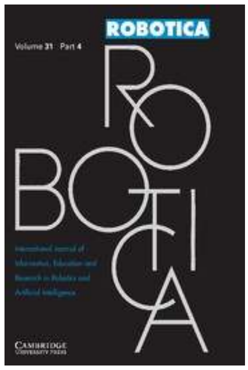

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Robotica / Volume 31 / Issue 04 / July 2013, pp 623 - 641
DOI: 10.1017/S0263574712000616, Published online: 16 November 2012

Robotica (2013) volume 31, pp. 623–641. © Cambridge University Press 2012
doi:10.1017/S0263574712000616

Application of statistical techniques in modeling and optimization of a snake robot

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(Accepted September 19, 2012. First published online: November 16, 2012)

SUMMARY

This paper provides a general framework based on statistical design and Simulated Annealing (SA) optimization techniques for the development, analysis, and performance evaluation of forthcoming snake robot designs. A planar wheeled snake robot is considered, and the effect of its key design parameters on its performance while moving in serpentine locomotion is investigated. The goal is to minimize energy consumption and maximize distance traveled. Key kinematic and dynamic parameters as well as their corresponding range of values are identified. Derived dynamic and kinematic equations of n-link snake robot are used to perform simulation. Experimental design methodology is used for design characterization. Data are collected as per full factorial design. For both energy consumption and distance traveled, logarithmic, linear, and curvilinear regression models are generated and the best models are selected. Using analysis of variance, ANOVA, effects of parameters on performance of robots are determined. Next, using SA, optimum parameter levels of robots with different number of links to minimize energy consumption and maximize distance traveled are determined. Both single and multi-criteria objectives are considered. Webots and Matlab SimMechanics software are used to validate theoretical results. For the mathematical model and the selected range of values considered, results indicate that the proposed approach is quite effective and efficient in optimization of robot performance. This research extends the present knowledge in this field by identifying additional parameters having significant effect on snake robot performance.

KEYWORDS: Snake robot; DOE; Serpentine gait; Dynamic; Energy; Distance; Regression; Optimization; Simulated annealing.

1. Introduction

Snake robots were first introduced by Hirose.[1] Many snake robots have been designed since the initial study done by Hirose.[1] Existing snake robot designs have different physical configurations and purpose. They mostly attempt to mimic locomotion of real snakes; however, some use non-snake-like gaits.[7–10] For example, some snake robots use powered wheels or tank treads, while others use passive wheels or no wheels. Some designs are able to travel on ground or in water environments and some can travel in both environments. Such capabilities enable snake robots to have many applications, from search and rescue missions to inspection and exploration.

Many researchers have attempted to identify body curves of real snakes.[1] However, Hirose[1] offered a new curve called serpenoid curve and proved that this curve is the most similar curve to the snake body. Hirose[1] examined the snake robot from the biological viewpoint and attributed the path that the snake moves to the serpenoid curve. He also showed that by applying torques on joints, the snake robot could develop a progressive force for forward motion. Dowling[2] used a table look-up method to determine locomotive patterns. He made the suggestion of using Fourier series coefficients as parameters for the functional form of the structure's body. Ostrowski[3] studied snake robot locomotion using geometric mechanics. Kinematic constraints were used and three types of gait locomotions were demonstrated. Ma[4] studied creeping motion of real snakes and proposed the serpentine curve; he showed that this curve better approximates real snake locomotion than previously suggested curves. Ma *et al.*[5] also studied the locomotion of snake-like robot along symmetrical and unsymmetrical serpenoid curves on inclined surfaces. The effect of unsymmetrical parameter on the required joint torques was also investigated and optimal parameters for creeping locomotion were obtained. Saito *et al.*[6] studied snake-like robot kinematic equations using Lagrange's method. In his study, they examined snake robot using both viscous and coulomb friction models. Hasanzadeh and Akbarzadeh[7,8] presented a novel gait, forward head serpentine (FHS), for a 2D snake robot. They used Genetic Algorithm to find FHS gait parameters and performed experiments to validate their results. Relationship between FHS gait parameters and friction coefficients of the ground were also developed. Akbarzadeh and Kalani[9,10] used serpenoid curve and obtained the dynamics of a worm-like locomotion. Motion stability was also investigated. Nilsson[14] showed that unlike a common view, serpentine locomotion also occurs on surfaces with uniform friction. He demonstrated this both theoretically and experimentally. Liljebäck *et al.*,[15] for the first time, presented a set of fundamental properties of the velocity of a snake robot moving in serpentine locomotion. They showed relationship between average forward velocities of a snake with some of the serpentine motion

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parameters. Nakhaee Nejad *et al.*[16–18] developed dynamics of a planar snake robot with and without wheels in serpentine locomotion on both flat and inclined surfaces. Since solving dynamic equations was time-consuming, they used Nueral Network and performed real-time optimal speed control of snake robot. Chirikjian and Burdick[20] presented a framework for kinematics and motion planning of snake robots. They modeled the snake-inspired robot as a continuous backbone curve, and analyzed the kinematics of gaits that used both "stationary," similar to inchworm locomotion, and "traveling" waves, similar to rectilinear snake locomotion. The model does not consider the dynamics of the system.

Many researchers have incorporated statistical techniques to investigate product design and characterization. Hu *et al.*[21] presented experimental and theoretical investigations of slithering of snakes. They observed kinematics of snake locomotion experimentally, measured the friction coefficients of snakeskin and presented a theoretical model for slithering locomotion. Tesch *et al.*[22] showed that the response surface methodology could be effectively used to maximize expected improvement by choosing subsequent experiments in optimization problems involving real snake-like robots. They applied this technique to optimize open-loop gait parameters for snake robots and showed improved locomotive capabilities. This method enables optimizing existing gaits for snake robots. Rout and Mittal[23] considered parametric design and performance optimization of a 2-Degree of Freedom (DOF), two revolute joints (R–R), planar manipulator. They used Design of Experiment (DOE) approach and Analysis of Variance (ANOVA) technique to identify statistical significance of six kinematic and dynamic parameters (mass, link length, and torque for each link) on performance of a manipulator. In their next study, additional kinematic and dynamic factors were considered and the fractional factorial DOE approach was used to screen the factors influencing performance of a manipulator.[24] Wu *et al.*[26] applied the Taguchi DOE technique to determine and optimize a robot's accuracy and repeatability at different operational factor settings. Seven operational factors (load, speed, design, orientation, direction, height, and starting point) were tested for significance using this technique, and robot process capability for path following was determined.

The contribution of this paper is a set of fundamental performance properties of a planar snake robot traveling with serpentine locomotion that are useful in snake robot design and potentially control algorithms. In particular, this paper identifies a practical range of values for key kinematic and dynamic parameters and contributes by (1) providing a general framework based on statistical design and SA optimization techniques for the development, analysis, and performance evaluation of forthcoming snake robot designs; (2) obtaining regression models for both energy consumption and distance traveled as a function of key kinematic and dynamic parameters; (3) providing statistical significance testing of key kinematic and dynamic parameters; (4) confirming existing broad knowledge as reported by Hopkins *et al.*[13] that (a) link length should be maximized, (b) number of segments should be maximized, and (c) weight of each link does not affect distance traveled but significantly effects energy consumption, and hence it should be minimized; and (5) identifying that (a) changes in initial winding angle significantly affect the distance as well as energy consumption. These relationships are positive, and (b) changes in the number of waves significantly affect distance traveled and energy consumption. These relationships are positive for energy consumption and mostly negative for distance traveled. Finally, the derived dynamic equations are verified using both Webots[31] and Matlab SimMechanics. To the best of authors' knowledge, findings of this paper, i.e., the above items 1–5 and parts of 4, have never before been reported before. It should also be noted that the results obtained in this paper might not apply to snake robots in general. This is because a specific mathematical model and a specific range for key kinematic and dynamic parameters are considered. See end of Section 9 for more detailed discussion on this subject.

2. Organization

This paper is organized as follows. First, in Section 3, serpentine locomotion is discussed and kinematics and dynamics of an n-link snake robot traveling using this gait is briefly shown. In Section 4, key parameters affecting robot performance are identified. In Section 5, statistical design methodologies are applied and two regression models, energy consumption and distance traveled, are generated. In Section 6, the SA algorithm and regression equations are used to find optimum parameter settings. This section also investigates snake models having specific number of links, i.e., 8, 12, and 16. In Section 7, Webots software[31] is used to validate obtained theoretical results. In Section 8, results of DOE are compared with the performance of existing snake robots. Finally, in Sections 9 and 10, concluding remarks are made. The details of the study presented in Sections 5 and 6 and DOE and SA algorithm are shown in Fig. 1.

3. Serpentine Locomotion

Serpentine locomotion, also known as lateral undulation, is the most frequently used form of snake locomotion that real snakes use on ground and in water environments. Hirose[1] showed that the key property of snakes in mimicking serpentine locomotion is the difference in friction coefficients for tangential and normal directions with respect to the body. This concept is necessary and important in serpentine movement. He accomplished this by placing small wheels at the bottom of snake robot's links. Therefore, the friction coefficients for the tangential direction will be higher than for normal directions (see Fig. 2).

3.1. The definition of snake's body curve

Researchers have defined certain curves to fit the body curve of snakes. However, Hirose[1] showed that serpenoid is the best curve for mimicking snakes' motion. He demonstrated that locomotion can be accomplished by propagating a wave in the form of serpenoid throughout snake's body. The curvature function,[5,6] for a serpenoid curve is defined as follows:

$$\rho(s) = \frac{-2K_n\pi\alpha}{L}\sin\left(\frac{2K_n\pi s}{L}\right), \tag{1}$$

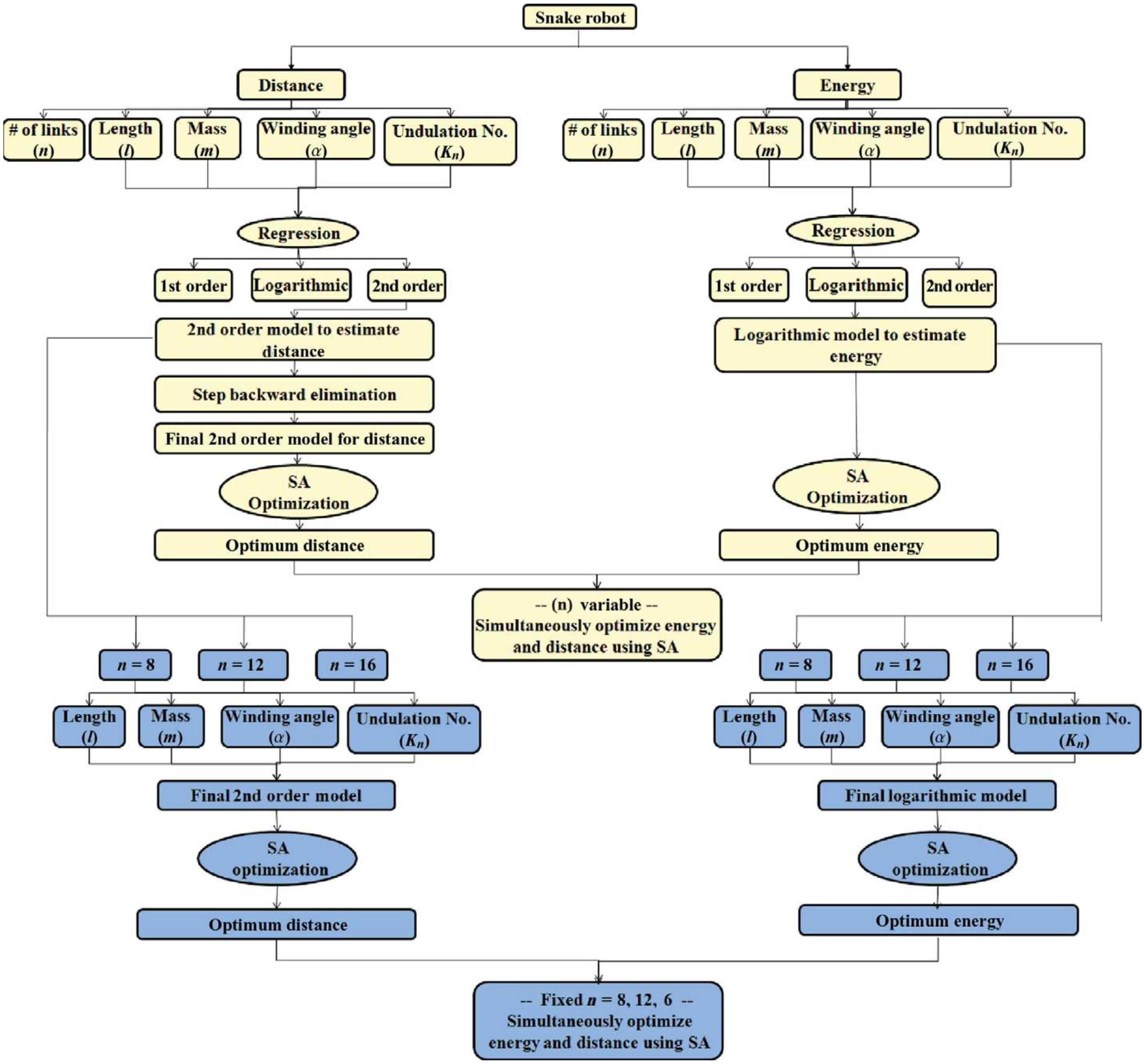

Fig. 1. (Colour online) Presentation of the paper.

where L is the total length of snake robot, K_n is the number of undulations, and α is the initial winding angle. In addition, parameter s is a measure of distance along the serpenoid curve at time t with respect to the fixed reference state at time t_0. Parameter s is also referred to as body length along the body curve as well as the arc length of the backbone curve. Figure 3 shows a fixed reference frame, the serpenoid curve, and the snake robot traveling along the serpenoid curve.

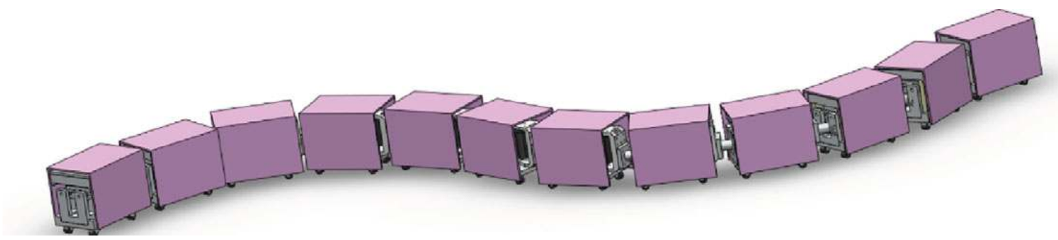

Fig. 2. (Colour online) Placing wheels at the bottom of a snake robot for serpentine locomotion.

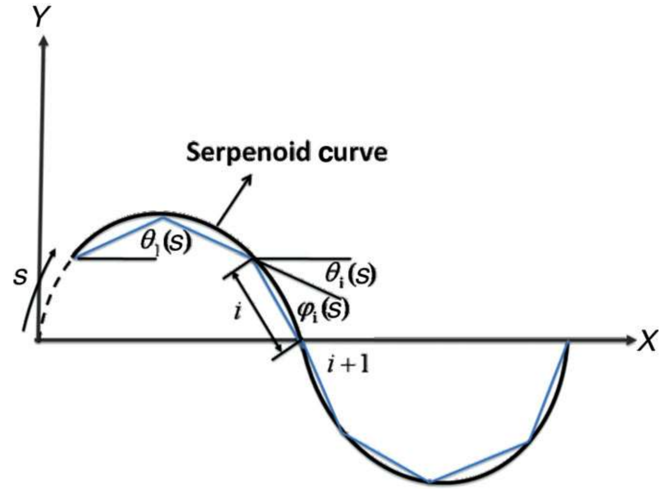

Fig. 3. (Colour online) Scheme for fitting the serpenoid curve.

We can write

$$s = \frac{1}{\rho}\varphi \rightarrow d\varphi = \rho ds \rightarrow \varphi = \int_{s+(i-1)l}^{s+il} \rho\,(u)\,du$$
$$= \int_{s+(i-1)l}^{s+il} \frac{-2K_n\pi\alpha}{L}\sin\left(\frac{2K_n\pi u}{L}\right)du. \quad (2)$$

After simplifying, the relative angles are obtained as

$$\varphi_i\,(s) = -2\alpha\sin\left(\frac{K_n\pi}{n}\right)$$
$$\times\sin\left(\frac{2K_n\pi s}{L} + \frac{2K_n\pi i}{n} - \frac{K_n\pi}{n}\right), \quad (3)$$

where n is the number of links and l is the unit length of a link. According to Fig. 3, relation between relative angle and absolute angle, θ_i, is obtained as

$$\theta_i = \theta_1 + \sum_{k=1}^{i-1}\varphi_k. \quad (4)$$

Relative and absolute angular velocity and angular acceleration may be obtained by direct differentiation of Eqs. (3) and (4) respectively.

3.2. Kinematic and dynamic models

A planar snake robot consisting of n-links connected through $n - 1$ joints is depicted in Fig. 4. Each link is rigid with uniformly distributed mass and is equipped with a torque actuator (motor). Each link is of mass m_i, length l_i, and moment of inertia I_i. Let (x_{ci}, y_{ci}) and θ_i define the center of gravity and the angle between each link and the x-axis respectively. Values of d_i represent distance from beginning of ith link to its mass center. The coordinates of tail link end are represented by (x_b, y_b). Then, the position and velocity of any point along the snake robot body can be simply calculated. Specifically, for the end of each link we can write,

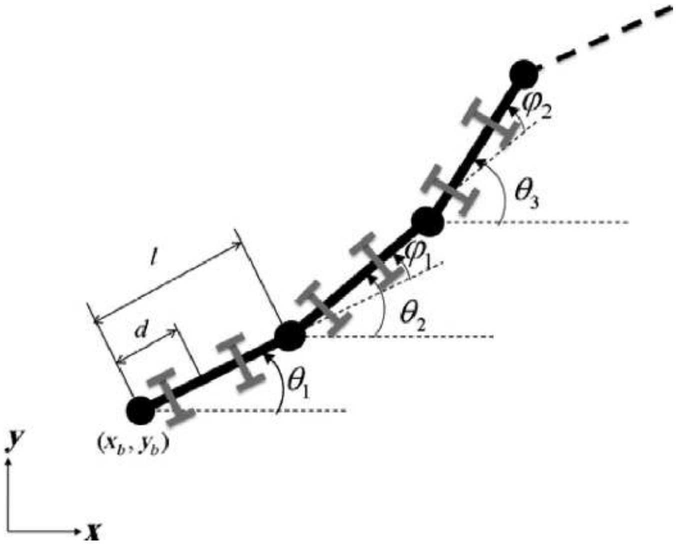

Fig. 4. Schematic of a snake robot.

$$(x_i, y_i) = \left(x_b + \sum_{j=1}^{i-1} l_j\cos\theta_j,\ y_b + \sum_{j=1}^{i-1} l_j\sin\theta_j\right), \quad (5)$$

where $j = 1,\ 2,\ \ldots,\ n$. Similarly, position of ith link of the center of gravity is obtained as follows:

$$(x_{ci}, y_{ci}) = \left(x_b + \sum_{j=1}^{i-1} l_j\cos\theta_j + d_i\cos\theta_i,\ y_b\right.$$
$$\left. + \sum_{j=1}^{i-1} l_j\sin\theta_j + d_i\sin\theta_i\right). \quad (6)$$

By specifying the position and the absolute angle of the tail link and actuated motor commands, the configuration of the snake robot is determined. In addition, velocity and acceleration of the end and center of gravity of each link can be obtained by direct differentiation of Eqs. (5) and (6) respectively. The generalized coordinates are selected as

$$q_j = [\theta_1, \theta_2, \theta_3, \ldots, x_b, y_b]. \quad (7)$$

Also, the Lagrangian formula is defined as

$$\frac{d}{dt}\left(\frac{\partial K}{\partial q_i}\right) + \frac{\partial K}{\partial q_i} - Q_i^{nc} = 0\ (i = 1,\ 2,\ \ldots,\ n + 2), \quad (8)$$

where K is the kinetic energy, Q_i^{nc} are the non-conservative forces, and V is the potential energy. Actuator torques and frictional forces are the non-conservative forces. Note that in this locomotion the potential energy is zero. By solving the Lagrangian formulation, i.e., Eq. (8), the dynamic model for snake robot is derived,[7,8]

$$B\tau = M\,(\theta)\,\ddot{q}\, + H\left(\theta, \dot{\theta}\right) + F(\theta), \quad (9)$$

where $M(\theta)$ is the $(n+2) \times (n+2)$ positive definite and symmetric inertia matrix, $H(\theta, \dot{\theta})$ is the $(n+2) \times 1$ matrix related to centrifugal and Coriolis terms, $F(\theta)$ is the $(n+2) \times 1$ matrix related to frictional forces, and B is the $(n+2) \times (n-1)$ constant matrix. τ is the $(n-1) \times 1$ matrix of input torques and $q, \dot{q}$, and $\ddot{q}$ are the $(n-1) \times 1$ matrices of generalized coordinates and their respective derivatives. $\theta, \dot{\theta}$, and $\ddot{\theta}$ are the $n \times 1$ matrices of links of absolute angles and their respective derivatives. The dynamic model is coded in Matlab software and verified using Matlab SimMechanic. See Appendix B.

Subject of dynamics with environment constraints is the calculation of interaction forces between the environment and the snake body.[1,6,15,19] A key property of biological snakes is related to the ground friction. What causes forward direction propulsion is that friction in the normal direction is much higher than in the tangential direction. In snake-like robots traveling in serpentine locomotion, the difference in friction is achieved by placing wheels under the robot. In this paper, a simple anisotropic Coulomb friction model[7] for the interaction of snake robot with the ground is used as

$$f_{ei} = -m_i g \mu_e sign(v_i^e), \tag{10}$$

where $e = t$, n (t and n represent tangential and normal directions respectively), g is the gravity constant, and μ_t and μ_n are tangential and normal Coulomb friction coefficients respectively. Suffix i corresponds to the ith link, f_{ti} and f_{ni} are friction forces in tangential and normal directions respectively, and v_i^t and v_i^n are the velocities of the center of mass of the ith link. The signum function is denoted by $sign(x)$, i.e., $sign(x)$ i = 1 if $x > 0$, 0 if $x = 0$, and –1 if $x < 0$. The friction coefficients of the Coulomb friction model in this paper are selected as $\mu_t = 0.1$ and $\mu_n = 0.55$.

The power consumption E can be calculated using Eq. (11) as

$$E = \sum_{1}^{n} \int_{0}^{T} |\tau_i \omega_i| \, dt, \tag{11}$$

where T is the simulation time, and τ_i and ω_i represent the torques and the absolute velocity angle of the ith link respectively.

4. Parameter Selection and Initial Design

The goal of this paper is to find optimum key design parameters to reduce power consumption and increase distance traveled. Referring to the dynamic equation, i.e., Eq. (9), the key parameters affecting the performance are s, K_n, α, l, m, n, coefficient of tangential μ_t, and coefficient of normal friction μ_n. To perform a DOE, an allowable range for each parameter must be identified. In addition, initial values for each of the parameters must be determined. The initial values determine the best possible initial snake robot design that a designer will select based on the best available scientific knowledge. Normally, this design represents the first prototype and requires further tests and optimization once the snake is actually constructed. The purpose of the Design for Six Sigma (DFSS), which includes application of DOE to product design, is to build robustness and performance into the initial product design.[24,25,27,28] Once DOE and subsequent optimization is completed, the initial and final design structures will be simulated and distance traveled and energy consumption will be calculated. The comparison of the values for before and after optimization will determine the percentage improvement. In this paper all simulations are performed in a fixed amount of time. Therefore, by optimization of the distance traveled, essentially the average speed of the locomotion is optimized.

Parameter s: Among the key parameters specified by the user, s directly affects robot speed. Higher the s, higher the speed. As stated earlier, parameter s is a measure of the displacement of tail along the serpenoid curve at time t. This displacement when measured in time determines the frequency of the snake body curve. Then the frequency changes as parameter s changes. Consequently, parameter s affects the robot speed. To get a better sense of the parameter s, assume that snake robot sits still in a tube shaped like a serpenoid curve. Next, imagine one pushes the snake forward in this tube. Then parameter s determines the speed in which the snake travels through this tube. Aside from effecting the speed, s does not have any effect on snake's physical shape. In this study s is selected to be s = 0.4 t. Therefore, excluding the dynamic effects, after 1 sec, we would expect the snake robot to travel about 0.4 m along the tube.

Module mass (m): The optimal choice for this parameter is clear, the less the better. Any undue increase represents inefficiency in design. To determine the initial value for m, first, Dynamixel AX-12A DC motor from Robotis is selected. Using the SolidWorks software the minimum module weight plus motor is calculated to be 0.28 kg. Therefore, 0.28 kg was selected for the initial design. To specify a range for this parameter, 0.28 kg and 0.32 kg were selected as the lowest and the highest values respectively.

Module length (l): Given a joint angle displacement for any link, center of mass of the link with the higher length is more displaced. This simple fact motivates us to select as high a value for length as possible. On the other hand, increase in length negatively effects link mass and required motor torques, and results into snake robot that does not resemble natural form of real snakes. Based on the size of the Dynamixel AX-12 motor, the initial module length is estimated to be 0.1 m. However, it is mechanically possible to further decrease module length to 0.08 m. Therefore, the mid value of 0.09 m is selected for the initial design (see Fig. 5).

Initial winding angle (α): The effect of this parameter on distance traveled and energy consumption is not known. Consider Fig. 6. As α increases, the body curve contracts. A design that is neither too contracted nor too stretched better represents a natural body form of a real snake. Therefore, a value of $\alpha = 0.5$ rad was selected as an initial design. To specify a range for this parameter, 0.4 and 0.6 were selected as the lowest and the highest values respectively.

Number of wave (K_n): The effect of this parameter on distance traveled and energy consumption is also unknown. Consider Fig. 6. As K_n increases, width of the snake body remains unchanged, while its height decreases. A high value

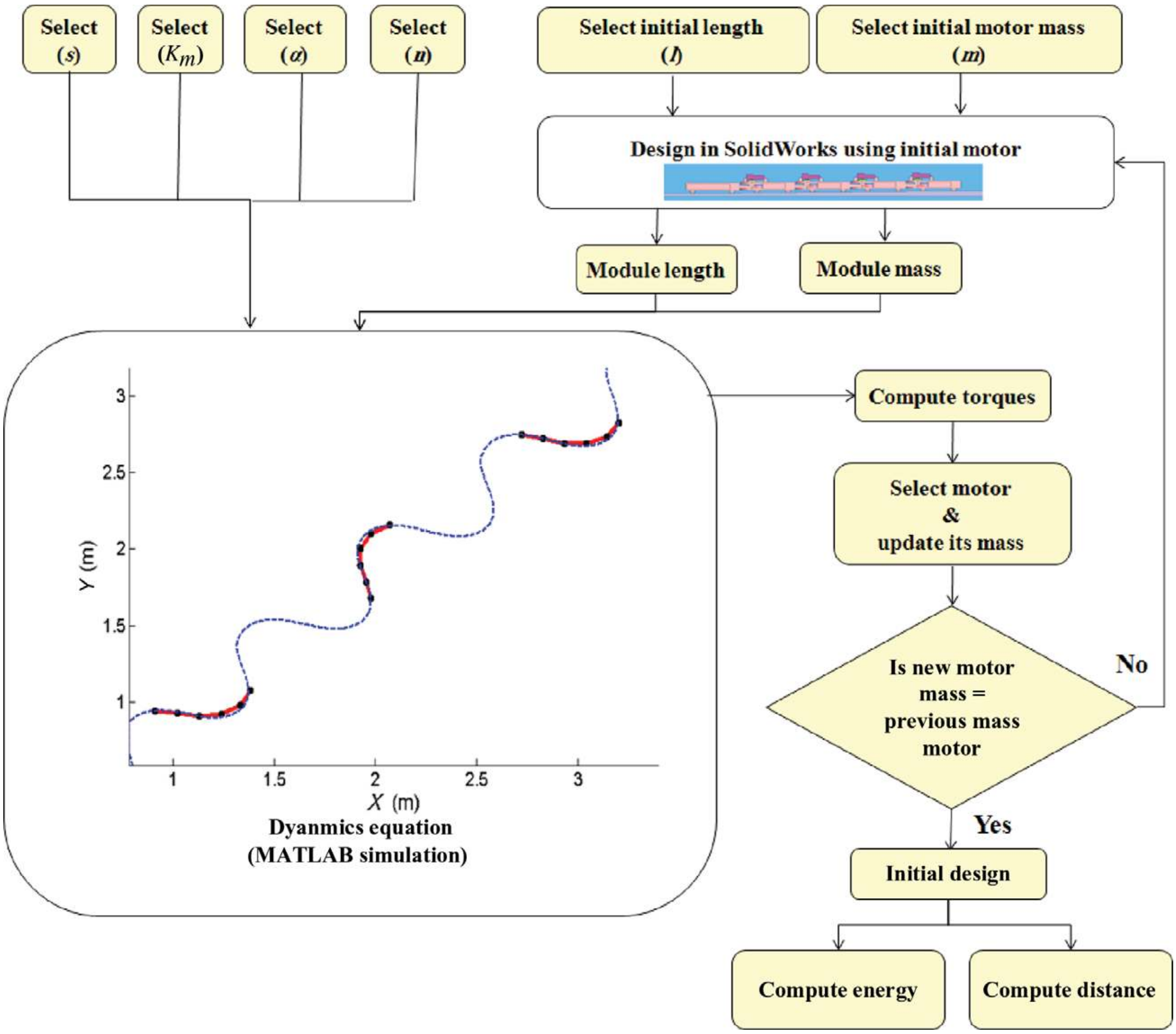

Fig. 5. (Colour online) Initial design flowchart.

for body curve does not represent most natural form of a real snake body. In this study, considering the range of parameters selected for n and α, a value of $K_n = 1$ to 3 with its midpoint of $K_n = 2$ were selected as range and the initial design value respectively.

Number of links (n): The number of links is another critical parameter affecting the performance of a snake robot. Real snakes are made of 200–400 vertebrae that are connected by joints. Clearly, as the number of links increases and link length decreases, a snake robot better resembles a real snake. However, the number of links does not influence the overall shape of a snake robot. A recent comprehensive study shows that the average number of links for existing snake robots is about seven, with maximum number being 20 links for the ACM III robot.[11,13] Consequently, the effect of 8, 12, and 16 links is chosen for this study.

The simultaneous effect of parameters, number of links, winding angle, and undulation number are graphically shown in Fig. 6. When in serpentine gait, a snake moves all of its vertebrae in a sinusoidal pattern. Therefore, body shapes that have sawtooth patterns or are either too contracted, or too stretched are not desirable. These criteria are used to graphically evaluate relations shown in Fig. 6. Results are summarized in Table I.

Table I. Relation between n, α, and K_n.

			K_n	
			High	Low
n = High	α	High	Not desirable	Not desirable
		Low	Desirable	Desirable
n = Low	α	High	Not desirable	Not desirable
		Low	Not desirable	Desirable

As can be seen in Table I, (1) with lower value for n, the best selections are lower values for K_n and α, (2) with higher value for n, best selection regardless of the value for K_n, is lower value of α.

These results also confirm our selected range for parameters K_n, α, and n. The selected initial settings for the key design parameters are tabulated in Tables II and III. Next, DOE is applied to identify relations between input and output parameters.

Table II. Selected parameters and their levels.

No.	Factor	Symbol	Units	Initial settings	Level 1	Level 2	Level 3
1	Length	l	M	0.09	0.08	0.09	0.1
2	Angle	α	Rad	0.5	0.4	0.5	0.6
3	Mass	M	kg	0.28	0.28	0.30	0.32
4	Number of waves	K_n	NA	2	1	2	3
5	Number of links	n	NA	8	8	12	16

5. Design of Experiment (DOE)

In a designed experiment, the engineer makes deliberate changes in the input variables to evaluate the changes in the output variables. Using this approach, the number of needed experiments reduces significantly. Table II shows the selected parameters and their levels.

The remaining parameters needed for the simulation are shown in Table III.

Table III. Simulation parameters.

s (m)	0.4 t
Simulation time, t (sec)	20
Terrain properties	
Cof. of tangential friction, μ_t	0.1
Cof. of normal friction, μ_n	0.55

Considering three levels for each of the five input parameters, 3^5 or 243 experiments are needed to construct a full factorial DOE table. For each entry of this table, dynamic equations are simulated in Matlab software, and energy consumption as well as distance traveled are also calculated. As the simulation time is constant for all experiments, distance traveled essentially corresponds to the average speed of locomotion. Results are tabulated in Table IV. In addition, the appendix A includes all entries for this experiment.

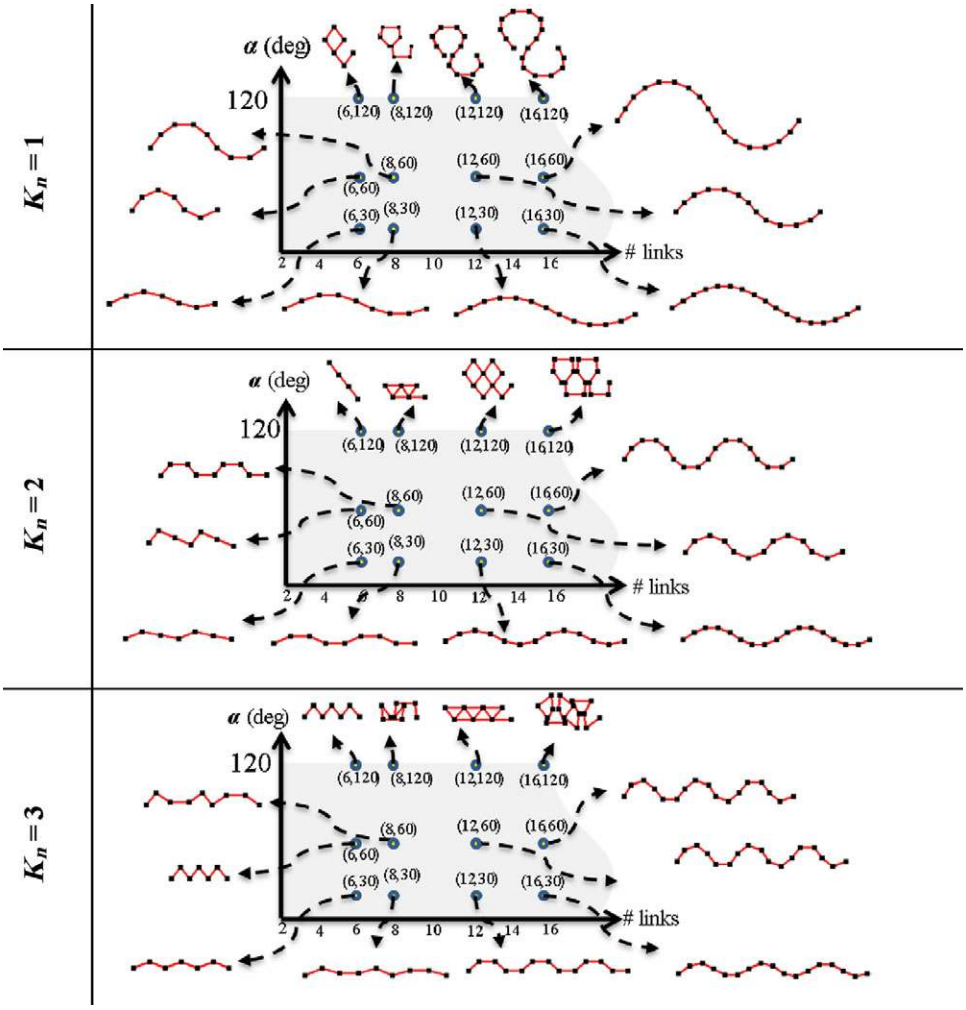

Fig. 6. (Colour online) Relation between number of links, winding angle, and undulation number.

Table IV. Full factorial design of experiments and the results.

Parameters					Number of links					
					$n = 8$		$n = 12$		$n = 16$	
Exp number	l (m)	α (rad)	m (kg)	K_n	Distance (m)	Energy (j)	Distance (m)	Energy (j)	Distance (m)	Energy (j)
1	0.09	0.5	0.3	2	1.61	275.07	2.67	276.26	3.39	278.65
2	0.1	0.4	0.32	2	0.49	136.39	1.39	136.41	1.92	139.79
3	0.08	0.6	0.28	1	2.94	68.27	3.42	58.72	3.69	54.90
4	0.09	0.5	0.28	1	2.02	38.85	2.33	32.98	2.57	30.55
5	0.08	0.4	0.32	1	0.71	29.42	0.99	24.76	1.31	22.89
	.							.		
	.		.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.
77	0.09	0.4	0.3	2	0.45	141.61	1.30	142.45	2.07	145.38
78	0.1	0.4	0.3	1	0.81	22.16	1.13	18.56	1.38	17.15
79	0.08	0.5	0.28	2	1.47	284.84	2.95	288.44	3.25	291.83
80	0.09	0.4	0.3	3	0.25	324.10	0.48	356.43	1.29	354.01
81	0.09	0.6	0.3	3	0.58	1340.03	2.23	1411.36	3.59	1359.10

5.1. *Modeling*

Regression is a statistical technique that investigates the relationships between input and output variables. It also constructs a mathematical model to relate input with output variables. In this research, linear, quadratic, and logarithmic regression equations using the data collected as per full factorial DOEs are developed. All these regressions are able to relate input with output variables. However, the accuracy of these models is different. Therefore, to select the best model for energy consumption and distance traveled, F-value, P-value, and R^2_{adj} for these regressions are compared using 95% confidence level. Results are shown in Table V.

The fundamental statistic used in comparing all possible models and to evaluate predictive ability of a model is called the Coefficient of Determination, also known as ordinary R^2. It measures the percentage of the total variability in the model output that is explained by the factors and variables in the model. If this number is large, it suggests a substantial predictive ability. A potential problem with this statistic is that it increases as factors are added to the model. A better statistic to use is the adjusted R^2 , R^2_{adj}, as it adjusts the ordinary R^2 for the size of the model, that is, the number of factors. As shown in Table V, R^2_{adj} for the 2nd order model is 96%, which is relatively large and suggests substantial predictive ability of the distance model. It indicates that 96% of the total variation in distance traveled can be accounted for by this line and only 4 is unaccountable. Another statistic used in statistical significance testing is called P-value. This value represents the test statistic at least as extreme as the one that was actually observed,[26] as shown in Table V. The model for distance has a P-value near zero. This means that the probability of obtaining an F-value of 293.09 is almost zero. This clearly indicates significance of the model. In general, higher F-value, higher R^2_{adj}, and lower P-value indicate that the model is better and more accurate. Similarly, the logarithmic model for energy consumption is found to be the superior model.

Next, ANOVA is used to calculate P-values for predictor variables of the selected models. These results are shown in Table VI.

As shown in Table VI, mass or any interaction terms that contain mass of the robot are not significant, have P-values greater than 0.05, and do not influence distance traveled.

Table VI. P-value for models.

	P-value			P-value	
Parameters	Distance Model	Energy Model	Parameters	Distance Model	Energy Model
$\boldsymbol{l}$	0.608	0.000	$\boldsymbol{mK_n}$	1.000	NA
$\boldsymbol{\alpha}$	0.000	0.000	$\boldsymbol{nl}$	0.073	NA
$\boldsymbol{m}$	1.000	0.000	$\boldsymbol{n\alpha}$	0.000	NA
$\boldsymbol{n}$	0.073	0.001	$\boldsymbol{nm}$	1.000	NA
$\boldsymbol{K_n}$	0.000	0.000	$\boldsymbol{nK_n}$	0.000	NA
$\boldsymbol{l\alpha}$	0.994	NA	$\boldsymbol{n^2}$	0.020	NA
$\boldsymbol{lm}$	1.000	NA	$\boldsymbol{l^2}$	0.616	NA
$\boldsymbol{lK_n}$	0.472	NA	$\boldsymbol{\alpha^2}$	0.022	NA
$\boldsymbol{\alpha m}$	1.000	NA	$\boldsymbol{m2}$	1.000	NA
$\boldsymbol{\alpha K_n}$	0.000	NA	$\boldsymbol{K_n^2}$	0.000	NA

Table V. P-value, F-value, and R^2 before modification.

	P-value			F-value			R^2_{adj}		
	1st order	2nd order	Logarithmic	1st order	2nd order	Logarithmic	1st order	2nd order	Logarithmic
Distance	0.000	0.000	0.000	256.73	**293.09**	185.63	84.1	96.0	79.2
Energy	0.000	0.000	0.000	186.45	802.51	**7278.11**	79.3	98.5	99.3

Table VII. F-value, P-value, and R^2 of the initial and final models.

		P-value		F-value		R^2_{adj}	
	Objective function	Initial	Final	Initial	Final	Initial	Final
Distance	2nd order	0	0	293.09	605.43	96.0	96.2
Energy	Logarithmic	0	0	7278.11	7278.11	99.3	99.3

Table VIII. Comparison of regression and dynamic models.

	Predicted (ANOVA)	Theoretical (dynamic equation)	Difference
Energy (j)	244.4624	256.73	4.6%
Distance (m)	1.8043	1.61	11.8%

These and other insignificant terms are eliminated from the regression model using the step backward elimination method. Stepwise regression is a procedure where a complex model can be simplified. In the step backward method, predictors are removed from the model one at a time, starting with the all-predictor model. At each step, the predictor that contributes the least to the model fit is removed. The procedure stops when only significant predictors remain to be removed.[26] The final modified models are as follows:

$$\text{Energy} = e^{4.74} \times n^{-0.0806} \times l^{-0.975} \times \alpha^{2.96} \times \text{m}^{1.00} \times K_n^{2.74}, \quad (12)$$

$$\text{Distance} = -7.9572 - 0.13728\, n + 24.885\, a + 1.7737 K_n + 0.2887\, nl + 0.45116\, na + 0.085873\, nK_n - 2.1427\, aK_n - 0.004579\, n^2 - 15.892\, a^2 - 0.52696\, K_n^2. \quad (13)$$

The initial and final F-value, P-value and R^2_{adj} for the 2nd order model distance traveled are shown in Table VII. As can be seen, the F-value of the modified model for distance traveled is increased. It should be noted that the step backward elimination method is not applied to the logarithmic model as all of its parameters have low P-values.

The initial variable settings are next placed into the two regression models, Eqs. (12) and (13), as well as the dynamic equation i.e., Eq. (9). Results are shown in Table VIII. As shown in this table, the two models agree closely.

Using Minitab software,[30] matrix plot of the dependent variables against all other independent variables for the two multiple regression models are generated and shown in Fig. 7. This figure shows that α and K_n are the two most influential variables of both energy consumption and distance traveled. This result was expected since α and K_n affect the body shape of the snake. Variables l and n only act as scale factors, while variable m does not affect the shape and scale of the snake robot.

The next step in this analysis aims to identify the parameter settings that result in optimized energy consumption and distance traveled. For real and large-size optimization problems, the traditional optimization methods are often inefficient and time-consuming. With the advent of computer technology and computational capabilities in the last few decades, the applications of heuristic algorithms are widespread. These techniques, such as SA, are usually based on physical or natural phenomena. In this paper, the SA algorithm is used as an optimization technique.

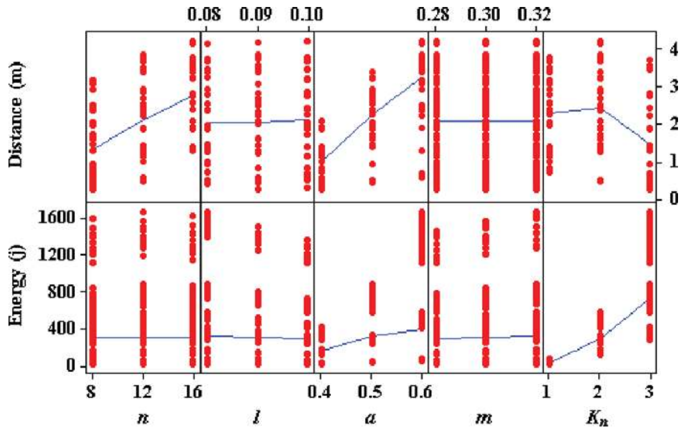

Fig. 7. (Colour online) Matrix plots of averages for distance traveled and energy consumption.

6. Simulated Annealing (SA) Algorithm

In 1953, Metropolis[32] proposed a procedure used to simulate the cooling of a solid for reaching a new energy state. The annealing process, used in metal working, involves heating the metal to a high temperature and then letting it gradually cool down to reach a minimum stable energy state. If the metal is cooled too fast, it will not reach the minimum energy state. Later, Kirkpatrick and his colleagues[29] used this concept to develop a search algorithm called Simulated Annealing. Among different heuristic algorithms, SA is one of the most powerful optimization methods that simulates the cooling process of a molten metal. The general stages of an SA algorithm are as follows:

1. Initialize the temperature parameter T_0, the cooling schedule r ($0 < r < 1$), and the termination criterion (e.g., number of iterations $k = 1 \ldots K$). Generate and evaluate an initial candidate solution (perhaps at random); call this the current solution, c.
2. Generate a new neighboring solution, m, by making a small change in the current solution and evaluate this new solution.
3. Accept the new solution as the current solution if

   (a) the objective value of the new solution, $f(m)$, is better than that of the current solution $f(c)$;

   (b) the value of acceptance probability function given by $(\exp\,(f(m) - f(c))/T_k)$ is greater than a uniformly

Table IX. SA results – single objective.

	Parameters				
	l (m)	α (rad)	m (kg)	K_n	n
Optimized distance	0.0994	0.5995	0.28	1.9984	16
Optimized energy	0.0998	0.4002	0.28	1.0012	16
Initial settings	0.09	0.50	0.2800	2	8

generated random number "rand"; where $0 < \text{rand} < 1$.

4. Check the termination criterion, update the temperature parameter (i.e., $T_k = r \times T_{k-1}$) and return to step 2.

The main advantages of SA are its flexibility, its fewer tuning parameters, and its ability to escape local optima and approach global optimality. In SA, there are only two major tuning parameters – the initial temperature and the cooling schedule. As a result, SA can easily be "tuned" with minimum trial runs. SA can also avoid local optima by occasionally taking downward steps. The details of this technique and its various applications are well documented in related literature.[12]

6.1. Optimization, n-link

In order to optimize robot performance, the SA algorithm is used to find optimum parameter settings. Clearly, parameter settings resulting in simultaneous increase in distance traveled and decrease in energy consumption are desired. However, first optimum values are obtained separately. The optimum parameters to optimize energy consumption and distance traveled separately are shown in Table IX. It took approximately 1 sec for SA to find these values.

It is concluded from Table IX that lower value of m and higher values of l and n will result in decreasing energy consumption and increasing distance traveled. However, to increase the distance traveled, $K_n = 2$ and the highest value for α should be used. Conversely, to decrease energy consumption, $K_n = 1$ and the lowest value of α should be used. Therefore, multi-criteria optimization seems necessary.

The findings of SA that, for example, the lower value of m is better should be taken with care. Clearly, as m approaches zero, the gait becomes less effective as the frictional forces on the wheels, which are the source of propulsion, also approach zero. Therefore, the results refer to the range of the values selected by the experiment.

The single objective optimization results for energy consumption and distance traveled with settings given in Table IX are shown in Table X. The initial settings are those that are shown in Table II.

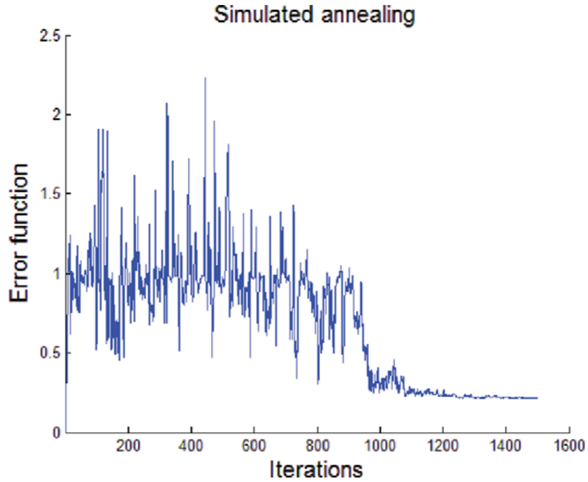

Fig. 8. (Colour online) SA convergence curve.

Next, both energy consumption and distance traveled are considered and optimized simultaneously. Mean squared error is used and multi-criteria fitness function is defined as follows:

$$\text{Fitness} = w_1 \frac{(\text{Energy}_d - \text{Energy})^2}{\text{Energy}^2} + w_2 \frac{(\text{Distance}_d - \text{Distance})^2}{\text{Distance}^2}, \qquad (14)$$

where Energy_d and Distance_d are desired values of energy consumption and distance traveled, which are obtained using the previous single objective optimization step. In addition, w_1 and w_2 are desired weight factors for energy consumption and distance traveled respectively. The algorithm along with its objective function is coded in Matlab software. In this study, the relative importance of energy consumption and distance traveled, their weight, are assumed to be equal. In practice, these weights may be set at any relative values as desired.

The Simulated Annealing parameters are as follows: *initial temperature*: 1000, *cooling rate*: 0.99, and *termination criterion*: 1000 iterations. These values are selected based on author's experience and trial and error. Similar to the single objective case, it took less than 1 sec to complete the simulation. The convergence curve of SA is shown in Fig. 8.

Results after running SA as well as using the initial setting are shown in Table XI.

It can be concluded from Table XI that to simultaneously optimize energy consumption and distance traveled, lower values of m and K_n along with higher value of l and n as well as $\alpha = 0.55$ are required. Using settings obtained in Table XI and regression models, Eqs. (12) and (13), optimized energy

Table X. Single objective optimization results.

	Predicted (ANOVA)	Theoretical (dynamic equation)	Initial settings	Improvement
Optimized energy (j)	16.3075	16.0794	256.73	93%
Corresponding distance (m)	1.2054	1.2556	1.61	–22%
Optimized distance (m)	4.29	4.3363	1.61	169%
Corresponding energy (j)	357.7085	410.2936	256.73	–83%

Table XI. SA results (simultaneous optimization vs. initial settings).

	Parameters				
	l (m)	α (rad)	m (kg)	K_n	n
Optimized settings	0.10	0.5503	0.2802	1	16
Initial settings	0.09	0.50	0.2800	2	8

consumption and distance traveled are predicted. In addition, these results are confirmed by placing the optimized values into the robot dynamic equation, i.e., Eq. (9). Lastly, the initial settings selected by the design team, Table II, are also placed into the robot dynamic equation (Eq. (9)) and percentage improvement due to simultaneous optimization is calculated. Results are tabulated in Table XII.

As Table XII shows, there is a significant improvement in the results of simultaneous optimization versus initial settings selected by the team. Energy consumption is reduced by 86%, while distance traveled is increased by 105%.

6.2. Snake models, 8-link, 12-link, and 16-link

Results of simultaneous optimization also suggest that a 16-link robot has the best performance. However, as the number of links increases, so does the cost. Furthermore, there is a lot that can be learned from building a snake robot with fewer links. In fact all dynamic equations and optimization results could be verified using an 8-link robot. The team decided to first build an 8-link snake, followed by a 12-link and finally a 16-link robot, which is shown to have the best performance. Therefore, determining optimum value of parameters for 8-link and 12-link robots are desired.

To do so, $n = 8$ and 12 are placed into the derived regression equations, Eqs. (12) and (13). In order to maintain consistency of the equations, $n = 16$ is also placed in Eqs. (12) and (13). From here on these resulting equations are used first for the single and then for the multi-criteria optimization studies.

8-link snake robot:

$$\text{Energy} = 0.845 \times e^{4.74} \times l^{-0.975} \times \alpha^{2.96} \times \int \text{m}^{1.00} \times K_n^{2.74}, \quad (15)$$

$$\text{Distance} = -9.349 + 28.4943\,\alpha + 2.4435\,K_n + 2.309\,l - 2.143\,\alpha K_n - 15.89\,\alpha^2 - 0.5269\,K_n^2. \quad (16)$$

12-link snake robot:

$$\text{Energy} = 0.818 \times e^{4.74} \times l^{-0.975} \times \alpha^{2.96} \times m^{1.00} \times K_n^{2.74}, \quad (17)$$

$$\text{Distance} = -10.276 + 30.298\,\alpha + 2.808\,K_n + 3.464\,l - 2.142\,\alpha K_n - 15.892\,\alpha^2 - 0.526\,K_n^2. \quad (18)$$

16-link snake robot:

$$\text{Energy} = 0.799 \times e^{4.74} \times l^{-0.975} \times \alpha^{2.96} \times m^{1.00} \times K_n^{2.74}, \quad (19)$$

$$Distance = -11.324 + 32.103\,\alpha + 3.148\,K_n + 4.6192\,l - 2.1427\,\alpha K_n - 15.892\,\alpha^2 - 0.5269\,K_n^2. \quad (20)$$

Single objective results are shown in Table XIII.

As shown in Table XIII, to increase distance traveled and decrease energy consumption, regardless of the number of links, l should be at the highest level and m at the lowest level. To increase distance traveled, α should be at the highest level. However, to decrease energy consumption α should be at the lowest level. Therefore, multi-criteria optimization seems necessary. Optimum value of K_n increases as the number of links increases. The optimized parameters for energy consumption and distance traveled from Table XIII are placed into Eqs. (15)–(20). Results are tabulated in Table XIV and are also shown in Fig. 9. Please note that Fig. 9(a) and (b) use significantly different scales to better highlight the effect of energy consumption.

As shown in Fig. 9, regardless of the optimization criteria, increasing the number of links increases the distance traveled. Next, because both energy consumption and distance traveled are important, mean squared error is used and optimization is performed simultaneously. Once again, the relative

Table XII. Predicted and theoretical values for simultaneously optimized energy and distance.

	Predicted (ANOVA)	Theoretical (dynamic equation)	Initial settings	Improvement
Optimized energy (j)	41.9651	35.1145	256.73	86%
Optimized distance (m)	3.4430	3.3007	1.61	105%

Table XIII. SA results (single objective).

	Number of links					
	$n = 8$		$n = 12$		$n = 16$	
Parameters	Distance	Energy	Distance	Energy	Distance	Energy
l (m)	0.0998	0.0999	0.0998	0.0998	0.0998	0.0998
α (rad)	0.5993	0.4002	0.5978	0.4010	0.5998	0.4003
m (kg)	0.2800	0.2801	0.2800	0.2807	0.2800	0.2804
K_n	1.0020	1.0016	1.3729	1.0011	1.9936	1.0006

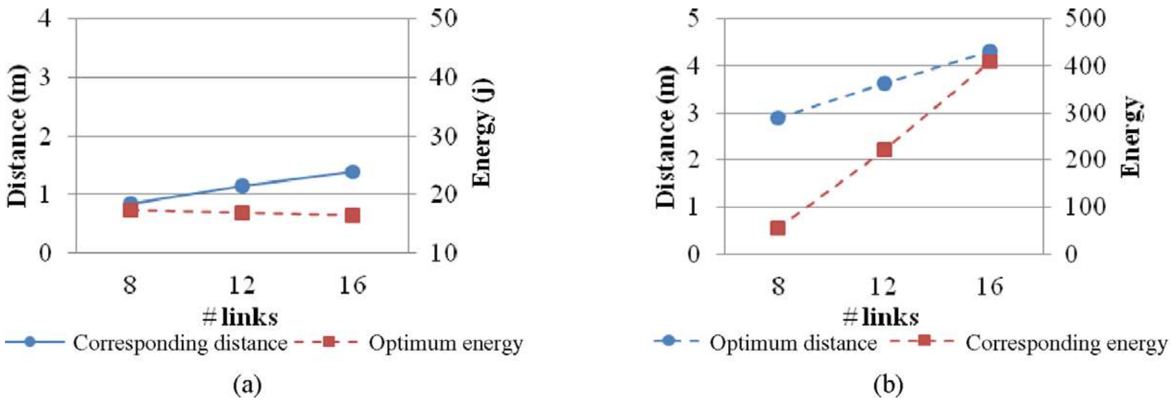

Fig. 9. (Colour online) Effect of number of links on distance and energy – single objective.

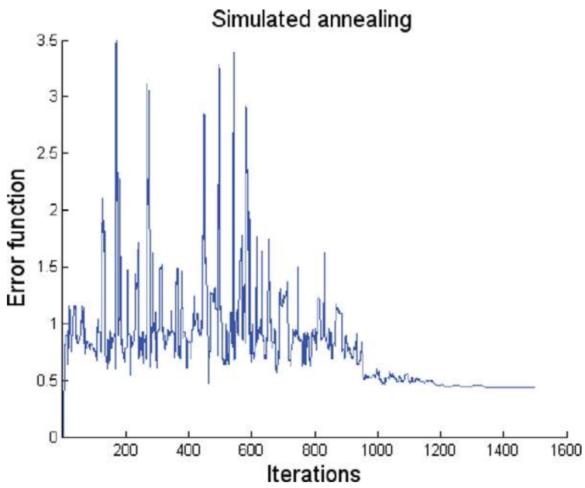

Fig. 10. (Colour online) Convergence curve of SA for $n = 8$.

Table XIV. SA results (single objective).

	Number of links		
	$n = 8$	$n = 12$	$n = 16$
Optimum energy (j)	17.2367	16.9275	16.5221
Corresponding distance (m)	0.8519	1.1472	1.3900
Optimum distance (m)	2.9020	3.6318	4.3202
Corresponding energy (j)	54.7965	221.6088	408.4790

importance of energy consumption and distance traveled are set to unity. The SA settings are the same as above. The convergence curve for SA is shown in Fig. 10.

The optimized parameter settings are shown in Table XV.

It can be concluded from Table XV that to simultaneously optimize energy consumption and distance traveled, regardless of the number of links, lower values of m and K_n, higher values of length, and relatively higher values of α are required. Furthermore, regardless of the number of links, optimum values for l, α, m, K_n remain practically unchanged. Therefore, a designer can use the same set of values for these parameters regardless of the number of links. Also, note that the results for $n = 16$ are the same as that for earlier simultaneous optimization when n was considered to be a variable. Using the settings obtained in Table XV, optimized energy consumption and distance traveled using regression

Table XV. SA results (simultaneous optimization).

	Number of links		
Parameters	$n = 8$	$n = 12$	$n = 16$
l **(m)**	0.1000	0.0998	0.1000
α **(rad)**	0.5313	0.5301	0.5463
m **(kg)**	0.2802	0.2803	0.2802
$\boldsymbol{K_n}$	1.0019	1.0012	1.0010

models, Eq. (15)–Eq. (20) and the theoretical model Eq. (9), are calculated and shown in Table XVI. It should be noted that the initial settings used in Table XVI are all the same as in Table II, except for the number of links where corresponding values of 8, 12, and 16 are used.

As shown in Table XVI and Fig. 11, regardless of single or multi-criteria optimization, as the number of links increases, the distance traveled increases but the optimum energy consumption remains mostly unchanged. Therefore, the best snake performance is with $n = 16$. Table XVI also shows that both energy consumption and distance traveled have improved when compared with initial settings, except for the distance traveled when $n = 16$.

One may note different results when comparing Table XII and XVI. For example, Table XII reports 105% improvement, while Table XVI reports –2% degradation for distance traveled. This difference is because the initial settings used for Table XII are 8 links, while Table XVI uses 16 links. This small degradation, –2%, after multi-criteria optimization, demonstrates that the team exercised good engineering judgment when selecting its desired number of links, which is to ultimately build a 16-link snake robot.

To separately investigate the effect of each factor, for the case when the number of links is 8, Fig. 12(a)–(d) are generated. In these figures, the optimum values for variables are used, while the corresponding remaining factor is varied.

These figures, once again, verify that maximum values of l, minimum values of m and K_n are the optimum values. However, for α the lowest level is the best for energy consumption and the highest for distance traveled.

7. Simulation of Snake Robot in Webots

Webots™ software is used for simulation.[28] Webots™ is a popular commercial software used for mobile robotics simulation and provides a rapid prototyping environment for

Table XVI. Predicted and theoretical values for energy and distance (simultaneous optimization).

		Number of links		
		$n = 8$	$n = 12$	$n = 16$
Energy (j)	Predicted (ANOVA model)*	38.4496	38.3638	40.9011
	Theoretical model (dynamic equation)*	**40.5838**	**34.4325**	**34.5306**
	Initial settings (dynamic equation)**	256.73	257.84	260.08
	Improvement	84%	86%	86%
Distance (m)	Predicted (ANOVA model)*	2.2758	2.8235	3.3830
	Theoretical model (dynamic equation)*	**2.4614**	**2.8509**	**3.3167**
	Initial settings (dynamic equation)**	1.61	2.67	3.39
	Improvement	34%	6%	−2%

*Using optimized values given in Table XIII.
**Table I, except for number of links.

Table XVII. Percentage improvement in distance.

	Dynamic equation	Webots software	Matlab SimMechanics
8 links to 12 links	17%	14%	20%
8 links to 16 links	37%	29%	32%

modeling, programming, and simulation. This software is useful for considering robot behavior in physically realistic world. In this section, three snake robots with different number of links are simulated. Each link is equipped with four wheels. As shown in Fig. 13, the snake robot with 16 links progresses more than the robot with 8 and 12 links. These findings verify the DOE results.

Comparison of percentage improvement when increasing the number of links from 8 to 12 as well as from 8 to 16 using the derived dynamic equations and the two simulation packages, Matlab SimMechanics and Webots, is shown in Table XVII. All three results agree on general improvement when the number of links is increased. See Webots specifics in Appendix C.

8. Comparing Results with Existing Snake Robots

In this section, results obtained in this study are compared with the results of various existing snake-inspired robot designs. In a recent study, Hopkins[13] investigated the relationship between snake-inspired robot dimensions, performance, and velocity, regardless of gait type. He found that snake robots share many common characteristics, which allow them to be easily grouped under a general classification (see Table XVIII). Three of his main findings are as follows:

- Faster robots have longer length. By plotting velocity versus robot length, a general positive relation between robot length and speed is observed.
- Robots with higher number of segments generally have higher speed.
- Robots having similar lengths and velocity but higher weights may indicate higher capability or inefficiency. The higher weight may be due to additional sensors, degrees of freedom, power supplies, etc. Otherwise, the additional weight may represent design deficiency.

Following similar conclusions are made upon completing the experimental design presented in this paper:

- Link length, l, should be maximized.
- Number of segments, n, should be maximized.
- Weight of each segment, m, does not affect distance traveled but significantly affects energy consumption, and thus should be minimized.

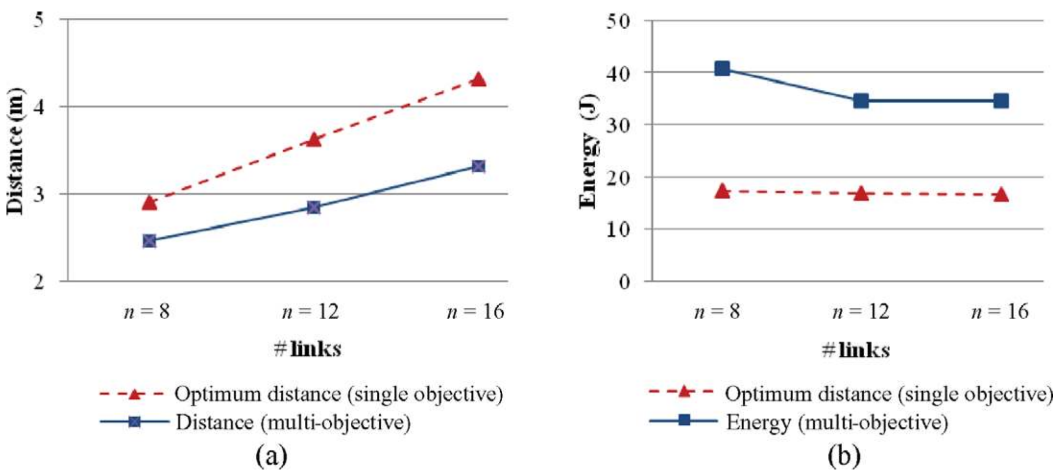

Fig. 11. (Colour online) Effect of the number of links on distance and energy (single and multi-criteria optimization).

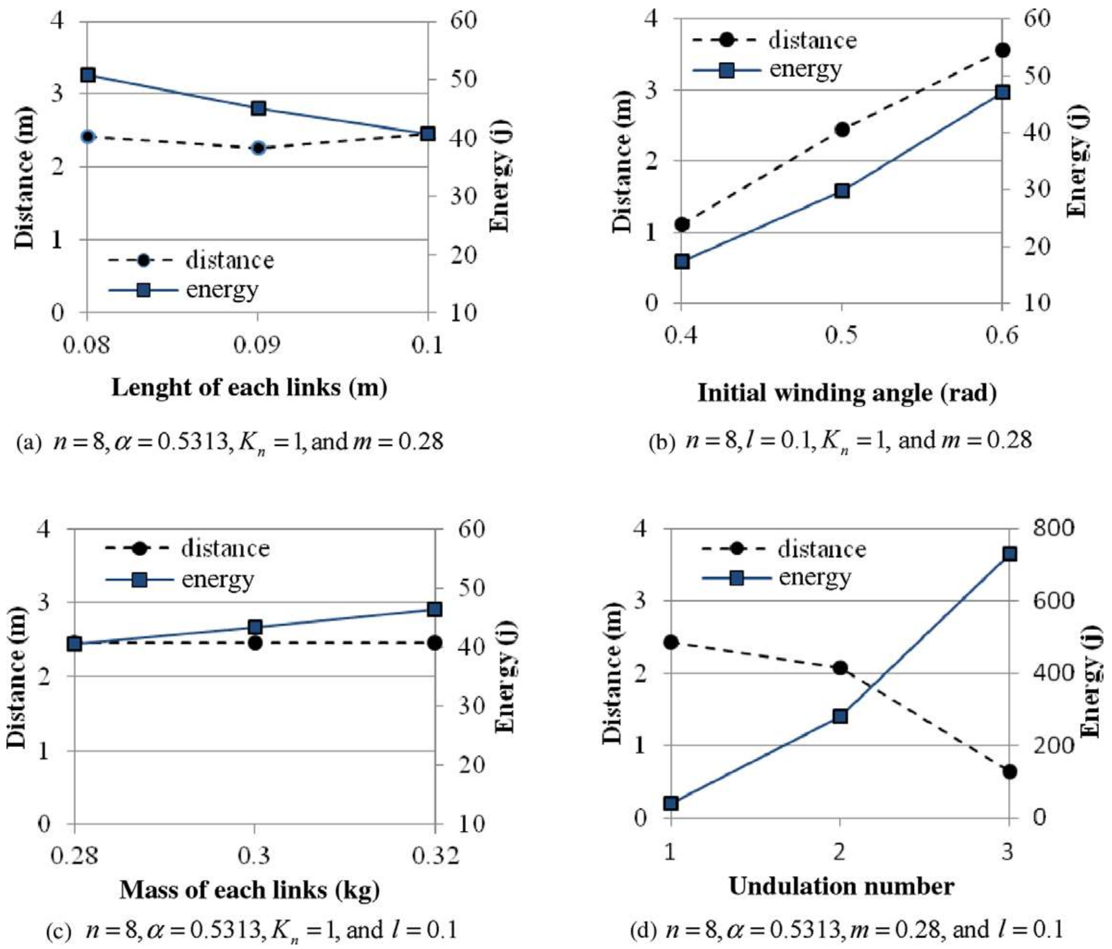

Fig. 12. (Colour online) Effect of individual factors on distance and energy – using optimum values.

9. Discussion and Summary of Results

The ultimate goal of many snake-like robots is to better mimic real snakes in nature. To reach this goal, a designer needs to have a deeper understanding of how key parameters affecting snake design influence snake performance such as speed and energy consumption. In this paper, the first formulation for kinematics and dynamics of an n-link snake robot traveling with serpentine gait on a flat terrain is briefly presented. Key parameters selected are mass and length of each link, number of links as well as the parameters affecting body shape, number of waves, and initial winding angle. Experimental design methodology is used to study the effect of key

Table XVIII. Snake-inspired robot dimensions and performance[13].

Robot	Length of each link (mm)	Overall length (mm)	Cross section (m^2)	Overall weight (kg)	Velocity (mm/s)	No. of links or modules
ACM III	144	2000	0.023	28	400	20
AmphiBot I (AB I)	70	490	0.002	–	35	8
AmphiBot II (AB II)	94	770	0.002	–	400	8
KR-II	–	3300	0.497	370	500	7
KR-I	–	1390	0.081	27.8	266	6
OmniTread (OT-8)	200	1270	0.034	13.6	100	5
OmniTread (OT-4)	–	940	0.007	3.6	150	7
JL-I	350	1050	0.038	21	180	3
Kotay's Inchworm I (KIR-1)	–	250	–	0.455	4	–
Kotay's Inchworm II (KIR-2)	–	330	–	0.566	13	–
CMU (M1)	–	840	0.003	1.26	102	–
Slim Slime Robot (SSR)	177.6	730	0.013	12	60	6
Planar Inchworm (PI)	–	710	–	6	1	2
FUM Snake-2	**100**	**1600**	**0.48**	**4.48**	**150***	**16**

*Values obtained by simulating dynamic equations.

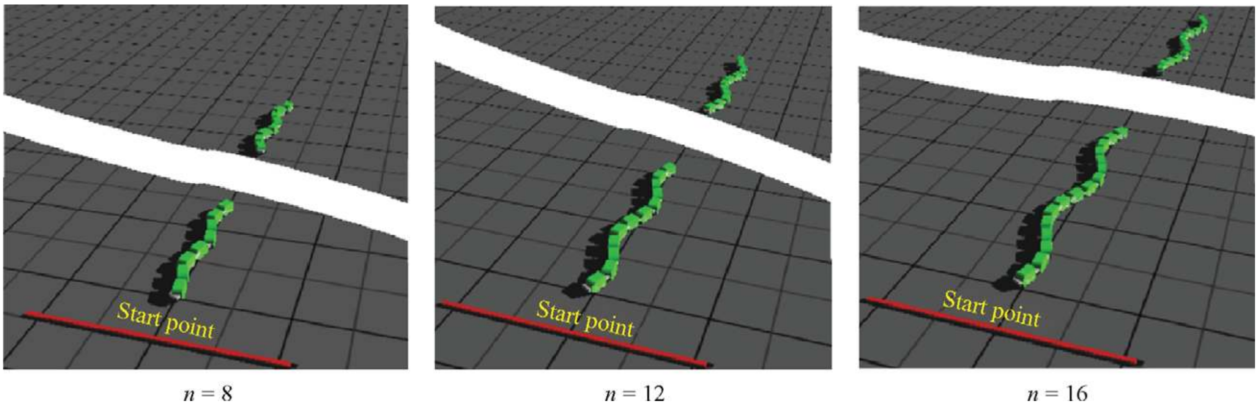

Fig. 13. (Colour online) Simulation of snake robot in Webots software (simulation time = 20 sec).

parameters on distance traveled and energy consumption. Derived dynamic equations are used to simulate snake robot and calculate distance traveled and energy consumption. The dynamic equations are also verified using Matlab SimMechanic. Using regression modeling, mathematical relationships between inputs and outputs are established. Three different models are considered. Second order and logarithmic models with high R^2, 96% and 99.3%, as well as high F-value, 293 and 7278, are found to best predict distance traveled and energy consumption respectively. The DOE concluded is as follows:

- α and K_n are the two most influential variables, followed by l and n, while m does not significantly affect distance traveled but affects energy consumption. Intuitively, these results were expected since α and K_n affect the body shape of the snake, while l and n act as a scale factor and m does not affect shape and scale of the snake robot.
- Maximize n and l and minimize m to achieve the highest speed. These results confirm the existing broad knowledge as reported by Hopkins *et al.*[13] that longer snakes have higher speed.

The SA algorithm, both single and multi-criteria, is next used to determine optimum parameter settings. Results of single and multi-criteria optimizations are as follows:

- The lowest α will result in minimum energy consumption, while the highest α will result in the highest distance traveled.
- The highest values for l and n, the lowest values for m and K_n, and approximately midpoint value for α are needed to minimize energy consumption and maximize distance traveled.
- Both distance traveled and energy consumption are improved by 105% and 86% versus initial settings respectively.

Because the goal is to first build a 8-link robot, three separate models (8, 12, and 16 links) were generated and both single and multi-criteria optimizations were performed. Following are the results:

- Regardless of single or multi-criteria optimization, as the number of links increases, distance traveled increases but the optimum energy consumption remains by and large unchanged. Therefore, the best snake performance is with $n = 16$.
- Regardless of the number of links, optimum values for l, α, m, K_n are practically unchanged.
- Webots software is used for further verification. Results confirm that as the number of links increases so does the distance traveled.

Finally, based on the DOE results, a new snake robot link is designed. Upon completion of construction, experiments will be performed to further validate the theoretical and simulation results, as well as to perform additional research.

It should be noted that the derived mathematical model is not general and does not include all physical effects of a real snake robot. Consequently, if a physical snake robot is constructed, then it is quite possible that its behavior may be slightly different from the simulated robot of this paper. This is because the theoretical model does not include effects of many parameters such as specific geometrical and mass properties of links, friction in joints and other moving parts, ground flatness, employed friction model, and even the effect of control parameters.

Furthermore, it should be noted that the results obtained in this study apply to the considered range of values of input parameters. A great deal of effort is made in determining the nominal and range of values for key kinematic and dynamic parameters. Several considerations are made when selecting these values such as resembling a natural form of a snake body, being practical, and studying the literature on the existing physical snake robots. However, still these values represent a finite range and are not general.

10. Conclusion

This paper has provided a framework based on statistical design techniques for the development, analysis, and performance evaluation of forthcoming snake robot designs. Using the DOE approach, a general model for distance traveled and energy consumption is developed. These models are shown to be statistically significant and are verified by both dynamics model and Webots simulation software. These models are in convenient form and may be readily used by optimization routines. Next, the SA optimization technique is applied. It is shown that (1) α and K_n are the two most influential variables, followed by l and n, while m does not significantly affect distance traveled but affects energy consumption; (2) maximizing n and l, as well as minimizing m will result in the highest speed. Using the optimum values, distance traveled and energy consumption are improved by 105% and 86% respectively. It should also be mentioned that the above conclusions apply to snake robots whose behavior is identical to the mathematical model as well as the range of values for the parameters considered in this paper.

The paper contributes by first using the idea of applying Statistical DOE techniques to identify key design parameters, applying the SA algorithm to obtain optimum settings, discovering the effect of input parameters on snake robot performance, confirming the existing literature's broad knowledge that increasing the number of links increases robot performance, and finally obtaining regression models for robot performance. By following the methodology outlined in this paper, future researchers can develop their own specific models and evaluate performance of their snake robot design. Future work will focus on completion of construction of FUM Snake-2 robot and performing experiments to further validate results. It is hoped that this paper can inspire other researchers working on snake robot design and overcome some of the inherent energy inefficiencies of these robots.

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Appendix A

Full factorial design of experiments (using dynamics model)

	Parameters				Number of links					
					$n = 8$		$n = 12$		$n = 16$	
No.	l (m)	α (rad)	m (kg)	K_n	Distance (m)	Energy (j)	Distance (m)	Energy (j)	Distance (m)	Energy (j)
1	0.09	0.5	0.3	2	1.61	275.07	2.67	276.26	3.39	278.65
2	0.1	0.4	0.32	2	0.49	136.39	1.39	136.41	1.92	139.79
3	0.08	0.6	0.28	1	2.94	68.27	3.42	58.72	3.69	54.9
4	0.09	0.5	0.28	1	2.02	38.85	2.33	32.98	2.57	30.55
5	0.08	0.4	0.32	1	0.71	29.42	0.99	24.76	1.31	22.89
6	0.1	0.6	0.3	2	2.39	437.06	3.69	438.73	4.24	439.11
7	0.1	0.4	0.28	1	0.81	20.68	1.13	17.32	1.38	16
8	0.08	0.5	0.3	1	1.98	46.68	2.54	39.71	2.4	36.93
9	0.09	0.6	0.32	3	0.58	1429.37	2.23	1505.45	3.59	1449.71
10	0.1	0.5	0.32	3	0.48	672.61	1.87	726.98	2.75	702.98
11	0.08	0.6	0.28	3	1.3	1398.25	1.9	1461.29	3.47	1426.27
12	0.1	0.4	0.3	3	0.29	291.11	0.58	321.8	1.39	318.6
13	0.1	0.6	0.32	1	3.19	62.69	3.47	53.63	3.8	50.14
14	0.09	0.5	0.3	1	2.02	41.63	2.33	35.33	2.57	32.73
15	0.08	0.6	0.32	2	2.37	572.87	3.75	583.09	4.17	583
16	0.1	0.6	0.3	3	0.69	1203.6	2.44	1274.12	3.71	1223.3
17	0.1	0.5	0.3	2	1.71	248.35	2.86	248.01	3.22	251.21
18	0.09	0.4	0.3	1	0.8	24.6	1.22	20.65	1.3	19.02
19	0.1	0.6	0.3	1	3.19	58.77	3.47	50.28	3.8	47.01
20	0.1	0.5	0.32	2	1.71	264.91	2.86	264.55	3.22	267.96
21	0.08	0.5	0.32	3	0.91	837.2	1.38	893.34	2.79	878.15
22	0.1	0.5	0.28	2	1.71	231.8	2.86	231.48	3.22	234.46
23	0.1	0.6	0.32	2	2.39	466.19	3.69	467.97	4.24	468.39
24	0.08	0.6	0.28	2	2.37	501.26	3.75	510.21	4.17	510.12
25	0.1	0.6	0.28	1	3.19	54.85	3.47	46.93	3.8	43.87
26	0.09	0.4	0.28	2	0.45	132.17	1.3	132.95	2.07	135.69
27	0.08	0.5	0.28	1	1.98	43.57	2.54	37.06	2.4	34.46
28	0.1	0.4	0.32	3	0.29	310.51	0.58	343.25	1.39	339.84
29	0.1	0.5	0.3	1	2.05	37.5	2.25	31.74	2.78	29.51
30	0.09	0.5	0.28	2	1.61	256.73	2.67	257.84	3.39	260.08
31	0.08	0.4	0.32	3	0.39	386.5	0.46	421.8	1.29	424.51
32	0.1	0.5	0.28	1	2.05	35	2.25	29.63	2.78	27.54
33	0.09	0.6	0.28	1	3.08	60.88	3.52	52.25	3.77	48.66
34	0.09	0.6	0.32	1	3.08	69.58	3.52	59.71	3.77	55.62
35	0.09	0.5	0.32	1	2.02	44.4	2.33	37.69	2.57	34.91
36	0.09	0.4	0.28	3	0.25	302.5	0.48	332.67	1.29	330.41
37	0.08	0.6	0.3	1	2.94	73.15	3.42	62.91	3.69	58.82
38	0.08	0.4	0.28	3	0.39	338.18	0.46	369.07	1.29	371.45
39	0.1	0.5	0.28	3	0.48	588.54	1.87	636.11	2.75	615.1
40	0.1	0.4	0.28	2	0.49	119.35	1.39	119.36	1.92	122.32
41	0.09	0.5	0.3	3	0.42	702.05	1.43	754.93	2.84	732.24
42	0.08	0.6	0.32	3	1.3	1598	1.9	1670.05	3.47	1630.03
43	0.1	0.6	0.28	2	2.39	407.92	3.69	409.48	4.24	409.84
44	0.09	0.6	0.28	3	0.58	1250.7	2.23	1317.27	3.59	1268.5
45	0.09	0.5	0.28	3	0.42	655.25	1.43	704.6	2.84	683.43
46	0.08	0.4	0.28	1	0.71	25.74	0.99	21.66	1.31	20.03
47	0.09	0.6	0.28	2	2.5	451.83	3.88	456.1	4.2	454.61
48	0.1	0.4	0.32	1	0.81	23.64	1.13	19.8	1.38	18.29
49	0.08	0.6	0.3	2	2.37	537.07	3.75	546.65	4.17	546.56
50	0.09	0.4	0.32	3	0.25	345.71	0.48	380.19	1.29	377.61
51	0.08	0.5	0.28	3	0.91	732.55	1.38	781.67	2.79	768.38
52	0.1	0.5	0.3	3	0.48	630.58	1.87	681.55	2.75	659.04
53	0.08	0.5	0.3	2	1.47	305.19	2.95	309.05	3.25	312.68
54	0.08	0.5	0.32	1	1.98	49.79	2.54	42.36	2.4	39.39
55	0.08	0.4	0.28	2	0.51	146.65	1.26	148.74	1.85	152.25

Appendix A

Continued

	Parameters				Number of links					
					$n = 8$		$n = 12$		$n = 16$	
No.	l (m)	α (rad)	m (kg)	K_n	Distance (m)	Energy (j)	Distance (m)	Energy (j)	Distance (m)	Energy (j)
56	0.1	0.4	0.3	2	0.49	127.87	1.39	127.89	1.92	131.06
57	0.08	0.5	0.3	3	0.91	784.88	1.38	837.5	2.79	823.27
58	0.1	0.6	0.32	3	0.69	1283.85	2.44	1359.06	3.71	1304.86
59	0.09	0.4	0.28	1	0.8	22.96	1.22	19.27	1.3	17.75
60	0.1	0.5	0.32	1	2.05	40	2.25	33.86	2.78	31.47
61	0.09	0.4	0.32	2	0.45	151.05	1.3	151.95	2.07	155.07
62	0.09	0.5	0.32	2	1.61	293.41	2.67	294.67	3.39	297.23
63	0.08	0.4	0.3	2	0.51	157.13	1.26	159.37	1.85	163.13
64	0.08	0.5	0.32	2	1.47	325.53	2.95	329.65	3.25	333.53
65	0.08	0.4	0.3	3	0.39	362.34	0.46	395.44	1.29	397.98
66	0.08	0.6	0.3	3	1.3	1498.13	1.9	1565.67	3.47	1528.15
67	0.08	0.4	0.32	2	0.51	167.61	1.26	169.99	1.85	174
68	0.09	0.6	0.32	2	2.5	516.37	3.88	521.26	4.2	519.55
69	0.1	0.4	0.28	3	0.29	271.7	0.58	300.35	1.39	297.36
70	0.09	0.4	0.32	1	0.8	26.24	1.22	22.03	1.3	20.29
71	0.09	0.6	0.3	1	3.08	65.23	3.52	55.98	3.77	52.14
72	0.09	0.6	0.3	2	2.5	484.1	3.88	488.68	4.2	487.08
73	0.08	0.6	0.32	1	2.94	78.03	3.42	67.11	3.69	62.74
74	0.08	0.4	0.3	1	0.71	27.58	0.99	23.21	1.31	21.46
75	0.1	0.6	0.28	3	0.69	1123.36	2.44	1189.18	3.71	1141.75
76	0.09	0.5	0.32	3	0.42	748.86	1.43	805.26	2.84	781.06
77	0.09	0.4	0.3	2	0.45	141.61	1.3	142.45	2.07	145.38
78	0.1	0.4	0.3	1	0.81	22.16	1.13	18.56	1.38	17.15
79	0.08	0.5	0.28	2	1.47	284.84	2.95	288.44	3.25	291.83
80	0.09	0.4	0.3	3	0.25	324.1	0.48	356.43	1.29	354.01
81	0.09	0.6	0.3	3	0.58	1340.03	2.23	1411.36	3.59	1359.1

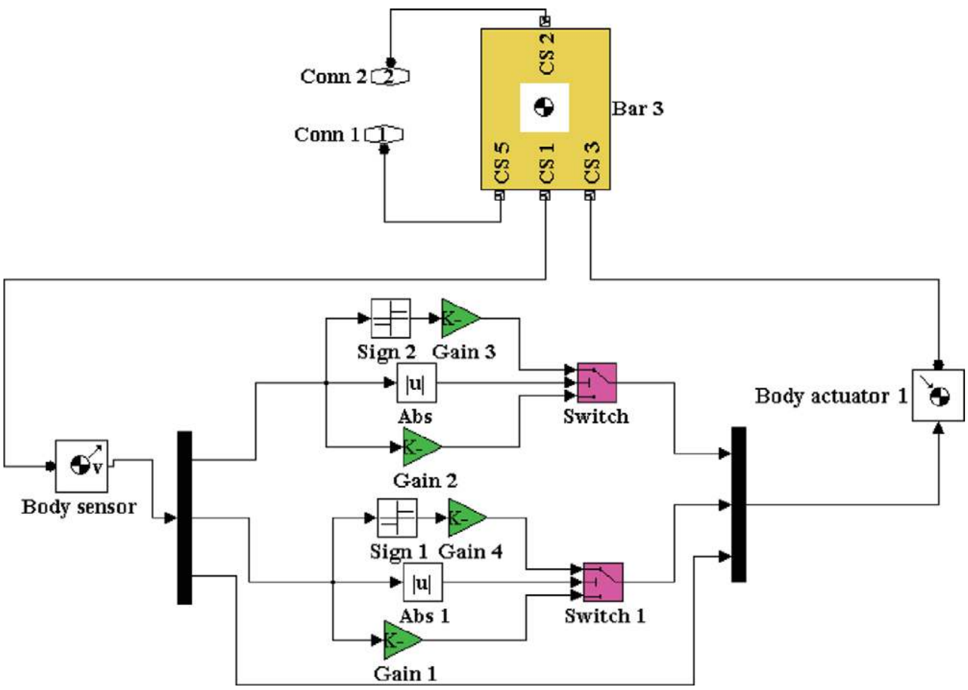

Fig. 14. (Colour online) Coulomb friction model of each link.

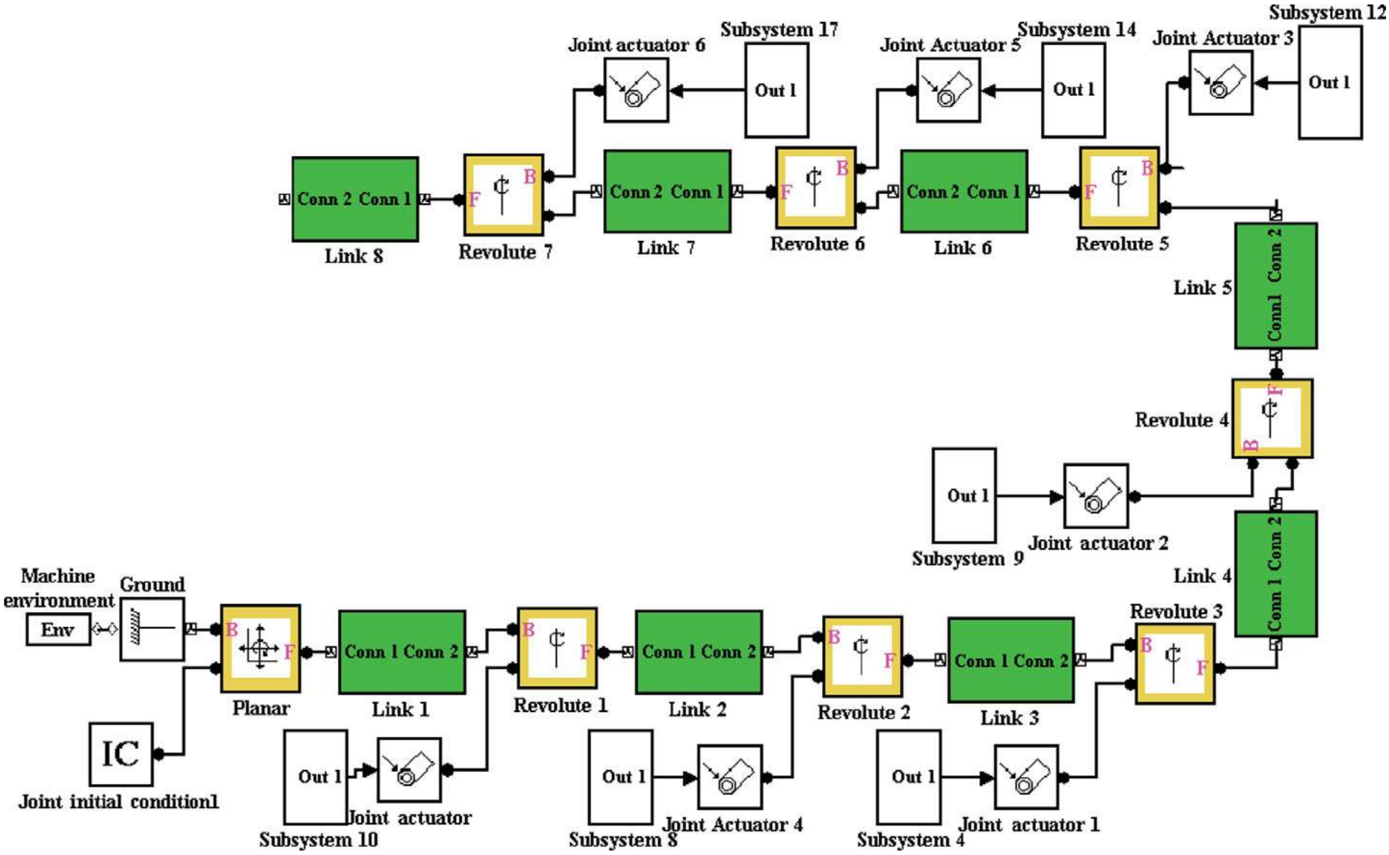

Fig. 15. (Colour online) Model of 8-link snake robot in SimMechanics.

Appendix B

In order to verify the derived dynamic equations, three snake robots (8, 12, 16 links) are modeled using SimMechanics toolbox in Matlab. SimMechanics is a block diagram modeling environment for the engineering design and simulation of rigid body machines and their motions, using the standard Newtonian dynamics of force and torques. The blocks used in SimMechanics toolbox are the elements necessary to model mechanical systems consisting of any number of rigid bodies, connected by joints representing translational and rotational degrees of freedom. One can impose kinematic constraints, apply force/torques, integrate Newton's equations, and measure resulting motions. As an example, consider an 8-link snake robot with identical links which is modeled using SimMechanics. As illustrated in Fig. 15, there is a revolute joint between each link of the model except for the first link. A planar joint is used to connect the first link, tail link, with *ground block*. Each *link block* is used to model the actual snake link with a uniform slender rod. The *link block* contains the following information (l_i = 0.1 m, m_i = 0.28 kg, I_i = 2.33E-4). In order to calculate friction forces, Eq. 10, it is necessary to measure velocity of center of mass of each link. As illustrated in Fig. 14, velocity measurement for each link is carried out using *body sensor* block. The *gain blocks* define the friction with ground. Gains1 = Gains2 = μ_n = 0.1 and Gains3 = Gain4 = μ_t = 0.55. The *sign blocks* are defined by $sign(v)$, i.e., $sign(v) = 1$ if $v > 0$, $sign(v) = 0$ if $v = 0$, and $sign(v) =$ -1 if $v < 0$. The *bar block* shown in Fig 14 is a subsystem of *link block* shown in Fig 15 and represents the actual link, a slender rod.

Appendix C

Webots software is used for modeling. Three snake robots are modeled with different number of modules n (8, 12, 16). Each module, link, is defined having a mass = 0.1 kg and link length = 28 cm. The inertia is calculated by the software using geometry of the link. Each link is connected to the next link using a rotational Servo, which defines a 1-DOF joint. This joint is an active joint and acts as a motor. The servo motors are position-controlled according to Eq. 3. Each of the four passive wheels at the bottom of the robot are modeled using a Servo but with maxForce = 0, which results in a passive joint. Contact properties are used to define contact properties between bodies. The following values are used: bounce = 0.5, and Coulomb friction = 0.1. Default values for all other environmental parameters, such as g = –9.8, are used.