. The results clearly indicate that there is consistent gain with respect to first-order entropy of the image data. In addition, there is tremendous gain with respect to the original source entropy for the visible bands,

Table 2 Correlation matrix for the Chernobyl image.

TM Bands (μm)	1 0.45–0.52	2 0.52–0.60	3 0.63–0.69	4 0.76–0.90	5 1.55–1.75	7 2.08–2.35
1	1.00					
2	0.96	1.00				
3	0.93	0.98	1.00			
4	0.64	0.73	0.70	1.00		
5	0.74	0.83	0.86	0.86	1.00	
7	0.78	0.86	0.91	0.75	0.97	1.00

Table 3 Correlation matrix for the Butler image.

TM Bands (μm)	1 0.45–0.52	2 0.52–0.60	3 0.63–0.69	4 0.76–0.90	5 1.55–1.75	7 2.08–2.35
1	1.00					
2	0.89	1.00				
3	0.96	0.92	1.00			
4	−0.38	−0.23	−0.47	1.00		
5	0.77	0.66	0.79	−0.30	1.00	
7	0.85	0.78	0.90	−0.46	0.93	1.00

Table 4 Correlation matrix for the Crescent Lake image.

TM Bands (μm)	1 0.45–0.52	2 0.52–0.60	3 0.63–0.69	4 0.76–0.90	5 1.55–1.75	7 2.08–2.35
1	1.00					
2	0.98	1.00				
3	0.96	0.98	1.00			
4	0.50	0.56	0.55	1.00		
5	0.81	0.85	0.89	0.76	1.00	
7	0.90	0.93	0.97	0.64	0.96	1.00

Table 5 Correlation matrix for the Wyoming image.

TM Bands (μm)	1 0.45–0.52	2 0.52–0.60	3 0.63–0.69	4 0.76–0.90	5 1.55–1.75	7 2.08–2.35
1	1.00					
2	0.98	1.00				
3	0.97	0.98	1.00			
4	0.18	0.24	0.16	1.00		
5	0.86	0.87	0.89	0.19	1.00	
7	0.88	0.88	0.92	0.02	0.95	1.00

Table 6 Correlation matrix for the Omaha image.

TM Bands (μm)	1 0.45–0.52	2 0.52–0.60	3 0.63–0.69	4 0.76–0.90	5 1.55–1.75	7 2.08–2.35
1	1.00					
2	0.94	1.00				
3	0.93	0.97	1.00			
4	−0.31	−0.28	−0.38	1.00		
5	0.44	0.50	0.47	0.34	1.00	
7	0.75	0.79	0.80	−0.11	0.81	1.00

Table 7 Correlation matrix for the Washington DC image.

TM Bands (μm)	1 0.45–0.52	2 0.52–0.60	3 0.63–0.69	4 0.76–0.90	5 1.55–1.75	7 2.08–2.35
1	1.00					
2	0.92	1.00				
3	0.86	0.92	1.00			
4	0.07	0.23	0.33	1.00		
5	0.36	0.50	0.56	0.79	1.00	
7	0.65	0.73	0.78	0.49	0.85	1.00

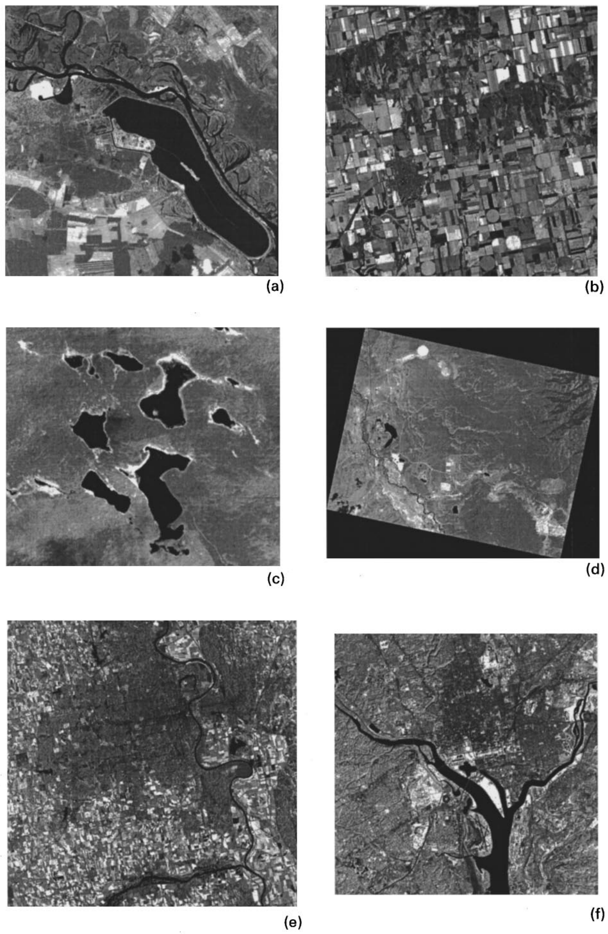

Fig. 2 Landsat TM band 4 (near-infrared) images used for testing our algorithm. (a) Chernobyl, (b) Butler, (c) Crescent Lake, (d) Wyoming, (e) Omaha, (f) Washington, D.C.

which is obtained by simply taking the linear order of the bands in a TM image.

It has been pointed out that a linear order may not yield the best possible compression. For example, Tate[9] has presented an algorithm for ordering multispectral images based on spanning trees, prior to applying predictive operators to spectrally ordered bands. He obtained better compression gain than just using the linear order of the bands. Wang, Zhang, and Tang[10] used correlation among the bands for ordering them. There are a few well-known metrics to measure how closely two permutations are related. These distance metrics are Spearman's ρ,[13–15] Hamming distance,[15] and exc and mse.[14,15] However, these well-known metrics did not indicate any meaningful relationship between the canonical sorting permutations of different bands. For example, Spearman's $\rho = 0.016$ for the canonical sorting permutations of bands 1 and 2 of the Washington D.C. image. The results obtained from hamming distance, exc, and mse are poor as well. While these results indicate that the canonical sorting permutations are not related, such conclusions are misleading. If the canonical sorting permutations are not related, we would not have obtained any positive results.

There are two possible sorting permutations for a given pair of bands. It is important to determine which one may yield more orderly data when applied onto the other band, and consequently may yield a higher compression ratio. To determine this, we tried each band's canonical sorting permutation on the other 6 bands, followed by the move-to-front coding. Empirical results indicated that for almost all the TM images, the canonical sorting permutation of band 2 generates more orderly data on bands 1 and 3. Also, the canonical sorting permutation of band 7 yields more orderly data on band 5 than the canonical sorting permutation of band 5 on band 7. Although the correlation among bands 5 and 7 is high (>0.8), and applying the canonical sorting permutation followed by the move-to-front coding yields compression, it does not always yield a higher compression ratio than predictive techniques, such as JPEG. This is because bands 5 and 7 have a spectral distance of 0.33 μm and a wider spectral range than the visible bands.

In Table 9, under the columns of band 1 and band 3, we list the results obtained in terms of first-order entropy when we applied the canonical sorting permutation of band 2 to band 1 and band 3, followed by move-to-front coding. Also, in Table 9, for comparison purposes we present the results of the best operator of the standard lossless compression technique JPEG, when applied to bands 1 and 3. As can be seen, for most of the images we obtained substantial gain over JPEG. By applying the canonical sorting permutation of band 1 to band 2, and band 2 to band 3 in linear order, improvements over JPEG technique were observed in most cases (Table 8). However, the results obtained by the canonical sorting permutation of band 2 to bands 1 and 3 yield more gain over JPEG.

One of the factors that makes band 2 the pivotal band is the number of gray values. Among the visible bands, band 2 has the lowest range of gray values. For test images, the number varies between 40 and 80, while for bands 1 and 3 the values range between 75 and 200. The canonical sorting permutation of band 2 then has 40 to 80 blocks (B_i), as opposed to 75 to 200. Therefore, the sorting permutation of band 2 creates less perturbation on the data of bands 1 and band 3, and forms more orderly data. The other factor is the distribution of the TM bands in the electromagnetic spectrum (EMS). The EMS is a continuum, and the TM bands have been selected as representative components of this continuum based on their utility for a variety of terrestrial applications. TM bands 1, 2, and 3 represent the visible portion of the EMS in almost a continuous manner and because band 2 is located between bands 1 and 3, it serves as a link or a transition between them. Therefore, the spectral distances between the bands are minimal. For example,

Table 8 Original entropy versus entropy obtained by applying our method.

TM Image	Band 2	Band 3	Band 4	Band 5	Band 6	Band 7
Chernobyl	4.83	5.53	6.08	7.10	5.04	6.54
	2.93	3.39	5.12	6.04	4.32	5.96
Crescent	4.91	5.50	5.54	6.49	4.91	6.05
Lake	2.63	3.02	4.57	5.62	4.07	5.16
Washington	3.85	4.31	5.43	5.68	3.44	4.76
DC	2.89	3.21	5.32	5.35	3.28	4.67

Table 9 Entropy of applying π_2^s to TM bands 1 and 3, followed by move-to-front coding, versus best JPEG.

Image	Band 1	JPEG	Band 3	JPEG
Butler	3.53	3.78	3.81	3.93
Chernobyl	3.49	3.77	3.39	3.83
Crescent Lake	3.28	3.58	3.02	3.52
Omaha	3.80	4.23	3.21	3.98
Washington D.C.	3.71	3.75	3.21	3.45
Wyoming	3.39	3.98	2.84	3.94
Average	3.49	3.79	3.24	3.76
Compression Ratio	2.29	2.11	2.46	2.13

the spectral separation between bands 1 and 2 is 0.00 μm and between 1 and 3—is 0.11 μm. However, the spectral separation between bands 2 and 3 is only 0.02 μm. With a minimal spectral distance between bands 1 and 2, and 2 and 3, band 2 would be the pivotal point where correlations among the three bands are maximized. This is also evident from the correlation matrices for the six images used in this research (Tables 2 to 7).

5 Conclusion and Future Research

In the past, predictive methods utilizing spatial and spectral characteristics and a combination of these characteristics have been used to decorrelate multispectral images. The only method that did not use predictive operators is the prediction tree method described by Memon, Sayood, and Magliveras.[8] This study introduced permutations as a new scheme for compression of multispectral images.

It was demonstrated that for the visible bands of a TM image, permutations can capture the spectral correlation and yield considerable improvement over the standard JPEG compression technique. Owing to the high correlation between the visible bands, their narrower spectral bandwidths, and their continuity in the EM spectrum, the canonical sorting permutation of band 2 can be used to increase the locality of reference of data in bands 1 and 3. Hence, the move-to-front coding generates a lower source entropy.

It is worth noting that although bands 5 and 7 are highly correlated, such correlation alone may not always yield a substantial gain over JPEG using permutations. This is obvious from the wavelength distances between the bands as described by Jensen.[16] The wavelengths for bands 5 and 7 are 1.55 to 1.75 μm and 2.08 to 2.35 μm, respectively. In this case, not only is the spectral range for band 7 larger (0.27 μm) than that for band 5 (0.20 μm), but the spectral distances are much greater than those for the visible bands (i.e., 0.33 μm compared with 0.0 μm between bands 1 and 2; 0.03 μm between bands 2 and 3; and 0.11 μm between bands 1 and 3). Therefore, even though we may obtain gains over JPEG, when applying the canonical sorting permutation of band 7 to band 5, the results are not always favorable.

We have demonstrated that canonical sorting permutations can capture spectral relationships. Indeed, there are many sorting permutations that can be derived, and it is possible that some of them may generate higher locality of reference than the canonical sorting permutation, which in turn may yield a higher compression ratio. This may be done with a small overhead, depending on the number of possible sorting permutations that can be chosen. Hence, the determination of which permutations may effectively capture such spectral relationships is worth pursuing as a future research problem.

Another possible research problem is to combine prediction with permutations. We mentioned that different bands have different numbers of blocks based on their range of BVs. Thus, their predictive operators may create prediction errors with higher variance. Because permutations can capture spectral relationships, one may construct a pseudo image of the previous band using the gray value frequencies of the current band (knowledge of gray frequencies of the current band requires small overhead; for example, for an image of size 512×512, the overhead is 0.02 bits/pixel) and the canonical sorting permutation of the previous band. Using the pixel values of the pseudo image, we may predict the current image to a higher degree of precision. This may minimize the variance of the prediction errors and may yield higher compression gain.

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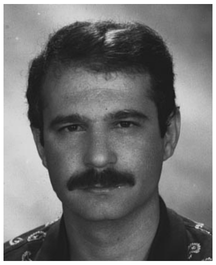

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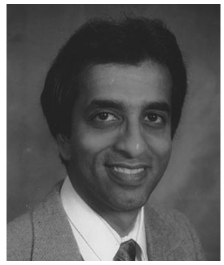

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