APPLICATIONS OF THE ROBUST INTERPOLATION FOR DTM DETERMINATION

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ABSTRACT:

Different data sources for the determination of a digital terrain model (DTM) are available. New measurement techniques like laser scanning (LS) and interferometric synthetic aperture radar (InSAR) allow a high degree of automation and dense sensing. They pushed the development of new methods for data modelling. This paper presents *robust* methods for the automatic determination of a DTM from point cloud data. These robust methods operate on the original data points and allow the elimination of off-terrain points and the modelling of the terrain surface within one process. After a short description of the algorithms the paper focuses on the results of this technique applied to datasets from different sensors (airborne LS, terrestrial LS, satellite LS, tacheometry and photogrammetry).

1. INTRODUCTION

For the determination of a digital terrain model (DTM) different methods for data capturing do exist. The choice of the most appropriate measurement system depends on the specific application. Apart from the "classical sensors" like photogrammetry and tacheometry, new measurement systems like laser scanning and interferometric synthetic aperture radar (InSAR) have been developed in the last years and offer new possibilities such as increasing measurement density and higher degree of automation. These systems pushed the development of new methods for DTM determination.

This article presents algorithms developed at our institute for the generation of DTMs from irregular distributed point cloud data (sec. 3). Robust methods for error reduction and elimination are integrated into the modelling process and are used for the classification into terrain and off-terrain points. Finally, the results of these techniques applied to datasets obtained by different sensors (airborne LS, terrestrial LS, satellite LS, tacheometry and photogrammetry) are presented. This demonstrates the generality of the proposed method (sec. 4). However, we expect that other algorithms developed for the elimination of off-terrain points in airborne laser scanner data (sec. 4.2) can be used in a more general context, too. In this section we also present different strategies for data densification and homogenisation in order to close data holes. Although it is an extrapolation of the measured points, it is necessary for many DTM applications.

It shall be made clear, that the algorithm for robust DTM generation has already been presented elsewhere (e.g. Kraus and Pfeifer, 1998), nevertheless the formulas are given in the appendix. Its versatile application possibilities have not been published so far and we expect that the general concept is very useful for all who have data with gross errors and want to interpolate a surface (not only a DTM) from it.

2. ERRORS WITH RESPECT TO THE DTM

In the context of this paper the term "error" is used for the disagreement between the z-value of the measured point and the "true" terrain surface. We speak of residuals if we mean the "true" height (t) minus the observed height (z). Under filter value (f) we understand the negative residual, thus: $t + f = z$. Because we restrict our applications to 2.5d-problems, which means that the surface is the graph of a bivariate function, the errors are measured in z-direction only.

As mentioned above, the data used is a cloud of points. This point of view can be characterized as data driven and has the advantage that it is very general and independent from the sensor type. However, the parameters for the model derivation as well as those of the derived models depend on the characteristics of the sensor, which will be seen in the examples section. On the other hand, the consideration of measurement errors with respect to the individual sensor components would allow a closer look on the realisation of all the measurements for one point observation. Modelling and filtering all sorts of errors (see below) in such a model would yield the most precise solution (approaches for the consideration of systematic errors in airborne laser scanner data can be found in Burmann, 2000; Filin, 2001 and Schenk, 2001).

We aspire a surface determination technique considering errors in the measured points in the modelling process (i.e. generating the DTM). Apart from random errors we have to consider gross and systematic errors in the z-values of our data points.

Random errors Depending on the measurement system, the points measured on the terrain surface have a more or less random distribution with respect to the "true" terrain surface. In general this random error distribution is characterized by the standard deviation of the measurement system and should be considered in the DTM generation process.

Gross errors Gross errors can occur in all datasets. Especially automatic systems like laser scanners produce a high number of gross errors with respect to the terrain surface. In the case of

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laser scanning these errors can be split in "real" measurement errors (e.g. so-called long ranges caused by multi-path effects) and errors caused by reflecting surfaces above the terrain (e.g. on vegetation or houses). A method for the automated elimination or at least reduction of gross errors is necessary.

Systematic errors In the case of systematic errors we have to distinguish between gross systematic errors, which have similar characteristics like gross errors, and small systematic errors (e.g. in the case of LS too short ranges caused by reflection in low vegetation or even in grass rather than on the soil). The influence of these small errors will be small in magnitude and it will be difficult to eliminate these errors without any further information. Systematic gross errors have the property that they appear with the same magnitude at many points. One example for such an error is a shift in the orbit of a LS platform.

3. ALGORITHMS

In the following, algorithms considering the 3 types of measurement errors (random, gross and systematic) are presented. If possible, systematic errors in the data should be avoided in the measurement process or corrected with suitable models before the surface model generation. However, it can be seen in the examples section that we are able to eliminate gross systematic errors (sec. 4.4) if enough error-free data is given. Small systematic errors can not be excluded from the DTM derivation.

Our method for the interpolation of randomly distributed point data - the linear prediction - has quite a long history, but still plays a centre role. The technique of robust interpolation developed for the generation of a DTM from airborne laser scanner data in wooded areas and its extension in a hierarchical set-up have stand a lot of tests for the DTM generation from laser scanner data. In the following, these algorithms are summarized and their consideration of errors are presented. The formulas can be found in the appendix.

3.1 Interpolation

For the interpolation of the DTM we use linear prediction, which is very similar to kriging (Kraus, 1998). This approach considers the terrain height as a stochastic process. Depending on the data its covariance function (corresponding to the variogramm of kriging) is determined automatically. This function describes the co-variance of measured point heights depending on the horizontal Euclidian point distance. In the algorithm used (Gaussian covariance) it attenuates monotonously. The interpolation is applied patch wise to the data which results in an adaptive (i.e. patch wise) setting of the covariance function.

The variance of measured heights (covariance at point distance zero) contains the variance of terrain heights plus the variance of the measurement errors. Subtracting the variance of the measurement error, which is a prior knowledge of the measurement, yields the variance of the terrain. Details on the computation of the covariance and the linear prediction (also known as surface summation with Gaussian basis functions) can be found in (Kraus, 2000, sec. H.3). The covariance functions are centred on each data point and factors for these functions are determined for each point in a linear system of equations. The sum of the (vertically) scaled functions is the interpolated surface. With the variance of the measurement errors the smoothness of the resulting surface can be influenced.

3.2 Robust Interpolation

This method was originally developed for the generation of a DTM from laser scanner data in wooded areas. For this purpose a solution was found, which integrates the elimination of gross errors and the interpolation of the terrain in one process. The aim of this algorithm is to compute an individual weight for each irregularly distributed point in such a way that the modelled surface represents the terrain.

It consists of the following steps:
1. Interpolation of the surface model considering individual weights for each point (at the beginning all points are equally weighted).
2. Calculate the filter values[1] (oriented distance from the surface to the measured point) for each point.
3. Compute a new weight for each point according to its filter value.

The steps are repeated until a stable situation is reached (all gross errors are eliminated) or a maximum number of iterations is reached. The results of this process are a surface model and a classification of the points in terrain and off-terrain points.

The two most important entities of this algorithm are the functional model (step 1) and the weight model (step 3). For the functional model linear prediction (sec. 3.1) considering an individual weight (i.e. individual variance for the measurement error) for each point is used. The elimination of the gross errors is controlled by a weight function (fig. 3, 6 and 12). The parameter of this function is the filter value and its "return value" is a (unit less) weight. The weight function is a bell curve (similar to the one used for robust error detection in bundle block adjustment) controlled by the half width value (h) and its tangent slope (s) at the half weight. Additionally it can be used in an asymmetric and shifted way (fig. 6) in order to allow an adaptation to the distribution of the errors in respect to the "true" surface. The asymmetry means, that the left and right branch are independent, and the shift means, that the weight function is not centred at the zero point. With the help of two tolerance values (t_- and t_+, fig. 3) points with a certain distance to the computed surface can be excluded from the DTM determination process. In general it can be said that the algorithm relies on a "good" mixture of points with and without gross errors in order to iteratively eliminate the off-terrain points. Finally, the classification into accepted and rejected points is performed by tolerance values (thresholds for the filter value). A detailed description of this method can be found in (Kraus and Pfeifer, 1998).

3.3 Hierarchic Robust Interpolation

As mentioned before the robust interpolation relies on a "good mixture" of points with and without gross errors. Therefore this algorithm is not able to eliminate gross errors, which occur clustered in large areas. To cope with this problem we use the robust interpolation in a hierarchic set-up, which is similar to the use of image pyramids in image processing. With the help of the data pyramids we provide the input data in a form that we are able to eliminate all gross errors with this coarse-to-fine approach. The coarse level surfaces obtained from the coarse level point sets are used for densification (i.e. adding finer level point data). The resulting fine level DTM consists of all measured terrain points.

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[1] In previous publications the term residual was used instead of filter value.
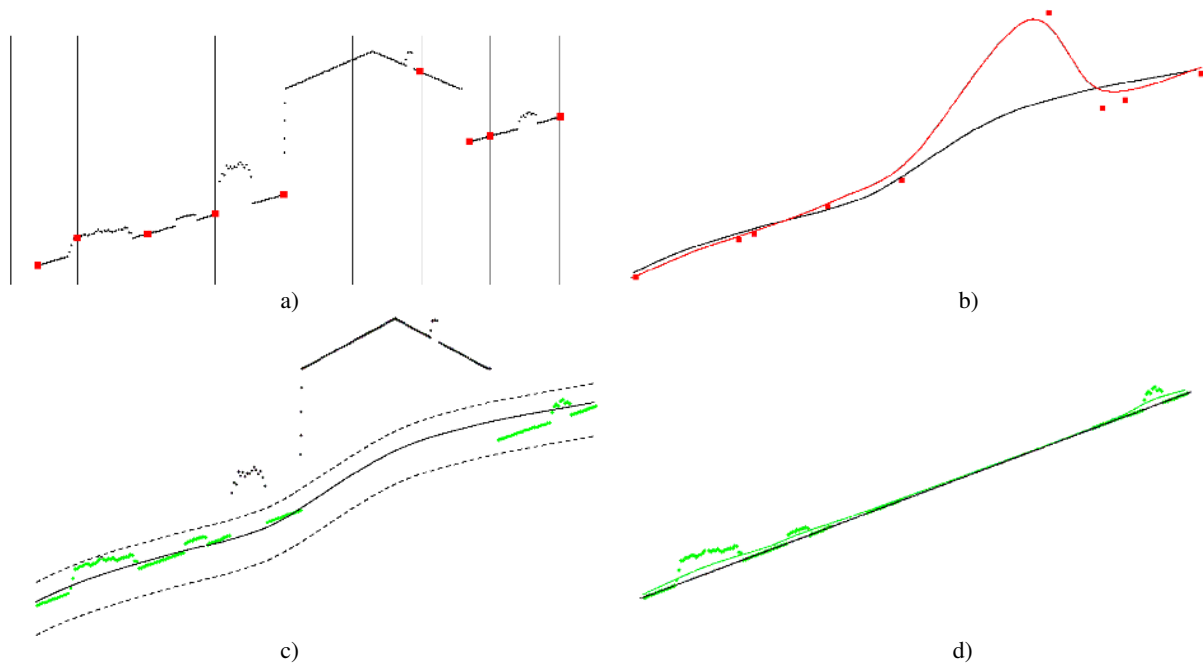
Figure 1: Sequence of the hierarchic robust interpolation
  a) Creation of a data pyramid, small points: original data, thick points: data pyramid (lowest point in a regular 5m interval).
  b) DTM generation in the coarse level by robust interpolation, the remaining point on the house is eliminated with an asymmetric and shifted weight function. The surface in the first and last iteration is shown.
  c) Coarse DTM with a tolerance band, all original points within the tolerance band are accepted.
  d) DTM generation in the fine level by robust interpolation using an asymmetric and shifted weight function. Again, the first and the last iteration is shown.

The hierarchic robust interpolation proceeds like the following:
1. Create the data pyramids with the lower resolutions.
2. Apply robust interpolation to generate a DTM, starting at the coarsest level.
3. Compare the DTM to the data of the higher resolution and accept points within a certain tolerance band.

The steps 2 and 3 are repeated for each finer level of detail. The sequence of the hierarchic robust interpolation on a synthetic laser scanner profile in a build-up area is presented in fig. 1.

Further details about hierarchical robust interpolation, its implementation in the software package SCOP++ and the results for an OEEPE dataset can be found in (Pfeifer et al., 2001).

4. EXAMPLES

In the following the results of these algorithms applied to different datasets are presented. As it will be seen the procedure is adapted to the characteristics of each dataset.

4.1 Robust Interpolation of Tacheometric and Photogrammetric Data

The Municipality of Vienna ordered a test project for the determination of a DTM from available digital map data. In the streets the data was captured with total stations Photogrammetric measurements were used for eaves, park areas and a few other features. The initial classification of terrain points was performed by point attributes stored in a database. The resulting dataset with all classified terrain points was very inhomogeneous due to the fact that there were a lot of points along the streets and only a few points in backyards and park regions. Therefore we decided to densify the data to get a complete DTM over the whole test area. The densification was performed by exporting a regular 5m raster after a triangulation of the data. The DTM was computed by linear prediction considering different point class accuracy. Therefore we were able to give the originally measured points a higher accuracy in contrast to the densified data (a small measurement variance of 25cm² vs. 1m² for the densification points).
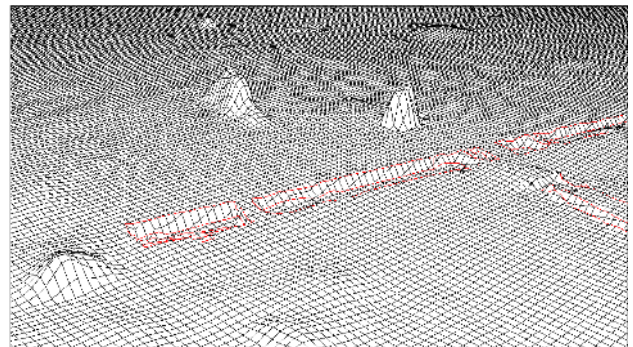

Figure 2: Perspective view of the DTM (5m grid with) from the selected terrain points (from the database) after densification

This leads to a DTM, which is mainly influenced by the measured data. The interpolated grid is only used to get a complete surface model. A perspective view of a part of this model can be seen in fig. 2. The terrain in this area is rather flat, but a few spots indicate data errors caused by false point attributes. A closer look at the data in combination with a geo-referenced vector map of the building blocks showed, that there are some misclassified points on the houses due to a false attribute and there where also a few points with obviously false z-values. Therefore it was necessary to eliminate these gross errors.

The first idea was to use a terrain shading and contour lines to locate these errors and to eliminate these false points manually, but it took quite a long time to go through this dataset with more than 300,000 points. To automate this process we applied the robust interpolation technique presented in sec. 3.2. The use of the hierarchic set-up was not necessary, because of the low point density and the low number of gross errors. However we had to adapt the weight function to the error distribution of this dataset. Unlike to laser scanning this dataset includes gross errors above and below the terrain and therefore we had to use a symmetric weight function to eliminate points with positive and negative filter values f. This weight function $p(f)$ with a sharp decline in the positive and negative branch is displayed in fig. 3. With the help of this weight function and a maximum number of three iterations the algorithm was able to eliminate these gross errors and a refined classification in terrain and off-terrain points with the help of tolerance values of ±0.3m was possible.
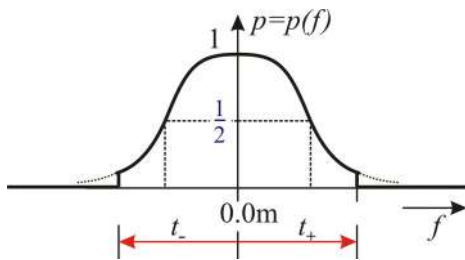

Figure 3: Symetrical weight function for the elimination of points with high positive and negative residuals with a half width value of 0.2m
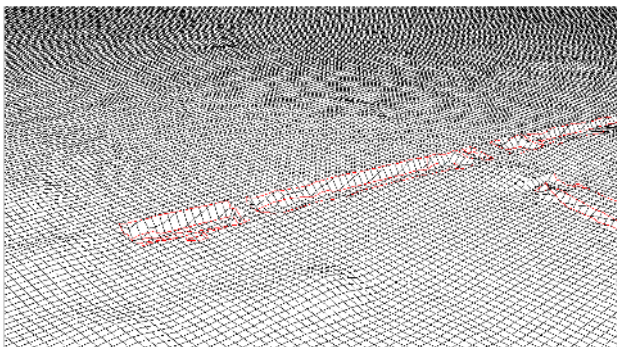

Figure 4: Perspective view of the DTM after robust Interpolation (automated elimination of off-terrain points, 5m grid with)

Photogrammetric measured break lines were included in the modelling process, which improved the automatic result. These break lines are treated as gross error free, which means that only the random measurement error is filtered. For the robust filtering this means that they have the weight 1 for all iterations. A perspective view of this DTM is presented in fig. 4. A comparison of this automated result with the manually corrected DTM by a difference model showed that the automatic process was very successful und led to similar results. Currently the complete data of the digital map (13 mio. points, ~ 400km²) is processed in this way.

4.2 Hierarchic Robust Interpolation of Airborne Laser Scanner Data

The generation of a DTM from airborne laser scanner (ALS) data is the "classical" use of the robust interpolation technique. A lot of different algorithms for this task do exist (e.g. Axelsson, 2000; Elmqvist et al, 2001; Vosselmann and Maas, 2001). As mentioned before, this algorithm was originally designed for the determination of a DTM in wooded areas (Kraus and Pfeifer, 1998). A pilot project with a dataset in the city of Vienna showed the limitations of this technique in build-up areas where large areas without terrain points do exist. Therefore we developed the hierarchical robust interpolation (sec. 3.3), which is also very useful in dense vegetated areas, which have similar characteristics for the DTM generation (large areas without ground points) like build-up areas. In the meantime the use of the hierarchical set-up, which strengthens the robustness and reduces computation time with two to three data pyramid levels, proved to be very useful in many test projects (see Pfeifer et al., 2001).

In this section the results of the DTM generation process in the Vienna test suite are presented (fig. 5). The accuracy of the DTM was checked by 816 tacheometric measured control points. The root mean square error (RMS) of the DTM was 0.11m.

For the robust interpolation in each data pyramid level an asymmetric shifted weight function must be used (fig. 6) in order to give high weights to points on the terrain surface, whereas the influence of off-terrain points (e.g. on the vegetation, houses and cars) is iteratively reduced by low weights.
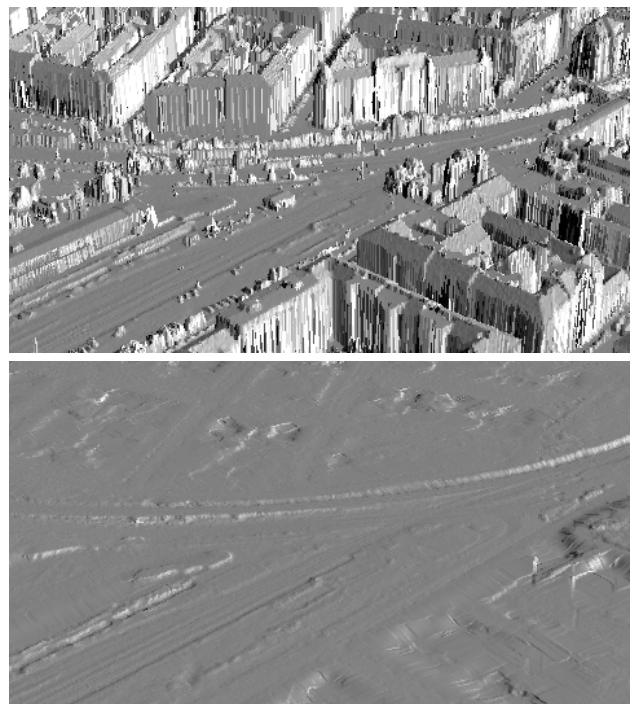

Figure 5: Perspective views of the digital surface model (DSM) and the DTM computed by hierarchical robust interpolation of the Vienna test suite
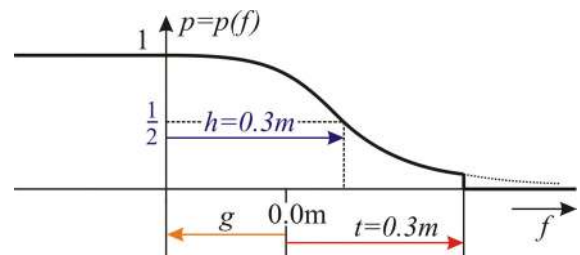

Figure 6: Typical weight function for the elimination of off-terrain points above the terrain. The value of g is determined automatically.

The effect of the elimination of off-terrain points can also be seen in the comparison of the empiric variogramms computed from all measured laser scanner points and from the classified ground points. In the geostatistical literature the variogramm is defined as the variance of the height difference between the heights z_i and z_j ($Var[z_i-z_j]$) at the locations $\mathbf{x}_i$ and $\mathbf{x}_j = \mathbf{x}_i + \mathbf{h}$. Under the hypothesis of 2nd order stationarity of the heights (considering them as the realisation of a random function), the variogramm does only depend on $\mathbf{h}$ and not on $\mathbf{x}_i$ (Journel and Huijbregts, 1978). This has also been described in sec. 3.1 for the covariance function.

An example of such an empiric variogramm computed from a dataset in a wooded area around Vienna (point distance of ~3m) is presented in fig. 7. For the classified terrain points we get, as expected, a horizontal tangent in the origin, which corresponds to a smooth (differentiable) surface and a nugget effect (measurement variance) close to zero. On the other hand the empiric variogram from all points shows a large nugget effect of 135m², corresponding to a standard deviation of ±12m for the height difference of very close neighboured points. Additionally, the tangent at the origin is not horizontal, indicating a continuous but not differentiable surface.
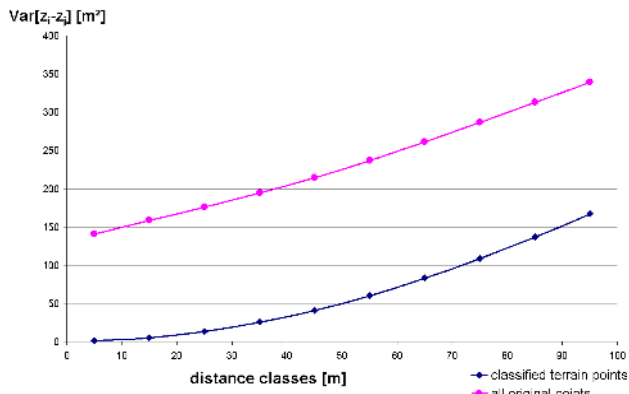

Figure 7: Empiric variograms of all original points and of the classified terrain points

4.3 Hierarchic Robust Interpolation of Terrestrial Laser Scanner Data

The generation of a DTM from terrestrial laser scanner (TLS) data proceeds similar to ALS data. Again the task was to eliminate points above the terrain surface and therefore the weight function has to be asymmetric and must be shifted. The difference to the ALS data lies in the point density. In the neighbourhood of the sensor we have a point density of nearly 1000points/m² whereas for larger distances this density is 4points/m². The laser scanner used is the Riegl LMS-Z210 (Riegl, 2002). Therefore the generation of data pyramids for homogenisation is necessary.

The parameters for the hierarchical robust interpolation are similar to the ALS case. The results from a test dataset of the company ArcTron (2002) are presented in the figures 8 and 9. The surface of this countryside area (~0.2km²) consists of wooded and open terrain. A visual inspection and a difference model showed that the algorithm did quite a good job. In the centre of this test suite the DTM is rather rough, which can be explained by the high point density, which allows a very detailed description of the terrain.

For the computation of the DTM in the last step we used a conditional data densification with the help of a distance transformation (chamfering) (Borgefors, 1986). Therefore, the ground plan locations of the terrain points are set as feature pixels in a digital image with a pixelsize of 0.5m. The distance transformation assigns each pixel the distance to its nearest feature pixel (i.e. the nearest measured point). In areas within a certain distance interval [1m,10m] we densified the terrain point set to close data holes. The heights of these densification points (1m grid) were sampled from a DTM, which was computed from a thinned out (low resolution) data set of the classified ground points. Therefore some extrapolation band exists around the terrain points and small areas without data are closed in the DTM of fig. 9.
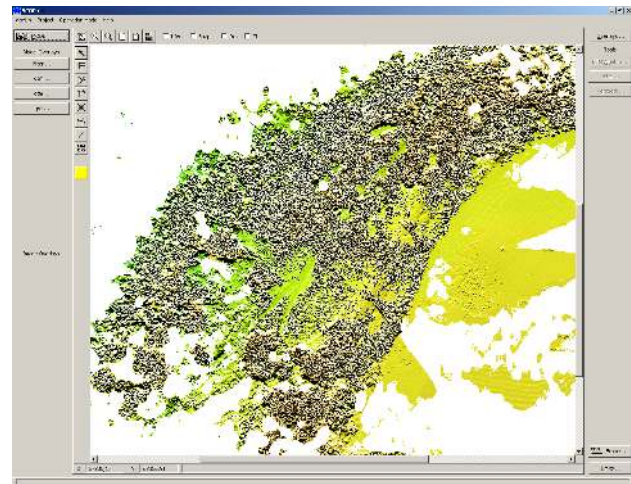

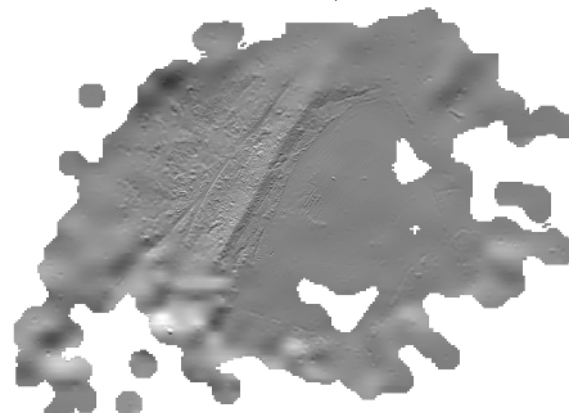Figure 8: DSM of the thinned out TLS data (lowest point in 0.2m raster)

Figure 9: Shading of the DTM of the thinned out classified terrain points with conditional densification

4.4 Elimination of Scan Errors in Satellite Laser Scanner Data from Mars

The Institute of Photogrammetry and Remote Sensing is participating as co-investigator in the "Mars Express" project of the European Space Agency (ESA). Within this project, several computations on the global Mars data from NASA's MOLA (Mars Orbiter Laser Altimeter) sensor were performed. This sensor can be characterized as a laser scanner profiler without a deflection unit for the laser beam. Therefore only a very inhomogeneous point distribution is available.

Presently the delivered MOLA data consists of about 640 mio. surface points, which contain scan errors due to referencing errors of the spacecraft. A DTM shading from a small part (250000 points) of this dataset is presented in fig. 10, where
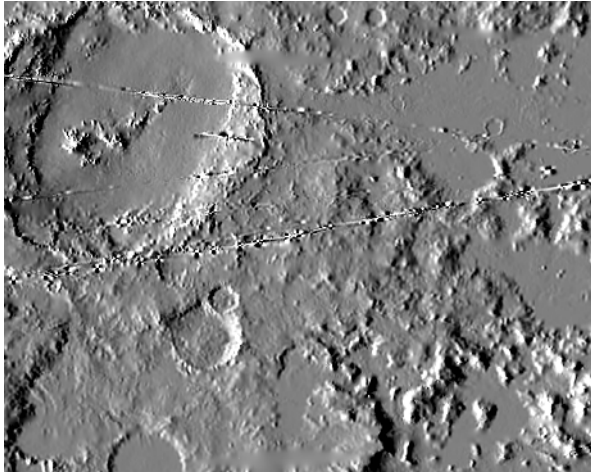
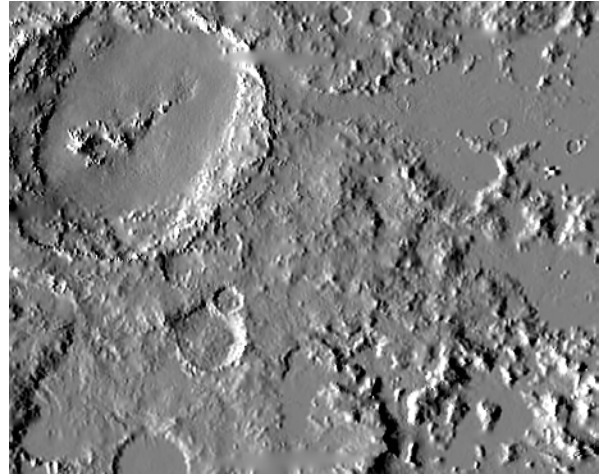Figure 10: Shading of the DTM with gross errors in the MOLA dataset

Figure 11: Shading of the DTM after automatic gross error elimination

these gross errors can be recognized. Of course, the generation of a DTM from the whole Mars surface by manual correction is not practicable and therefore an automatic procedure for error elimination is required.

In the first test we tried to use hierarchical (due to the very inhomogeneous point density) robust interpolation to eliminate these errors. The elimination of the scan errors was possible with this technique, but due to the roughness of the Mars surface we also eliminated points in regions without scan errors. The rough surface did not fit to our functional model of linear prediction, which is able to generate very smooth surfaces.

Better results were obtained by analysing scan line segments instead of the residuals of each individual point. It showed up that the average filter value of a segment (i.e. RMS of the residuals of the points belonging to this segment) could be used to eliminate those segments with gross errors. Due to the fact that correct scan line segments next to a segment with gross errors also get a higher RMS it was necessary to apply this gross error elimination in an iterative manner. Because not all points along a scan line segment are affected by gross errors we analysed the discrepancies of all eliminated points in respect to a DTM computed without these gross error segments and accepted all previous eliminated points within a certain user defined threshold value.

This iterative method proceeds like the following:
1. Compute a DTM with all points.
2. Compute the RMS per scan line segment and eliminate lines with a high RMS.
3. Compute a DTM with the accepted scan line segments.
4. Former eliminated points are accepted if they are within a certain tolerance to the DTM .
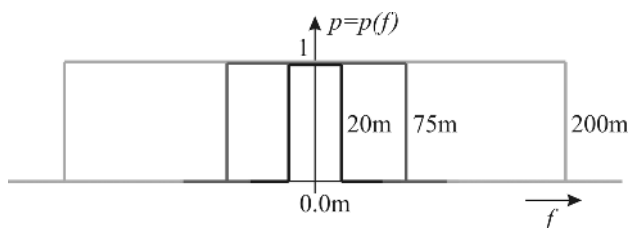5. Compute a new DTM.

Figure 12: Symmetric box weight function with decreasing extend in each iteration step for the elimination of scan line segments

The steps 2 to 5 are repeated with iteratively decreasing tolerance values until the tolerance of the RMS per line reaches a user defined threshold value. The results of this process can be seen in fig. 11.

This method corresponds to the robust interpolation with a box weight function with decreasing extend in each iteration step (fig. 12). In contrast to the previous examples, the weight function is not applied to the filter values of single points but for complete scan line segments.

5. CONCLUSIONS

We have presented a very general concept of gross error elimination for DTM generation (surface computation) and achieved good results for the presented datasets. What can be seen in the example section is that the general concept for gross error elimination is the same for all projects. Only a few parameters must be adapted to the characteristics of the specific datasets (weight function, number of hierarchical levels). This adaptation is performed manually. We have derived standard parameters, which are adapted to the characteristics of each project. This is performed in a trial and error basis with 2 or maximal 3 repetitions of the computation. Of course only the interpretation of the intermediate results requires human resources, the computation itself is performed totally automatically. In our experience most of the adaptations are necessary in the coarse levels. The functional model for surface computation is in the presented examples linear prediction, but any model can be used, which is capable of considering individual weights for the given points.

Additionally two methods of data densification have been presented (sec. 4.1 and 4.3).

In the future we will have to consider new measurement systems, which will provide further information of the sensed objects like laser scanner systems, which allow now the registration of multiple echoes and the intensities of the returned signal and in future even full waveform capture. An other important topic will be sensor combination. Nowadays sensor systems, which combine laser scanner sensors with digital line cameras, already do exist. The big task for the future will be to integrate all these information sources into one modelling process in order to achieve better results.

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APPENDIX A. ROBUST LINEAR PREDICTION

In the following the basic functions for linear prediction with individual weights are presented. This means that a surface is interpolated from a cloud of points, but each point has an individual accuracy. It can be used for robust interpolation by modulating the weights (accuracies) depending on the filter value (negative residual) of the observations in an iterative manner. The filter values of one iteration are used to determine the weights for the next surface interpolation. More details can be found in (e.g. Kraus and Pfeifer, 1998 and Kraus, 2000).

A.1 Linear prediction with individual weights

Given are n points P_i with the heights z_i, which have been reduced by subtracting a trend surface (e.g. a plane). After this reduction the expectancy of the observed heights is zero. The height z at a position P is determined by Eq. (1):

$$z = \mathbf{c}^T \mathbf{C}^{-1} \mathbf{z} \tag{1}$$

with:

$$\mathbf{c} = (C(PP_1), C(PP_2), \ldots C(PP_n))^T \tag{2}$$

$$\mathbf{z} = (z_1, z_2, \ldots z_n)^T \tag{3}$$

$$\mathbf{C} = \begin{pmatrix} V_{zzp_1} & C(P_1P_2) & \cdots & C(P_1P_n) \\ & V_{zzp_2} & & C(P_2P_n) \\ & & & \vdots \\ & & & V_{zzp_n} \end{pmatrix} \tag{4}$$

The function $C(P_iP_k)$ describes the covariance between two points on the surface in the following way (Gaussian model):

$$C(P_iP_k) = C(0)e^{-(\frac{P_iP_k}{c})^2} \tag{5}$$

with:

$C(0)$ = covariance for a distance of zero
P_iP_k = horizontal Euclidian distance between the two surface points P_i and P_k
c = factor (estimated from the given points) for controlling the steepness of the covariance function

V_{zzp_i} in Matrix $\mathbf{C}$ of Eq. (4) is the variance of the given points, which is the sum of $C(0)$ and the variance of the measurement σ_i^2. The points are considered to have the same σ_0^2 (a priori accuray), but different weights p_i. The accuracy σ_i^2 of a point P_i is obtained from:

$$\sigma_i^2 = \frac{\sigma_0^2}{p_i} \tag{6}$$

The variance of each point P_i can be computed by:

$$V_{zzp_i} = C(0) + \sigma_i^2 \tag{7}$$

A.2 Robust weight function

The weight p_i depends on the filter value f_i, which is the oriented distance from the prior computed surface (in the first step all points are equally weighted) to the measured point. The weight function (a bell curve), which can have different parameters (a, b) for the left and right branch, is given by:

$$p_i = \frac{1}{1 + (a(f_i - g)^b)} \tag{8}$$

f_i = filter value
g = shift value, determined automatically from the filter values
a = $1/h$
b = $4h + s$

with the half width value h and the slant s at the half weight.

Additionally thresholds are used to set the weight of a point with a higher or lower filter value to zero so that it is excluded completely from the modelling process.