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APPLIED GENERAL RELATIVITY

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I. INTRODUCTION

In this paper we discuss important relativistic effects and issues which must be considered in the interpretation of current measurements such as ranging measurements to LAGEOS and to the moon, in the implementation of the Global Positioning System, in the synchronization of clocks near the earth's surface, and in the adoption of appropriate scales of time and length for the communication of scientific results.

II. FEATURES OF GENERAL RELATIVITY

We restrict our discussion to Einstein's General Theory of Relativity (GR). GR describes events in a four-dimensional, space-time manifold (with coordinates X^μ), in which the invariant interval ds between events separated by dX^μ is given in terms of the metric tensor $G_{\mu\nu}$ by:

$$-ds^2 = G_{\mu\nu} dX^\mu dX^\nu \qquad \text{(summed on repeated indices).} \tag{1}$$

The interval ds has the following interpretations. First, a test particle in free fall, such as an earth-orbiting satellite, follows a geodesic path, along which ds is an extremum. Thus, equations of motion of bodies in free fall can be derived from a knowledge of the metric tensor. Second, the proper time elapsed on a standard clock (such as an atomic clock) in free fall through the interval dX^μ will be given by $|ds|$. Third, a pulse of electromagnetic radiation will travel along a null geodesic, ds = 0; this is another way of stating the constancy of the speed of light.

Also, ds is a scalar quantity, invariant with respect to arbitrary transformations of coordinates. Much of the calculation entailed in working out the implications of GR involves finding coordinates in which observations can be readily interpreted. Thus, along with the knowledge of a metric tensor, in a particular coordinate system, goes a procedure for interpretation of the theory.

III. BARYCENTRIC COORDINATES

Except for small structure effects to be mentioned later, solar system bodies can be described by an approximate point mass metric derived from Einstein's field equations by Eddington and Clark (1938). The metric tensor can be written in the following interesting form, with G_{00} accurate to order (V^4) :

$$G_{00} = -1 + 2\sum_A \frac{M_A}{\left|\vec{X} - \vec{X}_A\right| \left\{1 - \hat{N}\cdot\vec{V}_A\right\}}\Bigg|_{\text{ret}} - 2\left(\sum_A \frac{M_A}{\left|\vec{X} - \vec{X}_A\right|}\right)^2$$

$$-2\sum_A \frac{M_A}{\left|\vec{X} - \vec{X}_A\right|} \sum_B{}' \frac{M_B}{\left|\vec{X} - \vec{X}_B\right|} + 3\sum_A \frac{M_A V_A^2}{\left|\vec{X} - \vec{X}_A\right|} \tag{2}$$

where the retarded time is $T' = T - \left|\vec{X} - \vec{X}_A(T')\right|/c$. Thus, in the radiation gauge used for the solution of the field equations by Eddington and Clark, a retarded Lienard-Wiechert potential (Jackson 1975) appears. The other components are:

$$G_{0i} = -4\sum_A \frac{M_A V_A^i}{\left|\vec{X} - \vec{X}_A\right|}; \quad G_{ij} = \delta_{ij}\left(1 + \sum_A \frac{M_A}{\left|\vec{X} - \vec{X}_A\right|}\right), \tag{3}$$

where $\vec{X}_A$, $\vec{V}_A$, and $\vec{A}_A$ are the position, velocity, and acceleration of the A^{th} mass, and a prime on a summation symbol means that terms which are indefinitely large are to be omitted. M_A is the Schwarzschild mass parameter of the A^{th} mass; velocities are measured in units of c. We make use of the abbreviation $R = \left|\vec{X}_A - \vec{X}_B\right|$ which is a function of X^0 because both $\vec{X}_A$ and $\vec{X}_B$ depend on X^0. We introduce the "normalized" or external part of the metric by defining the negative of the potential in the neighborhood of the earth's mass M_E due to external sources (the subscript E denotes quantities describing the earth):

$$U^{(e)} = \sum_A{}' \frac{M_A}{\left|\vec{X} - \vec{X}_A\right|} \equiv \sum{}' M_A/R_A; \quad U_e \equiv U^{(e)}\left(\vec{X} = \vec{X}_E\right). \tag{4}$$

Using retarded time, expanding the retarded potential gives three retardation corrections in addition to the static potential:

$$\sum_A \frac{M_A}{\left|\vec{X} - \vec{X}_A\right| \left\{1 - \hat{N}\cdot\vec{V}_A\right\}}\Bigg|_{\text{ret}} =$$

$$\sum_A \frac{M_A}{\left|\vec{X} - \vec{X}_A\right|} \left\{1 + \frac{1}{2}\left[V_A^2 - (\vec{X} - \vec{X}_A)\cdot\vec{A}_A - (\vec{V}_A \cdot \hat{N}_A)^2\right]\right\} \tag{5}$$

where the unit vector $\hat{N}_A$ is given by: $\hat{N}_A = \left(\vec{X} - \vec{X}_A\right) / \left|\vec{X} - \vec{X}_A\right|$.

One can see the origin of terms of interest arising from retardation of gravitational signals in this coordinate system. Observation of such terms would be of great interest as it would provide indirect evidence for the existence of gravitational waves.

The equations of motion have been derived in detail by Moyer (1981 a,b) and others and may be written:

$$A_A^k = -\sum_B{}' \frac{M_A X_{BA}^k}{R_{AB}^3}\left(1-4U_A+V_A^2-\sum_C{}'\frac{M_C}{R_{CB}}+2V_B^2-4\vec{V}_B\cdot\vec{V}_A\right.$$

$$\left.-\frac{3}{2}\frac{\left(\vec{X}_{AB}\cdot\vec{V}_B\right)^2}{R_{AB}^2}-\frac{1}{2}\vec{X}_{AB}\cdot\vec{A}_B\right)+4\sum_B{}'\frac{M_B\left(\vec{X}_{AB}\cdot\vec{V}_A\right)\left(V_A^k-V_B^k\right)}{R_{AB}^3} \tag{6}$$

$$-3\sum_B{}'\frac{M_B\left(\vec{X}_{AB}\cdot\vec{V}_B\right)\left(V_A^k-V_B^k\right)}{R_{AB}^3}+\frac{7}{2}\sum_B{}'\frac{M_B A_B^k}{R_{AB}},$$

where:

$$\vec{X}_{AB}=\vec{X}_A-\vec{X}_B\,;\quad U_A=U^{(e)}\left(\vec{X}=\vec{X}_A\right);\quad \hat{N}_{AB}=\vec{X}_{AB}/R_{AB}\,. \tag{7}$$

For the case of an earth-orbiting satellite such as LAGEOS, significant perturbations arise from the term with the coefficient of $-3/2$ (which arises from retardation in this picture). This particular term could give rise to accelerations as large as 4.1×10^{-8} meters/sec^2. The Newtonian solar tidal accelerations of LAGEOS are only 10^5 larger.

IV. LOCAL INERTIAL FRAME

It is natural to do numerical computations of planetary ephemerides in solar system barycentric coordinates, using the above equations of motion that include effects due to all solar system bodies and to post-Newtonian relativistic effects. However, most observations are made from the earth — a locally inertial, freely falling platform — and do not reveal the existence of such effects. The proper interpretation of barycentric coordinates in terms of measurable quantities is key.

Our approach is to construct a transformation of coordinates from barycentric to local inertial coordinates — normal Fermi coordinates (Manasse and Misner 1963; Ashby and Bertotti 1984, 1986). In these coordinates, apart from the gravitational effects due to the Earth's mass itself, one can demand that time be measured by a standard clock near the position where the observations are made. In contrast, the coordinate time variable in the Eddington-Clark metric is measured by a standard clock at rest at infinity. Also, one can demand that the spatial coordinates be measured by standard rods or, in other words, that lengths and times be related in such a way the electromagnetic signals propagate with the defined value of the speed of light, c = 299, 792, 458 meters/sec.

Referring to Figure 1, $X^\mu(s)$ specifies a world line G, a timelike solution of the geodesic equations of the mass M_E in the metric given by Eqs. (2 and 3), with infinite self-interaction terms omitted. Four mutually orthonormal vectors $\Lambda^\mu_{(\alpha)}$, $\alpha = 0,1,2,3$ are introduced which are carried parallel to themselves along G. The zero[th] member of the orthonormal tetrad is the tangent vector to the geodesic. Given a field point $P(X^\mu)$ near G, a spacelike geodesic S is constructed which passes through P and intersects G orthogonally at proper time $x^0 = s$ as measured on a standard clock falling along G. The tetrad forms the basis for coordinates in the (almost) locally inertial frame. The first stage of the construction is finding these basic vectors.

The second stage is calculation of the transformation equations themselves by means of:

$$X^\mu(P) = X^\mu(P_0) + \Lambda^\mu_{(i)} x^i - \frac{1}{2}\Gamma^\mu_{\alpha\beta}(P_0)\Lambda^\alpha_{(i)}\Lambda^\beta_{(j)} x^i x^j$$
$$-\frac{1}{6}\Gamma^\mu_{\alpha\beta,\gamma}(P_0)\Lambda^\alpha_{(i)}\Lambda^\beta_{(j)}\Lambda^\gamma_{(k)} x^i x^j x^k + O(x^4)\ . \tag{8}$$

Lower case letters x^i are used to denote coordinates in the local frame. The coefficients of x^i in Eq. (8) are evaluated at P_0 and are functions of s (or x^0) only. The coefficients of the third and higher order terms in Eq. (8) are obtained from the equation of the space-like geodesic S. Cubic and quartic terms are necessary in order to verify that the field equations are satisfied in the local frame.

The leading terms in the resulting coordinate transformations have simple physical interpretations and can be written as follows:

$$X^0 = \int K\ dx^0 + \vec{V}_E \cdot \vec{r} = X^0(P_0) + \vec{V}_E \cdot \vec{r} + \ldots \tag{9}$$

$$X^k = X^k_E(X^0(P_0)) + x^k\left(1 - U_e - \vec{A}_E \cdot \vec{r} - \frac{1}{6}U_{e,mn}x^m x^n\right)$$
$$+\frac{1}{2}V^k_E(\vec{V}_E \cdot \vec{r}) + \Omega^{kj}\delta_{jm}x^m + \frac{1}{2}r^2 A^k_E + \ldots \tag{10}$$

In the first term in Eq. (9), the factor K is given by:

$$K^{-1} = ds/dX^0\Big|_G = \sqrt{\left(-G_{\mu\nu}dX^\mu dX^\nu\right)} \simeq 1 - U_e - V_E^2/2 \tag{11}$$

and gives the rate at which proper time elapses on a standard clock falling along with the origin of coordinates, with respect to barycentric coordinate time. Both special and general relativity effects are incorporated in K. U_e, the negative of the potential at the position of the falling clock due to all sources except the earth, gives rise to a gravitational frequency shift, and the term in V_E represents the second-order Doppler shift of the moving clock.

The second term in Eq. (9) arises from the well-known breakdown of simultaneity. This term plays an extremely important role in the coordinate transformation. The factor $(1-U_e)$ in Eq. (10) represents an overall length scale change, due to the external gravitational potential. The term quadratic in the Earth's velocity in Eq. (10) represents Lorentz contraction, and the term involving the antisymmetric quantity Ω^{ij} represents a slow rotation (geodetic precession) of the inertial frame axes with respect to distant stars (of about 19 milliarcsecs per year).

To describe gravitational effects in the local frame, one must perform a tensor transformation of the components of the metric, where the necessary partial derivatives, $\delta X^{\alpha}/\delta x^{\mu}$, are obtained from the coordinate transformations. One must also substitute the transformations into the expressions upon which $G_{\mu\nu}$ depends. For example, in the quantity $\left|\vec{X}-\vec{X}_E(X^0)\right|$, which occurs in the earth's potential, it is assumed the field point $\vec{X}$ and the source point $\vec{X}_E(X^0)$ are evaluated simultaneously in barycentric coordinates. The substitution must be done with exceptional care because of the relativity of simultaneity. In performing this substitution, one finds that Lorentz contractions and the relativity of simultaneity give rise to similar corrections, but that due to simultaneity is twice as large at that due to Lorentz contraction and is of the opposite sign. One finds:

$$\frac{2M_E}{\left|\vec{X}-\vec{X}_E(X^0)\right|}=\frac{2M_E}{r}\left\{1+U_e+\vec{r}\cdot\vec{A}_E/2+\left(\vec{r}\cdot\vec{V}_E/r\right)^2/2\right\} . \qquad (12)$$

The term in U_e arises from rescaling of lengths, the term in V_E from a combination of simultaneity breakdown and Lorentz contraction, and the term in A_E from spatial curvature (quadratic terms in the transformation of coordinates).

There are a number of additional contributions to g_{00} which are proportional to $2M_E/r$. These are as follows:

(1) An overall multiplicative factor K^2 contributes an additional $(K^2-1)\times 2M_E/r \simeq (2U+V^2)\,2M_E/r$;

(2) Nonlinear velocity correction terms to the masses which remain in G_{00} in addition to the velocity terms in the retarded potential of $3M_E V_E^2/r$;

(3) A cross-term between the external potential and the earth's potential in the squared potential term in G_{00} of $-4M_E U_e/r$;

(4) Nonlinear interaction terms in G_{00} of $-2M_E U_e/r$;

(5) Contributions from "magnetic" terms, G_{0i} of $-8M_E V_E^2/r$; and

(6) Contributions from G_{ij} of $+2M_E V_E^2/r$.

Collecting all the terms, we obtain:

$$\frac{2M_E}{r}\left\{1-\frac{1}{2}\left[V_E^2-\vec{r}\cdot\vec{A}_E-\left(\vec{r}\cdot\vec{V}_E/r\right)^2\right]\right\} . \tag{13}$$

This result must now be combined with the expansion of the retarded potential, Eq. (5). It is easily seen that all corrections cancel, leaving just the static potential term $2M_E/r$ in the local frame. Thus, what appears to be a retarded potential from a moving source in barycentric coordinates appears as a static potential in a local inertial frame falling along with the mass.

Obtaining the metric tensor in the local frame by transformation is complicated; it has been illustrated above for a term of one type. This has only been carried out for a restricted class of models because of the complexity of the calculations. In the general case, one can calculate the terms linear in x^k in the local frame and find that there are about 3 dozen terms, of 16 different types, all of which cancel out. This is as one would expect on the basis of the principle of equivalence, according to which gravitational forces due to distant bodies can be transformed away locally (*i.e.*, at the origin of local coordinates), by transforming to a freely falling inertial frame.

For a model in which one considers the earth to be falling around the sun (of mass $M_\odot$) in a circular orbit, the terms quadratic in local spatial coordinates can be calculated. The contributions to g_{00} consist of: a Minkowski term, the Newtonian potential term, a contribution from the nonlinear Schwarzschild field of the earth, Newtonian solar tidal terms, nonlinear earth-sun interaction terms, nonlinear solar tides due to interaction between the sun and the earth, and higher order solar tides. The spatial part of the metric in the local frame has the usual spatial curvature term due to the earth, plus solar tidal corrections. The g_{0i} terms contribute magnetic effects.

Calculations have also been performed using the PPN metric with parameters β, γ, $\zeta 1$, and $\zeta 2$ (Shahid-Saless and Ashby, in preparation). These give rise to some interesting effects involving shifts of the center of mass of the earth-sun system. The elliptical orbit case of the earth has also been treated, assuming, however, that the earth can be treated as a test particle so certain types of nonlinear interactions can be neglected. To verify the cancellations in more realistic cases requires considerable calculation. Computer algebra programs help some, but there is a tendency for such programs to fill up memory and fail in working on this problem.

V. EQUATIONS OF MOTION

Equations of motion can be derived from the local metric in a straightforward way. Here, we shall give only estimates of the orders of magnitude of the resulting accelerations of an earth-orbiting satellite caused by post-Newtonian effects. In evaluating the orders of magnitude, it is assumed that the satellite orbit satisfies $v^2 \simeq c^2 M_E/r$. For LAGEOS $r/10^9$ cm $\simeq 1.2$.

TABLE 1

ESTIMATES OF ACCELERATIONS OF AN EARTH SATELLITE DUE TO RELATIVITY THE MAGNITUDES OF THE ACCELERATIONS DUE TO NEWTONIAN FORCES OF ATTRACTION TO EARTH, AND THE SOLAR TIDES, ARE GIVEN FOR COMPARISON (N. ASHBY AND B. BERTOTTI 1984).

SOURCE	MAGNITUDE	
Newtonian potential	c^2M_E/r^2	$\simeq 4\times10^2[(10^9\text{ cm})/r]^2\text{ cm/sec}^2$
Solar tides	$c^2M_\odot r/R^3$	$\simeq 4\times10^{-5}[r/(10^9\text{ cm})]\text{ cm/sec}^2$
Nonlinear earth field	$c^2M_E^2/r^3$	$\simeq 2\times10^{-7}[(10^9\text{ cm})/r]^3\text{ cm/sec}^2$
Nonlinear solar tides	$c^2M_\odot^2r/R^4$	$\simeq 4\times10^{-13}[r/(10^9\text{ cm})]\text{ cm/sec}^2$
Solar "magnetic terms"	$c^2\sqrt{[M_\odot^3M_Er/R^7]}$	$\simeq 8\times10^{-13}\sqrt{[r/(10^9\text{ cm})]}\text{ cm/sec}^2$
Earth-sun interaction	$c^2M_\odot M_E/R^3$	$\simeq 2\times10^{-14}\text{ cm/sec}^2$

Note the relativistic perturbations are many orders of magnitude smaller than first calculated in the barycentric frame. This is because of the cancellations which occur upon transforming to a freely falling coordinate system. Rubincam (1977) has studied the effect of the nonlinear earth field.

The above relativistic orbital perturbations are so small that they cannot be expected to significantly affect LAGEOS's orbit. However, the nonlinear solar tidal term grows with distance from the earth and is much larger at the orbit of the moon. Combining the effects of this perturbing term on semimajor axis of the moon, eccentricity of the moon, and on the moon's mean motion, gives rise to a net perturbation on the distance between earth and moon of :

$$\delta r \simeq 4.4\text{ cm} \times \cos(2f_M + 2\omega - 2f_E) \tag{14}$$

where f_M and f_E are the true anomalies of the moon and of the earth, respectively. This effect is unfortunately obscured by its high correlation with effects due to the solar tides themselves.

A more interesting set of effects, which have not yet been fully explored analytically, arise from multipole contributions to the earth's field. The leading quadrupole contribution is about 10^{-3} of the main monopole term and relativistic corrections arising when this quadrupole is viewed as moving in the barycentric frame can be expected to be 10^{-8} smaller, thus such relativistic accelerations can be estimated to be about $4 \times 10^{-9}\text{ cm/sec}^2$ for LAGEOS, which is significant. Inclusion of such effects results in significant improvement in fitting the ranging data for LAGEOS (Ries, private communication).

VI. CHOICE OF TIME COORDINATE—TDT

Further transformations must be made since time standards laboratories having clocks used to define the SI second are on earth's surface, subject to additional motions due to earth rotation, and to the full gravitational potential of the earth including higher order multipole contributions to the potential.

Since the geoid is a surface of gravitational equipotential in the earth-fixed rotating frame, atomic clocks at rest on the geoid on the rotating earth all beat at the same rate. If one compares a clock near the pole with one on the equator, one finds that the one nearer the pole is close to the earth's center and is therefore beating more slowly due to a gravitational redshift effect. However it is also closer to earth's rotation axis and moving more slowly due to earth's rotation, and is subject to less time dilation (second-order Doppler shift). These effects cancel on the geoid.

The rate of a standard reference clock falling along the geodesic G, with rate correction given by the factor K in Eq. (11), does not incorporate the effects due to earth's mass and rotation. An additional correction is needed to obtain a new coordinate time x^0_{SI}, corresponding to the definition of the SI second. This is given by:

$$\frac{dx^0_{SI}}{dx^0} = 1 - \frac{1}{2}\frac{M_E}{a_1}(1 + J_2/2) - \frac{1}{2}(\omega a_1/c)^2 \tag{15}$$

where a_1 is the equatorial radius of the earth. Then, apart from periodic terms,

$$\frac{dx^0_{SI}}{dX^0} = 1 - \frac{M_E}{2a_1c^2}(1 + J_2/2) - (\omega a_1/c)^2/2 - U_e - V_E^2/2 \equiv 1 - L \tag{16}$$

where $L \simeq 1.55 \times 10^{-8}$; Hellings (1986) has given a more complete discussion of this. Barycentric Dynamical Time (TDB) runs at the same average rate as the SI second, so $X^O_{TDB} = (1 - L)X^O$. Thus TDB clocks beat at the same average rate as earth-borne clocks, from the point of view of an observer in the barycentric system. The TDB clocks beat more slowly, by the factor 1–L, than coordinate clocks in the Eddington-Clark metric. Therefore, to maintain a universally defined numerical value for the speed of light c, the length unit in TDB coordinates must be physically longer than the length unit in EC coordinates. Then since $(GM/c^2)_{TDB}$ represents a physical length as measured using a TDB meter stick, the numerical value of $(GM/c^2)_{TDB}$ will be less than it is in EC coordinates. But the speed of light, c, is the same in the two unit systems, so $(GM)_{TDB}$ $(1 - L)\,(GM)_{SI}$.

An outstanding issue is the definition of TDT — "terrestrial dynamical time." The problem is to define a terrestrial time scale which is as closely related to TAI in rate as is possible. Problems include how to specify the position of the master reference clock for the TDT (the geocenter is preferred), how to specify the rate of TDT in relation to TAI, and how to initialize the TDT clock since its position is not that of any clock contributing to TAI. Some synchronization convention must be adopted to resolve ambiguities.

VII. RELATIVISTIC EFFECTS IN THE GLOBAL POSITIONING SYSTEM

This area of application has been discussed elsewhere (Hellings 1986) so only a summary of the relativistic effects is given here. For clocks in GPS satellites or on the earth's surface, it is useful to synchronize to agree with hypothetical clocks synchronized in the local inertial frame. There are three residual relativistic effects: second-order Doppler frequency shifts of moving clocks, gravitational frequency shifts, and the Sagnac effect.

To the readings of each atomic clock, systematic corrections can be applied based on the known positions and motions of the clocks, such that, at each instant, the coordinate time thus produced agrees with the reading of a fictitious standard clock with which it instantaneously coincides and which is at rest in the local inertial frame. GPS coordinate time is thus generated by systematically modifying the proper time elapsed on standard clocks.

Standard clocks on the earth's surface provide the reference rate. Due to gravitational potential and motional effects, clocks in the satellites require a fractional rate offset correction having the value:

$$\frac{3GM_E}{2ac^2} - \frac{GM_E}{a_1c^2}(1 + J_2/2) - (\omega a_1/c)^2/2 \simeq -4.465 \times 10^{-10} . \tag{17}$$

In order for the SV clock to appear to an observer on the ground to beat at the chosen frequency of 10.23MHz, the SV clocks must be offset so that, to an observer in the SV rest frame, the frequency in $10.23 \times (1-4.465 \times 10^{-10})$MHz = 10.22999999543MHz.

An additional variable correction term, arising from a combination of gravitational frequency and second-order Doppler shifts in case the SV orbit is not circular, must be applied to the SV cock to yield GPS coordinate time:

$$\Delta t_{SV} = +4.4428 \times 10^{-10} \frac{\mathrm{sec}}{\sqrt{\mathrm{meter}}} e\sqrt{a} \sin E(t) \tag{18}$$

where e is the orbit eccentricity and E is the eccentric anomaly.

For a standard clock transported near the earth's surface, the following correction must be applied:

$$\Delta t' = \int_{path} (ds/c) \left[1-(\phi-\phi_0)/c^2 + (v'/c)^2/2 \right] + 2\omega A'_E/c^2 \tag{19}$$

where primed quantities are measured in the earth-fixed rotating frame. This gives the prescription for correcting the atomic clock reading $\int ds/c$ to account respectively for gravitational frequency shifts, second-order Doppler shifts, and the Sagnac effect. For clocks near the Earth's surface, $\phi-\phi_0 \simeq gh$, where g is the acceleration of gravity and h is the height above the geoid. The term $(v'/c)^2/2$ corrects for time dilation of clocks moving relative to the ground, and the last term expressing the Sagnac effect is equal to $2\omega A'_E/c^2$, where A'_E is the equatorial projection of the area swept out by the position vector $\vec{r}\,'$ of the clock in the rotating frame.

For synchronization by means of an electromagnetic signal, ds vanishes along the path. The elapsed coordinate time during propagation of the signal is:

$$\Delta t' = \int_{path} d\sigma' \left[1-(\phi-\phi_0)/c^2\right] + 2\omega A'_E/c^2 , \tag{20}$$

$$\Delta t' = \int_{path} d\sigma' \left[1-(\phi-\phi_0)/c^2\right] + 2\omega A'_E/c^2 , \tag{20}$$

where $d\sigma'$ is the increment of proper distance along the path of the signal and A'_E is the equatorial projection of the area swept out by the position vector of the electromagnetic pulse in the rotating frame. In the above expressions,

$$\begin{aligned}\phi-\phi_0 = &-\frac{M_E G}{r}\left[1- J_2 \,(a_1/r)^{2} p_2(\cos\,\theta)\right] - (\omega r' \sin\,\theta)^{2}/2 \\ &+\frac{GM_E}{a_1}(1 + J_2/2) + (\omega a_1)^{2}/2 \, .\end{aligned} \tag{21}$$

In GR, the same Sagnac correction terms arise whether synchronizing clocks by slow transport of portable clocks or by transmission of electromagnetic signals in the rotating frame.

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DISCUSSION

NIETO: In the case that you have a monopole plus J_2 representing the earth, what is the motion of the geocenter about the geodesic?

ASHBY: The geocenter wobbles about a timelike geodesic with an amplitude of about a meter.

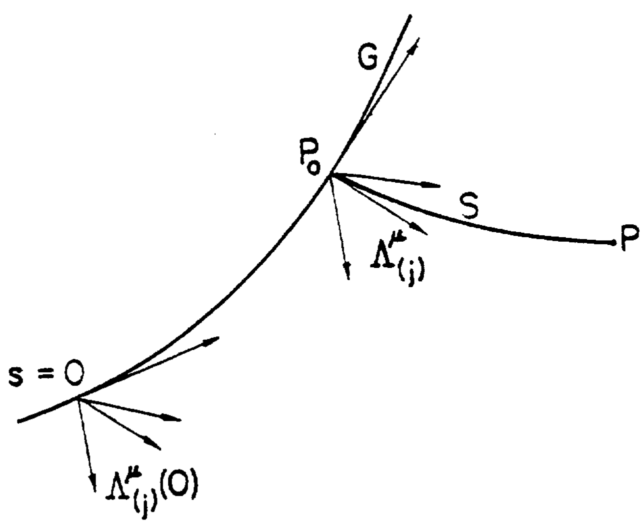

FIG. 1.—Diagram showing the local inertial frame falling freely along the geodesic G. The space-like geodesic S is constructed by dropping a geodesic from the field point P to P_0, that intersects G orthogonally.