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Application of the Biot Model to Ultrasound in Bone: Inverse Problem

Naima Sebaa, Zine E. A. Fellah, Mohamed Fellah, Erick Ogam, Farid G. Mitri, *Member, IEEE*, Claude Depollier, and Walter Lauriks

(Invited Paper)

***Abstract*—This paper concerns the ultrasonic characterization of human cancellous bone samples by solving the inverse problem using experimentally measured signals. The inverse problem is solved numerically by the least squares method. Five parameters are inverted: porosity, tortuosity, viscous characteristic length, Young modulus, and Poisson ratio of the skeletal frame. The minimization of the discrepancy between experiment and theory is made in the time domain. The ultrasonic propagation in cancellous bone is modelled using the Biot theory modified by the Johnson–Koplik–Dashen model for viscous exchange between fluid and structure. The sensitivity of the Young modulus and the Poisson ratio of the skeletal frame is studied showing their effect on the fast and slow waveforms. The inverse problem is shown to be well posed, and its solution to be unique. Experimental results for slow and fast waves transmitted through human cancellous bone samples are given and compared with theoretical predictions.**

I. Introduction

Osteoporosis is a disease caused by biochemical and hormonal changes affecting the equilibrium between the resorption and deposition of new bony tissue [1]. It leads to modification of the structure (porosity and thickness of trabeculae) and composition (mineral density) of this material. There has been much discussion of changes in trabecular pattern due to osteoporosis, but general indications are that the trabeculae grow thinner, possibly disappearing, and are therefore more widely spaced. Early clinical detection of this pathology via sound characterization would be of fundamental interest.

The primary method currently used for clinical bone assessment is based on X-ray absorptiometry, and measures total bone mass at a particular anatomic site [2]. Because other factors, such as architecture, also appear to have a role in determining an individual's risk of fracture, ultrasound, as an alternative to X-rays, has generated much attention [3], [4]. In addition to their potential for conveying the architectural aspects of bone, ultrasonic techniques also may have advantages in view of their use of nonionizing radiation and inherently lower costs, compared with X-ray densitometric methods. Although ultrasonic methods [5]–[15] appear promising for noninvasive bone assessment, they have not yet fulfilled their potential. Unfortunately, a poor understanding of the ultrasound interaction with bone has become one of the obstacles preventing it from being a fully developed diagnostic technique. Despite extensive research on the empirical relationship between ultrasound and the bulk properties of bone, the mechanism of how ultrasound physically interacts with bone is still unclear.

Since trabecular bone is an inhomogeneous porous medium, the interaction between ultrasound and bone is a highly complex phenomenon. Modelling ultrasonic propagation through trabecular tissue has been considered using porous media theories, such as Biot's theory [16], [17]. The Biot theory is an established way of predicting ultrasonic propagation in an inhomogeneous material and was originally applied to fluid saturated porous rocks for geophysical testing. The Biot model treats both individual and coupled behavior of the frame and pore fluid. Energy loss is considered to be caused by the viscosity of the pore fluid as it moves relative to the frame. The model predicts that the sound velocity and attenuation in a two-phase medium will depend on frequency, the elastic properties of the constituting materials, porosity, permeability, tortuosity, and effective stress. This method should allow us to relate the physical parameters of our porous medium to ultrasonic velocity and attenuation. The Biot theory has been applied to trabecular bone with varying degrees of success [18]–[24]. This theory predicts two compressional waves: a fast wave, whereby the fluid (blood and marrow) and solid (calcified tissue) move in phase, and a slow wave whereby the fluid and solid move out of phase. McKelvie [18], [21] predicted qualitatively the dependence of attenuation upon ultrasound frequency in cancellous bone; the attenuation values were of the right order of magnitude, but did not reproduce the full range of experimental values observed in natural tissues. However, McKelvie [18], [21] was unable to predict correctly the trends in ultra-

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sound velocity. Hosokawa and Otani [23] obtained better results by comparing the theoretical predictions using Biot theory and experiments for the wave velocities (fast and slow) than for acoustic attenuation. Williams [19] used a limited formulation of the Biot theory to calculate velocities alone and found good agreement for the fast wave velocity to predict experimental values obtained from tibial and femoral bovine cancellous bone samples. An excellent review of the application of Biot theory to ultrasound propagation through cancellous bone is given by Haire and Langton [22].

In this paper, the ultrasonic characterization of human cancellous bone is investigated using the modified [25] Biot theory. The inverse problem is solved in the time domain using experimentally measured signals. Five parameters are inverted: porosity, tortuosity, viscous characteristic length, Young's modulus, and Poisson's ratio of the skeletal frame. Experimental results are compared with theoretical predictions, giving a good correlation.

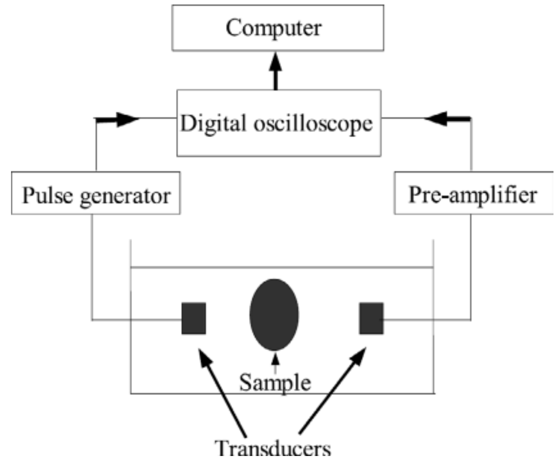

Fig. 1. Experimental setup for ultrasonic measurements.

II. Biot Theory

The equations of motion of the frame and fluid are given by the Euler equations applied to the Lagrangian density. Here, $\overrightarrow{u}$ and $\overrightarrow{U}$ are the displacements of the solid and fluid phases, respectively. The equations of motion are [17], [26]

$$\widetilde{\rho}_{11}\frac{\partial^2 \overrightarrow{u}}{\partial t^2} + \widetilde{\rho}_{12}\frac{\partial^2 \overrightarrow{U}}{\partial t^2} = P\overrightarrow{\nabla}.(\overrightarrow{\nabla}.\overrightarrow{u}) + Q\overrightarrow{\nabla}(\overrightarrow{\nabla}.\overrightarrow{U}) - N\overrightarrow{\nabla}\wedge(\overrightarrow{\nabla}\wedge\overrightarrow{u}), \quad (1)$$

$$\widetilde{\rho}_{12}\frac{\partial^2 \overrightarrow{u}}{\partial t^2} + \widetilde{\rho}_{22}\frac{\partial^2 \overrightarrow{U}}{\partial t^2} = Q\overrightarrow{\nabla}(\overrightarrow{\nabla}.\overrightarrow{u}) + R\overrightarrow{\nabla}(\overrightarrow{\nabla}.\overrightarrow{U}), \quad (2)$$

where P, Q, and R are generalized elastic constants which are related, via gedanken experiments, to other measurable quantities, namely, ϕ (porosity), K_f (bulk modulus of the pore fluid), K_s (bulk modulus of the elastic solid), and K_b (bulk modulus of the porous skeletal frame). N is the shear modulus of the composite as well as that of the skeletal frame. The equations that explicitly relate P, Q, and R to ϕ, K_f, K_s, K_b, and N are given by

$$P = \frac{(1-\phi)\left(1-\phi-\frac{K_b}{K_s}\right)K_s + \phi\frac{K_s}{K_f}K_b}{1-\phi-\frac{K_b}{K_s}+\phi\frac{K_s}{K_f}} + \frac{4}{3}N,$$
$$Q = \frac{\left(1-\phi-\frac{K_b}{K_s}\right)\phi K_s}{1-\phi-\frac{K_b}{K_s}+\phi\frac{K_s}{K_f}},$$
$$R = \frac{\phi^2 K_s}{1-\phi-\frac{K_b}{K_s}+\phi\frac{K_s}{K_f}}.$$

The Young modulus and the Poisson ratio of the solid E_s, ν_s and of the skeletal frame E_b, ν_b depend on the generalized elastic constants P, Q, and R via the relations:

$$K_s = \frac{E_s}{3(1-2\nu_s)},\quad K_b = \frac{E_b}{3(1-2\nu_b)},\quad N = \frac{E_b}{2(1+\nu_b)}. \quad (3)$$

The ρ_{mn} are the "mass coefficients," which are related to the densities of solid (ρ_s) and fluid (ρ_f) phases by $\rho_{11}+\rho_{12} = (1-\phi)\rho_s$ and $\rho_{12}+\rho_{22} = \phi\rho_f$, respectively. The coefficient ρ_{12} represents the mass coupling parameter between the fluid and solid phases and is always negative $\rho_{12} = -\phi\rho_f(\alpha-1)$, α being the tortuosity of the medium. To express the viscous exchanges between the fluid and the structure, which play an important role in damping the acoustic wave in porous material, the tortuosity α becomes a function of frequency, called the dynamic tortuosity $\alpha(\omega)$ [25]–[28]. The parts of the fluid affected by this exchange can be estimated by the ratio of a microscopic characteristic length of the medium, for example, pore size, to the viscous skin depth thickness $\delta = (2\eta/\omega\rho_f)^{1/2}$ (η: fluid viscosity, ω: angular frequency). This domain corresponds to the region of the fluid in which the velocity distribution is disturbed by the frictional forces at the interface between the fluid and the frame. At high frequencies, the viscous skin thickness is very thin near the radius of the pore r. The viscous effects are concentrated in a small volume near the surface of the frame $\delta/r \ll 1$. In this case, the expression of the dynamic tortuosity $\alpha(\omega)$ is given by [25]

$$\alpha(\omega) = \alpha_\infty\left(1+\frac{2}{\Lambda}\left(\frac{\eta}{j\omega\rho_f}\right)^{1/2}\right), \quad (4)$$

where α_∞ is the tortuosity, and Λ the viscous characteristic length.

For a slab of cancellous bone occupying the region $0 \le x \le L$ (Fig. 1), the incident $p^i(t)$ and transmitted $p^t(t)$ fields are related in the time domain by the transmission scattering operator $\tilde{T}$:

$$p^t(x,t) = \int_0^t \tilde{T}(\tau) p^i \left(t - \tau - \frac{(x-L)}{c_0} \right) d\tau, \tag{5}$$

where c_0 is the velocity outside the porous material. For the linear ultrasound propagation, the transmission operator is independent of the incident signal and depends only on the properties of the cancellous bone. In the frequency domain, the expression of the transmission coefficient $\mathcal{T}(\omega)$, which is the Fourier transform of $\tilde{T}$, is given by [26]

$$\mathcal{T}(\omega) = \frac{j\omega 2\rho_f c_0 F_4(\omega)}{(j\omega\rho_f c_0 F_4(\omega))^2 - (j\omega F_3(\omega) - 1)^2}, \tag{6}$$

where $F_4(\omega)$ and $F_3(\omega)$ are given in Appendix A.

In the next section, we solve the inverse problem and recover the physical parameters describing the propagation, using the theoretical expression of the transmission coefficient and the experimentally measured waves propagating through human cancellous bone samples.

III. Inverse Problem

The propagation of ultrasonic waves in a slab of cancellous bone is conditioned by many parameters: porosity ϕ, tortuosity α_∞, viscous characteristic length Λ, fluid viscosity η, the Young modulus of the elastic solid E_s, the Young modulus of porous skeletal frame E_b, the Poisson ratio of the elastic solid ν_s, the Poisson ratio of the porous skeletal frame ν_b, the solid density ρ_s, the bulk modulus of the saturating fluid K_f, and the fluid density ρ_f. It is therefore important to develop new experimental methods and efficient tools [29] for their estimation. The basic inverse problem associated with the slab of cancellous bone may be stated as follows: from measurements of the signal transmitted outside the slab, find the values of the medium's parameters. Solving the inverse problem for all of the Biot parameters using only the transmitted experimental data is difficult, if not impossible. To achieve this task requires more experimental data for obtaining a unique solution. For this reason, in this contribution we limit the inversion to the five parameters: E_b, ν_b, ϕ, α_∞, and Λ. In a previous paper [26] we reported the sensitivity of transmitted wave forms to variations of E_b, ν_b, ϕ, α_∞, and Λ. The solution of the direct problem involves the transmission coefficient expressed as a function of the physical parameters. The inversion algorithm for identifying the values of the slab parameters in the transmitted mode is based on the procedure: find the values of the parameters E_b, ν_b, ϕ, α_∞, and Λ such that the transmitted signal describes the scattering problem in the best possible way (e.g., in the least-square sense). The inverse problem is to find the parameters E_b, ν_b, ϕ, α_∞, and Λ which minimize the discrepancy function

$$U(E_b, \nu_b, \phi, \alpha_\infty, \Lambda) = \sum_{t_i} [s_{\text{theo}}(t_i) - s_{\text{exp}}(t_i)]^2, \tag{7}$$

where s_{exp} is the signal acquired using an oscilloscope, and s_{theo} is the simulated signal. The best set of parameters is that which minimizes the function F. The program of minimization is carried out under Matlab (The MathWorks, Inc., Natick, MA), using the "fminsearch" function which employs the method of the simplex to calculate the solution.

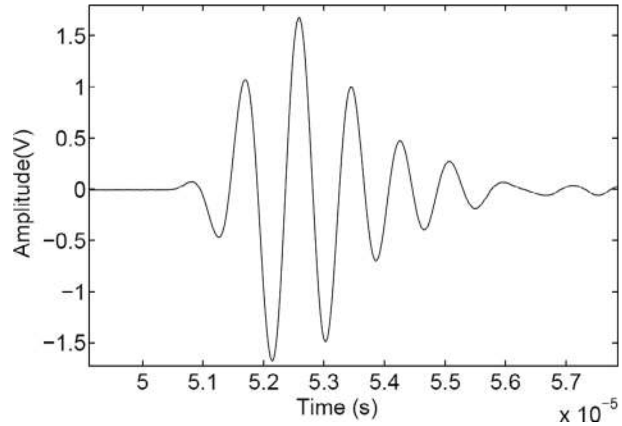

Fig. 2. Incident signal (hydroxyapatite).

IV. Ultrasonic Measurements

As an application of this model, some numerical simulations are compared with experimental results. Experiments are performed in water using two broadband Panametrics A 303S plane piezoelectric transducers (Panametrics, Waltham, MA) with a central frequency of 1 MHz in water, and a diameter of 1 cm; 400 V pulses are provided by a 5058 PR Panametrics pulser/receiver. The spectral composition (e.g., −6 dB bandwidth) of the ultrasonic pulse is 800–1200 kHz. Electronic interference is removed by averaging 1000 acquisitions. The experimental setup is shown in Fig. 1. The parallel-faced samples were machined from hydroxyapatite (a substitute for bone) and the femoral heads and femoral necks of human cancellous bone. The size of the ultrasound beam is very small compared to the size of the specimens. The emitting transducer insonifies the sample at normal incidence with a short (in time domain) pulse. When the pulse hits the front surface of the sample, a part is reflected, a part is transmitted as a fast wave, and a part is transmitted as a slow wave. When any of these components, travelling at different speeds, hit the second surface, a similar effect takes place: a part is transmitted into the fluid, and a part is reflected as a fast or slow wave. The fluid characteristics [26] are bulk modulus $K_f = 2.2$ GPa, density $\rho_f = 1000$ kg m^{-3}, and viscosity $\eta = 10^{-3}$ kg m s^{-1}. Consider a sample of hydroxyapatite: the pore size is approximately 100 μm), the thickness 12.5 mm, and solid density $\rho_s = 1700$ kg m^{-3}. The Young modulus $E_s = 13$ GPa and the Poisson ratio $\nu_s = 0.3$ of the solid are taken from the literature. Figs. 2 and 3 show the experimental, incident and transmitted signals, respectively. The inverse problem is solved by minimizing the function $U(E_b, \nu_b, \phi, \alpha_\infty, \Lambda)$. A large variation range is applied to each estimating parameter value in solving the

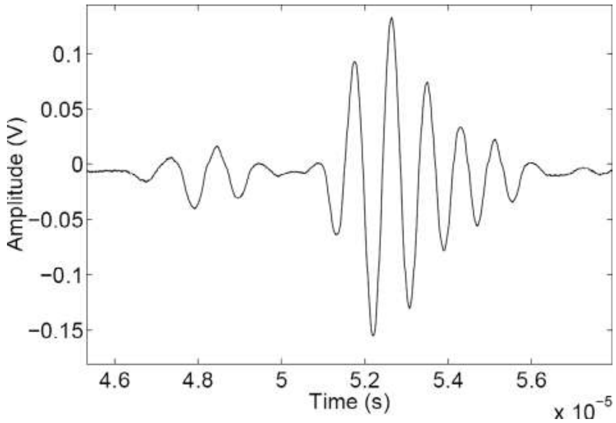

Fig. 3. Transmitted signal (hydroxyapatite).

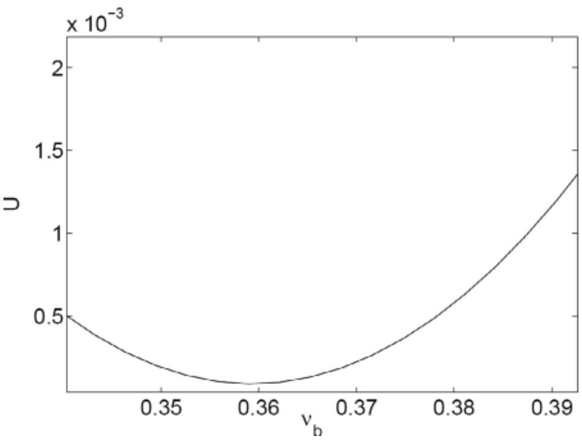

Fig. 4. Variation of the minimization F with the Poisson ratio of the skeleton frame (hydroxyapatite).

inverse problem. The variation range of the parameters is $\alpha_\infty \in [1, 3]$, $\phi \in [0.5, 0.99]$, $\Lambda \in [1, 300]\mu$m, $\nu_b \in [0.1, 0.5]$, and $E_b \in [0.5, 5]$ GPa. The variations of the cost function with the physical parameters present a single clear minimum corresponding to the mathematical solution of the inverse problem. This shows that the inverse problem is well posed mathematically, and that the solution is unique. The minimum, corresponding to the solution of the inverse problem, is clearly observed for each parameter. After solving the inverse problem, we find the following optimized values: $\phi = 0.79$, $\alpha_\infty = 1.06$, $\Lambda = 6.65$ μm, $\nu_b = 0.36$, and $E_b = 2.4$ GPa. Using these values, we present in Figs. 4–8 the variations in the discrepancy function U with respect to the values of the inverted parameters. For showing clearly the solution of the inverse problem, the variation of U in Figs. 4–8 is given only around the minima values of the inverted parameters. In Fig. 9, a comparison is made between the experimentally measured signal (dashed line) and the transmitted signal (solid line) simulated using the reconstructed values of E_b, ν_b, ϕ, α_∞, and Λ. The difference between the two curves is small, which leads us to conclude that the optimized values of the physical parameters are correct. The fast and slow waves predicted by the Biot theory are clearly visible in the transmitted signal.

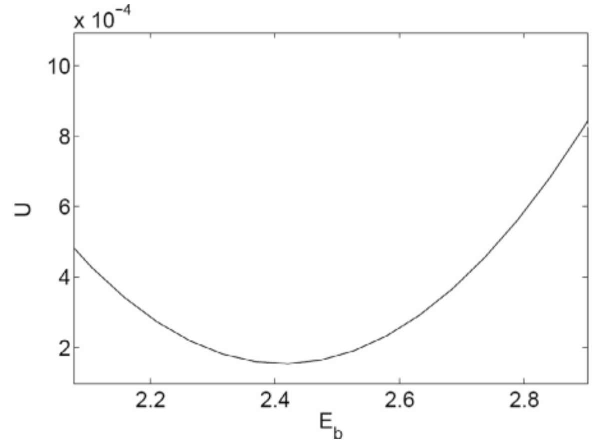

Fig. 5. Variation of the minimization F with the Young modulus of the skeleton frame (hydroxyapatite).

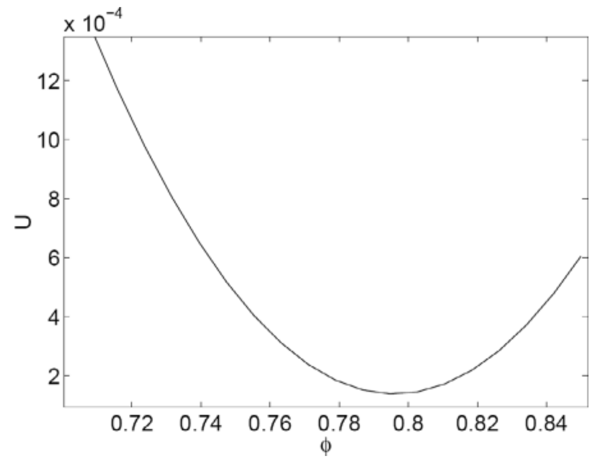

Fig. 6. Variation of the minimization F with the porosity (hydroxyapatite).

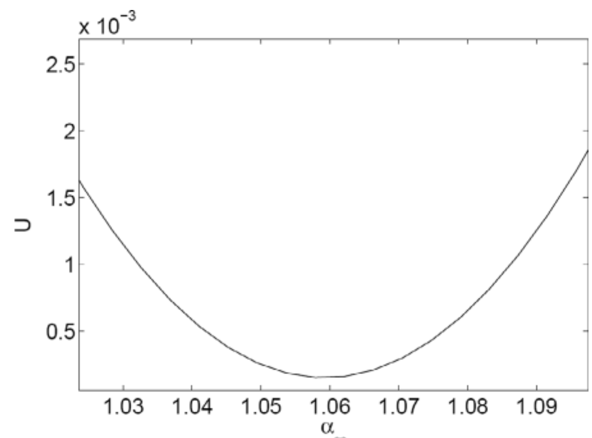

Fig. 7. Variation of the minimization F with the tortuosity (hydroxyapatite).

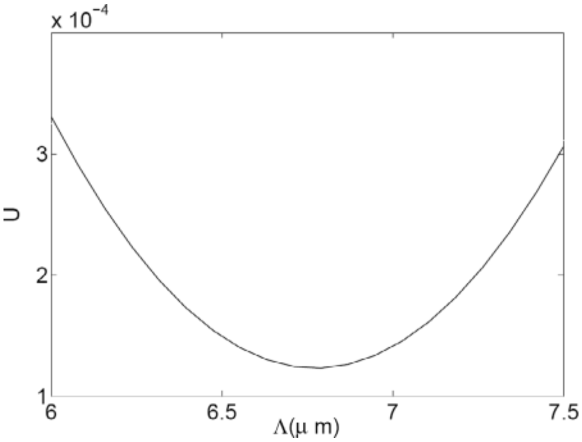

Fig. 8. Variation of the minimization F with the viscous characteristic length (hydroxyapatite).

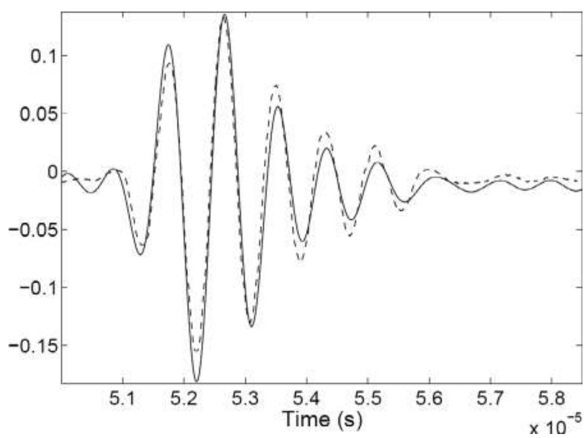

Fig. 9. Comparison between the experimentally measured (dashed line) and the simulated (solid line) transmitted signals (hydroxyapatite).

Let us now solve the inverse problem for samples of cancellous bone. The liquid in the pore space (blood and marrow) is removed from the bone sample and substituted by water. The experimentally measured wave forms travel through the cancellous bone in the same direction as the trabecular alignment (x direction). Consider a sample of human cancellous bone (femoral neck) of thickness 10.2 mm and solid density $\rho_s = 1990$ Kg.m^{-3}; the Young modulus $E_s = 13$ GPa and the Poisson ratio $\nu_s = 0.3$ of solid bone. By solving the inverse problem, the optimized values obtained are: $\phi = 0.72$, $\alpha_\infty = 1.14$, $\Lambda = 14.25$ μm, $\nu_s = 0.3$, and $E_b = 2.89$ GPa. Fig. 10 shows a comparison of experimentally measured signal and a simulated signal obtained by optimization after solving the inverse problem. Here, again, the correlation between theoretical predictions and experimental data is satisfactory. The different parameters have been calculated after minimization with different sample of cancellous bone and they are given in Table I.

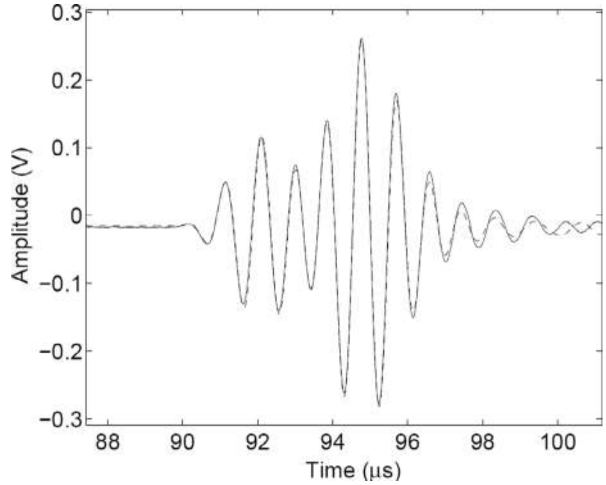

Fig. 10. Comparison between the experimentally measured (solid line) and the simulated (dashed line) transmitted signals (cancellous bone).

TABLE I
INVERTED PARAMETERS.

	L (mm)	ϕ	α_∞	Λ (μm)	ν_b	E_b (GPa)
M1	9.7	0.78	1.1	18.66	0.28	2.27
M2	11.1	0.8	1.1	24.54	0.3	1.97
M3	12	0.79	1.05	10.12	0.26	2.47

V. Conclusion

In this paper, the characterization of cancellous bone is treated by solving the inverse problem numerically using experimentally measured signals. The Biot theory modified by Johnson *et al.* model [25] is used to describe the viscous interaction between fluid and structure. Five physical parameters (porosity, tortuosity, viscous characteristic length, the Poisson ratio, and the Young modulus of the skeletal frame) are inverted. The modified Biot model is a well-adapted form for the analysis of the direct and inverse scattering problems.

Appendix A
Expression of the Transmission Coefficient

The expression of the transmission coefficient is given by [26]

$$\mathcal{T}(\omega) = \frac{j\omega 2\rho_f c_0 F_4(\omega)}{(j\omega\rho_f c_0 F_4(\omega))^2 - (j\omega F_3(\omega) - 1)^2},$$

where F_3 and F_4 are given by (8) (see next page).
The functions $\lambda_1(\omega)$ and $\lambda_2(\omega)$ are given by (9) (see next page), with

$$
\begin{aligned}
F_i(\omega) &= (1+\phi(\Im_i(\omega)-1))\sqrt{\lambda_i(\omega)}\frac{\Psi_i(\omega)}{\sinh\left(l\sqrt{\lambda_i(\omega)}\right)}\frac{2}{\Psi(\omega)}, \qquad i=1,2.\\
F_3(\omega) &= \rho_f c_0\left(F_1(\omega)\cosh\left(l\sqrt{\lambda_1(\omega)}\right)+F_2(\omega)\cosh\left(l\sqrt{\lambda_2(\omega)}\right)\right),\\
F_4(\omega) &= F_1(\omega)+F_2(\omega).
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
\lambda_1(\omega) &= \frac{1}{2}\left(-\tau_1\omega^2+\tau_2(j\omega)^{3/2}-\sqrt{\left(\tau_1^2-4\tau_3\right)\omega^4+2\left(\tau_1\tau_2-2\tau_4\right)(j\omega)^{7/2}+\tau_2^2(j\omega)^3}\right),\\
\lambda_2(\omega) &= \frac{1}{2}\left(-\tau_1\omega^2+\tau_2(j\omega)^{3/2}+\sqrt{\left(\tau_1^2-4\tau_3\right)\omega^4+2\left(\tau_1\tau_2-2\tau_4\right)(j\omega)^{7/2}+\tau_2^2(j\omega)^3}\right),
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
\Im_1(\omega) &= \frac{\left(2\tau_5-\tau_1\right)\omega^2+\left(\tau_2-2\tau_6\right)(j\omega)^{3/2}-\sqrt{\left(\tau_1^2-4\tau_3\right)\omega^4+2\left(\tau_1\tau_2-2\tau_4\right)(j\omega)^{7/2}+\tau_2^2(j\omega)^3}}{2\left(-\tau_7\omega^2-\tau_6(j\omega)^{3/2}\right)},\\
\Im_2(\omega) &= \frac{\left(2\tau_5-\tau_1\right)\omega^2+\left(\tau_2-2\tau_6\right)(j\omega)^{3/2}+\sqrt{\left(\tau_1^2-4\tau_3\right)\omega^4+2\left(\tau_1\tau_2-2\tau_4\right)(j\omega)^{7/2}+\tau_2^2(j\omega)^3}}{2\left(-\tau_7\omega^2-\tau_6(j\omega)^{3/2}\right)},
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\tau_1 &= R'\rho_{11}+P'\rho_{22}-2Q'\rho_{12},\\
\tau_2 &= A\left(P'+R'+2Q'\right),\\
\tau_3 &= \left(P'R'-Q'^2\right)\left(\rho_{11}\rho_{22}-\rho_{12}^2\right), \text{ and}\\
\tau_4 &= A\left(P'R'-Q'^2\right)\left(\rho_{11}+\rho_{22}-2\rho_{12}\right).
\end{aligned}
$$

Coefficients R', P', and Q' are given by

$$
\begin{aligned}
R' &= \frac{R}{PR-Q^2},\\
Q' &= \frac{Q}{PR-Q^2}, \text{ and}\\
P' &= \frac{P}{PR-Q^2}.
\end{aligned}
$$

The functions $\Im_1(\omega)$ and $\Im_2(\omega)$ are given by (10) (see above), where

$$
\begin{aligned}
\tau_5 &= \left(R'\rho_{11}-Q'\rho_{12}\right),\\
\tau_6 &= A\left(R'+Q'\right),\\
\tau_7 &= \left(R'\rho_{12}-Q'\rho_{22}\right).
\end{aligned}
$$

The coefficients $\Psi_1(\omega)$, $\Psi_2(\omega)$, and $\Psi(\omega)$ are given by

$$
\begin{aligned}
\Psi_1(\omega) &= \phi Z_2(\omega)-(1-\phi)Z_4(\omega),\\
\Psi_2(\omega) &= (1-\phi)Z_3(\omega)-\phi Z_1(\omega),\\
\Psi(\omega) &= 2(Z_1(\omega)Z_4(\omega)-Z_2(\omega)Z_3(\omega)),
\end{aligned}
$$

and the coefficients $Z_1(\omega)$, $Z_2(\omega)$, $Z_3(\omega)$, and $Z_4(\omega)$ by

$$
\begin{aligned}
Z_1(\omega) &= (P+Q\Im_1(\omega))\lambda_1(\omega),\\
Z_2(\omega) &= (P+Q\Im_2(\omega))\lambda_2(\omega),\\
Z_3(\omega) &= (Q+R\Im_1(\omega))\lambda_1(\omega),\\
Z_4(\omega) &= (Q+R\Im_2(\omega))\lambda_2(\omega).
\end{aligned}
$$

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