Application of the finite element method to the design of permanent magnets

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ABSTRACT

A new method of determining the lengths of magnets in a magnetic circuit by using the finite element method has been developed. This method has the advantage that the lengths of magnets which produce the prescribed flux distribution can be directly calculated.

In this paper, the error of this method is discussed at first,and then an example of application determining the shape of a magnet is shown. This method is effective for the design of magnetic circuits consisting of several permanent magnets and the determination of the shapes of magnets.

1. INTRODUCTION

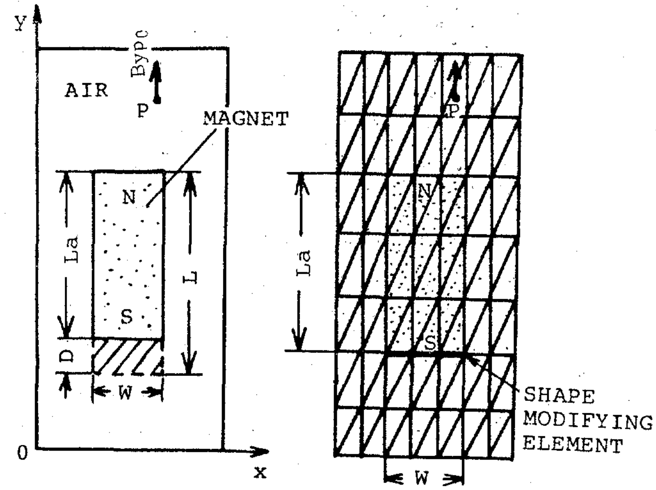
If the flux densities at several points in a magnetic field produced by magnets are prescribed and the lengths of magnets which produce such a flux distribution are unknown, many iterative modifications of assumed lengths of magnets are necessary in order to determine the lengths of magnets by using the usual finite element analysis.

A new method which is called the "finite element method with shape modification" [1] has been developed in order to solve the above-mentioned problems. However, the assumptions of this method lead to some errors. Then, the error is examined. The applicability of this newly developed method of determining the lengths of magnets with complicated shapes is also investigated.

2. ANALYSIS

When we want to produce a magnetic field, in which the flux densities at several points are prescribed, by using magnets, the problem is to determine the positions and sizes of the magnets. The positions and the widths of the magnets are often prescribed in advance. In such a case, usually the lengths of the magnets are assumed and the flux distribution is analyzed numerically. If the assumed length {La} is different from the true length {Lt}, iterative modifications of the assumed length {La} are necessary until satisfactory values are obtained by using a numerical method. {La} and {Lt} denote the column matrices of the lengths of magnets.

A new method by which the lengths of magnet producing the given flux densities at some prescribed points can be directly calculated is developed. This method is explained by an example shown in Fig. 1. For simplicity, the width W and the magnetization of the magnet are given. The hatched part in Fig. 1(a) shows the difference between the assumed magnet and the obtained magnet. D is the length of this part and it is directly calculated by our new method. The method is conceived from the idea that the hatched part can be equivalently denoted by line elements represented by thick lines in Fig. 1(b), because the flux lines in this part pass nearly perpendicular to the line elements. These line elements are called the "shape modifying element" [1],[2]. This element has the same energy as the hatched part in Fig. 1(a). If the calculated energy of the shape modifying element is positive, it means that the magnet should be lengthened. The final shape of the magnet is obtained by moving the side of the assumed magnet, on which the shape modifying element is set.



(a) model of a magnetic circuit (b) finite element subdivision

Fig. 1. Shape modifying element.

In this method, the following assumptions are necessary [2].

(a) The energy in the hatched part can be concentrated on the line element.

(b) The magnetization of the line element is equal to that of the adjacent magnet.

When the flux densities at some points, and the positions and the widths of magnets are prescribed, the lengths of magnets can be calculated by using the finite element method. The following equation is derived from the above-mentioned idea[2]

$$\begin{bmatrix}[H] & [C]\\ [F] & \end{bmatrix}\begin{Bmatrix}\{A\}\\ \{D\}\end{Bmatrix}=\begin{Bmatrix}\{Q\}\\ \{B_0\}\end{Bmatrix} \qquad (1)$$

where {A} and {B0} are the column matrices of vector potentials and the prescribed flux densities. {Q} is a function of equivalent magnetic current densities and the known vector potentials on the Dirichlet boundaries [2]. [H] is the so-called coefficient matrix, and [C] and [F] are the constant matrices,respectively [2].

Although the modified values {D} are directly obtained by the process denoted by the solid lines in Fig. 2, these results are not satisfactory because of the error caused by the above-mentioned assumptions. Therefore, usually a few repetitions are necessary until the desired results are obtained by modifying the subdivision as shown by the broken lines in Fig. 2.

Only the linear analysis is dealt with in this paper. The non-linear analysis will be reported later.

3. FUNDAMENTAL STUDY OF ERRORS

Our method contains the error due to the assumptions mentioned in section 2. Then, the error of this method is examined by a linear simplified model shown in Fig. 3. In Fig. 3, α-β-γ-δ-α denotes the

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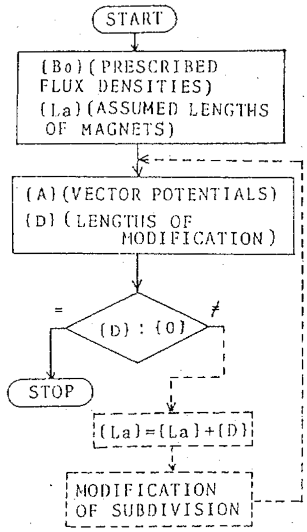


Fig. 2. Process of calculation.

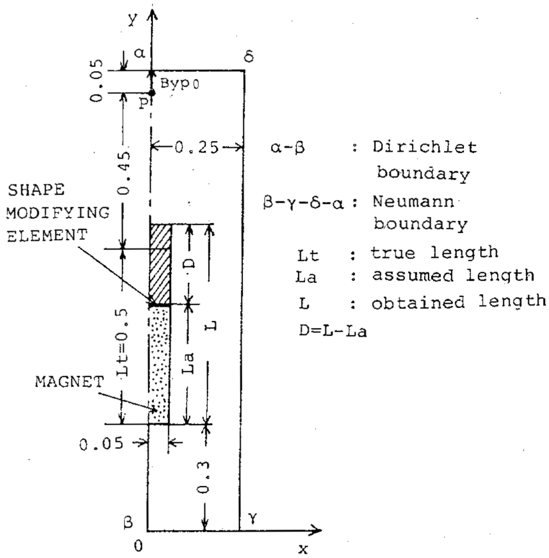


Fig. 3. Analyzed model.

analyzed region which is half of the whole region. The y-component Bypo of the flux density at a point P is prescribed. Figure 3 shows the case when the shape modifying element which is denoted by the thick line is set on the same side of the magnet as the prescribed point P.

Figure 4 shows the error ε_L of the length of the magnet and the error ε_B of the flux density. ε_L and ε_B are defined by the following equations.

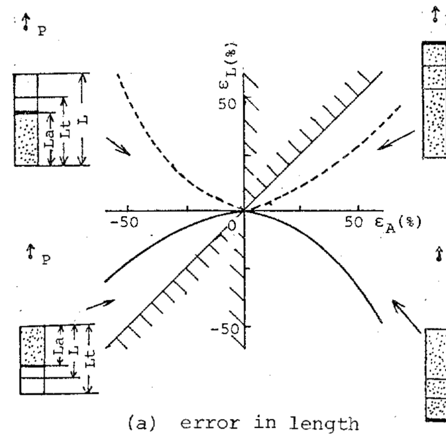
$$\varepsilon_L = \frac{L - Lt}{Lt} \times 100(\%) \tag{2}$$

$$\varepsilon_B = \frac{Byp - Bypo}{Bypo} \times 100(\%) \tag{3}$$

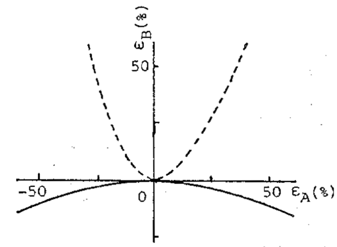
Where L and Byp are the obtained magnet length and the calculated flux density, respectively. The error ε_A of the assumed length is defined by the following equation.

$$\varepsilon_A = \frac{La - Lt}{Lt} \times 100(\%) \tag{4}$$

The relationships among La, Lt and L in each region are also shown in Fig. 4(a). There is no curve in the hatched parts from the physical point of view. Each upper half part of Fig. 4 (a) and (b) corresponds to Fig. 3. The shape modifying element is set on the same side of the magnet as the prescribed point P in



(a) error in length



(b) error in flux density

Fig. 4. Errors in length and flux density.

Fig. 3. When the shape modifying element is set on the opposite side of the magnet, the sign of the error ε_L is always negative and the lower half of Fig. 4 corresponds to this case. The error is due to the assumption (a) mentioned in section 2. As the analysis is a linear one, the assumption (b) does not cause the error.

It is concluded from Fig. 4 that the shape modifying element should be set on the opposite side to the prescribed point, and the assumed length {La} should be smaller than {Lt}.

4. AN EXAMPLE OF APPLICATION

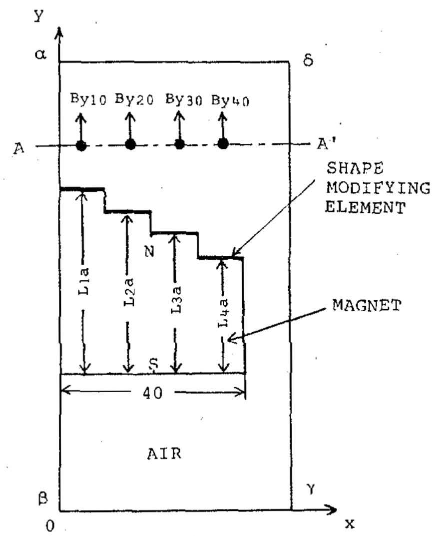
An example determining the shape of a magnet which produces a prescribed flux distribution is analyzed by using our method. The rotor of a synchronous generator with permanent magnets can be designed by a similar method.

Figure 5(a) shows the analyzed model whose flux distribution on the line A-A' is prescribed to be spatially sinusoidal as shown in Fig. 5(b). Four shape modifying elements are set on the same sides as the points at which the flux densities are prescribed as shown by the thick lines. The final shape of the magnet can be obtained by moving the upper side of the magnet by D.

The errors ε_L and ε_B in the length of magnet and the flux density are investigated when the initial values L_{1a}, L_{2a}, L_{3a}, L_{4a} of the lengths of magnets are 40, 35, 30, 25 mm, respectively. Figure 6 shows the errors ε_L and ε_B at each step of iterations denoted by the broken lines in Fig. 2. ε_L and ε_B after two iterations are both within about 0.8 %. The obtained magnet lengths L_1, L_2, L_3, L_4 are 37.3, 36.0, 33.5, 30.1 mm, respectively. Even if the maximum error of the initial assumed magnet length is about 16 %, it is possible to obtain the satisfactory lengths of the magnet by only a few iterations.

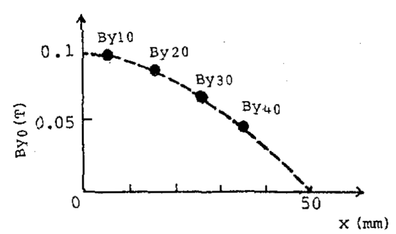
Figure 7 shows the flux distribution of the obtained magnet.

It is clear that our method is applicable to determining the size of magnet having a complicated shape.



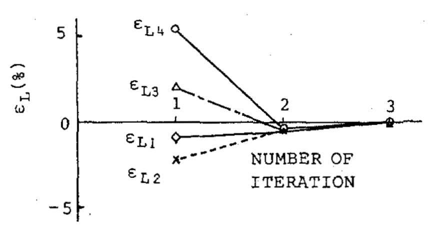
$B_{y10}, \cdots, B_{y40}$: prescribed flux densities
α-β : Dirichlet boundary
β-γ-δ-α : Neumann boundary

(a) analyzed model

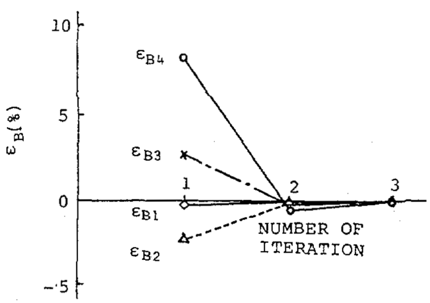


(b) prescribed flux distribution on the line A-A'

Fig. 5. Assumed shape of magnet and prescribed flux distribution.



(a) error in length



(b) error in flux density

Fig. 6. Errors at each iteration.

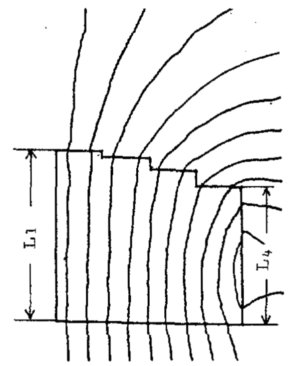


Fig. 7. Flux distribution of the obtained magnet.

5. CONCLUSIONS

The new method enables us to design the most suitable magnetic circuit composed of several magnets with complicated shapes.

The analysis of the following problems will be reported later: (a) error which occurs when there are more than two magnets, (b) analysis of the case when the directions of the prescribed flux densities do not coincide with the axes, (c) non-linear analysis, (d) modification of the width of magnet, (e) the optimum design of magnets having the minimum volume, (f) experimental study.

Our method will be also extended to the optimum design of electric fields.

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