Applied Demand Analysis: A Survey — Source link

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1. INTRODUCTION

Applied demand analysis is usually taken to refer to the microeconomic analysis of consumer and producer behaviour, especially the nature of input demand and consumer demand equations. In this paper, we shall confine ourselves to consumer demand and provide a review of the relevant theory and empirical results. According to Stigler's (1954) survey of early research, applied demand analysis has a long history going back to the 1790s in England with the budget studies of David Davies and Sir Frederick Morton Eden. Regarding the role of prices in determining consumption, Stigler (1954, p. 103) argues that "systematic and cumulative work ...... began only in the 1870s ...... when Fleeming Jenkin, Jevons, Walras, Marshall and others began to develop the demand function or curve as an integral part of the theory of price formation." The earliest empirical analysis known to Stigler of the relationship between prices and quantities consumed was carried out in 1861 by Ernst Engel (whose 1857 famous budget study led to what is now known as "Engel's law" -- the proportion of income devoted to food declines with increasing income). Engel's 1861 research dealt with harvests and prices of rye in Prussia for the period 1846-61.

The whole area of applied demand analysis is now so extensive that a comprehensive survey would not be possible within the confines of a single paper. Fortunately, a number of books on the topic are now available, including Bewley (1986), Deaton and Muelbauer (1980a), Goldberger (1987), Phlips (1974), Pollak and Wales (1992), Powell (1974), S. Selvanathan (1993), Theil (1975/76), Theil (1980a) and Theil and Clements (1987). Additionally, recent major survey articles are Blundell (1988), Blundell et al. (1993) and Deaton (1986). It is against this background that we have decided to provide in this paper a selection of material that we judge to be useful, neglected and/or misunderstood by many economists. The selection of material is necessarily subjective and we are sure that not everyone will agree with it. Our only defence in choosing what to highlight is our own experience of working in consumer demand. One further characteristic of our paper should be noted. The literature is often plagued with claims and counter claims regarding the benefits of one approach over others. For those who do not work in the field and have to rely on the literature for guidance, it is thus often difficult to assess these claims. Accordingly, our paper attempts to provide nonspecialists with some perspectives and direction.

In Section 2 we provide a brief presentation of the well-known economic theory of the consumer and tests of that theory. Section 3 discusses how the structure of preferences can be used to simplify demand equations, while Section 4 deals with the demand for groups of particular goods and the demand for commodities within groups. Subsequent sections cover alternative functional forms; the often-confused topic of how the behaviour of individual consumers can be sensibly aggregated to yield macro demand functions; the role of variables other than income and prices, such as advertising, in demand equations; and some useful empirical regularities. In various parts of the paper, we draw on Clements and S. Selvanathan (1994) and E. A. Selvanathan and Clements (1995).

2. THE ECONOMIC THEORY OF THE CONSUMER

Let $q_1, \ldots, q_n$ be the quantities consumed of n goods and let $p_1, \ldots, p_n$ be the corresponding prices. Then, $M = \Sigma_{i=1}^{n} p_i\, q_i$ is total expenditure, which we shall refer to as "income" for short. The consumer chooses the quantity vector $\mathbf{q} = [\, q_i \,]$ to maximise the utility function u ($\mathbf{q}$), subject to the budget constraint $\mathbf{p}'\mathbf{q} = M$, where $\mathbf{p} = [\, p_i \,]$ is the vector of prices. The solution to this problem leads to a Marshallian demand equation for good i:

$$(2.1) \qquad q_i = q_i\,(M,\; p_1,\; \ldots,\; p_n).$$

To be more concrete, suppose that (2.1) is log-linear:

$$(2.2) \qquad \log q_i = \alpha_i + \eta_i \log M + \sum_{j=1}^{n} \eta_{ij} \log p_j\,,$$

where α_i is an intercept; η_i is the i^{th} income elasticity; and η_{ij} is the $(i,j)^{th}$ uncompensated price elasticity.

The Marshallian (or money-income-constant) demand equation can be transformed into its Slutsky (or real-income-constant) counterpart by using the Slutsky decomposition for the uncompensated price elasticity, $\eta_{ij} = \eta'_{ij} - w_j\, \eta_i$, where η'_{ij} is the $(i,j)^{th}$

compensated price elasticity and $w_j = p_j q_j / M$ is the budget share of good j, the proportion of income spent on j. This yields, after a little algebra, the Slutsky demand equation for good i:

$$\text{(2.3)} \qquad \log q_i = \alpha_i + \eta_i \log Q + \sum_{j=1}^{n} \eta'_{ij} \log p_j ,$$

where $\log Q = \log M - \log P$, with $\log P = \sum_{j=1}^{n} w_j \log p_j$ the Divisia price index. That is, $\log Q$ is money income deflated by the price index, or a measure of the consumer's real income. Note that real income and the compensated price elasticities η'_{ij} appear on the right-hand side of (2.3), while money income and the uncompensated elasticities η_{ij} are in (2.2).

The consumer's maximisation problem implies three testable constraints on the demand equations. The first is demand homogeneity which states that an equi-proportional change in prices has no effect on the quantities demanded when real income is held constant. On the 13th of February 1966 when decimal currency was introduced into Australia, all prices and money incomes doubled overnight; demand homogeneity assures us that this change in the unit of account would have had no effects on consumption. In the context of equation (2.3) for i = 1, ... , n goods, homogeneity is expressed as the sum of the own- and cross-price elasticities for each good being zero:

(2.4) $$\sum_{j=1}^{n} \eta'_{ij} = 0 \qquad i = 1, \dots, n.$$

The second constraint is symmetry of the substitution effects, or Slutsky symmetry. This states that when real income is held constant, the effect of a \$1-rise in the price of a bottle of wine on beer consumption is exactly equal to the effect on wine consumption of a \$1-rise in the price of a bottle of beer. Going back to equation (2.3), η'_{ij} is the compensated price elasticity and $(q_i / p_j)\,\eta'_{ij}$ is the corresponding slope. Symmetry states that this slope is the same when we interchange the i and j subscripts, $(q_i / p_j)\,\eta'_{ij} = (q_j / p_i)\,\eta'_{ji}$, or, multiplying both sides by $p_i\, p_j / M$:

(2.5) $$w_i\, \eta'_{ij} = w_j\, \eta'_{ji} \qquad i, j = 1, \dots, n.$$

Early tests of homogeneity and symmetry by Barten, Byron and others (see Barten, 1977, for references) used F-test to test constraints (2.4) and (2.5) and variants thereof. As these hypotheses seemed to be plausible to most, it came as a major surprise that many of the early tests led to rejections. What exactly was the problem? Should the theory of the utility-maximising consumer really be abandoned, as advocated by those who took the results literally (such as Christenson et al., 1975)? Or was the source of the problem the lack of dynamics or other variables excluded from the demand equations?

This puzzle was finally resolved by Theil and his students. Using Monte Carlo simulation experiments, they showed convincingly that the rejections were just an artefact of the tests employed. The test statistics have an asymptotic justification, but with sample sizes (the number of observations) and model sizes (n, the number of commodities distinguished) typically used, the tests are seriously biased against the null, i.e., they overreject. When appropriate adjustments are made to the test statistics, their finite-sample performance for homogeneity and symmetry is perfectly satisfactory. For details, see Laitinen (1978), Meisner (1979) and Theil (1987).

The third constraint is the law of demand, viz., that demand curves slope down when real income remains constant. This is sometimes referred to as the <u>negativity condition</u>, as the $n \times n$ matrix of compensated price elasticities $[\eta'_{ij}]$ must be negative semi-definite (it is only semi-definite as this matrix is subject to the homogeneity constraint). In other words, $[\eta'_{ij}]$ has $n - 1$ negative characteristic roots, plus one which is zero. The most frequent way of dealing with this condition in applied work is to simply verify the signs of the roots of the estimate of $[\eta'_{ij}]$, although there are more sophisticated approaches (see, e.g., Barten and Geyskens, 1975).

A further set of constraints on the system of n demand equations is the <u>adding-up restrictions</u> which are implied by the budget constraint. In terms of (2.3), these take the form $\sum_{i=1}^{n} w_i \alpha_i = 0$, $\sum_{i=1}^{n} w_i \eta_i = 1$, $\sum_{i=1}^{n} w_i \eta'_{ij} = 0$, $j = 1, \ldots, n$. As the data used to estimate demand equations are constructed to satisfy the budget constraint, these restrictions are not testable.

3. THE STRUCTURE OF PREFERENCES

Consider demand equation (2.3) for $i = 1, \dots, n$ goods. In this system of n equations there are n intercepts, $\alpha_1, \dots, \alpha_n$, n income elasticities, $\eta_1, \dots, \eta_n$, and n^2 price elasticities η'_{ij}, $i, j = 1, \dots, n$, so that the total number of coefficients is $n+n+n^2 = n(2+n)$. For a moderate-sized system of $n = 10$ goods, this total equals 120, which is an impossibly large number of coefficients to be estimated in an unrestricted fashion. Even when we take account of the homogeneity, symmetry and adding-up restrictions, the number of coefficients is still of the order n^2.

One way of proceeding is to set to zero some of the cross-price elasticities (η'_{ij} for $i \neq j$) in equation (2.3), perhaps on the basis of the intrinsic nature of the commodities involved or on the basis of prior evidence. A more systematic approach is to pattern the $n \times n$ elasticity matrix $[\eta'_{ij}]$ by further structuring the nature of the consumer's utility function $u(q_1, \dots, q_n)$. Important early contributions in this area include Barten and Turnovsky (1966), Goldman and Uzawa (1964), Gorman (1959, 1968), Leontief (1947), Pearce (1961, 1964), Sono (1961) and Strotz (1957).

Suppose the n goods are broad aggregates such as food, clothing, housing and so on. It is then not unreasonable to view the demand for each good as representing the desire for some characteristic(s) unique to each good: Food provides nutrition and taste, clothing warmth and style and housing provides shelter. These unique, or basic, characteristics represent fundamental desires which generate utility. Moreover for them to be truly basic characteristics, it should be the case that there is little or no interaction between them in the utility function, so that utility is generated by the consumption of food and clothing and housing, with the emphasis on the "ands" representing the notion of additivity.

These ideas can be formalised by an additive utility function, whereby utility is the sum of n sub-utility functions, one for each good:

$$u(q_1, \ldots, q_n) = \sum_{i=1}^{n} u_i(q_i), \tag{3.1}$$

where $u_i(q_i)$ is the sub-utility function for good i. According to (3.1), each marginal utility is a function only of the good in question, $\partial u/\partial q_i = d u_i/d q_i$, and is independent of the consumption of all other goods, $\partial^2 u/\partial q_i \partial q_j = 0, i \neq j$. Accordingly, (3.1) is also known as preference independence (PI). An example of (3.1) is the Cobb-Douglas utility function, $\prod_{i=1}^{n} q_i^{\alpha_i}$. As monotonic transformations of the utility function leave the demand equations unaffected, we can express the Cobb-Douglas in logarithmic form, $\sum_{i=1}^{n} \alpha_i \log q_i$, which is now additive in $\log q_1, \ldots, \log q_n$.

To analyse the implications of PI, let δ_{ij} be the Kronecker delta ($\delta_{ij} = 1$ if i = j, 0 otherwise) and ϕ be the reciprocal of the income elasticity of the marginal utility of income [i.e., $\phi^{-1} = \partial(\log\lambda)/\partial(\log M)$, where λ is the marginal utility of income, the Lagrangian multiplier for the budget constraint]; for brevity, we shall refer to ϕ as the "income flexibility". If preferences are of the form (3.1), the $(i, j)^{th}$ price elasticity then becomes (see, e.g., Clements et al., 1995):

(3.2) $$\eta'_{ij} = \phi\, \eta_i \left(\delta_{ij} - w_j\, \eta_j\right),$$

where η_i is the i^{th} income elasticity and w_j is the budget share of j, as before. As $\eta_j = \left(\partial q_j / \partial M\right)\left(M / q_j\right)$, it follows that the term $w_j\, \eta_j$ on the right-hand side of (3.2) equals $\theta_j = \partial\left(p_j\, q_j\right)/\partial M$, which is the marginal share of good j. This θ_j answers the question, how much of a \$1-rise in income is spent on good j? The marginal share is to be contrasted with the budget share, $w_j = p_j\, q_j / M$, which relates to pre-existing, or average, expenditure on j. The ratio of the marginal share to the corresponding budget share is the income elasticity, $\eta_j = \theta_j / w_j$.

If we use (3.2) in the demand equation (2.3), the substitution term then becomes

$$\sum_{j=1}^{n} \eta'_{ij} \log p_j = \phi\, \eta_i \left(\log p_i - \log P'\right), \tag{3.3}$$

where $\log P' = \sum_{i=1}^{n} \theta_i \log p_i$ is the Frisch price index. In contrast with the Divisia price index, which uses budget shares as weights, the Frisch index uses marginal shares as weights. As luxuries ($\eta_i > 1$) have marginal shares in excess of their budget shares, it follows that these goods are more heavily weighted in the Frisch index than in the Divisia index.

Using (3.3) in (2.3), the i^{th} demand equation under preference independence takes form

$$\log q_i = \alpha_i + \eta_i \log Q + \phi\, \eta_i \left(\log p_i - \log P'\right). \tag{3.4}$$

This shows that PI implies that only the own-relative price appears in each demand equation and that the own-price elasticity is $\phi\, \eta_i$. Accordingly, under PI there are no specific substitutes or complements (Houthakker, 1960). Additionally, as ϕ and the own-price elasticity $\phi\, \eta_i$ are both negative, PI implies that each income elasticity η_i is positive, so that inferior goods are ruled out. A further implication of PI is that the own-price elasticities ($\phi\, \eta_i$) are proportional to the corresponding income elasticity (η_i), with the income flexibility (ϕ) the (negative) proportionality factor. In other words, luxuries are more price elastic than necessities.

The own-price elasticity in equation (3.4), $\phi\,\eta_i$, is a Frisch elasticity which holds constant the marginal utility of income. By contrast, the compensated and uncompensated elasticities hold constant real and money income, respectively. If C, F and U are the compensated, Frisch and uncompensated elasticities, then under PI the relationship between them is

$$C = F\,(1 - \theta_i), \qquad U = C - \theta_i,$$

where θ_i is the marginal share of good i. As all goods are normal under PI, $\theta_i > 0$ and it follows that $|U| > |F| > |C|$. It also follows that if the marginal share is small, $C \approx F \approx U$. Similar considerations also apply to the cross-price elasticities.

The final feature of preference independence to note is the reduction in the number of unknown parameters in the demand equations. As stated above, in the unrestricted demand equation (2.3) for i = 1, ..., n there are n^2 unknown price elasticities. By contrast, in (3.4) for i = 1, ..., n there is only one free parameter in the substitution term, the income flexibility ϕ.

The hypothesis of PI can be tested by comparing the fit of the restricted and unrestricted demand equations, equations (3.4) and (2.3) for i = 1, ..., n, or variants thereof. Most of the earlier tests of PI rejected the restriction (see Barten, 1977, for a survey). In a highly-influential paper, Deaton (1974) analyses the implications of PI indirectly by testing whether unrestricted own-price elasticities are proportional to the corresponding income elasticities. Using UK data for n = 37 and 8 commodities, he finds no relationship between income and price elasticities and concludes

> "that *the assumption of additive preferences is almost certain to be invalid in practice and the use of demand models based on such an assumption will lead to severe distortion of measurement.*"
>
> (Deaton, 1974, p. 346, his emphasis)

The earlier tests of PI all had only an asymptotic justification, and in view of the difficulties with the asymptotic tests of homogeneity and symmetry discussed above in Section 2, there is reason to believe that these tests also have problems of overrejection. S. Selvanathan (1987, 1993) developed a Monte Carlo test of PI which avoids possible problems associated with asymptotics and finds a good deal of support for the hypothesis with data from 18 OECD countries. S. Selvanathan (1993) also estimates double-log demand equations for n = 10 goods in each of 18 OECD countries. These equations have income and the own-relative price on the right-hand side, but the price elasticities are otherwise unconstrained. Table 3.1 (from Clements and S. Selvanathan, 1994) and Figure

3.1 present the joint frequency distribution of the $10 \times 18 = 180$ elasticities (this is only the approximate number as there are minor differences in the number of goods in different countries). As can be seen, $34/55 = 62$ percent of the necessities ($\eta_i \leq 1$) have own-price elasticities less than one half (in absolute value), while $30/45 = 67$ percent of the luxuries ($\eta_i > 1$) possess price elasticities larger than one half. The conclusion is that luxuries are indeed more price elastic than necessities, which supports the PI hypothesis. Accordingly, in light of this more recent evidence, Deaton (1974) may have been premature in rejecting preference independence.

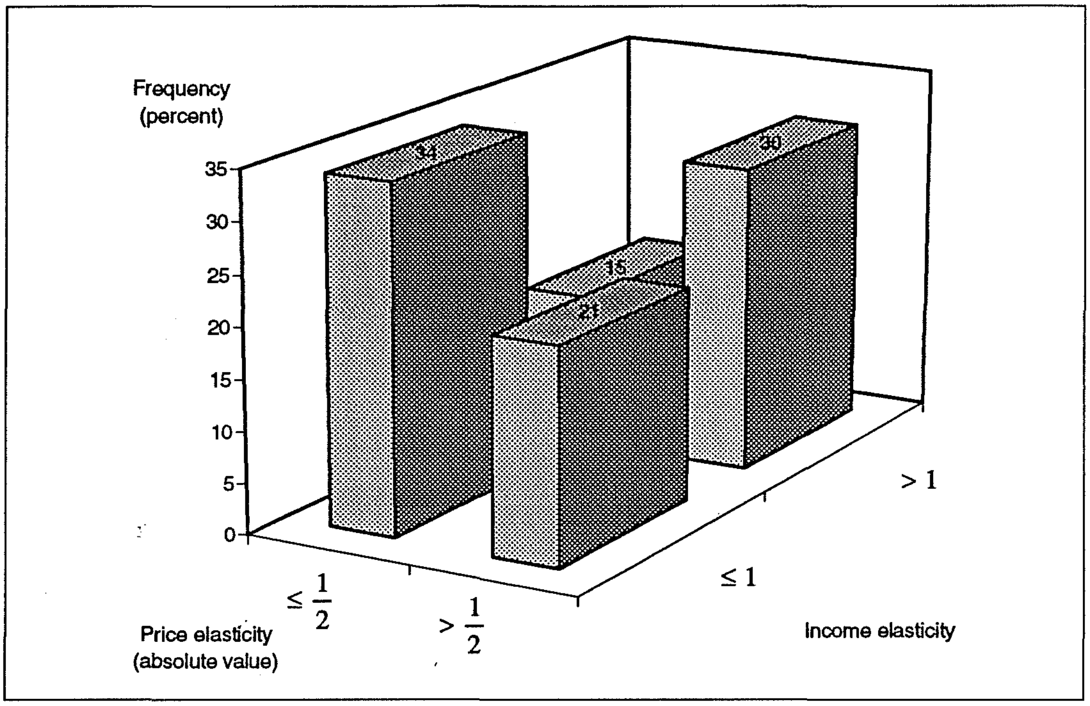
TABLE 3.1

JOINT FREQUENCY DISTRIBUTION OF INCOME AND PRICE ELASTICITIES
(Percentages)

Income elasticity	Absolute value of price elasticity		
	$\leq \frac{1}{2}$	$> \frac{1}{2}$	Total
≤ 1	34	21	55
> 1	15	30	45
Total	49	51	100

Figure 1

JOINT FREQUENCY DISTRIBUTION OF INCOME AND PRICE ELASTICITIES



A weaker version of preference independence is block independence whereby the consumer's utility function is additive in groups of goods, rather than individual goods. Let the n goods be divided into G < n groups, written S_1, ..., S_G, such that each good belongs to only one group. Then, preference are of the block independence form when the utility function is the sum of G group utility functions, each involving the quantities of only one group,

$$(3.5) \qquad u\left(q_1, \ldots, q_n\right) = \sum_{g=1}^{G} u_g(\mathbf{q}_g),$$

where $\mathbf{q}_g$ is the vector of the q_i's that fall under S_g. Thus if alcoholic beverages make up one block-independent group and all other goods another, the marginal utility of, say, beer would then be affected by the consumption of wine and spirits, but not by consumption of any good outside of the group.

The demand equation for good $i \in S_g$ implied by (3.5) is (see, e.g., Clements et al., 1995)

$$\text{(3.6)} \qquad \log q_i = \alpha_i + \eta_i \log Q + \sum_{j \in S_g} \eta''_{ij}\left(\log p_j - \log P'\right),$$

where η''_{ij} is the $(i, j)^{th}$ price elasticity (strictly speaking, this is also a Frisch price elasticity which holds constant the marginal utility of income). The Frisch elasticities are subject to the restriction

$$\text{(3.7)} \qquad \sum_{j \in S_g} \eta''_{ij} = \phi\, \eta_i \qquad i \in S_g.$$

Equation (3.6) is to be compared with (3.4), the corresponding demand equation under preference independence. The assumption of PI implies that only the own-price affects consumption, while block independence means that the prices of goods in the same group as the commodity in question play a role. If the alcoholic beverages group comprises beer, wine and spirits, and if these form a block-independent group, then only the prices of the three beverages affect the consumption of each beverage and the prices of other goods play no role. It can therefore be seen that there is an appealing unification between preferences and demand equations.

4. GROUP DEMAND AND CONDITIONAL DEMAND

Consider again the demand for beer, wine and spirits. One way to analyse these beverages would be to postulate that they form a block-independent group and then use demand equation (3.6). One disadvantage of this approach is that equation (3.6) involves real income and, through their influence on the Frisch price index $\log P'$, the prices of other (i.e., non-alcohol) goods. Conditional demand equations deal only with alcohol and thus avoid the problem. Accordingly, a system with a large number of commodities can be transformed into a number of smaller, independent sub-systems, one for each group of goods. In this section, we set out details of this approach.

Recall from Section 2 that the budget share w_i is the proportion of total expenditure devoted to good i, while the marginal share θ_i measures the increase in expenditure on i as a result of a one-dollar increase in income. The budget and marginal shares for the group S_g are

$$W_g = \sum_{i \in S_g} w_i , \qquad \Theta_g = \sum_{i \in S_g} \theta_i .$$

It then follows that $w'_i = w_i / W_g$ is the conditional budget share of $i \in S_g$, while $\theta'_i = \theta_i / \Theta_g$ is the corresponding conditional marginal share.

We return to (3.6), the demand equation for good i under block independence. If we multiply both sides of that equation by $w'_i = w_i / W_g$ and sum over $i \in S_g$, we obtain

$$\text{(4.1)} \qquad \sum_{i \in S_g} w'_i \log q_i = \sum_{i \in S_g} w'_i \alpha_i + \frac{1}{W_g} \sum_{i \in S_g} w_i \eta_i \log Q$$
$$+ \frac{1}{W_g} \sum_{i \in S_g} \left[w_i \sum_{j \in S_g} \eta''_{ij} \left(\log p_j - \log P' \right) \right].$$

We define the Divisia volume index and the Frisch-price index for group S_g as

$$\text{(4.2)} \qquad \log Q_g = \sum_{i \in S_g} w'_i \log q_i , \qquad \log P'_g = \sum_{i \in S_g} \theta'_i \log p_i .$$

The sum on the left-hand side of equation (4.1) is then just the Divisia volume index of the group $\log Q_g$. If we use the definition of Θ_g and $w_i \eta_i = \theta_i$, then the term

$(1/W_g)\Sigma_{i \in S_g} w_i \eta_i$ on the right-hand side of (4.1) is the group income elasticity $\eta_g = \Theta_g / W_g$.

We write the last term on the right-hand side of (4.1) as

$$
\begin{aligned}
(4.3)\qquad & \frac{1}{W_g} \sum_{i \in S_g} \sum_{j \in S_g} w_i \eta''_{ij} \left(\log p_j - \log P'\right) \\
& \qquad = \frac{1}{W_g} \sum_{j \in S_g} \left(\sum_{i \in S_g} w_i \eta''_{ij} \right) \left(\log p_j - \log P'\right).
\end{aligned}
$$

To simplify this, we multiply both sides of equation (3.7) by w_i to give $\Sigma_{j \in S_g} w_i \eta''_{ij} = \phi \theta_i$, $i \in S_g$. As the Frisch price slopes of the demand functions are symmetric, $w_i \eta''_{ij} = w_j \eta''_{ji}$, so that $\Sigma_{i \in S_g} w_i \eta''_{ij} = \phi \theta_j$, $j \in S_g$. Consequently, the right-hand side of (4.3) can be expressed as

$$\frac{1}{W_g} \sum_{j \in S_g} \phi \theta_j \left(\log p_j - \log P'\right).$$

Recalling that $\theta'_i = \theta_i / \Theta_g$, we can then write the above term as

$$\frac{1}{W_g} \sum_{j \in S_g} \phi \theta_j \left(\log p_j - \log P'\right) = \frac{\phi \Theta_g}{W_g} \left(\sum_{j \in S_g} \theta'_j \log p_j - \log P' \right)$$

$$= \phi\, \eta_g \left(\log P'_g - \log P'\right),$$

where the second step is based on (4.2) and the definition of the income elasticity for S_g, $\eta_g = \Theta_g / W_g$.

Therefore, equation (4.1) simplifies to

$$\text{(4.4)} \qquad \log Q_g = \alpha_g + \eta_g \log Q + \phi\, \eta_g \left(\log P'_g - \log P'\right),$$

where $\alpha_g = \sum_{i \in S_g} w'_i\, \alpha_i$. This is the composite demand equation for S_g as a group. As can be seen from the right-hand side of this equation, only the relative price of the group, $\left(\log P'_g - \log P'\right)$, and income affect the demand for the group as a whole under block independence. This relative price is the Frisch-deflated Frisch price index of the group. It is also to be noted that the relative prices of goods outside the group do not play any role in the composite demand equation. The own-price elasticity of demand for the group takes the form $\phi\, \eta_g$; this is the elasticity of the Divisia volume index of the group with respect to the Frisch-deflated Frisch price index of the group. In the terminology of the previous section, this is also a Frisch elasticity as the marginal utility of income is held constant. Note also that if there are g = 1, ... , G groups of goods, the own-price elasticities $\phi\, \eta_1, \ldots, \phi\, \eta_G$ are proportional to the corresponding income elasticities $\eta_1, \ldots, \eta_G$, with the income flexibility as the factor of proportionality. Comparing equations (4.4) and (3.4), the demand equation under preference independence, it can be seen that the former is an upper-case version of the latter. This reflects the fact that under block-independence, groups of goods are preference independent.

The composite demand equations given by (4.4) for g = 1, ... , G show that the allocation of income to each of these G groups depends only on income and the G relative prices of each group. In other words, equation (4.4) explains the first stage of a two-stage budgeting process. In the second stage, group expenditure is then allocated to commodities within the group. Conditional demand equations deal with the second stage of this budgeting process.

To obtain the conditional demand equation for good $i \in S_g$, we rearrange (4.4) as

$$\log Q = \frac{1}{\eta_g}\left(\log Q_g - \alpha_g\right) - \phi\left(\log P'_g - \log P'\right).$$

Substituting the right-hand side of this equation for $\log Q$ in (3.6), we obtain

$$\log q_i = \alpha_i + \frac{\eta_i}{\eta_g}\left(\log Q_g - \alpha_g\right) - \phi\, \eta_i \left(\log P'_g - \log P'\right)$$

$$+ \sum_{j \in S_g} \eta'_{ij}\left(\log p_j - \log P'\right).$$

Let $\alpha_i^g = \alpha_i - \alpha_g\, \eta_i^g$, where $\eta_i^g = \eta_i / \eta_g$ is the conditional income elasticity of demand for $i \in S_g$. Then using (3.7), we obtain

$$\text{(4.5)} \qquad \log q_i = \alpha_i^g + \eta_i^g \log Q_g + \sum_{j \in S_g} \eta''_{ij}\left(\log p_j - \log P'_g\right).$$

Equation (4.5) is the demand equation for $i \in S_g$, given the demand for the group as a whole, $\log Q_g$. As the right-hand side of this equation is exclusively concerned with variables pertaining to the group S_g to which the i^{th} commodity belongs, this is known as the <u>conditional demand equation</u>. Note that the price elasticities in (4.5), η''_{ij}, are exactly the same as those in the <u>un</u>conditional demand equation (3.6). In other words, Frisch elasticities are invariant to the level of aggregation. Note also that although equation (4.5) is based on the assumption of block-independent preferences (or strong separability), it also holds under the weaker condition of blockwise dependence (weak separability). For details of these and other matters dealt with in this section, see Theil (1975/76).

5. ALTERNATIVE FUNCTIONAL FORMS

In the previous sections we used log-linear (or double-log) demand equations. In this section we provide a brief overview of four other popular functional forms.

The linear expenditure system (LES) has been widely used in demand analysis, commencing with Stone (1954). The i^{th} equation of this model states that expenditure on commodity i is a linear function of the n prices and income:

$$(5.1) \qquad p_i q_i = p_i \gamma_i + \beta_i \left(M - \sum_{j=1}^{n} p_j \gamma_j \right),$$

where $\beta_i > 0$ and $\gamma_i < q_i$ are constants. If the coefficients γ_i are nonnegative, then LES has the following interpretation: The consumer initially buys γ_i units of commodity i spending $p_i \gamma_i$ dollars; this is called "subsistence consumption" of that commodity. Total subsistence consumption of all goods costs $\sum_{j=1}^{n} p_j \gamma_j$, which leaves $M - \sum_{j=1}^{n} p_j \gamma_j$ as "supernumerary income". Out of this supernumerary income, a fraction β_i is spent on commodity i.

The LES is derived from the Stone-Geary utility function $\sum_{i=1}^{n} \beta_i \log(q_i - \gamma_i)$. As this is of the preference independence form, discussed in Section 3, LES rules out inferior goods, as well as specific interactions between goods. But as argued in Section 3, such restrictions may not be as unreasonable as was once thought. The major attractions of LES are its linearity, transparency and economy of parameterisation. These features, together with the availability of LES estimates for a substantial number of countries in the book by Lluch et al. (1977), mean that it is probably the dominant model used for consumer demand in CGE models of developing countries (Robinson, 1989) and other applications.

A problem with LES relates to the movements in the income elasticities as income changes (Theil, 1983). In general, the i^{th} income elasticity (η_i) is the ratio of the marginal share (θ_i) to the corresponding budget share (w_i). Under LES, $\theta_i = \beta_i$, a constant, so that $\eta_i = \beta_i / w_i$. A rise in income with prices held constant causes the budget shares of necessities to fall and those of lúxuries to rise. It then follows that LES implies that increasing affluence causes the income elasticities of necessities to rise, while those luxuries fall. Take the case of food, a necessity. Under LES, as the consumer becomes richer, the income elasticity of food increases, causing food to become less of a necessity or more of a luxury. This behaviour of the income elasticity is implausible as food should be less of a luxury for a richer consumer.

Another popular functional form is the translog demand system due to Christensen et al. (1975). The i^{th} equation of this model has the budget share on the left-hand side:

(5.2)
$$w_i = \frac{\beta_i + \sum_{j=1}^{n} \beta_{ij} \log\left(\frac{p_j}{M}\right)}{\sum_{k=1}^{n} \beta_k + \sum_{k=1}^{n} \sum_{j=1}^{n} \beta_{kj} \log\left(\frac{p_j}{M}\right)},$$

where the β_i and β_{ij} are constants. This model is derived from a log-quadratic indirect utility function. The translog is less restrictive than LES as it obviously involves many more parameters. A major drawback of the translog is, however, that these parameters lack simple behavioural interpretations; the elasticities implied by (5.2), for example, are highly complex functions of the parameters.

Deaton and Muellbauer (1980b) suggested another flexible demand system, the almost ideal demand system (AIDS). The i^{th} equation of AIDS is

(5.3)
$$w_i = \alpha_i + \beta_i \log\left(\frac{M}{P}\right) + \sum_{j=1}^{n} \gamma_{ij} \log p_j,$$

where $\log P = \Sigma_{k=1}^{n} \alpha_k \log p_k + (1/2)\Sigma_{k=1}^{n} \Sigma_{j=1}^{n} \gamma^*_{kj} \log p_k \log p_j$ and $\gamma_{ij} = (1/2)\left(\gamma^*_{ij} + \gamma^*_{ji}\right)$, with the α_k, γ^*_{kj} and β_k all constants. This model is based on the consumer's cost function and expresses the budget share of commodity i as a linear function of the logarithm of real total expenditure and the logarithmic prices. The income and price elasticities implied by (5.3) are $\eta_i = 1 + \beta_i / w_i$ and $\eta'_{ij} = -\delta_{ij} + (1/w_i)[\gamma_{ij} + \beta_i \beta_j \log(M/P)] + w_j$, where δ_{ij} is the Kronecker delta.

We return to equation (3.6), the double-log demand equation for good $i \in S_g$ under block independence. The substitution term in this equation involves the summation over prices of goods belonging to the same group as good i, viz., S_g. If we now suppose that the n goods form only one block in the utility function, then preferences are unrestricted and this summation is over all n goods,

$$\log q_i = \alpha_i + \eta_i \log Q + \sum_{j=1}^{n} \eta''_{ij}\left(\log p_j - \log P'\right).$$

We take infinitesimal changes of all variables in this equation and then multiply both sides by w_i to yield

$$(5.4) \qquad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \nu_{ij}\left[d\left(\log p_j\right) - d(\log P')\right],$$

where $\theta_i = w_i \eta_i$ is the marginal share of good i and $\nu_{ij} = w_i \eta''_{ij}$ is the $(i, j)^{th}$ price coefficient. The above is the i^{th} equation of the <u>relative price version of Theil's (1980a) differential demand system</u>. The variable on the left-hand side of (5.4) has the dual interpretation as the quantity component of the change in the budget share of good i and the contribution of i to the Divisia volume index $d(\log Q)$. Note that the "coefficients" of (5.4), θ_i and ν_{ij}, are not necessarily constants.

We define the Slutsky coefficient as $\pi_{ij} = \nu_{ij} - \phi\,\theta_i\,\theta_j$, where ϕ is the income flexibility. Using $d(\log P') = \sum_{i=1}^{n} \theta_i d(\log p_i)$ in equation (5.4), as well as (3.7) in the form $\sum_{j=1}^{n} \nu_{ij} = \phi\,\theta_i$, we then have

$$(5.5) \qquad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij} d(\log p_j),$$

which is the <u>absolute price version of the differential demand equation for good i</u>.

The demand equations (5.4) and (5.5) are in terms of infinitesimal changes. The <u>Rotterdam model</u>, due to Barten (1964) and Theil (1965), is a finite-change version of those equations. We write $Dx_t = \log x_t - \log x_{t-1}$ for the finite log-change in a variable x from period t-1 to t and $\overline{w}_{it}$ for the arithmetic average of the budget share, $\overline{w}_{it} = (1/2)(w_{it} + w_{i,t-1})$. The finite-change version of equation (5.4) is then

$$(5.6) \qquad \overline{w}_{it}\, Dq_{it} = \theta_i\, DQ_t + \sum_{j=1}^{n} \nu_{ij}\left(Dp_{jt} - DP'_t\right),$$

where $DQ_t = \sum_{i=1}^{n} \overline{w}_{it}\, Dq_{it}$ and $DP'_t = \sum_{i=1}^{n} \theta_i\, Dp_{it}$. When the coefficients θ_i and ν_{ij} of (5.6) are treated as constants, it is known as the i^{th} demand equation of the <u>relative</u>

price version of the Rotterdam model. Under the same parameterisation, the finite-change version of equation (5.5) is

$$\overline{w}_{it}\, Dq_{it} = \theta_i\, DQ_t + \sum_{j=1}^{n} \pi_{ij}\, Dp_{jt}\,, \tag{5.7}$$

which is the ith demand equation of the absolute price version of Rotterdam model.

Recalling that the Frisch index DP'_t involves the unknown marginal shares, equation (5.6) is nonlinear in the parameters, whereas (5.7) is linear. For small values of n, the absolute price version is suitable for estimation but when n becomes large, it is cumbersome. For large n, it is better to use the relative price version and impose suitable restrictions on the ν_{ij} according to notions of separability. A weakness of the Rotterdam model is that both versions have constant marginal shares, a defect which is shared with LES as discussed above. It has also been argued that another weakness is that the assumption of constant coefficients implies that the model is consistent only with Cobb-Douglas utility. This criticism is originally due to McFadden (1964), but as indicated in the next section, more recent research has now established that the model holds under much weaker conditions.

The attraction of AIDS is that it gives an arbitrary first-order approximation to any demand system; satisfies the axioms of choice (almost) exactly; aggregates perfectly without invoking the assumption of parallel linear Engel curves; and has a functional form which is consistent with known household budget data. It is well-known that although many of the desirable properties of AIDS are possessed by one or other of the Rotterdam or translog models, neither posses all of them simultaneously. The AIDS model in its general form is non-linear. In practice, however, by a suitable approximation to the price index, it is made linear. As for the Rotterdam and translog models, AIDS can also be used to test the restrictions of homogeneity and symmetry. As Barnett (1984, p. 285) says

> "The question therefore naturally arises as to just what has been gained by the widespread adoption of flexible functional forms, at the expense of linearity, easy estimability, informative parameterisation, and well-behaved error structure that has long been available from the older Rotterdam model. If 'flexibility' is the answer, then only Monte Carlo studies could confirm the existence of that gain. However, it would be surprising if the Rotterdam model were to be found to be consistently 'less' flexible than, say, the currently fashionable translog model."

As each model has its strengths and weaknesses, one cannot be dogmatic in the choice of functional form. Pollak and Wales (1992, p. 23) offer the following pragmatic perspective:

> "...... we do not believe that there is a single 'one-size-fits-all' functional form that is ideal for all applications. Instead, we believe that the

characteristics that make a particular functional form suitable for one application may well make it inappropriate for another. For example, household budget data typically present the investigator with wide variation in observed levels of total expenditure but limited price variation. Time series data, on the other hand, typically offer less variation in expenditure and more variation in relative prices. Thus, it is not surprising that the parametric forms best suited for analyzing household budget data differ from those best suited for analysing per capita time series data."

6. AGGREGATION OVER CONSUMERS

The demand equations discussed in the previous sections are of micro nature as they are based on the utility-maximising behaviour of the individual consumer. As data in economics are usually available only in aggregate form (e.g., per capita or per adult), it is natural to ask, to what extent do the properties of the micro demand equations carry over to the aggregate (macro) or market demand functions?

It can be easily shown that under certain conditions, the micro demand equations, LES given by (5.1), the translog (5.2) and AIDS (5.3), can be aggregated into analogous macro forms; see, respectively, Theil (1975/76), Jorgensen et al. (1982) and Deaton and Muellbauer (1980b). The aggregation of the differential demand equations (5.4) and (5.5) is more complex. The aggregation issues of the Rotterdam (differential) demand system were considered by Barnett (1979, 1981), E. A. Selvanathan (1991b) and Theil (1971, 1975/76) using the convergence approach. As this is not well understood by many analysts, in this section we briefly outline the convergence approach.

Let us write the micro demand equation (2.1) for the c^{th} consumer as

$$(6.1) \qquad q_{ic} = q_{ic}(M_c, \mathbf{p}), \qquad i=1,\ldots,n, \quad c=1,\ldots N,$$

where we make the assumption that each consumer faces the same price vector **p**; and where N is the total number of consumers. For commodity i, if we sum (6.1) over c=1,...,N and divide both sides by N, we obtain

$$(6.2) \qquad \bar{q}_i = \frac{1}{N}\sum_{c=1}^{N} q_{ic}(M_c, \mathbf{p}) = f_i(M_1,\ldots,M_N,\mathbf{p}), \qquad i = 1, \ldots, n,$$

for some functions f_i. In (6.2), $\bar{q}_i$ is average consumption of commodity i per consumer, or per capita consumption of i for short. If exact aggregation is possible then for some functions g_i, i = 1, ..., n, we can write (6.2) in the form

$$(6.3) \qquad \bar{q}_i = g_i(\bar{M}, \mathbf{p}), \qquad i=1,\ldots,n,$$

where $\overline{M}$ is per capita income. Equation (6.3) is the per capita analogue, or the macro analogue, of the micro equation (6.1).

The problem is that the functions g_i do not, in general, exist. Let us consider (6.1) in linear form,

$$(6.4) \qquad q_{ic} = \alpha_{ic} + \beta_{ic} M_c, \qquad i=1,...n, \quad c=1,...,N,$$

where α_{ic} is a constant term; M_c is the income of the c^{th} consumer; and β_{ic} is the income parameter. For simplicity, prices have been omitted from (6.4). If we sum both sides of the micro relation (6.4) over c = 1, ..., N and divide by N, we obtain the macro relation,

$$\overline{q}_i = \overline{\alpha}_i + \frac{1}{N}\sum_{c=1}^{N} \beta_{ic} M_c,$$

or equivalently,

$$(6.5) \qquad \overline{q}_i = \overline{\alpha}_i + \frac{\sum_{c=1}^{N} \beta_{ic} M_c}{\sum_{c=1}^{N} M_c} \overline{M},$$

where $\overline{\alpha}_i = (1/N)\sum_{c=1}^{N} \alpha_{ic}$ and we have assumed that $\sum_{c=1}^{N} M_c \neq 0$. Generally, equation (6.5) cannot be considered as a linear macro relation in $\overline{q}_i$ and $\overline{M}$. To consider (6.5) as an approximate linear macro relation, we apply the Theil's (1971, 1975/76) convergence approach to aggregation in the following manner.

Let us assume that the N consumers are independent random elements of an infinite consumer population and that the coefficients $\beta_{i1}, \ldots, \beta_{iN}$ are identically independently distributed and come from a probability distribution having means β_i and finite variance σ^2. We shall also initially assume that $M_1, \ldots, M_N$ are nonstochastic. Then the coefficient of $\overline{M}$ in (6.5) is a random variable with expectation

$$E\left[\frac{\sum_{c=1}^{N} \beta_{ic} M_c}{\sum_{c=1}^{N} M_c}\right] = \beta_i$$

and has the following variance

$$E\left[\frac{\sum_{c=1}^{N} \beta_{ic} M_c}{\sum_{c=1}^{N} M_c} - \beta_i\right]^2 = \sigma^2 \frac{\sum_{c=1}^{N} M_c^2}{\left[\sum_{c=1}^{N} M_c\right]^2},$$

where we have used the fact the β_{ic} are identically independently distributed. As

$\overline{M} = (1/N)\sum_{c=1}^{N} M_c$ and $\sum_{c=1}^{N} (M_c - \overline{M})^2 = \sum_{c=1}^{N} M_c^2 - N\,\overline{M}^2$, the above variance can be written as

$$E\left[\frac{\sum_{c=1}^{N} \beta_{ic} M_c}{\sum_{c=1}^{N} M_c} - \beta_i\right]^2 = \frac{\sigma^2}{N}\left[\frac{\frac{1}{N}\sum_{c=1}^{N} (M_c - \overline{M})^2}{\overline{M}^2}\right].$$

When $(1/N)\sum_{c=1}^{N} (M_c - \overline{M})^2 < \infty$, we have

$$\lim_{N\to\infty} E\left[\frac{\sum_{c=1}^{N} \beta_{ic} M_c}{\sum_{c=1}^{N} M_c} - \beta_i\right]^2 \to 0.$$

Using Chebyshev's inequality, we conclude that

$$\frac{\sum_{c=1}^{N} \beta_{ic} M_c}{\sum_{c=1}^{N} M_c} \text{ converges in probability to } \beta_i.$$

Therefore, for sufficiently large N, we can write equation (6.5) as

$$\text{(6.6)} \qquad \overline{q}_i \approx \overline{\alpha}_i + \beta_i \overline{M},$$

which is a linear macro relation in per capita variables $\overline{q}_i$ and $\overline{M}$ with constant coefficients.

Next, consider the case when $M_1, \ldots, M_N$ are stochastic and redefine

$$\beta_i = \frac{E[\beta_{ic} M_c]}{E[M_c]}. \tag{6.7}$$

We assume that (i) M_c, c = 1, ..., N, are identically independently distributed; (ii) $\beta_{ic} M_c$, c = 1, ..., N, are also identically independently distributed at each point in time; and (iii) $E[M_c]$ and $E[\beta_{ic} M_c]$ exist and are finite. Applying Khinchine's theorem,

$$\overline{M} = E[M_c] + o_p(1),$$

where $o_p(1)$ denotes a random variable that converges in probability to zero as N tends to infinity. Assuming that $E[M_c] > 0$, applying Slutsky's theorem, we get

$$\frac{1}{M} = \frac{1}{E[M_c]} + o_p(1). \tag{6.8}$$

As $\beta_{ic} M_c$, c = 1, ..., N, are identically independently distributed, using Khinchine's theorem again, we obtain

(6.9) $$\frac{1}{N}\sum_{c=1}^{N} \beta_{ic} M_c = E\left[\beta_{ic} M_c\right] + o_p(1).$$

Therefore from equations (6.7), (6.8) and (6.9), we have

$$\frac{\frac{1}{N}\sum_{c=1}^{N} \beta_{ic} M_c}{\frac{1}{N}\sum_{c=1}^{N} M_c} = \frac{E\left[\beta_{ic} M_c\right]}{E[M_c]} + o_p(1) = \beta_i + o_p(1).$$

Now taking the limit as N tends to infinity of both sides of the above equation, we get

$$\lim_{N\to\infty}\left[\frac{\frac{1}{N}\sum_{c=1}^{N} \beta_{ic} M_c}{\frac{1}{N}\sum_{c=1}^{N} M_c}\right] = \beta_i,$$

where we have used $\text{plim}_{N\to\infty}\, o_p(1) = 0$. Thus for sufficiently large N, (6.5) becomes equivalent to (6.6) even when income is random.

Using the above convergence approach, under fairly strong assumptions about the macro parameters and variables, Theil (1971, 1975/76) shows that (5.4), the micro version of the differential demand equation in relative prices, can be aggregated into the corresponding macro form. Under weaker assumptions, Barnett (1979, 1981) aggregates (5.5), the demand equation in absolute prices, into the corresponding macro equation.

E. A. Selvanathan (1991b) extends Theil's work by deriving the macro form of the relative price version under the weaker assumptions of Barnett (1979,1981).

7. THE ROLE OF OTHER VARIABLES: ADVERTISING

In this section we consider the role of variables other than income and prices. Although we focus on the role of advertising, the analysis is applicable to other shift variables in demand functions. This section is based mostly on E. A. Selvanathan (1989, 1995).

Let a_i be the advertising of good i (i = 1, ..., n) and $\mathbf{a} = [\, a_i \,]$ be the corresponding vector. We assume that a_i is outside the control of the consumer, like p_i and M. Following Barten (1977), we postulate that the consumer's preferences can be described by means of a utility function of the form $u = u(\mathbf{q}, \mathbf{a})$. By setting up the usual utility-maximisation problem, we can derive a system of demand equations of the form $\mathbf{q} = \mathbf{q}(M, \mathbf{p}, \mathbf{a})$ or

$$(7.1) \qquad q_i = q_i(M, p_1, ..., p_n, a_1, ..., a_n), \qquad i = 1, ..., n.$$

This is an extended version of the demand system (2.1) for i = 1, ..., n.

Let μ_{ij} be the elasticity of the marginal utility of good i with respect to advertising of good j,

$$\mu_{ij} = \frac{\partial\left(\log \frac{\partial u}{\partial q_i}\right)}{\partial\left(\log a_j\right)}.$$

We can show that the elasticity of the quantity demanded of good i with respect to the advertising of j, τ_{ij}, is related to the price elasticities in the following manner: $[\tau_{ij}] = \partial(\log \mathbf{q})/\partial(\log \mathbf{a}') = -\eta'\mu$ or

$$(7.2) \qquad \tau_{ij} = \frac{\partial\left(\log q_i\right)}{\partial\left(\log a_j\right)} = -\sum_{k=1}^{n} \eta'_{ik}\, \mu_{kj}, \qquad i, j = 1, \ldots, n,$$

where $\eta' = [\eta'_{ij}]$ is the matrix of compensated price elasticities. Equation (7.2) is a simple result showing the link between the effects of advertising and the substitution effects. We now analyse the implications of two special cases of the matrix $[\mu_{ij}]$.

The First Special Case

Consider the special case when the elasticity of the marginal utility of each good with respect to its own advertising is unity and all the cross elasticities are zero. That is,

$$\mu_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise,} \end{cases}$$

so that $\mu = \mathbf{I}$, the $n \times n$ identity matrix. In this situation (7.2) implies

$$\tau_{ij} = -\eta'_{ij}, \qquad i, j = 1, ..., n.$$

In words, the elasticity of consumption of i with respect to advertising of j is the negative of the corresponding (compensated) price elasticity. This means that $\tau_{ii} > 0$ for $i = 1, ..., n$; $\tau_{ij} < 0$ if i and j are substitutes; and $\tau_{ij} > 0$ if i and j are complements. These results make sense since it is reasonable to expect that advertising will depress the sales of products which are substitutes for the good in question; and vice versa for complements.

The Second Special Case

Next, consider the slightly more general case,

$$\mu_{ij} = \begin{cases} \mu & \text{if } i = j \\ 0 & \text{otherwise,} \end{cases}$$

where μ is a scalar, so that $\boldsymbol{\mu} = \mu \mathbf{I}$. This kind of restriction has been considered by Theil (1980b). Here again all cross-advertising elasticities of the marginal utilities are zero, but now the own elasticities are non-unity. In this situation, the advertising elasticities of consumption are a constant multiple $-\mu$ of the corresponding price elasticities.

Demand Equations in Relative Prices

Equation (5.4) is the i^{th} equation of the relative price version of the differential demand system. By taking the differential of (7.1), E. A. Selvanathan (1989) extends (5.4) to incorporate advertising to yield

$$(7.3) \qquad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} v_{ij}\left[d\left(\log \frac{p_j}{P'}\right) - d\left(\log \frac{\tilde{a}_j}{\tilde{A}'}\right)\right],$$

where $d(\log p_j/P') = d(\log p_j) - d(\log P')$; and $d(\log \tilde{a}_j/\tilde{A}') = d(\log \tilde{a}_j) - d(\log \tilde{A}')$ is the change in relative advertising, with $d(\log \tilde{a}_j) = \sum_{k=1}^{n} \mu_{jk} d(\log a_k)$ and $d(\log \tilde{A}') = \sum_{j=1}^{n} \theta_j d(\log \tilde{a}_j)$. Equation (7.3) shows that the demand for good i depends on real income, the relative price and relative advertising of each good. When $\boldsymbol{\mu} = \mathbf{I}$, $d(\log \tilde{a}_j) = d(\log a_j)$ and $d(\log \tilde{A}') = d(\log A') = \sum_{j=1}^{n} \theta_j d(\log a_j)$. Consequently, in this case the j^{th} relative advertising term in equation (7.3) simplifies to $d(\log a_j / A')$, so that the last term in that equation becomes

$$\sum_{j=1}^{n} v_{ij}\left[d\left(\log \frac{p_j}{P'}\right) - d\left(\log \frac{a_j}{A'}\right)\right].$$

The j^{th} term in this sum involves only the relative price and advertising of good j. This is not true in equation (7.3) as in general $d(\log \tilde{a}_j)$ involves the advertising of all goods.

We can easily show that when $\mu = \mu\, \mathbf{I}$, $d(\log \tilde{a}_j) = \mu\, d(\log a_j)$, and $d(\log \tilde{A}') = \mu\, d(\log A')$. Hence the relative advertising index $d(\log \tilde{a}_j / \tilde{A}')$ becomes a multiple μ of $d(\log a_j / A')$. Consequently, the last term in equation (7.3) simplifies to

$$\sum_{j=1}^{n} v_{ij}\left[d\left(\log \frac{p_j}{P'}\right) - \mu\, d\left(\log \frac{a_j}{A'}\right)\right].$$

Again the j^{th} term in this sum involves only the j^{th} relative price and advertising.

Demand Equations in Absolute Prices

Equation (7.3) is formulated in terms of relative (or deflated) prices and advertising. In absolute (or undeflated) terms, this equation takes the form

$$(7.4) \qquad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij} d(\log p_j) + \sum_{j=1}^{n} \lambda_{ij} d(\log a_j),$$

where $\lambda_{ij} = w_i \tau_{ij}$ is the $(i,j)^{th}$ advertising coefficient satisfying $\sum_{j=1}^{n} \lambda_{ij} = 0$, $i = 1, \ldots, n$, implying that a proportionate change in all the advertising variables does not affect the demand for any good, income and prices remaining unchanged. The above is the extended version of equation (5.5).

When the advertising elasticity of the i^{th} marginal utility $\mu_{ij} = \delta_{ij}$ (the Kronecker delta), we have $\tau_{ij} = -\eta'_{ij}$. Therefore, $\lambda_{ij} = -w_i \eta'_{ij} = -\pi_{ij}$ in this case. In view of this result, demand equation (7.4) becomes

$$w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij} \left[d(\log p_j) - d(\log a_j)\right].$$

Here advertising acts as the deflator of each good's price change. When $\mu_{ij} = \mu\, \delta_{ij}$, we have $\tau_{ij} = -\mu\, \eta'_{ij}$. This implies that $\lambda_{ij} = -\mu\, w_i\, \eta'_{ij} = -\mu\, \pi_{ij}$, so that (7.4) becomes

$$w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij} \left[d(\log p_j) - \mu\, d(\log a_j)\right].$$

An Example

E. A. Selvanathan (1995) applies a conditional version of (7.4) to beer, wine and spirits consumption in the UK under the assumptions of (i) block independence and (ii) a scalar μ matrix (the second special case above). Assumption (ii) is tested and not rejected by the data. The estimate of μ is .52, which indicates that when advertising of a beverage is increased by ten percent, the marginal utility of that beverage increases by 5.2 percent.

Table 7.1 gives the results in elasticity form. The income elasticities are .5, 1.9, 1.7 for beer, wine and spirits, respectively. These estimates indicate that, within alcohol, beer is a necessity whereas wine and spirits are luxuries. The own-price elasticities are $-.2$, $-.3$ and $-.1$ (in the same order). Columns 6-8 of the table present the advertising elasticities. As can be seen from the first row, the own-advertising elasticity of beer is .09, indicating that a 10 percent increase in the advertising of beer raises beer sales by .9 percent. The elasticity of beer consumption with respect to advertising of wine is $-.05$, which implies that a 10 percent rise in wine advertising depresses beer sales by .5 percent, so that beer and wine are competitive, as expected. The other advertising elasticities are interpreted in the same way. Note that the largest elasticity is wine with respect to beer advertising ($-.22$). Note also that the row sums of three advertising elasticities are zero. This means that an equiproportional increase in the advertising of all beverages has a cancelling effect [see the discussion below equation (7.4)].

For related research on advertising and alcohol consumption, see Brown and Lee (1993), Duffy (1987, 1990, 1991a, 1991b) and Nelson and Moran (1994). We shall have something further to say about alcohol demand in the next section.

TABLE 7.1

CONDITIONAL DEMAND ELASTICITIES FOR ALCOHOLIC BEVERAGES: UK

Beverage	Conditional income elasticity	Conditional compensated price elasticities			Conditional advertising elasticities		
		Beer	Wine	Spirits	Beer	Wine	Spirits
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Beer	.45	-.17	.09	.08	.09	-.05	-.04
Wine	1.90	.41	-.28	-.13	- .22	.15	.07
Spirits	1.72	.16	-.06	-.10	-.09	.03	.05

Source: E. A. Selvanathan (1995).

8. USEFUL EMPIRICAL REGULARITIES

Applied economists need demand elasticities for trade practices issues, tax analysis, the construction of CGE models and so on. In this section we present several topics that could be useful in providing guidance for the values of these elasticities, as well as some related issues.

Constructing Price Elasticities from Income Elasticities

It is usually the case that income elasticities are more readily available than price elasticities. Income elasticities can come from cross-sectional analysis of expenditure surveys, or from simple judgements about the luxuriousness of goods.

Under the assumption of preference independence, as discussed in Section 3 all own- and cross-price elasticities can be derived from information regarding the budget shares, income elasticities and the value of one parameter, the income flexibility. The expression for the $(i, j)^{th}$ price elasticity is given by equation (3.2), which we repeat here for convenience:

$$\eta'_{ij} = \phi\, \eta_i \left(\delta_{ij} - w_j\, \eta_j\right), \tag{8.1}$$

where η'_{ij} is the $(i, j)^{th}$ compensated price elasticity; ϕ is the income flexibility; δ_{ij} is the Kronecker delta; w_j is the budget share of j; and η_j is the j^{th} income elasticity. According to (8.1), the n^2 price elasticities, η'_{ij}, $i, j = 1, \ldots, n$, can be constructed from a value of ϕ, the n income elasticities $\eta_1, \ldots, \eta_n$ and the n budget shares $w_1, \ldots, w_n$.

If the income elasticities and budget shares are available, we are still faced with the problem of what value of ϕ to use in equation (8.1). Frisch (1959) speculated that ϕ

would increase (in absolute value) with real income, but the available evidence does not indicate strong support for this income dependence. Clements and S. Selvanathan (1994) present a review of estimates of the income flexibility and recommend treating it as a constant with a centre of gravity value of $\phi = -1/2$.

We write equation (8.1) as $\eta'_{ij} = \phi\, \eta_i\, \delta_{ij} - \phi\, \eta_i\, w_j\, \eta_j$. As both ϕ and w_j are fractions and as η_j is on average unity, the second term on the right-hand side of this equation, $\phi\, \eta_i\, w_j\, \eta_j$, is of second order. Thus, we have the approximation $\eta'_{ij} \approx \phi\, \eta_i\, \delta_{ij}$, which together with $\phi = -1/2$, implies $\eta'_{ii} \approx -(1/2)\, \eta_i$, or that <u>the own-price elasticity is approximately equal to the corresponding income elasticity divided by minus two</u>. This rule of thumb is based on (i) the assumption of preference independence and (ii) the value of the income flexibility $\phi = -1/2$. As argued in Section 3, preference independence is likely to be reasonable, at least when applied to the broad aggregates, so that this rule of thumb is likely to be useful when there is no other information available.

<u>Alcohol Demand</u>

The demand for alcoholic beverages is of interest to economists for several reasons. At the individual level, alcohol consumption is subject to great idiosyncratic behaviour -- some people are addicted, some simply drink a lot, while others abstain completely. There is thus the intellectual challenge to determine whether this sort of behaviour is amenable to conventional economic analysis. Another reason for interest revolves around the public finance aspects of alcohol such as externalities (both positive and negative) and the appropriate levels of taxation. A recent example of this is the controversy generated by the Scales Report (Scales et al., 1995) into wine taxation in Australia. Finally, as in many cases data on alcohol consumption stem from taxation records, the quality of the data is above average. It is for these reasons that the alcoholic beverages group is perhaps the most studied of all commodity groups, especially in the last 10 years. It is also worth noting that based on our experience, the economics of alcohol consumption makes lively teaching material. In this sub-section, we set out some key results on alcohol demand which emerge from a number of countries.

Under the assumption of block independence, the composite demand equation for the group S_g is given by equation (4.4). Using Australian alcohol data, Clements and S. Selvanathan (1991) estimate the group income elasticity in this equation, η_g, to be close to unity. If we set $\eta_g = 1$, equation (4.4) can then be written as

$$\log Q_g - \log Q = \alpha_g + \phi\left(\log P'_g - \log P'\right),$$

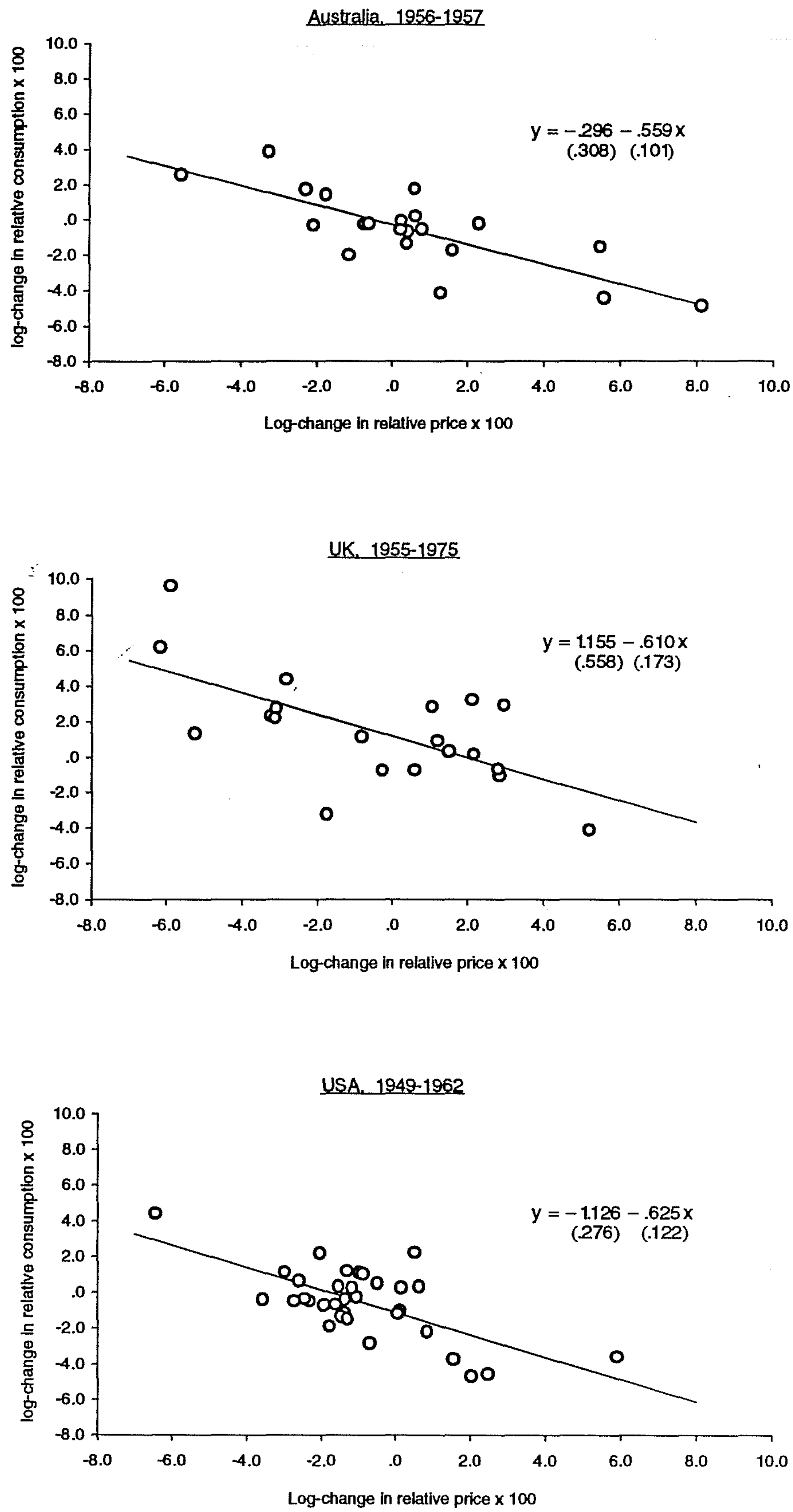
where α_g is an intercept and ϕ is simultaneously the own-price elasticity of demand for S_g and the income flexibility. E. A. Selvanathan and Clements (1988) use this equation in terms of changes to plot the growth in consumption of alcohol as a whole relative to income, $\Delta \log Q_g - \Delta \log Q$, against the change in its relative price, $\Delta \log P'_g - \Delta \log P'$, after replacing the Frisch price indexes (which involve unknown marginal shares) with their Divisia counterparts (which involve known budget shares); see also Clements and S. Selvanathan (1991).

Figure 8.1 gives the plots for Australia, the UK and the USA. As can be seen, the three solid lines (the LS regression lines) are more or less parallel with a slope of about $-1/2$. Accordingly, the price elasticity of alcohol as a whole is about $-1/2$, a result which is confirmed with more formal methods (see E. A. Selvanathan and Clements, 1988). We speculate that price elasticity of currently illegal drugs would also be of the order of $-1/2$.

E. A. Selvanathan (1991a) estimates conditional demand equations for beer, wine and spirits in a number of countries. One of his findings is that the three alcoholic beverages satisfy the homogeneity and symmetry restrictions, suggesting that drinkers are rational in their beverage choice. Selvanathan's conditional income elasticities are given in Table 8.1. Although there is a good deal of dispersion across countries, it is still the case that the elasticities show a distinct pattern: Except for Japan, in all countries the conditional income elasticity for beer is less than unity, while that for spirits is greater than unity. Accordingly, beer is a necessity and spirits a luxury. On the other hand, however, there seems to be no particular pattern among the wine income elasticities; and the same is true for the price elasticities (see E. A. Selvanathan, 1991a, for details).

FIGURE 8.1

CONSUMPTION OF ALCOHOL AGAINST RELATIVE PRICE



Source: E. A. Selvanathan and Clements (1988).

TABLE 8.1

CONDITIONAL INCOME ELASTICITIES FOR ALCOHOLIC BEVERAGES

Country	Sample period	Conditional income elasticities		
		Beer	Wine	Spirits
1. Australia	1955-85	.8	.7	1.9
2. Canada	1953-82	.7	1.0	1.3
3. Finland	1969-83	.4	1.6	1.3
4. Japan	1964-83	1.4	.3	.5
5. New Zealand	1965-82	.9	1.1	1.2
6. Norway	1960-86	.3	1.4	1.6
7. Sweden	1960-86	.2	.5	1.5
8. UK	1955-85	.5	1.3	1.8
9. US	1949-82	.7	.6	1.4

Source: E. A. Selvanathan (1991a).

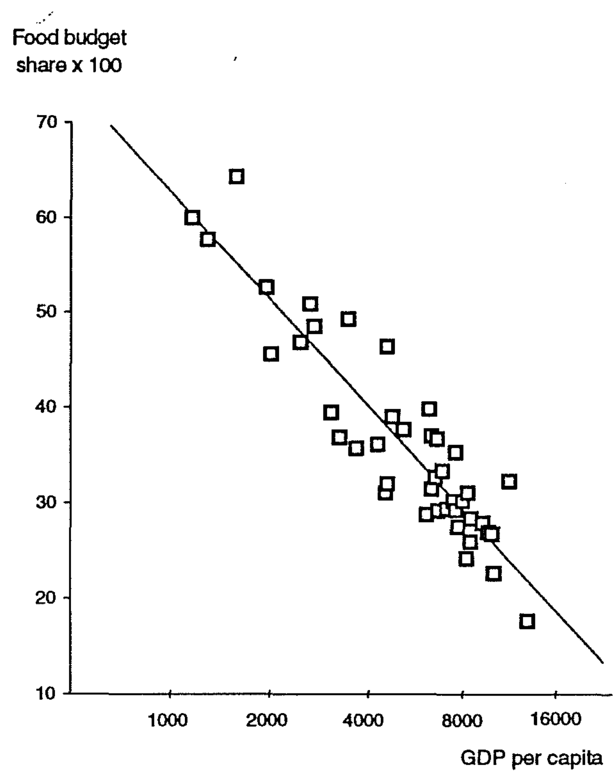
The final empirical regularity pertaining to alcohol consumption is the finding of some complementarity among beverages (see, e.g., Clements and S. Selvanathan, 1991). This can be understood in terms of the formal-dinner model in which all three beverages are consumed sequentially, so that one beverage reinforces the utility of the others, rather than being competitive. The BBQ model (which John Freebairn attributes to the Tasman Institute) also gives rise to the same prediction of complementarity.

The Strong Version of Engel's Law

Figure 8.2, from Chen (1993), is a scatter plot for 42 countries of the food budget share (w) against income (M) measured in the form of GDP per capita. The huge variability of the data should be noted -- the poorest countries spend about 60 percent of income on food, while this falls to below 20 percent for the richest (the USA). This decline in the food share is just Engel's law in its common form, but here we are concerned with the precise nature of the decline.

FIGURE 8.2

FOOD BUDGET SHARES AND GDP IN 42 COUNTRIES



Bearing in mind that the horizontal axis of Figure 8.2 is logarithmic, the data strongly suggest that the food budget share falls arithmetically as income grows geometrically. Note that the regression line accounts for 84 percent of the variability of the budget shares, which is impressively high in view of the great differences among the 42 countries. The slope of the LS regression line in the figure is –.15. This value is remarkably consistent with other cross-sectional studies (see Chen, 1993, and Theil et al., 1989, for reviews), a result which leads us to treat the value of this slope as something approaching a natural constant.

The model underlying Figure 8.2 is Working's (1943),

$$w = \alpha + \beta \log M, \tag{8.2}$$

where α and β are constants. To appreciate the implications of this model, consider moving from one country to another which is twice as affluent as the first, so that incomes are M and 2M. Then according to model (8.2), the change in the food budget share is $\Delta w = \beta \Delta \log M$, or, when $\beta = -.15$ and $\Delta \log M = \log 2 = .69$,

$$\Delta w = -.15 \times .69 = -.10.$$

Thus, <u>a doubling of income leads to the food budget share declining by 10 percentage points</u>. Theil et al. (1989) call this <u>the strong version of Engel's law</u>.

This law could be used as a short cut in making real-income comparisons. If, for example, the food budget share for some country (or group of countries) were 10 percentage points less than that of another, we could then conclude that, prima facie, the former is twice as affluent as the latter. When the compensated own-price elasticity of demand for food is approximately constant, a corollary of the above law is that a doubling of income leads to the uncompensated own-price elasticity falling in absolute value by .10 (Chen, 1993). This result could be useful when there is little information available about food elasticities.

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