Application of the Cauchy Method for Extrapolating/Interpolating Narrow-Band System Responses

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***Abstract*— In this paper, it is shown that Cauchy's method can be used effectively to interpolate/extrapolate narrow-band system responses. The given information can either be theoretical datapoints or measured experimental data over a band. For theoretical data extrapolation or interpolation, the sampled values of the function and, optionally, a few of its derivatives have been used to reconstruct the function. For measured data, only measured values of the parameter are used to create broad-band information from limited data as derivative information is too noisy. Cauchy's method assumes that the parameter to be extrapolated/interpolated, as a function of frequency, is a ratio of two polynomials. The problem is to determine the order of the polynomials and the coefficients therein. The method of total least squares (TLS) has been used to solve the resulting matrix equation involving the coefficients of the polynomials. Typical numerical results have been presented to show that reliable interpolation/extrapolation can be done for various system responses.**

***Index Terms*—Cauchy's method, extrapolation, interpolation.**

I. Introduction

IN A HOST of applications in engineering, it is necessary to obtain information about a system over a broad range. In most cases it is not possible to evaluate the parameter of interest in a closed form. However, either theoretical or experimental data is available in a narrow band. Generation of the data over the broad band is not possible or may be extremely time-consuming. In this paper, the principle of analytic continuation is utilized by the Cauchy method [1] to extrapolate/interpolate the data over a wide band.

The Cauchy method deals with approximating a function by a ratio of two polynomials. Given the values of the function and its derivatives at a few points, the order of the polynomials and their coefficients are evaluated. Once the coefficients of the two polynomials are known, they can be used to generate the parameter over the entire band of interest.

Rational polynomials have been used extensively to model frequency-domain responses. The key difference between the various methods is the approach used to evaluate the order of the two polynomials and the coefficients that define them. In [1], the authors also introduce the frequency derivative technique. The approach uses the derivatives of the parameter being modeled with respect to frequency. These derivatives are used to evaluate the coefficients. The order of the polynomials is determined by the available information.

The more popular rational polynomial approach is to use Padé approximations [2]. The approach is based on the Taylor expansion of the parameter as a function of frequency, i.e., the moments of the parameter around a given frequency point. In [3], Sanaie *et al.* use complex frequency hopping without poles such that moments at different frequency points can be used to model the parameter of interest. The method is dependent on choosing expansion points in frequency and then comparing either resulting poles or the value of the rational polynomial generated by these expansion points. In [4], the authors use frequency-shifted moments to obtain the Padé approximation. The performance of the model is strongly dependent on the choice of frequency points. In both cases, the order of the polynomials is determined by the available information. In [5], Sakata has extended the Padé approximation to two dimensions.

The approach presented in this paper is significantly different from Padé-type approximations. The model does not depend on a Taylor expansion around a set of frequency points. The coefficients are obtained directly using the singular value decomposition (SVD) of a data matrix. The SVD also allows one to estimate the required order of the polynomials used in the rational model. This is a significant advance over the earlier published methods. Further, total least squares (TLS) are used to solve for the coefficients. This allows for some suppression of the effects of noise in the data.

The basic difference between this paper and the work in [1] is that, here, the computations have been made automatic utilizing the SVD. This methodology also helps address the two critical issues as to when the Cauchy model is not valid for the data at hand and when is the data adequate for a reliable interpolation/extrapolation. This is accomplished from the distribution of the singular values and a decision can be made on the reliability of the extrapolated/interpolated results.

In this paper, Cauchy's method has been utilized to generate broad-band currents on conducting bodies. The currents are used to calculate the radar cross section (RCS) of the body as a function of frequency. The extrapolation is done from narrow-

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band calculations of the currents using the method of moments (MoM). Particularly in the MoM, generation of the response at each frequency point is very time consuming. However, the current and its derivatives with respect to frequency can be quickly calculated at a few points using the MoM. Then Cauchy's method can be used to extrapolate/interpolate the current over a broad frequency range from which the RCS can be calculated.

Another example is the characterization of optical systems. The time required to evaluate a response over a broad range of the size parameter would be prohibitive. The Cauchy method can be used to generate the response of interest over a broad band from the value of the function at some discrete points.

The Cauchy method has wide application. A third example is in the area of filter analysis. In the laboratory it is not always possible to make accurate broad-band measurements. This problem is especially severe in the case of measuring the transfer function of a filter in the stopband. The signal-to-noise ratio may be too low to be confident about the measurements of filter characteristics. Here, the Cauchy method can be used to generate broad-band information from measured narrow-band data.

Yet another area of application for the Cauchy method is that of device characterization. A very useful tool in automated circuit design would be an online description of the characteristics of many devices, but since each device may be used under different operating conditions (each with its own frequency characteristic) the memory required to describe all devices would be prohibitive. Here, the Cauchy method can be used to generate broad-band information while storing the measured data at only a few frequency points.

In an application of the Cauchy method, the choice of polynomial orders is restricted by the information at hand. While it is true that the more information given, the higher one can choose the orders, this is not always desirable. In filter analysis especially, the choice of the order of the polynomials proves to be very important.

In this paper, the Cauchy technique is used for the interpolation and extrapolation of frequency-domain responses. In each of the cases mentioned above, the Cauchy technique would save a significant amount of program execution time or computer memory while still producing accurate results over broad-band frequencies. The method is tested and numerical results are presented along with two examples illustrating its use as a time-saving device.

II. The Cauchy Method

Consider a system function $H(s)$. The objective is to approximate $H(s)$ by a ratio of two polynomials $A(s)$ and $B(s)$ so that $H(s)$ can be represented by fewer variables.

Consider

$$H(s) \simeq \frac{A(s)}{B(s)} = \frac{\sum_{k=0}^{P} a_k s^k}{\sum_{k=0}^{Q} b_k s^k}. \tag{1}$$

Here, the given information may be the value of $H(s)$ and its N_j derivatives at some frequency points s_j, $j = 1, \cdots, J$. If $H^n(s_j)$ represents the nth derivative of $H(s)$ at point $s = s_j$, the Cauchy problem is as follows:

Given $H^{(n)}(s_j)$ for $n = 0, \cdots, N_j$, $j = 1, \cdots, J$, find P, Q, $\{a_k, k = 0, \cdots, P\}$, and $\{b_k, k = 0, \cdots, Q\}$.

One solution to this problem has been outlined in [1]. Here a different computational approach is taken which is more reliable and gives us *a priori* a level of confidence in the results. This includes whether the given data is adequate and the more important problem of whether Cauchy's method can be applied to the data. These issues can be addressed by utilizing the SVD in the solution of the problem.

As seen from [1], the unknowns a_k and b_k can be put in the following form:

$$[\mathbf{A}]a = [\mathbf{B}]b \tag{2}$$

or

$$[\mathbf{A} - \mathbf{B}]\begin{bmatrix} a \\ b \end{bmatrix} = 0 \tag{3}$$

$$[a] = [a_0, a_1, a_2, \cdots a_P]^T \tag{4}$$

$$[b] = [b_0, b_1, b_2, \cdots b_Q]^T \tag{5}$$

where the matrices $[\mathbf{A}]$ and $[\mathbf{B}]$, of orders $N \times (P+1)$ and $N \times (Q+1)$, respectively, and T denoted transpose.

Define

$$\mathbf{A} = [A_{(j,n),0}, A_{(j,n),1}, \cdots A_{(j,n),P}] \tag{6}$$

$$\mathbf{B} = [B_{(j,n),0}, B_{(j,n),1}, \cdots B_{(j,n),Q}]. \tag{7}$$

For ease of notation, define $[\mathbf{C}] \equiv [\mathbf{A} - \mathbf{B}]$. $\mathbf{C}$ is of order $N \times (P+Q+2)$. A SVD of the matrix $\mathbf{C}$ will give us a gauge of the required values of P and Q [7]. A SVD results in the equation

$$[\mathbf{U}][\mathbf{\Sigma}][\mathbf{V}]^H \begin{bmatrix} a \\ b \end{bmatrix} = 0. \tag{8}$$

The matrices $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and Σ is a diagonal matrix with the singular values of $\mathbf{C}$ in descending order as its entries. The columns of $\mathbf{U}$ are the left singular vectors of $\mathbf{C}$ or the eigenvectors of CC^H. The columns of $\mathbf{V}$ are the right singular vectors of $\mathbf{C}$ or the eigenvectors of $C^H C$. The singular values are the square roots of the eigenvalues of the matrix $C^H C$. Therefore, the singular values of any matrix are real and positive. The number of nonzero singular values is the rank of the matrix in (3) and so gives one an idea of the information in this system of simultaneous equations. If $\mathbf{R}$ is the number of nonzero singular values, the dimension of the right null space of $\mathbf{C}$ is $P + Q + 2 - R$. The authors' solution vector belongs to this null space. Therefore, to make this solution unique, one needs to make the dimension of this null space 1 so that only one vector defines this space. Hence, P and Q must satisfy the relation

$$R + 1 = P + Q + 2. \tag{9}$$

The solution algorithm must include a method to estimate R. This is done by starting out with the choices of P and Q that are higher than can be expected for the system at hand. Then, one gets an estimate for R from the number of nonzero singular values of the matrix $\mathbf{C}$. Now, using (9) better estimates for P and Q are obtained. Letting P and Q stand for these new estimates of the polynomial orders, one can recalculate the matrices $\mathbf{A}$ and $\mathbf{B}$. Therefore, one comes back to the relation

$$[\mathbf{C}]\begin{bmatrix} a \\ b \end{bmatrix} \equiv [\mathbf{A} - \mathbf{B}]\begin{bmatrix} a \\ b \end{bmatrix} = 0. \tag{10}$$

where $[\mathbf{C}]$ is a rectangular matrix with more rows than columns. Many methods to solve (10) are well documented [7]. For reasons indicated in the Appendix, the authors choose the method of total least squares TLS [8]. The Appendix also outlines the technique of TLS.

How the singular values of matrix $[\mathbf{C}]$ are distributed tells the user if: 1) the Cauchy method is applicable to the data at hand and 2) if the data are adequate, that a curve fitting can be carried out. Both of these tests are carried out by observing the distribution of the singular values of matrix $[\mathbf{C}]$ and the ratio of the largest singular value to the smallest singular value.

If there is a reasonable spread of singular values and the ratio of the largest to smallest singular value is larger than the signal-to-noise ratio in the data, then it is reasonable to proceed with the Cauchy method. For example, if the data have five effective bits, the approximate signal to noise ratio is 30 dB (each bit increases the signal to noise ratio by about 6 dB). If the above conditions are not satisfied, the Cauchy method may not yield meaningful results.

III. Applications of the Cauchy Method

A. The Method of Moments

The method of moments (MoM) approximates the interactions of complicated bodies with a set of smaller, easily solvable interactions [6]. The currents are approximated by a linear combination of some known basis functions. The problem then of evaluating the current density, as a function of frequency, reduces to finding the coefficients in the linear combination. This approach allows the problem to be written as a matrix equation with the unknown coefficients as the solution to the equation. The major limitation of the MoM is that a large matrix equation has to be solved at every frequency point of interest. If a large system is to be studied, the program execution time may be as long as days.

The Cauchy method can partially solve this problem. The MoM program generates information over a limited band from which the Cauchy method generates broad-band information.

1) Interfacing with the MoM: The Cauchy method can easily be incorporated as part of a MoM analysis. The MoM converts a linear operator equation into a matrix equation of the form

$$[V] = [Z][I] \tag{11}$$

Here, $[I]$ is the vector of coefficients in the representation of the current as a linear combination of basis functions. $[V]$ is the known excitation to the system, while $[Z]$ is the matrix that describes the interaction of the currents and the excitation.

Differentiating the above equation with respect to frequency results in a binomial expansion

$$[V]' = [Z]'[I] + [Z][I]' \Rightarrow [I]' = [Z]^{-1}[[V]' - [Z]'[I]]. \tag{12}$$

In general,

$$\begin{aligned} [V]^{(n)} &= \sum_{i=1}^{n} {}^{n}C_i [Z]^{(n-i)}[I]^{(i)} \Rightarrow [I]^{(n)} \\ &= [Z]^{-1}\left[[V]^{(n)} - \sum_{i=1}^{n-1} {}^{n}C_i [Z]^{(n-i)}[I]^{(i)}\right]. \end{aligned} \tag{13}$$

In the above equations, $[V]^{(n)}$ is the vector with each element of $[V]$ differentiated with respect to frequency n times. Similarly, $[Z]^{(n)}$ is the matrix generated by differentiating each element of the matrix $[Z]$ with respect to frequency n times.

Hence, using a MoM program, one can generate all the information needed to apply the Cauchy method. The use of derivative information saves execution time because no new matrix inversions are required to generate the additional information. Hence, evaluation of a derivative at a frequency point required $O(N^2)$ operations as opposed to solving for the currents at a frequency point which takes $O(N^3)$ operations. Each element of the solution current $[I]$ vector is treated as the function $H(s)$. Given the current and its derivatives at some frequency points, one can use the Cauchy method to approximate the current at many more points.

2) Numerical Examples: To test the Cauchy method, RCS's of five different perfectly conducting three-dimensional (3-D) bodies were calculated over wide frequency bands. A program to evaluate the currents on an arbitrarily shaped closed or open body using the electric field integral equation and triangular patching as described in [9] was used. The triangular patching approximates the geometry of the surface of the body with a set of adjacent triangles. The program then uses these currents to evaluate the RCS of the body. It was modified to also calculate the first four derivatives of the currents with respect to frequency. This information was used as input to a Cauchy subroutine. The original MoM program was used to calculate the RCS without the Cauchy method. The two RCS plots were compared to show the accuracy of the Cauchy method.

The bodies chosen were a sphere, a square plate, a disk, a concave, and a convex hemisphere. In all cases, the currents and their first four derivatives were evaluated at five frequency points. Hence, the total information allows a maximum of $5\times(4+1) = 25$ coefficients combined in the two polynomials of (1). In the application of the Cauchy method to the MoM, it was found that no singular values of the original matrix $[A - B]$ are zero. This is to be expected, since the current as a function of frequency is not a ratio of two polynomials. Hence, the higher the polynomial orders chosen, the more accurate the approximation would be. Therefore, in this application, the step of estimating R, P, and Q, in (9), is bypassed. Given the 25 samples, the numerator polynomial was of order 11 while the denominator was a polynomial of order 12. Physically, one knows that for the polynomial approximation to be stable, the numerator polynomial must be of lower order than the denominator polynomial.

The motivation to apply the Cauchy method to the MoM is to save program execution time. To get an idea of how much time can be saved, the program was timed for two of the above bodies and compared to the original MoM program. The two bodies chosen were the sphere and the plate.

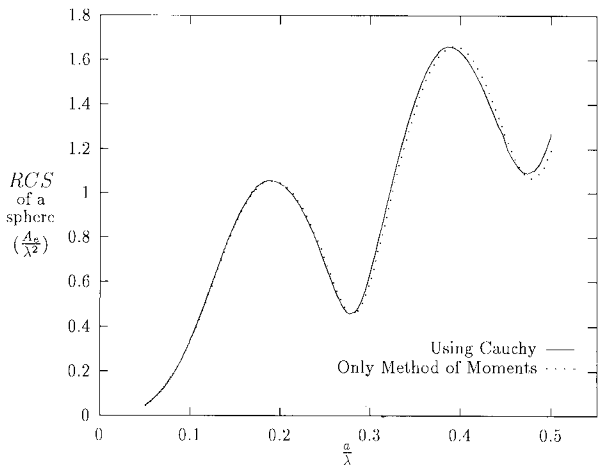


Fig. 1. Radar cross section of a sphere.

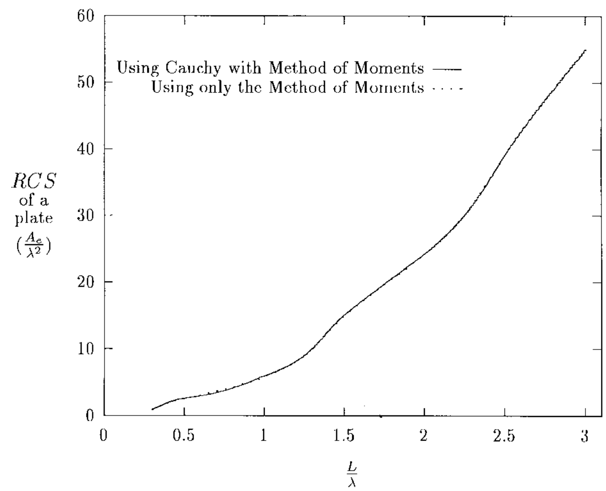


Fig. 2. Radar cross section of a square plate.

In the first example, a sphere of radius 0.3 m was analyzed. The sphere was triangularized using 182 nodes and 540 edges. Because the sphere is a closed object, this results in 540 unknowns in the expansion of the current in terms of the basis functions. The currents on the sphere and its first four derivatives, with respect to frequency, were evaluated at five frequency points. The points chosen were in the range $\lambda = 0.30$ m and $\lambda = 0.84$ m at a spacing of 0.135 m. Using this information and the Cauchy method, the current on the sphere was calculated for 51 points in the same frequency range. Using these currents, the RCS of the sphere was calculated at the 51 frequency points. The time taken for this calculation is compared to the time taken by the original MoM program to evaluate the RCS at five frequency points in the same range:

1) using the MoM only (for 5 points): 47 min 56 s;
2) interpolating with the Cauchy method (for 51 points): 57 min 57 s;
3) to generate the same information at 51 points, the MoM program would take approximately 8 h 8 min.

In Fig. 1, the results of applying the Cauchy method to the evaluation of the RCS of a sphere are seen. Here, the RCS is plotted over a decade bandwidth. This bandwidth was broken up into three ranges:

$$0.6 \text{ m} \leq \lambda \leq 1.0 \text{ m}$$
$$1.0 \text{ m} \leq \lambda \leq 1.8 \text{ m}$$
$$1.8 \text{ m} \leq \lambda \leq 6.0 \text{ m}.$$

In each of the three ranges the current and its first four derivatives were evaluated at five equally spaced points using the MoM program. Using this information, the polynomials in (1) were formed. This rational polynomial was used to evaluate the current at 51 points in each range. Also, the original MoM program was used to calculate the currents at a few points in the decade bandwidth. The currents were used to calculate the RCS of the sphere in this bandwidth. As can be seen from the figure, the agreement between the results from the use of the Cauchy program and the original MoM program is excellent.

As a second example, a square plate of side 0.3 m was analyzed. The plate was triangularized using 169 triangle nodes and 456 edges. In the MoM formulation the nodes on the boundary of an open object are not unknowns. Hence, the number of unknowns in this case was only 408. The procedure followed is similar to the analysis of the sphere. Here, the five frequency points chosen were in the range $\lambda = 0.15$ m and $\lambda = 0.30$ m at intervals of 0.0375 m. Using this information and the Cauchy method, the currents on the plate were evaluated at 201 frequency points. The time taken for this calculation is compared to the time taken by the original MoM program to evaluate the RCS at five frequency points in the same range:

1) using the MoM (for 5 points): 21 min 50 s;
2) interpolating with the Cauchy method (for 201 points) 27 min 47 s;
3) to generate the same information at 201 points, the MoM program would take approximately 14 h 38 min;
4) all programs were executed on an IBM RS6000 platform running AIX.

Fig. 2 shows the application of this technique to the evaluation of the RCS of a plate. Again, to evaluate the RCS over a decade bandwidth, three intervals were chosen and the two polynomials of (1) formed in each interval. The numerator polynomial had order 11 while the denominator polynomial had order 12. The intervals chosen were:

$$0.1 \text{ m} \leq \lambda \leq 0.15 \text{ m}$$
$$0.15 \text{ m} \leq \lambda \leq 0.3 \text{ m}$$
$$0.3 \text{ m} \leq \lambda \leq 1.0 \text{ m}.$$

The rational polynomial was used to evaluate the currents at 201 points in each range. The original MoM program was used to evaluate the RCS of the plate in this decade bandwidth. As can be seen from the figure, the agreement between the two results is excellent.

The third example is a disk of radius 0.3 m. The disk was triangularized using 142 nodes and 460 edges. Of these, only 440 were interior edges. Fig. 3 shows the RCS of the disk over a decade bandwidth. Here too, the decade bandwidth was broken up into three intervals and polynomials of order 11 and 12 formed in each interval. The rational polynomial was used to evaluate the currents and then the RCS of the disk at 51 frequency points in each range. The intervals chosen were:

$$0.6 \text{ m} \leq \lambda \leq 2.4 \text{ m}$$

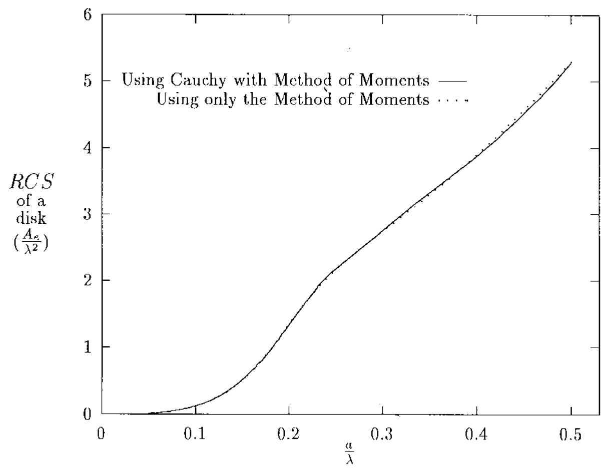


Fig. 3. Radar cross section of a disk.

$$2.4 \text{ m} \leq \lambda \leq 4.2 \text{ m}$$
$$4.2 \text{ m} \leq \lambda \leq 6.0 \text{ m}.$$

Fig. 4 shows the results of the final example of the Cauchy method applied to the MoM. The RCS of a convex and a concave hemisphere was calculated over a decade bandwidth. Fig. 4(a) shows the RCS of a convex hemisphere while Fig. 4(b) shows the RCS of a concave hemisphere. The radius of both hemispheres was 0.3 m. The convex hemisphere had 257 nodes and 736 edges. This resulted in a problem with 704 unknowns. The concave hemisphere had 316 nodes and 910 edges. Of these, 875 were interior nodes. The decade bandwidth was broken into the following ranges:

$$0.6 \text{ m} \leq \lambda \leq 1.0 \text{ m}$$
$$1.0 \text{ m} \leq \lambda \leq 2.6 \text{ m}$$
$$2.6 \text{ m} \leq \lambda \leq 6.0 \text{ m}.$$

As in the case of the disk, for the hemispheres too, the two polynomials of (1) were formed in each of the three ranges. In both cases, the MoM program evaluated the currents and its first four derivatives with respect to frequency at five points in each range. This information was used by the Cauchy subroutine to approximate the currents at 51 points in each range from which the RCS of the hemispheres were calculated at 51 points in each range. Also, the original MoM program was used to calculate the RCS over the decade bandwidth. As can be seen from Figs. 3 and 4, the agreement in each case is excellent.

B. Optical Computations

The calculation of either the scattering efficiency or the intensity is highly computationally intensive. If these parameters are desired over a broad range and at finely spaced points of the size parameter, the time required for the calculations may be prohibitive. The Cauchy method would solve this problem by needing the calculations to be done at a much coarser spacing and interpolate the parameter of interest.

This application was tested on the scattering efficiency of a sphere as a function of size parameter [10]. The sphere had an index of refraction of 2.0. The original data calculated the scattering efficiency at a spacing 0.002 in the size parameter.

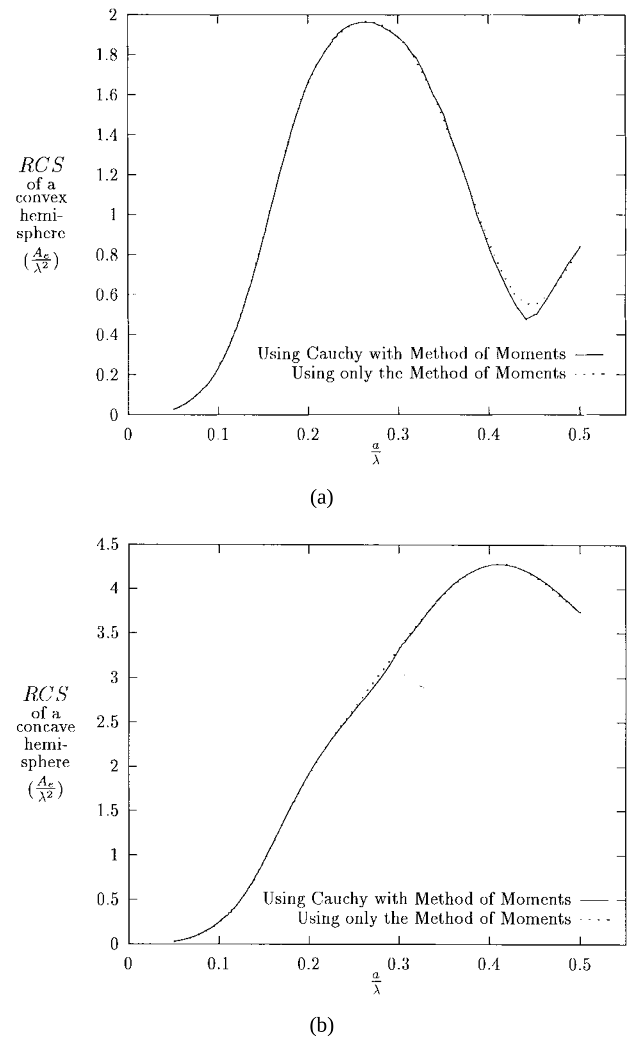


Fig. 4. Radar cross section of a hemisphere. (a) Convex hemisphere. (b) Concave hemisphere.

The range of the size parameter was from 7.0 to 8.0. Hence, the original data had 501 points. The Cauchy method needed a spacing of 0.01 in the size parameter, without any derivative information, to accurately calculate the scattering efficiency of the sphere at the original 501 points. This cuts down the program execution time by a factor of 5. The input to the Cauchy program is shown in Fig. 5(a).

Because all computer calculations suffer from roundoff error, most of the singular values returned from the SVD subroutine are not exactly zero. The choice of the threshold was such that a singular value was considered zero if it was 18 orders of magnitude lower than the largest singular value. This is because the data was in double precision. Only 72 singular values are above the chosen threshold, i.e., $R = 72$. Using this estimate for R and (9), the choice for the polynomial orders was reduced to 35 for the numerator and 36 for the denominator. Using these polynomials, the scattering efficiency was calculated at the original 501 points.

Fig. 5(b) shows the results of the application of the Cauchy method to optical computations. The dotted line represents the original data while the unbroken line represents the interpolated data. As can be seen, the two plots are nearly visually indistinguishable. Also, even though the input data to the

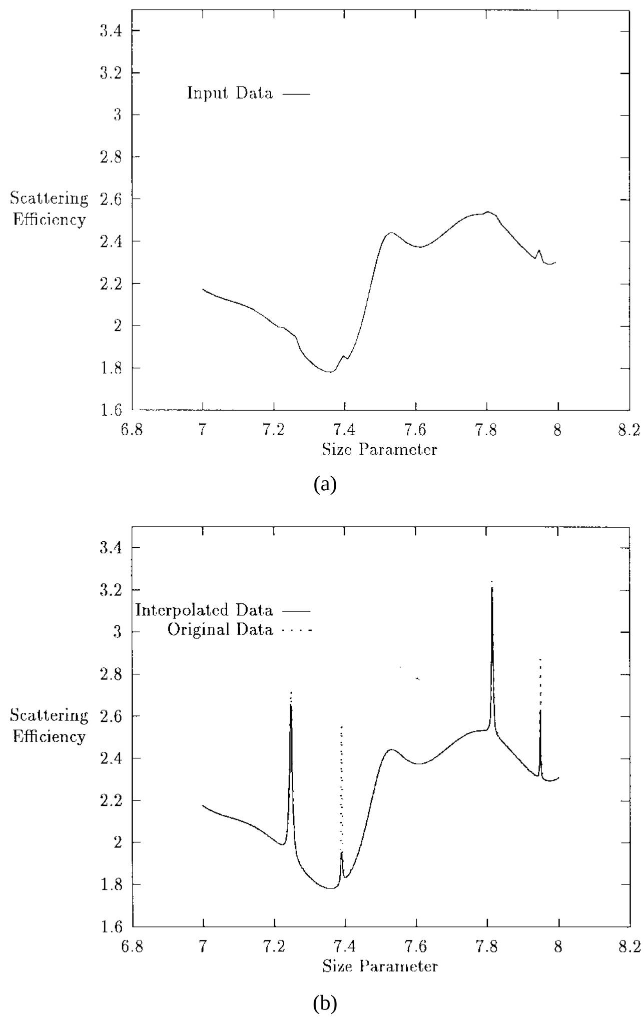


Fig. 5. Scattering efficiency as a function of size parameter. (a) Input to the Cauchy program. (b) Results of application of the Cauchy method to optical computations.

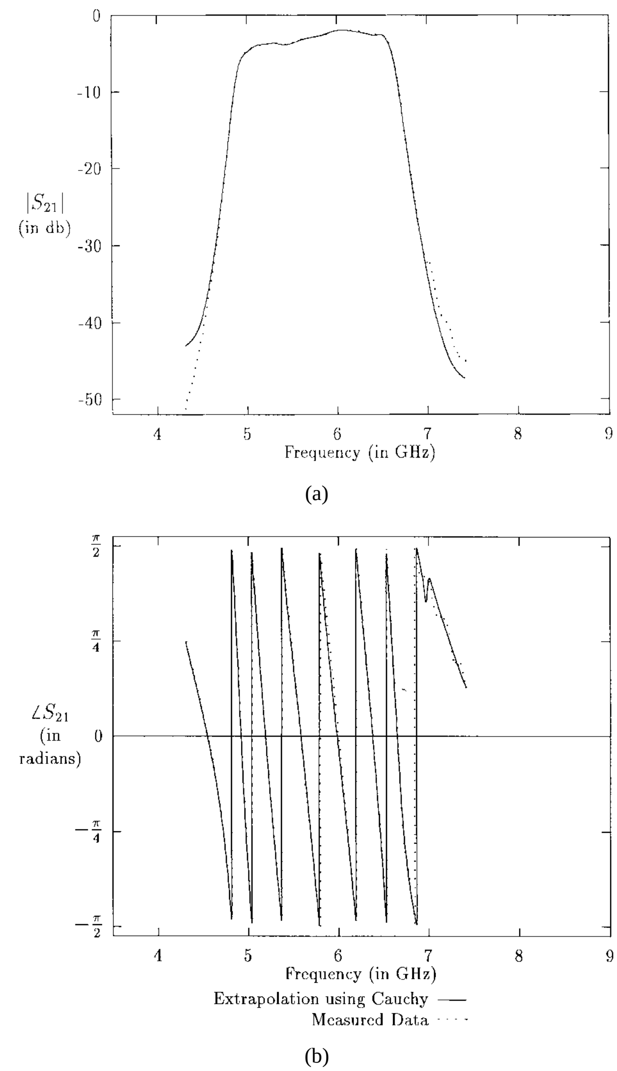


Fig. 6. Generation of stopband response using passband data. (a) Magnitude response. (b) Phase response.

program did not have the peaks of the scattering parameter, the Cauchy method was able to reproduce them.

C. Filter Analysis

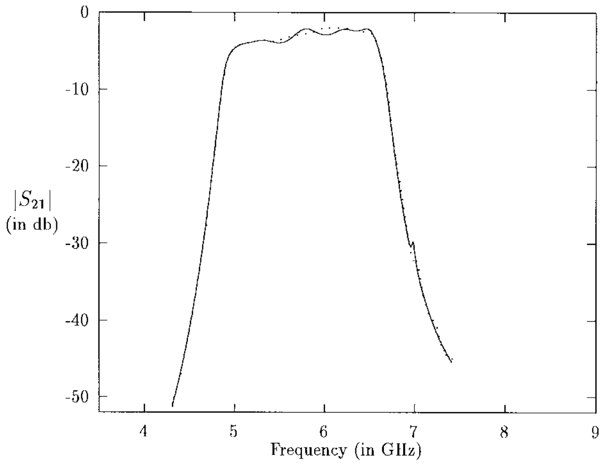
The Cauchy method can also be used in analysis of filters over broad frequency ranges. A filter response is a ratio of two polynomials and, hence, lends itself easily to the use of a Cauchy method. This has practical application to the problem of generating the stopband response given the passband response or the reverse, i.e., generating the passband response given some data from the stopband.

A filter transfer function (S_{21}) was measured using a network analyzer at frequency points in and out of the filter passband. The filter had its 3-dB points at 4.98 and 6.61 GHz, respectively. Hence, the filter had a passband of 1.63 GHz with a center frequency of 5.80 GHz. The filter response was measured at 415 equally spaced points in the frequency range 4.31–7.42 GHz.

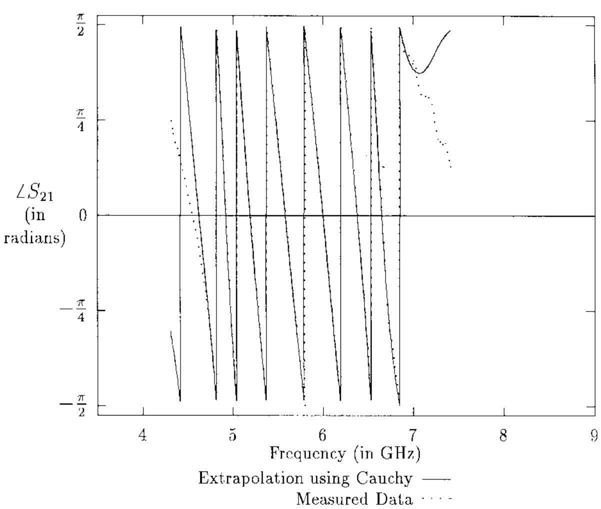
In the first application, the response over the entire band of measurement was recovered using mostly passband information. 51 equally spaced points, in the frequency range 4.79–6.96 GHz, were chosen as input to a Cauchy method. Because this is measured data, there is no information about the derivative of the transfer function with respect to frequency.

The threshold was chosen such that a singular value was considered zero if it was 14 orders of magnitude lower than the largest singular value. After the method checked for the number of nonzero singular values, the estimate for R was 16. The order of the numerator polynomial was chosen to be 7 while that of the denominator polynomial was chosen to be 8.

In Fig. 6(a) and 6(b) the results from the Cauchy program are described. Fig. 6(a) shows the magnitude response while Fig. 6(b) shows the phase response of the filter. As is often the case with filters, the magnitude response was considered more important. Hence, the phase response was allowed to show a poor agreement so as to maximize the agreement of the magnitude response. If a 10% error in the magnitude were acceptable, the extrapolation is valid for 0.39 GHz. This is 6.7% of the center frequency and 23.9% of the bandwidth. For frequencies beyond the passband, the extrapolation is accurate within 10% up to 7.42 GHz, the frequency until which data

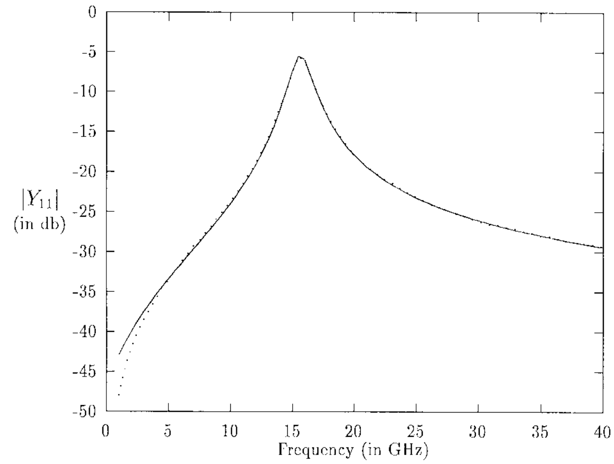


(a)

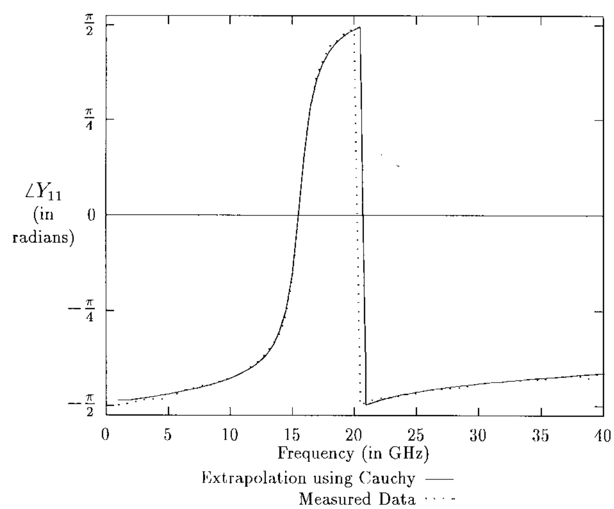


(b)

Fig. 7. Generation of passband response using stopband data. (a) Reconstructed magnitude response. (b) Reconstructed phase response.



(a)



(b)

Fig. 8. The Cauchy method applied to device characterization. (a) Magnitude Y_{11} reconstructed over broad frequency range. (b) Phase $\angle Y_{11}$ over same range.

was available. Hence, an accurate data over a bandwidth of 3.32 GHz starting with data over a bandwidth of 2.16 GHz is generated.

In the second application, data from the stopband and a little from the passband was used to interpolate into the passband. Here too, the choice of threshold is very important. Using the same threshold as in the first application, the estimate of R remained the same. Hence, in this case too, the numerator polynomial had order 7 while the denominator had order 8. In this case, 23 equally spaced points from 4.31 up to 5.35 GHz and 28 equally spaced points from 6.20 to 7.42 GHz were used to interpolate into the passband. This represents on interpolation of 0.85 GHz, which is 14.6% of the center frequency or 52.7% of the bandwidth. Fig. 7(a) and 7(b) show the results of this application. Fig. 7(a) is the reconstructed magnitude response and Fig. 7(b) is the reconstructed phase response. Again, since more attention is paid to the magnitude response, the phase response shows poorer agreement with the true response.

In both figures the dotted line represents the measured data while the continuous line represents the results of the Cauchy method.

D. Device Characterization

An application of the Cauchy method is in creating a database of many devices working in varying operating conditions. The Cauchy method would require the value of a parameter at a few frequency points and use this information to evaluate the parameter over a wide frequency band. Over many devices, and their operating conditions, this would yield significant savings in memory requirements.

To test this application, the Y-parameters of a pseudomorphic high electron mobility transistor (PHEMT) were measured over the range of 1.0–40.0 GHz. Just five of these points were used as input to the Cauchy method. The points chosen were at the frequency points 1.0, 10.0, 20.0, 30.0, and 40.0 GHz, respectively. This resulted in a numerator polynomial of order 1 and denominator polynomial of order 2. Here again, the step of estimating R, P, and Q is bypassed. Fig. 8(a) shows the magnitude $|Y_{11}|$ reconstructed over this broad frequency range. Fig. 8(b) shows the phase $\angle Y_{11}$ over the same range. As can be seen, the agreement with the measured values and the interpolated values is excellent.

IV. Conclusions

This paper has presented a technique with many practical applications. The Cauchy method starts by assuming that the parameter of interest, as a function of frequency, can be approximated by a simple rational polynomial function. The method presented here uses the singular value decomposition to evaluate the order of the polynomials and the coefficients that define them. Using this form, the parameter is evaluated at many frequency points. It is shown that the technique has applications to many practical problems. In this paper, the technique is applied to the MoM, optical systems, filter analysis, and device characterization. In all applications the Cauchy method has shown to save time and memory.

The examples presented here show that the Cauchy method places little restriction on the required data. In the case that the derivative of the parameter of interest with respect to frequency is available, this information is used. However, if the derivatives are not available, the values of the parameter at given frequency points is adequate.

A topic of further research is to investigate when and how the Cauchy method breaks down. As can be seen from the results of applying the Cauchy method to model filter responses, the method fails to yield extensive extrapolation or interpolation in the case of very low signal-to-noise ratios. Initial results into the investigation of the effect of noise in the data show that the method performs well for a signal-to-noise ratio of over 20 dB [11].

It must be pointed out that the Cauchy method is completely general and can be used to extrapolate or interpolate with respect to any variable other than frequency. However, in many applications in electromagnetics, frequency is the variable of interest.

Appendix

Many methods to solve (10) are known [7]. The usual approach is that of least squares (LS). In this, the equation is rewritten as

$$[\mathbf{A} - \mathbf{B}]^H[\mathbf{A} - \mathbf{B}]\begin{bmatrix} a \\ b \end{bmatrix} = 0. \tag{14}$$

The solution $\begin{bmatrix} a \\ b \end{bmatrix}$ is taken as the eigenvector corresponding to the zero eigenvalue of the resulting matrix. However, as has been seen, it is important to limit the rank of the null space of the matrix $[\mathbf{A} - \mathbf{B}]$ to one. But, this approach has an extra step of a matrix multiplication. Also, since the eigenvalues are not sorted, it is additional work to find the number of nonzero eigenvalues.

A better approach would be the TLS [8]. In the matrix of (10), the submatrix $\mathbf{A}$ is a function of the frequencies only and does not depend on the parameter measured. Hence, this matrix is not affected by measurement errors and noise. However, the submatrix $\mathbf{B}$ is affected by the errors. To take this nonuniformity into account, one needs a QR decomposition of the matrix $[\mathbf{A} - \mathbf{B}]$ up to its first $P+1$ columns. A QR decomposition of the matrix results in

$$\begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{0} & \mathbf{R}_{22} \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 0 \tag{15}$$

where, $\mathbf{R}_{11}$ is upper triangular and $\mathbf{R}_{22}$ is completely affected by the noise. Hence,

$$\Rightarrow [\mathbf{R}_{22}]b = 0 \tag{16}$$

and

$$[\mathbf{R}_{11}]a = -[\mathbf{R}_{12}]b. \tag{17}$$

A SVD of $\mathbf{R}_{22}$ results in the equation

$$[\mathbf{U}][\mathbf{\Sigma}][\mathbf{V}]^H b = 0. \tag{18}$$

By the theory of the TLS [8], the solution of the above equation is proportional to the last column of the matrix $\mathbf{V}$. Hence, one can choose

$$b = [\mathbf{V}]_{Q+1}. \tag{19}$$

This is the optimal solution even in the case that the matrix $\mathbf{R}_{22}$ does not have a null space. This was possible when the Cauchy method to the MoM was applied.

Using this solution for the denominator coefficients and using (22), the numerator coefficients using the conventional LS solution can be solved. The above TLS approach removes some of the errors of the conventional LS approach.

It can be shown [8] that for the case where $\mathbf{R}_{22}$, is contaminated by noise, the TLS is the optimum solution technique.

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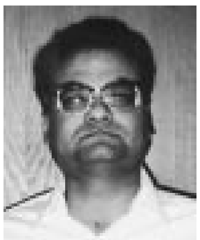

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