Application of the Linear Prediction filters in equatorial electrojet studies

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Abstract. Wiener filters are derived from the horizontal field data of two adjacent equatorial electrojet stations, Adis Ababa and Trivandrum from several sequences of quiet days. The time invariant property of the filter is established and the filter is applied to conditions marked by afternoon counter electrojet events. The prediction efficiency is shown to be consistently high. Possible uses of this technique in studies related to generating mechanisms of counter electrojet events and the day-to-day variability in electrojet currents are indicated. Inter-relationships and day-to-day variability of different components of the fields at the two stations are highlighted.

Keywords. Linear prediction filters; equatorial electrojet; counter electrojet

1. Introduction

The technique of Linear Prediction filtering was an innovation by Wiener (1942) which was adapted for application to discrete data by Levinson (1949). Much later, its usefulness in magnetospheric physics was demonstrated by Iyemori et al (1979) and Iyemori and Maeda (1980). The technique, based on least squares criteria, leads to an empirical determination of the governing linear relationship between an input function and a suitable output function. This is described in the form of a sequence of prediction filter weights which are derived from the auto-correlation coefficients of the input time series and the cross-correlation coefficients between the input and observed output. The tacit assumptions for the validity of the functional relationship are (Robinson 1967):

(i) The X and Y series are stationary and their errors are uncorrelated, where X may be considered input and Y the output series.
(ii) A linear relationship exists between X and Y.
(iii) The linear relationship is time-invariant.

In other words, filters derived from different epochs of X and Y data should exhibit nearly the same response characteristic. The Wiener prediction filter or the impulse response function so derived describes the link between input and output as a function of time lag.

The optimum length of the filter can be determined from the normalized mean square error sequence as it levels off indicating no improvement in the prediction efficiency with further increase in the number of filter coefficients. We can also define

a prediction efficiency parameter (PE) defined as

$$PE = (1 - \sigma_T^2/\sigma_o^2) \times 100$$

(Clauer *et al* 1983) which gives a quantitative measure of the 'goodness of fit'. σ_T^2 is the variance of the residuals and σ_o^2 that of the observed output. The impulse response proves useful in understanding the physical mechanism involved in the interaction processes governing the causative (input) and the resultant (output) parameters. An excellent review of the application of the technique in studies of solar wind-magneto-sphere coupling is given by Clauer (1986). McPherron *et al* (1986) have shown the effectiveness of the method in real time prediction of geomagnetic activity while Fay *et al* (1986) have applied the Wiener technique for the Dst index and polar cap potentials.

These and other similar studies have used parameters of the solar wind and inter-planetary magnetic fields as input and geomagnetic activity indices AE, AL, Am or Dst as outputs to derive the linear relationship. Instead of looking at such 'cause-and-effect' situations, we attempt to use the technique to predict the equatorial magnetic field variations at a station to the west from the data of a station to the east. It is stressed that the geomagnetic field at both the stations chosen are basically outputs of currents over the dip equator and need not be considered as 'cause-and-effect'. However, if the technique is shown to be successful, then the predicted variations can be compared with observed variations on a day-to-day basis to look for the specific physical mechanisms when the relationship fails. It can also be used to fill data gaps of the station to west from the computed values using observations at the station to the east. If the station separation is too large, the technique may not be successful as the spatial extent of correlated variations is rather restricted (Schlapp and Mann 1983). Our choice of the two equatorial electrojet stations are Adis Ababa (9°N 39°E) and Trivandrum (8·5°N 77°E). If there were an additional station in close proximity to Adis Ababa, the entire procedure could have been checked with the data from the third station. Unfortunately there is no possibility for this in the present case.

2. Data selection and analysis

The data used in the analysis are the mean hourly values of horizontal intensity at Trivandrum treated as input and that at Adis Ababa as output. For the Wiener technique to be applicable, the basic assumption that the filter is time invariant must be established. In order to ensure this, we chose our data sets to cover different phases of solar activity and different seasons. Five samples for each season in different years were identified when the index Ap of magnetic activity on 6 consecutive quiet days was < 7 indicative of the fact that the magnetospheric conditions were quiet. The computer programme, given by Robinson (1967), was used for deriving the impulse response function in each case, after verification with his test data. The filter lengths were varied between 20 and 60 and a choice of 50 was finally adapted, as the prediction efficiency levels off in the vicinity of this length and deteriorates somewhat beyond.

As we are not considering a system where the input is a causative factor and the output is the result but one in which the two share, perhaps, the same energy source, the concept of coherence functions could also be effectively used. This alternate

Table 1. List of 15 sequences of quiet days in different years, divided according to seasons and the corresponding daily mean Ap values.

Consecutive days	Mean daily Ap values					
Nov 30 – Dec 5, 1972	4,	6,	5,	6,	4,	2
Nov 16 – Nov 21, 1976	5,	7,	5,	9,	7,	4
Nov 16 – Nov 24, 1977	6,	4,	5,	3,	2,	2
Jan 19 – Jan 24, 1978	6,	3,	1,	2,	3,	6
Jan 18 – Jan 23, 1980	4,	4,	6,	6,	5,	5
May 19 – May 24, 1972	4,	4,	4,	5,	7,	5
Jun 22 – Jun 27, 1975	5,	4,	3,	4,	5,	4
Aug 12 – Aug 17, 1974	5,	6,	5,	4,	5,	6
Jul 26 – Jul 31, 1978	5,	4,	6,	4,	3,	2
May 16 – May 21, 1980	3,	3,	4,	8,	5,	4
Mar 10 – Mar 15, 1972	3,	7,	3,	7,	4,	5
Apr 12 – Apr 17, 1974	3,	7,	5,	4,	4,	6
Oct 22 – Oct 27, 1976	5,	6,	6,	5,	4,	7
Sep 13 – Sep 18, 1978	6,	5,	3,	6,	8,	3
Mar 10 – Mar 15, 1980	4,	4,	1,	7,	5,	2

approach could be carried out through cross spectral analysis and computing the coherence and phase difference as a function of frequency (Robinson 1967). However, the Linear Prediction technique derives the impulse response as a function of time lag from the data base itself with the least squares criterion so that splitting into frequency bands is not essential. All oscillatory effects are clubbed together in the output series derived from the filter. Whenever necessary, one can easily complement the impulse response information with appropriate coherence functions. As an alternative to the coherence function to complement the L.P. technique as also for a better understanding of the longitudinal dependence of the features of equatorial electrojet, we computed the amplitudes and phases of the diurnal and semi-diurnal components of the daily variation curves for each of the 90 days, at both stations and studied their day-to-day variability. These two periodicities are the most dominant ones in the quiet-day variations close to the dip equator.

3. Results and discussion

The impulse response functions for the 15 samples corresponding to a filter length of 50 are shown in figure 1a. The consistent common feature of the curves is the maximum between 2 and 6 hours. This is in conformity with the expected diurnal pattern at Adis Ababa which is about 40° to the west of Trivandrum. The response curves also clearly indicate that the features are not dependent on the phase of the solar cycle or the three seasons, though it is well known that the quiet-day field is stronger during equinoxes and during epochs of solar maximum (Rastogi and Iyer 1976). It is, thus, clearly established that the basic assumption of time invariance of the impulse response is valid in the present context, and therefore we can consider the use of Wiener filter in the envisaged studies.
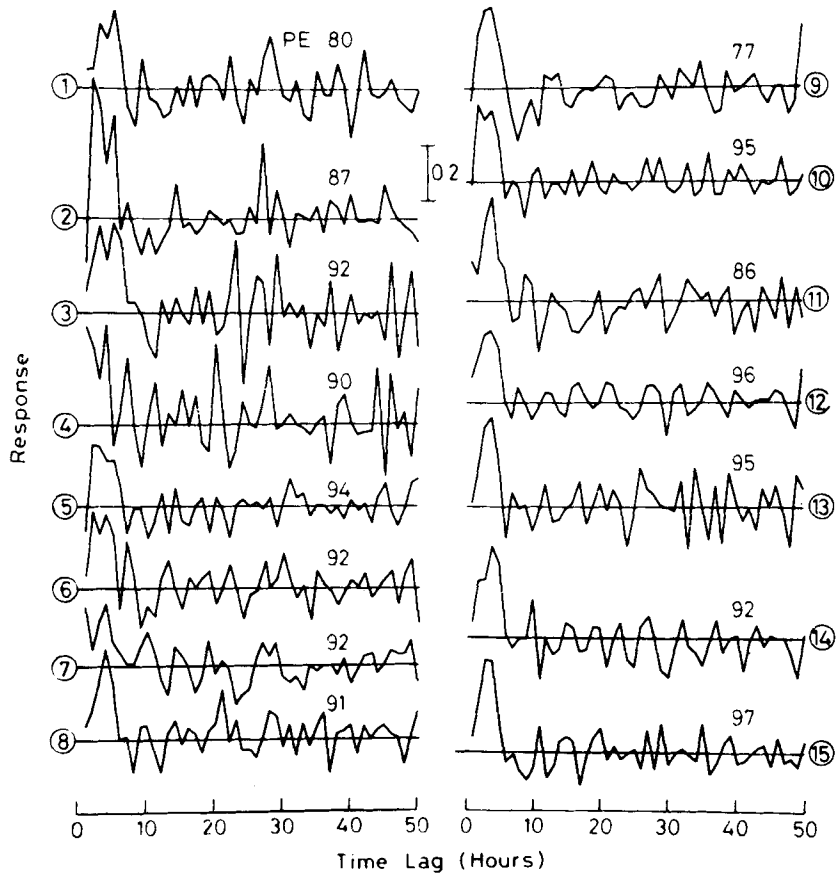
Figure 1(a). Impulse response functions relating daily variation on quiet days at Adis Ababa and Trivandrum for 15 samples listed in table 1. PE in the figure corresponds to the prediction efficiency percentage derived from variances of observed output and residuals.
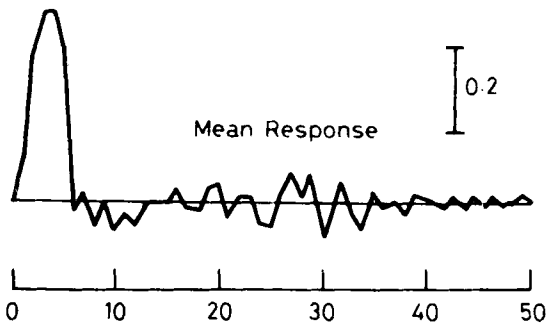

Figure 1(b). The average impulse response function derived from the 15 samples.

The prediction efficiency (PE) has a minimum percentage of 77. For 11 out of 15 cases, it exceeds 90%. It is, therefore, possible to consider an average curve based on these 15 functions to derive a Wiener filter, which can then be used to infer magnetic field at Adis Ababa from Trivandrum data. The average curve derived from the 15
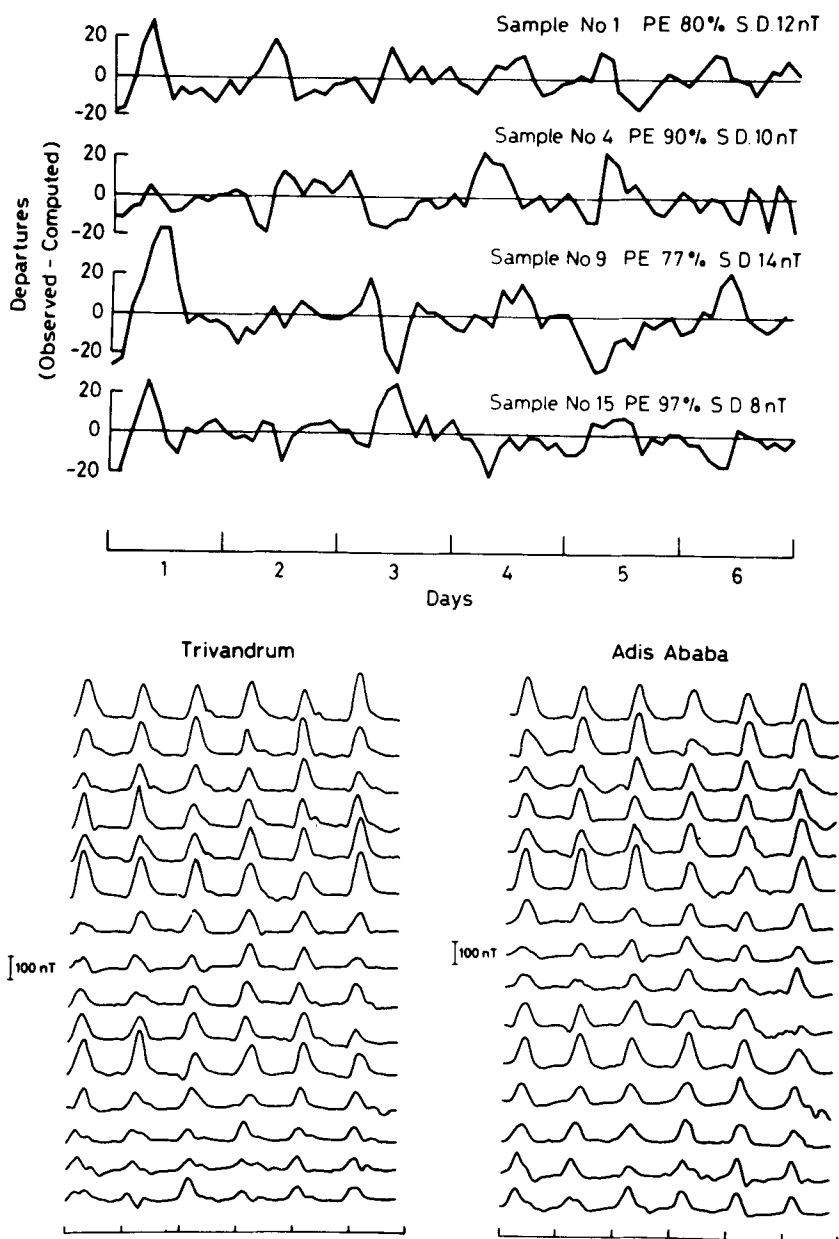
Figure 2. Mean hourly values of Horizontal intensity at Trivandrum and Adis Ababa for the 15 samples of 6 consecutive quiet days.

In the top panel are shown the residuals (computed-observed) of H at Adis Ababa for four typical instances. Sample number and the root mean square deviation of the residuals are indicated.

samples, is shown in figure 1b. As expected, the higher noise level seen in individual curves for lags beyond 10 is substantially diminished while the major feature in the earlier part is retained intact. Figure 2 shows the observed hourly values for consecutive 6 days at Trivandrum and Adis Ababa. Also shown, on the top, are the departures

of the computed values at Adis Ababa from observations for four typical instances where the prediction efficiency varied between the minimum and maximum. It can be immediately seen that apart from the high percentage of the prediction efficiency, individual predicted hourly values do not differ much from actually observed data and that when the differences are large enough, there is no relationship to the diurnal pattern of the field variations. The root mean square of the residuals has a maximum of 14 nT, as against the diurnal range in excess of 100 nT. The high level of prediction efficiency also implies that the spatial extent of correlated equatorial changes is at least 45° wide ($\sim$ 3 hours).
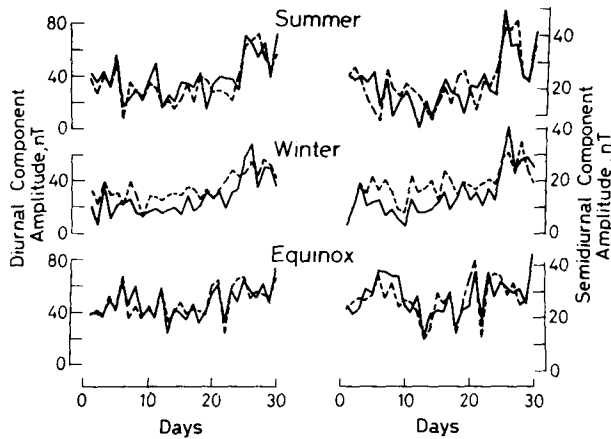

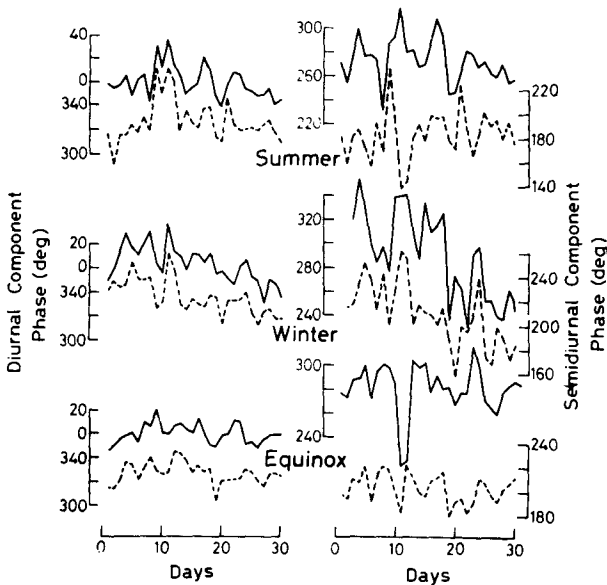**Figure 3(a).** Day-to-day variability in the amplitude.

Figure 3(b). Day-to-day variability in the phase of the diurnal and semi-diurnal components for the three seasons, for the two stations Trivandrum and Adis Ababa.

Table 2. Parameters of linear regression between various components of the H field at Trivandrum and Adis Ababa (Adis Ababa data treated as dependent variable).

		Slope	$\bar{T}$	$\bar{A}$	σ_T	σ_A	CC
Summer	C1	0·823	38 nT	36 nT	16	16	0·82
	ϕ_1	0·737	359°	326°	15	19	0·99
	C2	0·550	20 nT	23 nT	9	9	0·54
	ϕ_2	0·263	274°	184°	24	23	0·25
Winter	C1	0·832	27 nT	33 nT	18	1	0·60
	ϕ_1	0·715	4°	335°	16	15	0·65
	C2	0·764	9 nT	6	14	14	0·53
	ϕ_2	0·603	294°	212°	42	27	0·38
Equinox	C1	0·832	47 nT	48 nT	12	12	0·84
	ϕ_1	0·762	358°	327°	10·	9	0·27
	C2	0·762	28 nT	28 nT	8	8	0·77
	ϕ_2	0·293	276°	202°	23	13	0·16
CEJ	C1	0·572	30 nT	32 nT	10	9	0·54
	ϕ_1	0·553	12°	342°	15	15	0·53
	C2	0·511	21 nT	23 nT	7	7	0·58
	ϕ_2	0·653	293°	218°	29	20	0·59
	Per. 1	0·464	57	57	13	13	
	Per. 2	0·542	29	31	9	9	

T Trivandrum Mean Value   C1 Diurnal amplitude
Ā Adis Ababa Mean Value ϕ_1 Diurnal phase
CC Correlation Coefficient   C2 Semi-diurnal amplitude
σ_T, σ_A, Std. Deviations ϕ_2 Semi-diurnal phase
   Per 1: Percentage variance accounted for by the diurnal component
   Per 2: Percentage variance accounted for by the semi-diurnal component.

In determining the filter, we have used 6 consecutive days. It is, therefore, likely that the high percentage of efficiency may be attributed to the averaging processes in the least square fit. To examine whether the correlated changes are valid for even shorter time scales of one day, we took recourse to harmonic analysis of the mean hourly values for each day. The day-to-day variability in the amplitude of the diurnal and semi-diurnal components at both the stations are shown in figure 3a and that in the phases are in figure 3b, for 3 seasons separately. During equinox, the variability in amplitude is practically identical for both the components and the sense of variability continues to be the same in the other two seasons also, though the magnitudes do not match. Phase progression for the 24 hr component is similar for both stations with the expected phase difference of about 40° being constant from one day to the other. The 12 hr component on the other hand, exhibits rather random behaviour particularly during summer and equinox. The consistency in the winter is indeed surprising particularly when the Sq currents in the southern hemisphere tend to cut across the equator to intrude into the northern hemisphere and consequently distort the same. Regression parameters for the diurnal and semi-diurnal components at Trivandrum and Adis Ababa are given in table 2. These have been derived from the least squares linear best fit between the 30 days data for each season.

4. Inter-relationship during counter electrojet events

It is well known that the equatorial horizontal field is often marked by afternoon depression in the field. These are termed Counter Electrojet events and their morphology have been studied in great detail (Rastogi 1974). An objectively derived index of counter electrojet strength in the Indian zone by Bhargava *et al* (1983) categorised these into four classes: from index 1 for feeble depression below nighttime reference level to 4 for strong events with magnitudes of depression in excess of 40 nT. They reported that CEJ is not uncommon and that more than 50% of the days could be considered as days with afternoon CEJ event with index between 1 and 4.

The longitudinal extent of the CEJ events is not clearly known. We attempt to find out if the linear prediction filters derived earlier can also be applied to the data when the days are marked with CEJ. For this purpose we identified 10 sequences of 6 days each with the index of CEJ between 1 and 4 with as few ones as possible from the compilation of Bhargava *et al* (1983). The average prediction filter derived from the earlier analysis was used to derive the predicted hourly values of Adis Ababa
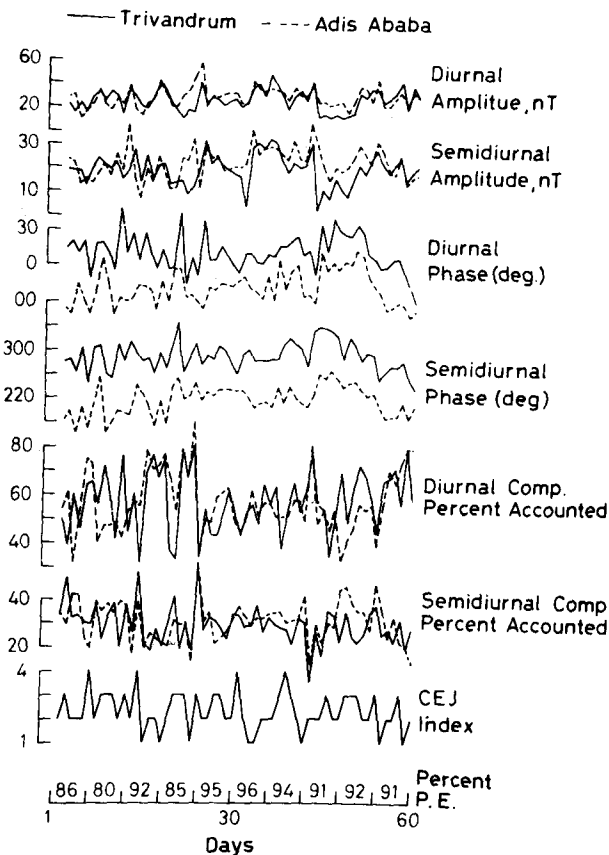

Figure 4. Prediction efficiency (PE) when the Linear filter is applied to Trivandrum H data to derive Adis Ababa data during periods marked by Counter Electrojet (CEJ) events. The day-to-day variability in the amplitudes and phases as well as the percentage of variance accounted for by them for all the days of CEJ are also shown.

and the prediction efficiency was computed. These values are again consistently greater than 90% with the least value of 80% as shown in figure 4. It is, thus, clearly demonstrated that even when the normal diurnal pattern is modified by significant afternoon depression of the field, Wiener technique is still applicable to derive the diurnal pattern of H field at a station to the west from the data of the station to the east.

Results of harmonic analysis for individual days are also plotted in figure 4. In addition to the amplitudes and phases of the 24 and 12 hour components we also computed the percentage of the total variance accounted for by the two dominant terms. It is well known that the semi-diurnal component gets significantly enhanced during periods of afternoon counter electrojet events (Bhargava and Sastri 1977; Somayajulu 1988). These parameters depicted in figure 4 are subjected to regression analysis and results are included in table 2. The day-to-day variability in phase of the diurnal and semi-diurnal components is fairly similar at both stations as seen from the moderate values of CC. The amplitudes, on the other hand, appear to be less well correlated. From these curves, it may be inferred that prediction efficiency using such pairs of stations could be significantly large when the diurnal patterns are similar in shape indicative of similarity in phase even if the amplitudes are different.

From an examination of magnetograms for 1964–65 at Adis Ababa and Kodaikanal, Rastogi (1973) suggested that the afternoon CEJ can be fairly localized and may not be concurrent at two stations even if they are separated only by 2 or 3 hours in longitude. Rao and Rao (1963) found that the significant depressions in H magnetogram of Adis Ababa found by Gouin (1962) on January 3 and 5, 1962 were significantly absent in the records at the Indian stations Trivandrum and Kodaikanal. These individual events clearly show that there could be significant interference in the current systems responsible for the CEJ over a short duration of time on some occasions. The mechanism(s) responsible for these are yet to be clearly understood.

The technique proposed here gives an opportunity to firstly catalogue the occasions when the predicted variations of Adis Ababa H differs significantly from the observations and then compare the ionospheric and wind parameters (if available) on these occasions at both the locations to isolate the causes. The statistics would also be useful in validating theories for the counter-electrojet phenomena and the day-to-day variability in the Sq and electrojet currents.

5. Conclusions

Wiener's Linear Prediction technique to derive the impulse response functions can be successfully applied to the data from a pair of equatorial electrojet stations so that variations of the field at the station in the western longitude can be derived from the data of a station located to the east. This, to our mind, is a new application in contrast to the demonstrated usage in solar wind-magnetosphere coupling studies where physical mechanisms were being sought to explain the observed changes in geomagnetic activity and has potential uses in understanding the causative mechanisms for equatorial CEJ, day-to-day variability of Sq and EJ etc. The impulse response function is clearly shown to be independent of season and solar activity thus satisfying the most rigorous criterion of the technique. Some characteristic features of the equatorial electrojet like the longitudinal extent of correlated changes, the inter-relationship of the day-to-day variability can be verified by adopting this technique

for suitable pairs of stations with increasing longitudinal separation. Days when the predicted diurnal pattern does not match closely with the observed variation could be examined in great detail for other influencing parameters like global Sq, equatorial electric field, F-region contributions, IMF or solar wind variability.

Acknowledgements

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