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APPLICATION OF TRAVELLING WAVE RESONATORS TO SUPERCONDUCTING LINEAR ACCELERATORS*

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Introduction

The energy gradient in a superconducting accelerator is sharply limited by the peak electric and magnetic fields present at the surface of the accelerating structure. Given this limitation, a travelling wave structure is desirable in order to achieve the maximum possible accelerating gradient. In the case of a superconducting structure, however, travelling wave operation can only be possible if the structure is incorporated into a travelling wave resonator (TWR). The TWR is a resonant device in the form of a closed loop or ring in which ideally power circulates in one direction only. The accelerating gradient for a structure used in the TWR configuration can be 20-30% greater than that of a standing wave structure with identical peak field limitations.

The general theory of TWR's has been developed previously by several authors.[1,2] The application of the TWR as a structure for superconducting electron linacs has also been considered.[3,4,5] This paper extends this earlier work to include the case of a high Q TWR accelerating structure, both with and without beam loading, when deviations from resonance and perfect matching are present. Considerable simplification is possible by assuming a low-loss TWR, as compared with the case for arbitrary loss usually presented. The TWR equations are shown to take the identical form, in the absence of a mismatch, as the expressions for the standing wave superconducting accelerator with beam loading.[6]

At SLAC a two-foot long TWR test accelerator (project Leapfrog) is currently under construction. A thorough understanding of TWR behavior is necessary for the successful design and operation of such a device. In the following sections of this paper the basic relations describing the behavior of the TWR accelerator are summarized. In the final section some implications of the theory for the Leapfrog accelerator are given. The description of TWR behavior as presented here is necessarily brief. Details of the derivations and a more extensive discussion are given in Ref. 7.

Definitions

A schematic diagram of a superconducting accelerator using a TWR configuration is shown in Fig. 1. The one-way field attenuation parameter, τ, is assumed to be small and the loss in the return waveguide portion of the loop is assumed to be negligible. The power coupling coefficient of the input directional coupler is C. Phase angles are taken as positive in the counter-clockwise direction, and the factor $e^{-j\omega t}$ is understood. The electrical length of the loop, after subtracting out multiples of 2π, is ϕ. The magnitude of the net voltage reflection coefficient for all perturbations in the loop is Γ. The coupling coefficient β, the mismatch parameter μ, and the tuning angle ψ are now defined as

$$\beta \equiv \frac{C}{2\tau} \qquad \mu \equiv \frac{2\beta}{1+\beta}\cdot\frac{\Gamma}{C}$$

$$\tan\psi \equiv \frac{2\beta}{1+\beta}\cdot\frac{\phi}{C} = 2Q_L\frac{\delta\omega}{\omega}$$

Critical coupling (β=1) is obtained when $C=2\tau$. The phase of the field in the resonator with respect to the resonant value in the absence of beam loading and mismatches is given by ψ. The unloaded Q is given as usual by $Q_0=(1+\beta)Q_L$, and the filling time for the structure by $T_F=2Q_L/\omega$. The mismatch parameter μ measures the strength of coupling between the forward and backward wave modes in the ring. Forward waves will be denoted by the superscript (+) and backward waves by the superscript (-).

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Summary of TWR Expressions

Forward wave.

In the absence of an electron beam the normalized forward wave and power flow in the loop are

$$\frac{\tilde{E}^+}{\tilde{E}_s} = \frac{1}{\sqrt{C}}\cdot\frac{2\beta}{1+\beta}\cdot\frac{1-j\tan\psi}{[1-j\tan\psi]^2+\mu^2}$$

$$\frac{P^+}{P_s} = \frac{1}{C}\cdot\frac{4\beta^2}{(1+\beta)^2}\cdot\frac{1+\tan^2\psi}{(1-\tan^2\psi+\mu^2)^2+4\tan^2\psi}$$

Here $\tilde{E}_s$ is the field produced by a power P_s flowing in the loop for $\mu=\psi=0$. For values of $\mu<0.486$, $\tilde{E}^+$ exhibits a single peak resonance at $\psi=0$. At $\mu=0.486$ the forward wave separates into a double resonance with peaks located at $\psi=\psi_m^+$ where $\psi_m^+=\pm\tan^{-1}[\mu(\mu^2+4)^{1/2}-1]^{1/2}$. For large μ, $\tan\psi_m\approx\pm\mu$. Figure 2 shows contours of constant E^+ in the ψ, μ plane. The broken line follows the locus of maximum field ($\psi=\pm\psi_m^+$) which approaches 0.5 for large μ.

In Fig. 3 the normalized forward wave amplitude, E^+/E_0, is plotted as a function of the mismatch parameter, μ. Here E_0 is the value of $|\tilde{E}^+|$ for $\mu=\psi=0$. We see also from the above equations that for large β and $\mu=\psi=0$, the power gain of the ring approaches 4/C.

Reverse wave.

The reverse wave experiences no energy exchange with the electron beam but it does cause a degradation in the accelerating wave. More critical for a superconducting accelerator, it enhances the peak electric and magnetic fields. The normalized reverse wave is given by

$$\frac{\tilde{E}^-}{\tilde{E}_s} = \frac{1}{\sqrt{C}}\cdot\frac{2\beta}{1+\beta}\cdot\frac{\mu}{(1-j\tan\psi)^2+\mu^2}$$

$$\frac{P^-}{P_s} = \frac{1}{C}\cdot\frac{4\beta^2}{(1+\beta)^2}\cdot\frac{\mu^2}{(1-\tan^2\psi+\mu^2)^2+4\tan^2\psi}$$

The reverse wave also exhibits a double resonance but not until μ exceeds unity. For $\mu>1.0$ the peaks are located at $\psi_m^-=\pm\tan^{-1}[\mu^2-1]^{1/2}$. There is a single peak at $\psi=0$ for $\mu<1.0$. The normalized reverse wave is plotted in Fig. 3. It is seen that $\tilde{E}^-/E_0$ is 0.5 on the resonant peak for $\mu>1.0$.

Peak field.

The normalized peak field is given by

$$\frac{|E_p|}{|\tilde{E}_0|} = \frac{|\tilde{E}^-|+|\tilde{E}^+|}{|\tilde{E}_0|} = \frac{\mu+\sqrt{1+\tan^2\psi}}{\sqrt{(1-\tan^2\psi+\mu^2)^2+4\tan^2\psi}}$$

The peak field reaches a maximum value of 1.21 at $\mu=0.414$ and separates into a double resonance at $\mu=0.486$. Also of interest is $|E^+/E_p|$, the maximum accelerating field for a given peak field

$$\frac{|E^+|}{|E_p|} = \frac{\sqrt[4]{\mu^4+4\mu^2}}{\mu+\sqrt[4]{\mu^4+4\mu^2}} \quad \text{for } \mu>0.486$$

For $\mu<0.486$ $|E^+|/|E_p|=1/(1+\mu)$. The accelerating to peak field ratio vs μ is also plotted in Fig. 3.

Field in load.

The wave to the load terminating the line from the generator (see Fig. 1) is given by

$$\tilde{E}_L/\tilde{E}_s = 1\sqrt{C}\,(\tilde{E}^+/\tilde{E}_s) = 1-(2\beta/1+\beta)\,Ae^{j\psi_e}$$

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and

$$P_L/P_s = 1-(4\beta/1+\beta)\,A\cos\psi_e + (4\beta^2/(1+\beta)^2)\,A^2$$

where

$$A(\mu,\psi) = \sqrt{\frac{1+\tan^2\psi}{(1-\tan^2\psi+\mu^2)^2+4\tan^2\psi}}$$

and

$$\tan\psi_e = \tan\psi\left\{\frac{1+\tan^2\psi-\mu^2}{1+\tan^2\psi+\mu^2}\right\}$$

Here $\psi_e(\mu,\psi)$ is also the phase angle between $\tilde{E}^+(\mu,\psi)$ and $\tilde{E}^+(0,0)$.

Wave reflected to the generator.

The RF source is subjected to varying mismatches during TWR tuning. The input reflection coefficient is given by

$$\tilde{\Gamma}_{in} = \sqrt{C}\,\frac{\tilde{E}^-}{\tilde{E}_s} = \frac{2\beta}{1+\beta}\,\frac{\mu}{(1-j\tan\psi)^2+\mu^2}$$

$$\frac{P_R}{P_s} = \frac{4\beta^2}{(1+\beta)^2}\cdot\frac{\mu^2}{(1-\tan^2\psi+\mu^2)^2+4\tan^2\psi}$$

For $\beta>>1$ and $\psi=\psi_m^-$, $\tilde{\Gamma}_{in}$ is nearly unity for all values of $\mu>1.0$. $\tilde{\Gamma}_{in}$ is also quite large for $\psi=\psi_m^+$.

The TWR Superconducting Accelerator with Beam Loading

The effects of beam loading can be treated most readily by making a superposition of the beam induced wave, $\tilde{V}_b^+$, and the wave in the loop, $\tilde{V}_g^+$, produced by the generator. For the off resonant case, $\tilde{V}_b^+$ will make an angle $(\psi_e+\pi)$ with the fundamental RF component of the bunched electron beam. Referring to Fig. 4, $\tilde{V}_c^+$ is the total voltage in the accelerator. The net accelerating voltage is given by

$$V_a = \mathrm{Re}[\tilde{V}_c^+] = |\tilde{V}_g^+|\cos(\theta+\psi_e) - |\tilde{V}_b^+|\cos\psi_e\ .$$

It can be shown[7] that

$$|\tilde{V}_g^+| = \sqrt{r\ell P_s}\,\frac{2\sqrt{\beta}}{1+\beta}\cdot A(\mu,\psi)$$

$$|\tilde{V}_b^+| = \frac{ir\ell}{1+\beta}\cdot A(\mu,\psi)$$

where i is the dc electron beam current for a tightly bunched beam, r is the shunt impedance per unit length of the accelerating structure and ℓ is the length of the structure.

The energy gain of the structure now becomes

$$V_a = \frac{2\beta}{1+\beta}A(\mu,\psi)\left[\sqrt{\frac{r\ell P_s}{\beta}}\cos(\psi_e+\theta) - \frac{ir\ell}{2\beta}\cos\psi_e\right]$$

It is seen from the preceding expression that, in the limit of large β, the energy gain is determined by $r\ell/\beta=(2\pi/C)(\ell/\lambda)(r\ell/Q_0)(c/v_g)$, where λ is the free space wavelength and v_g is the group velocity. This combination of parameters is determined solely by the geometry of the resonator and is independent of loss.

The interaction of phasing and tuning is illustrated in Fig. 5 for the case of optimum beam loading. The energy is seen to decrease very rapidly if θ and ψ_e have the same sign and quite slowly if they are opposite in sign.

For the unperturbed, on-resonant case ($\psi=\mu=0$) the energy gain is

$$V_a = \sqrt{r\ell P_s}\,\frac{2\sqrt{\beta}}{1+\beta}\left[1-\frac{K}{\sqrt{\beta}}\right]$$

where the beam loading parameter $K=(i/2)\sqrt{r\ell/P_s}$. The efficiency iV_a/P_s is

$$\eta = \frac{4K\sqrt{\beta}}{1+\beta}\left[1-\frac{K}{\sqrt{\beta}}\right]$$

and is optimum for $K/\sqrt{\beta}=1/2$, $\eta_{opt}=\beta/(1+\beta)$.

The power dissipated in the structure is also of interest, since it must be removed by the cryogenic refrigeration system. For $\psi=\mu=0$,

$$\frac{P_D}{P_s} = \frac{4\beta}{(1+\beta)^2}\left[1-\frac{K}{\sqrt{\beta}}\right]^2 .$$

At optimum efficiency this becomes $P_D/P_s=\beta/(1+\beta)^2$.

The power dissipated in the external load for $\psi=\mu=0$ is

$$\frac{P_L}{P_s} = \left[\frac{\beta-1-2K\sqrt{\beta}}{\beta+1}\right]^2$$

At optimum efficiency this reduces to $P_L/P_s=1/(1+\beta)^2$. The power dissipated in the load when a beam is passing through the structure and the RF source is off is calculated to be $P_L(P_s{=}0)=i^2r\ell\beta/(1+\beta)^2$.

The power reflected to the generator is proportional to the reverse power flow in the loop. For $\psi=\mu=0$ the reflected power is zero in the case of a long accelerating structure, since in an infinitely long structure only a forward wave is induced by the electron beam. In a finite structure consisting of N coupled cells, assuming that the structure is perfectly matched without a beam, a small backward wave will be induced which is given by $V_b^-=V_b^+/N$. This can be of concern for very short TWR accelerators such as the "Leapfrog" structure at SLAC.

Leapfrog Project

The 2-foot, 16-20 MeV test TWR superconducting accelerator will operate at 1.85°K with the design parameters

$C = 45.5$ db $= 2.8\times10^{-5}$	$\beta = C/2\tau = 24$
$Q_0 = 4\times10^9$	$r\ell/\beta = 3.7\times10^{11}$ ohms

The upper limit on the allowable mismatch is determined by the tolerable degradation in the accelerating field for a given peak field. If $\Delta(E^+/E_p)$ is to be held to less than 5% ($\mu<0.05$) then $\Gamma<7\times10^{-7}$. Similarly, if it is desirable to hold the field in the resonator to within 1% of the resonant value, then the phase of this field must be held to within 0.1 radians (6°) of resonance at the design current. The allowable phase error in the electrical length of the loop is then $\phi=(C\tan\psi)(1+\beta)/2\beta)=1.4\times10^{-6}$ radians.

Thus both the mismatch and phase length of the Leapfrog ring must be adjusted with extreme precision. A system of movable superconducting plungers, with fine adjustments made by feedback controlled piezo-electric bender disks, has been designed to accomplish this operation.

References

1. F. J. Tischer, IRE Transactions PGMTT, MTT5, 51-56 (1957).
2. K. Tomiyasu, IRE Transactions PGMTT, MTT5, 267 (1957).
3. R. B. Neal, Report No. SLAC-PUB-437 (1968).
4. P. B. Wilson, IEEE Trans. Nucl. Sci. NS-6, 1092 (1969).
5. W. R. Fowkes, Report No. SLAC-TN-69-6 (1968).
6. P. B. Wilson, Linear Accelerators (Ed. P. M. Lapostelle and A. L. Septier) (North Holland Publishing Co., Amsterdam, 1970); p. 1107.
7. W. R. Fowkes and P. B. Wilson, Report No. SLAC-TN-71-6 (1971).

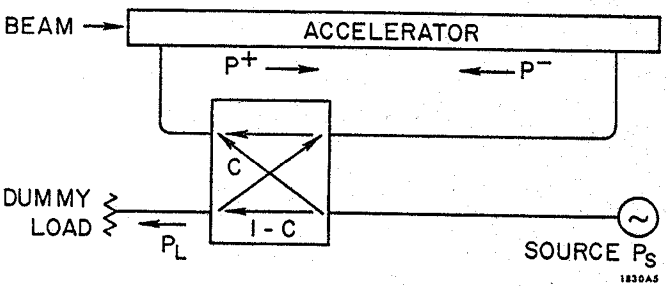

FIG. 1--TWR accelerator schematic.

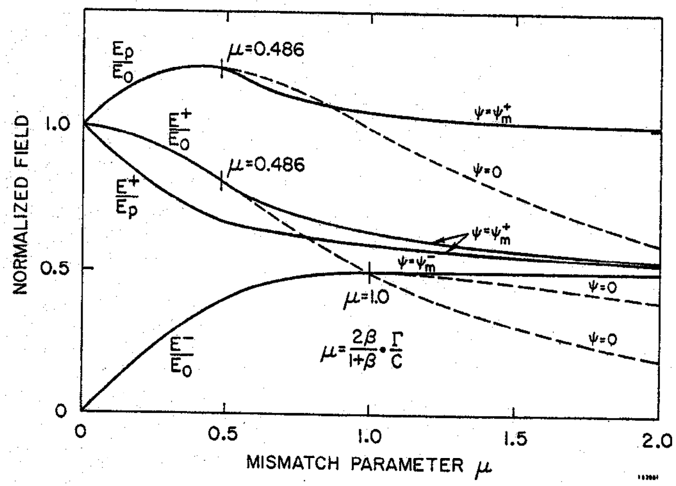

FIG. 3--Normalized forward, reverse and peak field waves as function of mismatch parameter. Vertical tics indicate beginning of double resonances.

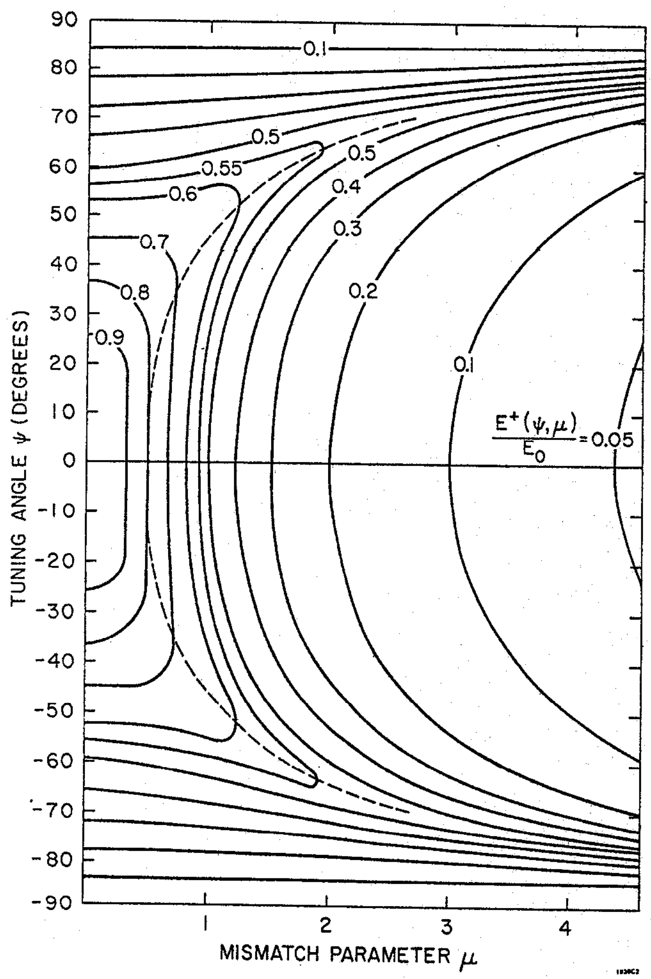

FIG. 2--Contour plot of forward wave amplitude as function of tuning angle and mismatch. Dashed curve shows locus of resonant peaks.

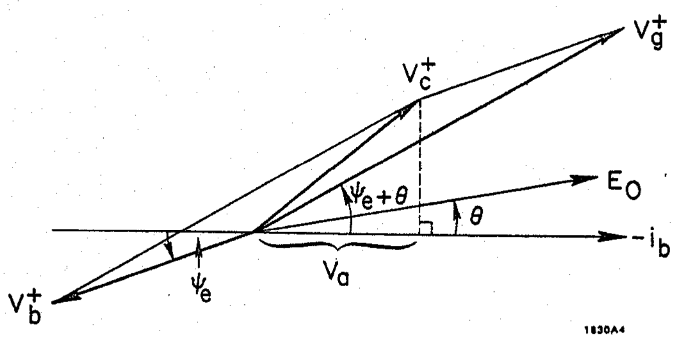

FIG. 4--Vector addition of voltages in TWR accelerator.

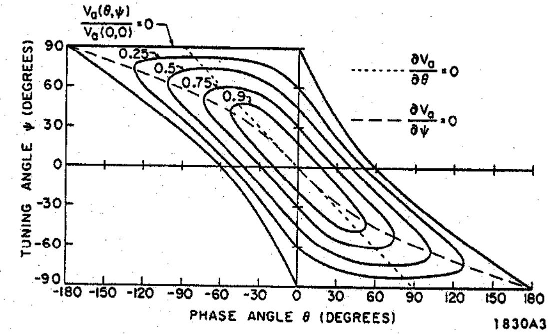

FIG. 5--Contour plot of relative energy gain as function of phasing and tuning at optimum beam loading.