APPLICATION OF SPARSE EIGENVALUE TECHNIQUES TO THE SMALL SIGNAL STABILITY ANALYSIS OF LARGE POWER SYSTEMS

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Abstract – This paper presents two sparsity-based eigenvalue techniques – simultaneous iterations and the modified Arnoldi method – and their application to the small signal stability analysis of large power systems.

Simultaneous iterations and the modified Arnoldi method are two recently developed methods for large, sparse unsymmetrical eigenvalue problems, and have been reported as very efficient in computing the partial eigensolution of several types of matrices, such as stochastic ones. It is shown in this paper that they can also be applied successfully to the matrices derived for small signal stability studies of power systems. An algorithm utilizing these two methods is proposed for calculating the eigenvalues around a fixed point which can be placed at will in various parts of the complex plane. The sparsity is fully preserved in the algorithm by using the augmented system state equations as the linearized power system small signal model and performing the corresponding sparsity-oriented calculations. Several applications of the algorithm are discussed and illustrated by numerical examples.

The proposed methods and algorithm have been tested on two test systems with 20 and 50 machines respectively. The results show that they are suitable for the eigenanalysis of large power systems.

Keywords: Small signal stability, Eigenvalues, Sparse methods.

INTRODUCTION

The evaluation of the small signal stability of power systems requires the calculation of the eigenvalues of a very large unsymmetrical and nonsparse matrix. The well-known QR method is robust and converges fast [1] but cannot be implemented with sparsity techniques, so that its application is limited to relatively small power systems. On the other hand, for a large power system with thousands of state variables, it is usually required to calculate only a specific set of eigenvalues with certain features of interest, for example, local mechanical modes, inter-area modes, etc. Therefore, significant effort has been expended to develop or apply new methods with the following three basic properties:

(a) Sparsity techniques can be used

(b) A specific set of eigenvalues can be found efficiently

(c) Mathematical robustness is guaranteed, i.e. good convergence characteristics and numerical stability.

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Among these (a) is of utmost importance since it provides the possibility to handle large power systems. Several sparsity-based methods have been proposed in recent years. PEALS [2] is mainly aimed at the computation of slow inter-area oscillatory modes; the S-Method [3] is most efficient for finding the unstable modes; STEPS [4] can be used for computing the eigenvalues belonging to a small study zone; [5] gives an implementation of the inverse iterations. In addition to these methods, [6] and [7] also report special methods to solve the eigenvalue problem of large power systems.

This paper presents two sparsity-based eigenvalue techniques – simultaneous iterations and the modified Arnoldi method – and their application to the small signal stability analysis of large power systems. These two methods are mathematically well-developed and both have been proved to be very efficient in computing the dominant eigenvalues of large, sparse, unsymmetrical matrices [8,9]. The former is an extension of the classical power method with a tactically designed interaction analysis which makes the method converge reliably. The latter is a method similar to the well-known Lanczos method, but more reliable by having better numerical properties after introducing appropriate modifications. Both simultaneous iterations and the modified Arnoldi method are successful in the eigenanalysis of power systems, as will be illustrated by various numerical examples.

For the small signal stability analysis, an algorithm is proposed to make the eigenvalue problem of power systems fit the two methods mentioned above. The sparsity is fully preserved in the algorithm by using the augmented system state equations as the linearized power system small signal model and performing the corresponding sparsity-oriented calculations. A simple spectral transformation – fractional transformation – is then applied to the augmented state matrix to make dominant the eigenvalues around a specified shift point, so that a group of eigenvalues near the shift point can be computed by either of the two methods. This algorithm is most suitable for calculating a desired number of eigenvalues nearest to or all eigenvalues within certain distance from the shift point. For example, if the local mechanical modes are of interest, shift points with typical frequencies between 1 to 2 Hz can be used to sequentially calculate the eigenvalues in this area.

Two test systems with 20 and 50 machines respectively have been chosen to test the performance of the proposed methods and algorithm. Comparisons are also made for the two eigenvalue methods with other formerly used techniques. Some means for improving the methods as well as experience with the application of the algorithm are discussed and illustrated by numerical examples.

SOLUTION METHODS

Sparsity-Based Eigenvalue Techniques

Since the eigenanalysis of modern power systems deals with matrices of very large dimension, sparsity techniques play a key role in the analysis. A survey of the available sparsity-based eigenvalue techniques for general unsymmetrical matrices results in the following four methods:

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(a) Power method and inverse iterations

(b) Simultaneous iterations

(c) Arnoldi method

(d) Lanczos method.

The application of (a) to the eigenanalysis of power systems is reported in [4] and [5]. A proposal of using (b) on vector and array processors is presented in [10] which, however, does not contain numerical results. (d) has also been applied to this problem in [3] and [11]. We note that (a) is good only for computing one eigenvalue, or at most a few with deflation, and this is not satisfactory in most cases. (d) is a very successful method for the symmetrical eigenvalue problem, but has serious flaws in the case of unsymmetrical eigenvalue problems as, for example, the phenomenon of 'breakdown' as pointed out in [12] and also experienced by the authors (see Appendix 1 for a brief discussion of the block Lanczos method). On the other hand, as far as we know, (b) has not been tried on ordinary computers for the eigenanalysis of power systems, and (c) has never been applied to these problems, but both (b) and (c) have been used successfully in some other applications such as the partial eigensolution of stochastic matrices. Since they have generally better numerical properties, it seems that they may be the best candidates for the eigenanalysis of power systems. This is the reason why we choose them as solution methods in this study.

It is interesting to note that all four methods mentioned above belong to a class of methods known as the Krylov method [13] in which the Krylov subspace $\{ x \; Ax \; \cdots \; A^{i-1}x \}$ is used to approach the dominant invariant subspace of a matrix A. There are two important and useful features for these methods. First, they are all aimed at finding a few of the dominant eigenvalues of A (here dominance refers to largeness in modulus). This corresponds to the requirement that usually only a few of the eigenvalues are needed in the eigenanalysis of large power systems, although some transformation is necessary to make the required eigenvalues dominant. Second, in these methods the only operation involving A is the matrix-vector multiplication Ay. Therefore, it is not necessary to form A explicitly, provided that Ay can be calculated easily. This allows us to use the augmented system state equations to preserve the full sparsity of the problem.

Simultaneous Iterations

The method of simultaneous iterations was originally proposed in [14] for the symmetrical eigenvalue problem. The extension of the method to general real unsymmetric matrices is first found in [15], and then fully analyzed in [16] and in [8] which also provides a practical algorithm of the lopsided simultaneous iterations. Although the matrices dealt with in the above references are all real, the method is also applicable to general complex matrices, as demonstrated below.

Let $A \in C^{n \times n}$ have eigenvalues λ_i, with

$$|\lambda_1| \geq |\lambda_2| \geq \cdots \geq |\lambda_n|$$

and

$$\Lambda = \begin{bmatrix} \Lambda_a & 0 \\ 0 & \Lambda_b \end{bmatrix}$$

where $\Lambda_a = \text{diag}(\lambda_1 \; \cdots \; \lambda_m)$ and $\Lambda_b = \text{diag}(\lambda_{m+1} \; \cdots \; \lambda_n)$. Denote the matrix of the right eigenvectors of A by

$$Q = [\, Q_a \;\; Q_b \,] = [\, q_1 \;\; \cdots \;\; q_m \mid q_{m+1} \;\; \cdots \;\; q_n \,]$$

where q_i is associated with λ_i. Then we have

$$AQ_a = Q_a\Lambda_a \quad \text{and} \quad AQ_b = Q_b\Lambda_b \tag{1}$$

Assuming that we start with m independent trial vectors

$$U = [\, u_1 \;\; u_2 \;\; \cdots \;\; u_m \,] \in C^{n \times m}$$

perform the multiplication

$$V = AU \tag{2}$$

Since U may be represented by

$$U = Q_aC_a + Q_bC_b \tag{3}$$

where $C_a \in C^{m \times m}$ and $C_b \in C^{(n-m) \times m}$ are coefficient matrices, it is clear that

$$V = AU = Q_a\Lambda_aC_a + Q_b\Lambda_bC_b \tag{4}$$

Note that in eqn.(4) the first term is more dominant than in eqn.(3), i.e. the components of Q_b have been somehow washed out in V. To further refine the eigenvalues in Λ_a, an interaction analysis is introduced by defining

$$G = U^HU \simeq U^HQ_aC_a \tag{5}$$

and

$$H = U^HV \simeq U^HQ_a\Lambda_aC_a \tag{6}$$

where the superscript H means conjugate-transpose. Assuming that U^HQ_a is non-singular, we obtain

$$G^{-1}H \simeq C_a^{-1}(U^HQ_a)^{-1}U^HQ_a\Lambda_aC_a = C_a^{-1}\Lambda_aC_a \tag{7}$$

or, if B is the solution of

$$GB = H \tag{8}$$

then we have

$$C_aB \simeq \Lambda_aC_a \tag{9}$$

which implies that Λ_a and C_a contain the approximate eigenvalues and left eigenvectors of B. If P is the matrix of the right eigenvectors of B,

$$P \simeq C_a^{-1}$$

then

$$W = VP \simeq Q_a\Lambda_a + Q_b\Lambda_bC_bC_a^{-1} \tag{10}$$

gives an improved set of right eigenvectors of A. Taking W as the new set of trial vectors, the above process can be iterated until all required eigenvalues are found. It can be readily shown (see, for example, [16] for a similar proof of the simultaneous bi-iteration method) that this method is convergent for the first i eigenvalues of A if

$$|\lambda_i| > |\lambda_{m+1}| \tag{11}$$

for $i = 1, 2, \cdots, m$ and the convergence rate for λ_i is $|\lambda_{m+1}| / |\lambda_i|$.

Locking Device

It may be noticed that matrix G in eqn.(5) is symmetrical positive definite. Therefore, the Cholesky decomposition can be used to solve eqn.(8). Moreover, when one eigenvalue (say, the ith one) has converged, the first i rows and columns of G will not change in all subsequent calculations, and the Cholesky decomposition of G up to the ith step will also remain unchanged. Thus we can 'lock' the decomposed matrix G up to the ith row and column, and only perform the Cholesky decomposition from the $(i+1)$th step. This is the so-called 'locking device' which helps to improve the efficiency of the algorithm.

Guard Vectors

In practice, if s dominant eigenvalues of A are required, an m larger than s is usually used in this method to obtain better convergence rate and to ensure the convergence of all s eigenvalues if $|\lambda_s| = |\lambda_{s+1}|$. The additional vectors are named guard vectors in [8]. A practical question is how to decide the number of guard vectors so as to have the best computational efficiency. Unfortunately, there is no theoretical answer available. The only way to explore this is by numerical tests. Some calculations have been done on our test systems for this problem. The results are reported later, in the section on numerical results.

Fast Iteration Cycles

In [8] the idea of fast iteration cycles is introduced and proved to be very efficient for a variety of large, sparse matrices. It is basically the iteration procedure (2) with the interaction analysis omitted for a number of iterations. For power system problems, however, it seems less attractive since the multiplication in eqn.(2) is quite expensive for large systems and, more seriously, the successive multiplications will force the vectors in U to become dependent so that the matrix G is no longer positive definite, which will make the subsequent calculations very inefficient (this did happen in our test calculations). For this reason, the fast iteration cycles are not considered in our algorithm.

General Procedure

We give the following algorithm which we used in our program, as a summary of the discussions on simultaneous iterations.

(a) Set up the initial trial vectors U^1 with independent columns; let $i = 1$

(b) Calculate V^i by eqn.(2)

(c) Calculate G^i by eqn.(5) and factorize it by the Cholesky decomposition

(d) Calculate H^i by eqn.(6)

(e) Solve for B^i from eqn.(8)

(f) Perform full eigenanalysis for B^i by the QR method, obtaining the eigenvalues $\Lambda_B^i = \mathrm{diag}(\lambda^i{}_{B_1} \cdots \lambda^i{}_{B_m})$ and the associated right eigenvectors P^i

(g) Compare Λ_B^i with Λ_B^{i-1} ($\Lambda_B^0 = 0$). If all required eigenvalues have been found, exit; otherwise go on to the next step

(h) Calculate the new trial vectors U^{i+1} by eqn.(10)

(i) Let $i = i + 1$ and go to (b) to perform the next iteration

Modified Arnoldi Method

The Arnoldi method was first presented in [17]. However, because of its poor numerical properties, it was not successful before implementing several modifications to it [9]. The main problems of the original Arnoldi method are loss of orthogonality and slow convergence if a number of dominant eigenvalues is needed. The latter entails in most cases the need for the full eigenanalysis of a relatively large Hessenberg matrix, which is expensive. These problems can be solved by using the complete reorthogonalization and the iterative process described in [9].

The Arnoldi method presented in [9] is also for general real matrices. The following extends it to general complex matrices with discussions on two modifications.

Let $A \in C^{n\times n}$ and $v_1 \in C^n$ the starting vector with $\|v_1\|_2 = 1$. The subsequent orthonormal vectors are produced by the recursive formula

$$h_{i+1,i} v_{i+1} = (I - V_i V_i^H) A v_i \qquad i = 1, \cdots, m \tag{12}$$

where $h_{i+1,i}$ is chosen such that $\|v_{i+1}\|_2 = 1$, and $V_i = [\, v_1 \ \cdots \ v_i \,]$. From eqn.(12) we can obtain

$$A v_i = V_i h_i{}' + h_{i+1,i} v_{i+1} \tag{13}$$

where $h_i{}' = V_i^H A v_i \in C^i$. For all m equations assembled, eqn.(13) becomes

$$A V_m = V_m H_m + h_{m+1,m} v_{m+1} e_m^T \tag{14}$$

where $e_m^T = [\, 0 \ \cdots \ 0 \ 1 \,]$ and H_m is an upper Hessenberg matrix with the ith column equal to

$$h_i = \begin{bmatrix} h_i{}' \\ h_{i+1,i} \\ 0 \end{bmatrix} \tag{15}$$

Eqn.(14) can be approximated by dropping the second term on the right hand side. Thus,

$$A V_m \simeq V_m H_m \tag{16}$$

which implies that the eigenvalues of H_m are the approximations of the eigenvalues of A. Clearly, the error depends on $h_{m+1,m}$ which vanishes when $m = n$. In fact, as m increases, eigenvalues of H_m with largest and smallest modulus will gradually converge to the eigenvalues of A. This is the well-known property of the Lanczos method, which also holds here. The approximate eigenvectors of A can be readily found as

$$W = V_m P \tag{17}$$

where P is the $m \times m$ matrix of the right eigenvectors of H_m.

Reorthogonalization

It has been found that the original Arnoldi method as mentioned above has numerically poor behavior because of the loss of orthogonality for the vector series v_i after a number of iterations. The natural remedy for this problem is to reorthogonalize every newly-produced vector v_{i+1}. A modified Gram-Schmidt method [18] is used for this purpose, in which we simply replace eqn.(12) by the iterative process

$$u_i^{k+1} = (I - V_i V_i^H) u_i^k \qquad k = 1, 2, \cdots \tag{18}$$

with $u_i^1 = A v_i$. This process continues until

$$\frac{\|u_i^k\|_2}{\|u_i^{k+1}\|_2} \le \theta \qquad \text{with } \theta > 1 \tag{19}$$

for some $k = k_r$. Then we take

$$h_{i+1,i} = \|u_i^{k_r+1}\|_2 \tag{20a}$$

and

$$v_{i+1} = u_i^{k_r+1} / h_{i+1,i} \tag{20b}$$

In [9] a scheme of incomplete reorthogonalization was proposed. From our experience, however, it is not of great benefit because, first, by using the iterative Arnoldi method a small m is used, and second, the reorthogonalization scheme (18)-(20) is very efficient so that in most cases only one iteration is enough.

Iterative Arnoldi Method

Since the original Arnoldi method converges usually for relatively large m, the complete eigenanalysis of a Hessenberg matrix of large dimension is necessary. To reduce the order of the Hessenberg matrix, the iterative Arnoldi method is introduced. Let m be fixed at a moderate value. We perform the original Arnoldi method with reorthogonalization to obtain the eigenvalue and eigenvector approximations λ_i and w_i for $i = 1, \cdots, m$. Then we repeat the same method but using a new starting vector, for example the one recommended in [9]:

$$v_1 = \alpha \sum_{i=1}^{s} \| (A - \lambda_i I) w_i \|_2 w_i \tag{21}$$

where α is a scalar to normalize v_1 and s is the number of eigenvalues to be found. The iteration continues until all required eigenvalues are found. It can be shown (see Appendix 2) that eqn.(21) is equal to

$$v_1 = \alpha' V_m P^* \bar{p} \tag{22}$$

where α' is again a normalizing scalar, $P^* = [\, p_1 \ \cdots \ p_s \,]$ and $\bar{p} = [\, |p_{m1}| \ \cdots \ |p_{ms}| \,]^T$. Here p_i is the ith right eigenvector of H_m and p_{mi} is the last element of p_i.

With the iterative Arnoldi method we will have the problem of choosing a proper m. Recommendations are given later by the numerical results which are based on our test systems.

General Procedure

In what follows, we give the algorithm for the modified Arnoldi method in which both the complete reorthogonalization and the iterative process are used.

(a) Set up the starting vector v_1; let $V_1 = v_1$ and $i = 1$

(b) Calculate $u_i^1 = Av_i$; let $k = 1$

(c) Calculate u_i^{k+1} by eqn.(18)

(d) If the condition (19) is satisfied, go on to the next step; otherwise let $k = k + 1$ and go to (c)

(e) Calculate $h_{i+1,i}$ and v_{i+1} by eqn.(20a) and (20b)

(f) Calculate $h_i' = V_i^H A v_i$ and form h_i by eqn.(15)

(g) If $i = m$, matrix H has been formed and go on to the next step; otherwise let $V_{i+1} = [\, V_i \;\; v_{i+1} \,]$, $i = i + 1$ and go to (b)

(h) Perform full eigenanalysis for H by the QR method. If all required eigenvalues have been found, exit; otherwise go on to the next step

(i) Calculate the new starting vector v_1 by eqn.(22); let $V_1 = v_1$, $i = 1$ and go to (b)

APPLICATION TO POWER SYSTEMS

Power System Modeling

The linearized power system model for the small signal stability analysis is easy to derive (for example, see [1]). However, since the state matrix of a power system is in general not sparse, the direct construction of the state matrix would be impossible for large systems. Various schemes have been proposed to implement sparsity techniques [2], [3], [4], [5]. Here we adopt the method of [4] in which the augmented system state equations are used:

$$\begin{bmatrix} px \\ 0 \end{bmatrix} = \begin{bmatrix} J_A & J_B \\ J_C & J_D \end{bmatrix} \begin{bmatrix} x \\ V \end{bmatrix} \tag{23}$$

where x is the vector of the state variables and V is the vector of the system voltages. J_A, J_B, J_C and J_D are sparse matrices which depend on the system parameters and the operating point. It can be seen that the state matrix A may be formed from eqn.(23) as

$$A = J_A - J_B J_D^{-1} J_C \tag{24}$$

which is only of theoretical significance in this work.

Spectral Transformation

For the small signal stability analysis of power systems, two types of the eigenvalues are of special interest: the weakly-damped local mechanical modes with frequencies between 0.8 to 2.0 Hz and inter-area modes with frequencies between 0.1 to 0.6 Hz. Unfortunately, these eigenvalues are usually much smaller in modulus than other eigenvalues (for example, the fast damped local modes), so that most of the sparsity-based eigenvalue algorithms can not be applied directly. The solution of this problem is to apply a spectral transformation to the original state matrix to shift the required eigenvalues so that they become dominant in modulus. The simplest way to do this is to use the fractional transformation

$$A_t = (A - \lambda_t I)^{-1} \tag{25}$$

which transforms the eigenvalue λ_i of A to

$$\lambda_{ti} = \frac{1}{\lambda_i - \lambda_t} \tag{26}$$

where λ_t is a fixed shift. It is easy to verify that the transformation (25) transforms the eigenvalues of A within the unit circle centered at λ_t to the eigenvalues of A_t outside the unit circle at the origin. Thus, if the eigenvalues around some point λ_t (say a fixed frequency) are required, the shift λ_t can be used in eqn.(25) to magnify the eigenvalues near λ_t. Sparsity-based eigenvalue techniques can then be applied to the transformed matrix A_t to find these dominant eigenvalues.

We would like to make a short comment here for the Cayley transformation used in [3]. An advantage of the Cayley transformation is that the transformed matrix A_t (or matrix S as in [3]) remains real, while for the fractional transformation it becomes complex in general. However, this will not substantially increase the storage requirement for the latter. For example, if we consider a system with 1200 buses, 1400 lines, 300 machines and 3000 state variables, the increase of the storage is about 370 KB for double precision calculations, which can easily be handled by modern computers. On the other hand, by using the Cayley transformation all complex eigenvalues will be calculated twice for each conjugate pair, and the real eigenvalues around the origin may also have to be calculated, even if only slow oscillatory modes are of interest. Therefore, the fractional transformation can provide more flexibility and better computational efficiency.

Practical Implementation

To apply the two eigenvalue techniques described in this paper, we only need to provide the matrix-vector product Ay, or $A_t y = (A - \lambda_t I)^{-1} y$ if the transformed matrix A_t is considered. The corresponding calculation in terms of the augmented state matrix is then to solve the equation

$$\begin{bmatrix} J_A - \lambda_t I & J_B \\ J_C & J_D \end{bmatrix} \begin{bmatrix} x \\ V \end{bmatrix} = \begin{bmatrix} y \\ 0 \end{bmatrix} \tag{27}$$

for x. An algorithm for solving the above equation with the sparsity techniques is given in Appendix 2 of [4].

Numerical Results

Description of the Test Systems

Two test systems are employed in order to examine the performance of the above two sparsity-based eigenvalue techniques applied to the eigenanalysis of large power systems. In each system, a 9th order model is used for the synchronous machine and its control systems. Loads are represented by constant impedances. The first system, T77, has 77 buses, 20 machines and 180 state variables, and the second one, T169, is a 169 bus, 50 machine system with 450 state variables. Complete eigenanalyses by the QR method have been performed for both systems to make sure the results from the new methods are correct. Fig. 1 and Fig. 2 show the eigenvalue distributions of the two systems (in both figures, eigenvalues with the real parts less than -30 and with negative imaginary parts are omitted).

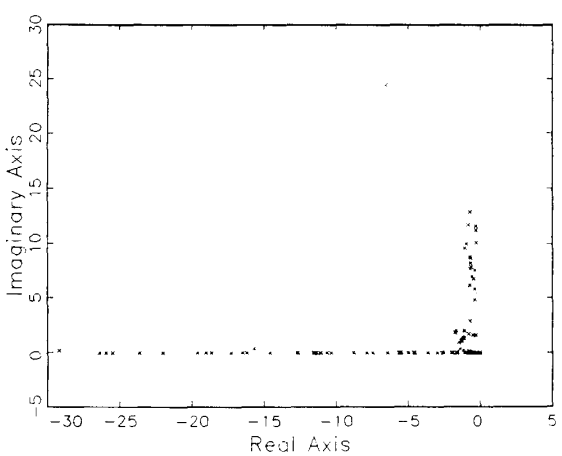

Fig. 1 – Eigenvalue Distribution of T77

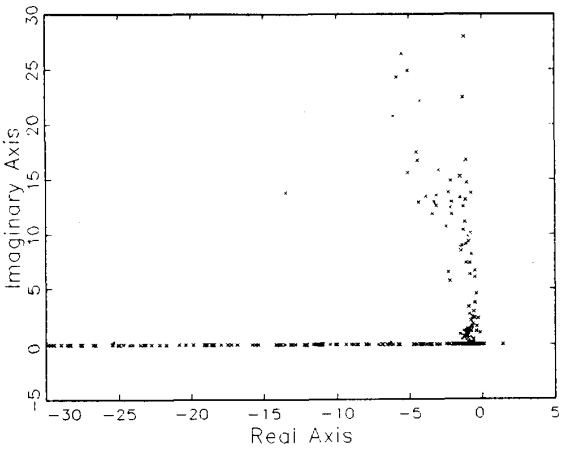

Fig. 2 – Eigenvalue Distribution of T169

The results by the QR method show that T77 is stable with all eigenvalues having negative real parts, and T169 is unstable with two unstable modes (eigenvalues): 0.089017 and 1.389242.

All calculations reported in this paper were performed with double precision (error tolerance = 10^{-8}) on an IBM4361 computer running CMS.

Guard Vectors

The effect of guard vectors in simultaneous iterations is explored mainly on T77 by using a shift $\lambda_t = -0.1+j11.0$ and sequentially calculating up to 5 eigenvalues nearest to λ_t. The 5 calculated eigenvalues are: –0.291205+j11.181740, -0.299078+ j11.558411, –0.273586+j10.069816, –0.820126+j11.686807 and –0.956709+j9.957278. The CPU times for these calculations are shown in Fig. 3 where s is the number of eigenvalues found in each calculation. It can be seen from the figure that for small s (i.e. only a couple of required eigenvalues), guard vectors are not helpful in improving the computational efficiency. For medium s, however, one guard vector seems the best. Several calculations on T169 indicate that one guard vector results in the optimal CPU time in most cases, however, two or more guard vectors are necessary to reach convergence within a proper number of iterations when (a) a large number of eigenvalues (> 5) are required and (b) the shift point λ_t is in an area where eigenvalues are densely distributed. In general, we recommend that one guard vector be used for $s < 5$, and two for $s \geq 5$. For the case when both a large number of eigenvalues are required and λ_t is in the area densely filled with eigenvalues (such as the area of inter-area modes), one should consider three or more guard vectors.

Order of the Hessenberg Matrix

To determine the proper order of the Hessenberg matrix defined in the modified Arnoldi method (i.e. the iterative Arnoldi method with complete reorthogonalization), we use the same systems as in the above section. The CPU times are given in Fig. 4 where s is again the number of eigenvalues found in each calculation, and the horizontal axis is the order of the Hessenberg matrix minus s (or the 'net additional order'). We see from the figure that the additional order is necessary to avoid divergence and/or slow convergence. The computational efficiency is basically the same over a wide range after the initial slow convergence period (i.e. after an additional order of about 4). This slow convergence period varies for different numbers of eigenvalues required and also depends on the system size. From our experience, a value of 10 to 20 for the additional order is appropriate in most cases: smaller value for fewer required eigenvalues and smaller systems.

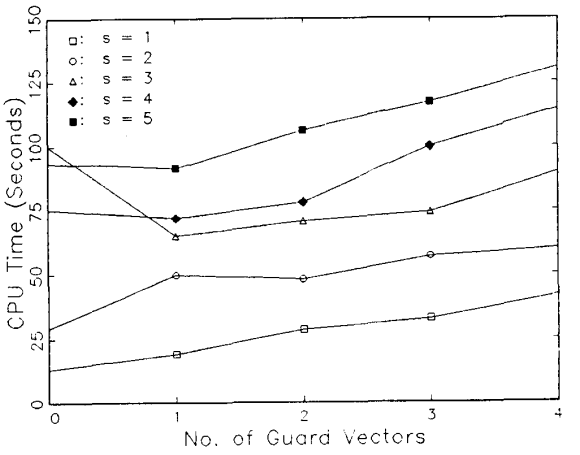

Fig. 3 – Effect of Guard Vectors

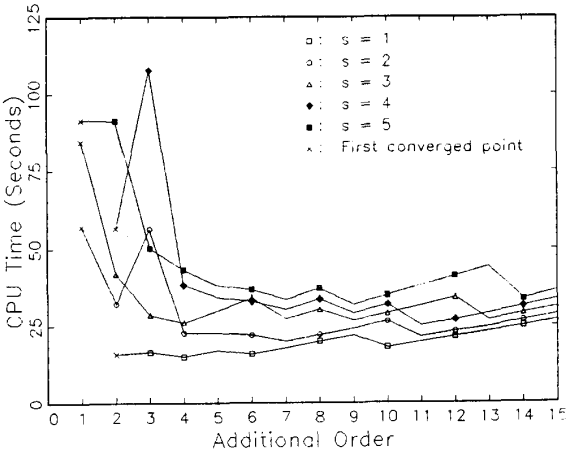

Fig. 4 – Effect of Order of the Hessenberg Matrix

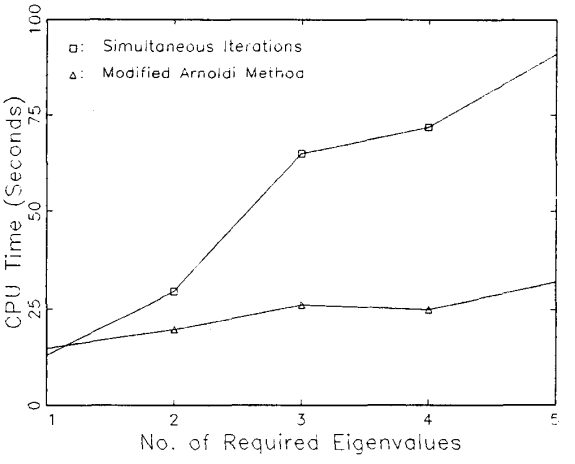

Fig. 5 – Comparison of the Two Methods

Comparison of the Two Methods

A comparison of the two methods – simultaneous iterations and the modified Arnoldi method – is made from Fig. 3 and Fig. 4 by taking the optimal CPU time for each s. The results are shown in Fig. 5. It is clear that, in this example the modified Arnoldi method has an overall faster computational speed than simultaneous iterations. This is because in simultaneous iterations the Krylov subspace is directly constructed on the basis of $\{ x \; Ax \; \cdots \; A^{i-1}x \}$, but in the modified Arnoldi method it is spanned on an orthogonal base which is superior. Therefore, in general the modified Arnoldi method is faster than simultaneous iterations at the price of more storage requirement because of the complete reorthogonalization and the additional order. Hence, for very large power systems, simultaneous iterations become more attractive than the modified Arnoldi method due to storage restrictions.

Comparisons with Other Methods

Comparisons were also attempted for the two eigenvalue methods with other formerly used methods. The inverse iterations with deflation can find up to 3 eigenvalues for the above problems within 500 iterations. However, its speed is nearly 2.7 times slower than of simultaneous iterations for the case when 3 eigenvalues are found. A block Lanczos algorithm similar to the one in [3] was tried to solve the same problems, but due to the severe numerical problems the methood did not converge. Two main difficulties were encountered: very rapid loss of orthogonality for the Lanczos vectors and 'breakdown' (see Appendix 1). The conclusion is that the methods presented here are more reliable and efficient.

Calculation of Local Mechanical Modes

In most applications of the eigenanalysis of modern power systems, two types of eigenvalues are of special interest: the weakly-damped local mechanical modes and inter-area modes. We first focus on the weakly-damped local mechanical modes which have typical frequencies of 0.8 to 2.0 Hz. For T77 we use two shift points with small negative real parts and frequencies around the two ends of the typical frequency range, i.e., $\lambda_{t1} = -0.1+j6.28$ and $\lambda_{t2} = -0.1+j12.57$. Both eigenvalue methods are applied to compute the first 10 eigenvalues for each shift point and they give the same results. Fig. 6 shows the distribution of the complex eigenvalues of T77, in which the eigenvalues within each of two circles are found in the calculations. Since there are two eigenvalues within the intersection of the two circles, we consider that all required eigenvalues have been found. From the figure we see that indeed all local mechanical modes with frequencies from 0.76 to 2.05 Hz have been circled. For T169, we can repeat the whole procedure, as shown in Fig. 7. However, with two shift points, we see that there is a frequency gap in which no eigenvalue has been found (this can be seen by noting that the highest frequency in the λ_{t1} circle is 1.3 Hz and the lowest frequency in the λ_{t2} circle is 1.65 Hz, so the frequency gap is 0.35 Hz). Therefore, we add an additional shift point $\lambda_{t3} = -0.1+j9.42$ in the middle between λ_{t1} and λ_{t2} and compute the first 10 eigenvalues near λ_{t3}. Now we see from Fig. 7 that the three circles cover most of the local mechanical modes with frequencies from 0.6 to 2.34 Hz, and the omitted local mechanical modes are all fast-damped ($\xi \geq 0.22$).

Calculation of Inter-area Modes

We now turn to the second type of interesting eigenvalues – the weakly-damped inter-area modes which have typical frequencies of 0.1 to 0.6 Hz. We use again two shift points with small negative real parts and frequencies of 0.2 and 0.5 Hz (here a slightly higher frequency of 0.2 Hz is used to avoid as much as possible the convergence to real eigenvalues). The results for both test systems are in Fig. 8 and Fig. 9 respectively, which are magnified from Fig. 6 and Fig. 7 to show the portion containing the inter-area modes. Both figures clearly indicate that two circles have covered the most weakly-damped inter-area modes, so that our purpose is fulfilled.

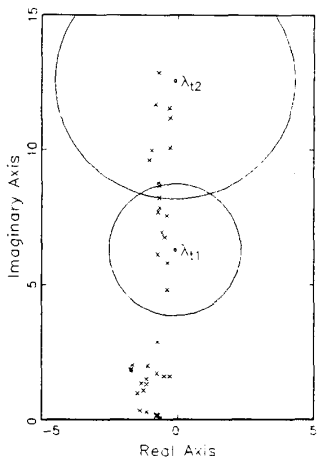

Fig. 6 – Oscillatory Modes of T77

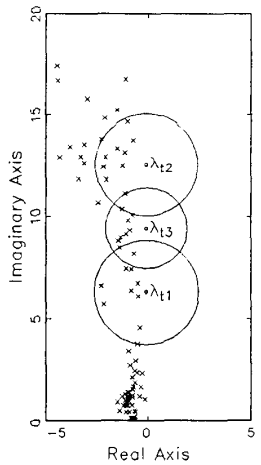

Fig. 7 – Oscillatory Modes of T169

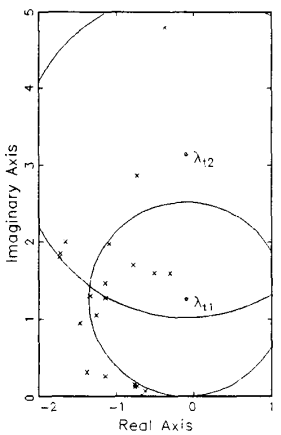

Fig. 8 – Inter-area Modes of T77

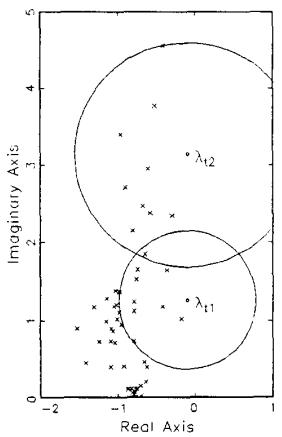

Fig. 9 – Inter-area Modes of T169

Calculation of Unstable Modes

If a power system is unstable due perhaps to improper parameters or the chosen operating point, it is desirable to calculate the unstable modes of the system. Basically, we can use the same procedure as described in the above two sections, except that the shift points should be placed in the right half of the complex plane with positive real parts and appropriate frequencies. Real unstable modes can be found by simply using a real shift point. For example, the two real unstable modes of T169 can easily be obtained by using the shift point $\lambda_t = 1.0$. Note that if a system contains unstable local mechanical modes and/or unstable inter-area modes, the procedure in the above two sections can probably find them since from Fig. 6 to 9 we see that each circle covers also portions of the unstable area. Therefore, the calculation of unstable modes can be considered to some extent as part of the calculation of the local mechanical and inter-area modes.

CONCLUSIONS

The paper has presented two sparsity-based eigenvalue techniques – simultaneous iterations and the modified Arnoldi method – and their application to the small signal stability analysis of large power systems. From the principles of the methods and the numerical results reported, the following conclusions can be drawn:

(a) Both methods have reliable convergence characteristics and are successful in the eigenanalysis of large power systems

(b) Generally speaking, the modified Arnoldi method is faster than simultaneous iterations, while the latter needs less storage space

(c) The algorithm based on the methods is suitable for calculating a specific group of eigenvalues (including multiple ones) such as the weakly-damped local mechanical modes and inter-area modes. In particular, the fractional transformation is very efficient in shifting the required eigenvalues to permit application of the above methods

(d) The program can be fully automated since it does not require any initial guess of eigenvalues. The selection of the nature of desired eigenvalues is achieved by the placement of the shift points

(e) Since the methods and the associated algorithm do not have any restriction in the modeling of power system components, any detailed models can be implemented in the program to make the results more practical.

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APPENDICES

Appendix 1 – Block Lanczos Algorithm

Let $A \in C^{n \times n}$, and $X_1 \in C^{n \times r}$ and $Y_1 \in C^{n \times r}$ the starting matrices, with the conditions that the columns in both X_1 and Y_1 are independent and $\det(Y_1^H X_1) \neq 0$. The following recursive formula produces the bi-orthogonal matrix series $\{ X_1, X_2, \cdots, X_{k+1} \}$ and $\{ Y_1, Y_2, \cdots, Y_{k+1} \}$:

$$X_{k+1} = AX_k - X_k\alpha_k - X_{k-1}\beta_{k-1} \qquad \text{(A-1a)}$$

$$Y_{k+1} = A^T Y_k - Y_k\alpha'_k - Y_{k-1}\beta'_{k-1} \qquad \text{(A-1b)}$$

where,

$$\alpha_k = (Y_k^H X_k)^{-1} (Y_k^H A X_k)$$

$$\beta_{k-1} = (Y_{k-1}^H X_{k-1})^{-1} (Y_{k-1}^H A X_k)$$

$$\alpha'_k = (X_k^H Y_k)^{-1} (X_k^H A^T Y_k)$$

$$\beta'_{k-1} = (X_{k-1}^H Y_{k-1})^{-1} (X_{k-1}^H A^T Y_k)$$

In the above equations, T means transpose and H means conjugate-transpose. The bi-orthogonal condition implies that

$$Y_k^H X_l = 0 \qquad \forall\, k \neq l \tag{A-2}$$

while

$$\det(Y_k^H X_k) \neq 0 \tag{A-3}$$

From eqn.(A-1a), we obtain

$$A[X_1\ X_2\ \cdots\ X_k] = [X_1\ X_2\ \cdots\ X_k]T_k + [0\ \cdots\ 0\ X_{k+1}] \tag{A-4}$$

where

$$T_k = \begin{bmatrix} \alpha_1 & \beta_1 & & & & \\ I & \alpha_2 & . & & & \\ & I & . & . & & \\ & & . & . & . & \\ & & & . & . & \\ & & & & \alpha_{k-1} & \beta_{k-1} \\ & & & & I & \alpha_k \end{bmatrix}$$

So if $X_{k+1} = 0$, eqn.(A-4) is reduced to

$$A[X_1\ X_2\ \cdots\ X_k] = [X_1\ X_2\ \cdots\ X_k]T_k \tag{A-5}$$

i.e. the columns of $[X_1\ X_2\ \cdots\ X_k]$ define an invariant subspace for A. Eqn.(A-4) can also be approximated by eqn.(A-5) if X_{k+1} is small, and then some iterative procedure (such as the one used in the modified Arnoldi method) can be used to refine the dominant eigenvalues of A. We note that $r = 2$ in the above algorithm corresponds to the method in [3].

There exist two main problems in the above block Lanczos algorithm. First, the bi-orthogonality which is fundamental for the algorithm is very rapidly lost on a finite-precision computer due to the round-off errors. The complete reorthogonalization for overcoming this problem needs both matrix series $\{X_1, X_2, \cdots, X_k\}$ and $\{Y_1, Y_2, \cdots, Y_k\}$. Second, if $Y_{k+1} = 0$, or $\det(Y_{k+1}^H X_{k+1}) = 0$ (this is the so-called breakdown), then the algorithm is terminated without giving any information about the invariant subspace of A. The algorithm must be restarted with the new starting matrices, but it is still uncertain whether or not the breakdown will occur again. In this sense, the Lanczos method for the unsymmetrical eigenvalue problem is not reliable, and also not economical if complete reorthogonalization is going to be used.

Appendix 2 – Derivation of Equation (22)

We proceed with equation (21):

$$v_1 = \alpha \sum_{i=1}^{s} \| (A - \lambda_i I)\, w_i \|_2 w_i \tag{A-6}$$

Since by eqn.(17)

$$w_i = V_m p_i \tag{A-7}$$

and by definition

$$(H_m - \lambda_i I) p_i = 0 \tag{A-8}$$

we have

$$\begin{aligned} \| (A - \lambda_i I) w_i \|_2 &= \| (A V_m - \lambda_i V_m) p_i \|_2 \\ &= \| (V_m H_m + h_{m+1,m} v_{m+1} e_m^T - \lambda_i V_m) p_i \|_2 \\ &= | h_{m+1,m} |\, | p_{mi} | \end{aligned} \tag{A-9}$$

where $| p_{mi} | = | e_m^T p_i |$. Let $\alpha' = \alpha / | h_{m+1,m} |$ (if $| h_{m+1,m} | = 0$, then all λ_i and w_i are the exact eigenpairs of A). Substituting eqn.(A-9) into eqn.(A-6) yields

$$v_1 = \alpha' \sum_{i=1}^{s} | p_{mi} |\, V_m p_i = \alpha' V_m P^* \bar{p} \tag{A-10}$$

where $P^* = [p_1\ \cdots\ p_s]$ and $\bar{p} = [\, | p_{m1} |\ \cdots\ | p_{ms} |\,]^T$. Eqn.(A-10) is the same as eqn.(22).

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