APPLIED LIFE DATA ANALYSIS

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In Grateful Memory
of my Mother and Father

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Preface

Many major companies spend millions of dollars each year on product reliability. Much management and engineering effort goes into evaluating risks and liabilities, predicting warranty costs, evaluating replacement policies, assessing design changes, identifying causes of failure, and comparing alternate designs, vendors, materials, manufacturing methods, and the like. Major decisions are based on product life data, often from a few units. This book presents modern methods for extracting from life test and field data the information needed to make sound decisions. Such methods are successfully used in industry on a great variety of products by many who have modest statistical backgrounds.

This book is directed to engineers and industrial statisticians working on product life data. It will also aid workers in other fields where survival is studied, for example, in medicine, biology, actuarial science, economics, business, and criminology. Also, this book may supplement texts for many statistics and engineering courses, since it gives a wealth of practical examples with real data, emphasizes applied data analysis, employs computer programs, and systematically presents graphical methods, the method of maximum likelihood, censored data analysis, prediction methods, and linear estimation.

Life data generally contain running times on unfailed units, which require special statistical methods. In the past, these rapidly developing methods were associated with aerospace applications, but they are more widely used for consumer and industrial products. This book presents many applications to diverse products ranging from simple dielectrics and small appliances to locomotives and nuclear reactors.

This book draws from my experience teaching courses on life data analysis throughout the General Electric Company and at Rensselaer Polytechnic Institute and Union College. These courses have been popular with practicing engineers and graduate students in engineering, statistics, and operations research.

This book is organized to serve practitioners. The simplest and most widely useful material appears first. The book starts with basic models and simple graphical analyses of data, and it progresses through advanced analytic methods. All preliminary material for a topic is stated, and each topic is self-contained for easy reference, although this results in some repetition. Thus this book serves as a reference as well as a textbook. Derivations are generally omitted unless they help one understand the material. Such derivations appear in advanced sections for those who seek a fundamental understanding and wish to develop new statistical models and data analyses.

Readers of this book need a previous course in statistics and, for some advanced material, facility in calculus or matrix algebra. While many methods employ new and advanced statistical theory, the book emphasizes how to apply them. Certain methods (particularly those in Chapters 8 and 12), while important, are difficult to use unless one has special computer programs, which are now available.

There is much literature on life data analysis. So I have selected topics useful in my consulting. However, I briefly survey other topics in the final chapter.

Chapter 1 describes life data analysis, provides background material, and gives an overview of the book in detail. Chapter 2 presents basic concepts and statistical distributions for product life. Chapters 3 and 4 present graphical methods for estimating a life distribution from complete and censored life data. Chapter 5 explains statistical models and analyses for data on competing failure modes and on series systems. Chapters 6, 7, and 8 provide analytic methods, mainly linear and maximum likelihood methods, for estimating life distributions from complete and censored data. Chapter 9 provides methods for analyzing inspection data (quantal-response and interval data). Chapters 10, 11, and 12 provide methods for comparing samples (hypothesis tests) and for pooling estimates from a number of samples. Chapter 13 surveys other topics.

The real data in all examples come mostly from my consulting for the General Electric Company and other companies. Many of these real data sets are messy. Proprietary data were protected by vaguely naming a product and by multiplying the data by a factor. So engineers are advised not to use examples as typical of any product.

For help on this book I am overwhelmed with a great feeling of gratitude to many. Dr. Gerald J. Hahn, my co-worker, above all others, encouraged me, helped me to obtain support from General Electric, generously contributed much personal time reading the manuscript, and offered many useful suggestions. Gerry is the godfather of this book. I am much indebted for support from management at General Electric Corporate Research and Development—Dr. Art Bueche, Mr. Stu Miller, Mr. Virg Lucke, Dr. Dick Shuey, Mr. E. Lloyd Rivest, Dr. Dave Oliver, Dr. Hal Chestnut, and Mr. Bill Chu. Professor Al Thimm, encouraged by Professor Josef Schmee, both of Union College, kindly provided me with an office, where I worked on this book, and a class that I taught from my manuscript during a leave from GE. Professor John Wilkinson of Rensselaer Polytechnic Institute gave me the original opportunity to teach courses and develop preliminary material for this book.

Colleagues have generously given much time reading the manuscript and offering their suggestions. I am particularly grateful to Paul Feder, Gerry Hahn, Joseph Kuzawinski, Bill Meeker, John McCool, Ron Regal, Josef Schmee, Bob Miller, Bill MacFarland, Leo Aroian, Jim King, Bill Tucker, and Carolyn Morgan.

Many clients generously let me use their data. They also inspired methods (such as hazard plotting) that I developed for their problems. Many students contributed suggestions. There are too many to name, unfortunately.

The illustrations are mostly the superb work of Mr. Dave Miller. The manuscript benefited much from the skillful technical typing of Jean Badalucco, Ceil Crandall, Edith White, and Ruth Dodd.

WAYNE NELSON

Schenectady, New York
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About the Author

Wayne Nelson privately teaches and consults for companies, technical societies, and universities in the United States and abroad. Formerly with General Electric Co. Corp. Research and Development in Schenectady, NY over 20 years, he consults on engineering and scientific applications of statistical methods for reliability data, accelerated testing, quality control, planned experiments, sampling, tolerancing, measurement errors, and data analysis. He is an adjunct Professor at Union College. He can be reached at 739 Huntingdon Dr., Schenectady, NY 12309.

For his contributions to Reliability, Accelerated Testing, and Reliability Education, he was elected a Fellow of the American Statistical Assoc. (ASA), the American Society for Quality Control (ASQC), and the Institute of Electrical and Electronics Engineers (IEEE). He has a B.S. in Physics from Caltech and an M.S. in Physics (NSF Fellow) and a Ph.D. in Statistics (NSF Fellow) from the University of Illinois. He has published over 90 articles on statistical methods for engineering applications. For his publications, he received the 1969 Brumbaugh Award, the 1970 Jack Youden Prize, and the 1972 Frank Wilcoxon Prize of the ASQC. He received Outstanding Presentation Awards of the ASA for presentations at the 1977, 1979, 1987, 1988, and 1989 Joint Statistical Meetings. General Electric presented him the 1981 Dushman Award for his contributions to research and applications of reliability data analysis and accelerated testing.

He also authored the book *Accelerated Testing: Statistical Models, Test Plans, and Data Analyses* (John Wiley & Sons, 1990). He contributed to *Handbook of Reliability Engineering and Management* (McGraw-Hill, 1988) and to *Practical Machinery Management* (Gulf Publ., 1983). He wrote booklets *How to Analyze Reliability Data*, *How to Analyze Data with Simple Plots*, and *How to Plan and Analyze Accelerated Tests* for the ASQC and contributed to several engineering standards on data analysis.

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1

Overview And Background

1. INTRODUCTION

This chapter presents (1) an overview of this book's contents and (2) background information for the rest of the book. To read this chapter and the rest of the book, one needs a basic statistics course. Although addressed mostly to engineering applications, this book applies to many other fields. A key characteristic of life data analysis distinguishes it from other areas in statistics: namely, data are usually censored or incomplete in some way. Like other areas, it is concerned with estimates and confidence limits for population parameters and with predictions and prediction limits for future samples. The following paragraphs describe applications and the history of life data analysis.

Applications. This book presents methods for analysis of product life data and gives many engineering applications. In this book, examples of applications to products include diesel engine fans, transformers, locomotive controls, generator field windings, material strength, generator retaining rings, locomotive reliability, electrical insulating fluids and oils, the strength of electrical connections, Class-B and Class-H motor insulations, appliance cords, fuses, turbine disks, shave dies, alarm clocks, batteries, toasters, capacitors, cryogenic cables, motor coils, engine cylinders and pistons, power lines, large motor warranty costs, turbine wheels, and distribution transformers.

The methods apply to other fields and types of data as the following examples show. Economists and demographers study the length of time people are in the work force (Kpedekpo, 1969). Employers are concerned with the length of time employees work before changing jobs. Mental health officials use tables of length of stay in facilities to predict patient load. Businessmen wish to know the shelf life of goods and the time it takes inventory to turn over; for example, one manufacturer wanted to know the distribution of time from manufacture to installation of a major appliance. Wildlife managers use mortality tables to predict wildlife population sizes and determine hunting seasons. Hoadley (1970) studied the length of time telephones remain disconnected in vacant quarters in order to determine which telephones to remove for use elsewhere and which to leave in for the next customer. Kosambi (1966) proposed that knowledge of the distribution of the time that coins remain in circulation can help the mint plan production. The success of medical treatments for certain diseases is measured by the length of patient survival (Gross and Clark, 1975). The distribution of time from prison release to committing a crime measures the success of prison programs. A trading stamp company estimated the proportion of stamps that would be redeemed; this was used to determine needed cash reserves to cover outstanding stamps. Potency of some pesticides (and chemicals) is bioassayed by observing the times to death (or other reaction) of a sample of insects or animals. Life insurance companies determine premiums from mortality tables. The life of TV programs has been evaluated (Prince, 1967). Jaeger and Pennock (1957) estimated service life of household goods. The Association for the Advancement of Medical Instrumentation (1975) has a proposed standard with methods for estimating the life of heart pacemakers. Zahn (1975) described a psychological experiment on the time a (planted) "stranded" motorist must wait for aid from a passerby. The durability of garments is studied by manufacturers (Goldsmith, 1968). Wagner and Altman (1973) studied the time in the morning when baboons come down from the trees.

This book presents engineering applications and uses mostly engineering and reliability terminology. Biomedical, actuarial, and other fields have their own terminology for many concepts; some of their terminology is mentioned. Differences in terminology may cause initial difficulties to those who read publications in other fields.

History. Todhunter (1949) describes the early human life table of Halley (Chapter 2) and Bernoulli's work on the effect of smallpox innoculation on the distribution of life. Insurance companies have long used actuarial methods for constructing human life tables. Early in this century, actuarial methods were used to estimate survivorship of (1) medical patients under different treatments and of (2) equipment, particularly on railroads. In the 1950s and 1960s, reliability engineering blossomed; this resulted from demands for more reliable equipment from military and space programs. In this period, engineering design methods for reliable equipment made great strides. However, reliability data analysis mostly employed the oversimple exponential and Poisson distributions. In the 1950s and 1960s, most advances in life data analysis came from biomedical applications. Now methods are widely being developed for engineering applications to many consumer and industrial products. This book brings together recent methods for life data analysis. This field continues to grow, although many important problems remain unsolved, as this book shows.

2. OVERVIEW OF THE BOOK

This section describes this book's contents, organization, and how to use the book. The types of data mentioned here are described in Section 3.

Chapter 1 gives an overview of the book and presents needed background. Chapter 2 describes distributions for life and failure data. Chapter 3 presents simple probability plots for analyzing complete and singly censored data. Chapter 4 presents hazard plots for multiply censored data. Chapter 5 describes models for and graphical analyses of data with a mix of failure modes. Chapters 6 through 9 give analytic methods for (1) estimates and confidence limits for distribution parameters, percentiles, reliabilities, and other quantities and for (2) predictions and prediction limits for future samples. Chapter 6 treats analysis of complete data. Chapter 7 gives linear methods for singly censored data. Chapter 8 gives maximum likelihood methods for multiply censored data. Chapter 9 gives maximum likelihood methods for quantal-response and interval data. Chapter 10 presents various methods for comparing samples of complete data by confidence intervals and hypothesis tests; such methods include 1-, 2-, and K-sample comparisons and estimation by pooling a number of samples. Chapter 11 presents such comparisons based on linear estimates from singly censored samples. Chapter 12 presents such comparisons based on maximum likelihood methods for multiply censored and other types of data. Chapter 13 surveys topics in reliability and life data analysis that are not presented in the book.

Figure 2.1 shows this book's chapters. They are organized by type of data (complete, singly censored, multiply censored, etc.) and by statistical method (elementary, linear, and maximum likelihood). The chapters are in order of difficulty. Early chapters present simple graphical methods, and later ones present advanced analytic methods. The arrows in Figure 2.1 show which chapters are background for later chapters. Also, each chapter introduction

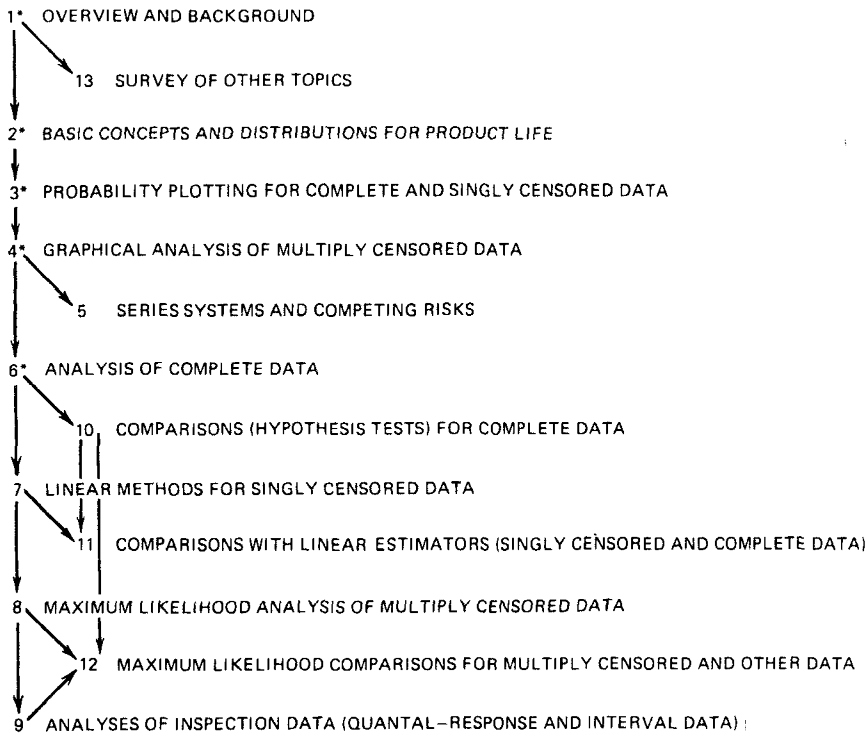

Figure 2.1. Book organization (asterisk denotes basic material).

refers to needed background and describes the difficulty of the chapter. Some section introductions do the same, and they state if a section is theoretical and can be skipped. Most sections are written for those who just wish to apply the methods. The first four chapters are simple and basic reading for all. The more advanced Chapters 5 through 9 are in order of difficulty. Chapters 10, 11, and 12 can be read after the corresponding Chapters 6 through 9.

Maximum likelihood methods (Chapters 8, 9, and 12) are versatile and apply to most distributions and types of data. Also, they have good statistical properties. If time is limited, one might skip the linear methods (Chapter 7) in favor of maximum likelihood methods.

The book employs the following scheme for numbering sections, equations, figures, and tables. Within each chapter, the sections are numbered simply 1, 2, 3, etc.; subsections are numbered 4.1, 4.2, etc. Equation numbers give the (sub)section number and equation number; for example, (2.3) is the third numbered equation in Section 2. Figure and table numbers include the section number; Figure 2.3 is the third figure in Section 2. Unless another chapter is stated, any reference to an equation, figure, or table is to one in the same chapter.

There are two types of problems at the end of a chapter. One type involves an analysis of data with the methods in that chapter; the other involves extending the results of the chapter to other problems. An asterisk (*) marks more laborious or difficult problems.

The book cites references by means of the Harvard system. A citation includes the author's name, year of publication, and his publications in that year. For example, "Nelson (1972b)" refers to Nelson's second referenced publication in 1972. All references are listed near the end of the book.

Basic statistical tables are in an appendix near the end of the book. Other tables must be obtained from the literature and are referenced.

The index of the book is detailed. It will be an aid to those who wish to use the book as a reference for selected methods. Also, to aid users, each section is written to be self-contained, thus repeating some material.

The book omits many derivations. Reasons for this are the following: (1) users can properly apply most methods, not knowing derivations, (2) many derivations are easy for a reader or instructor to supply, and (3) more time can be spent on methods useful in practice. Many derivations appear in Mann, Schafer, and Singpurwalla (1974), Gross and Clark (1975), Bain (1978), and Lawless (1982).

3. BACKGROUND MATERIAL

Background material useful for the rest of this book is briefly presented here. The topics are (1) statistical models, (2) population and sample, (3) valid data, (4) failure and exposure, (5) types of data, (6) nature of data analysis, (7) estimates and confidence intervals, (8) hypothesis tests, (9) predictions, (10) practical and statistical significance, (11) numerical calculations, (12) notation.

Statistical models. Supposedly identical units made and used under the same conditions usually have different values of performance, dimensions, life, etc. Variability of such a performance variable is inherent in all products, and it is described by a statistical model or distribution.

Population and sample. A statistical model describes some **population**. A manufacturer of fluorescent lamps is concerned with the future production of a certain lamp—an essentially infinite population. A manufacturer of locomotives is concerned with a small population of locomotives. A metallurgist is concerned with the future production of a new alloy—an essentially infinite population. A generator manufacturer is concerned with the performance of a small population of units to be manufactured next year. To obtain information, we use a **sample** (a set of units) from the population. We analyze the sample data to get information on the underlying population distribution or to predict future data from the population.

Valid data. There are many practical aspects to the collection of valid and meaningful data. Some are described below. Throughout, this book assumes that such aspects are properly handled.

Most statistical work assumes that the sample is from the population of interest. A sample from another population or a subset of the population can give misleading information. For example, failure data from appliances on a service contract may overestimate failure rates for appliances not on contract. Also, laboratory test data may differ greatly from field data. Data on units made last year may not adequately predict this year's units. In practice, it is often necessary to use such data. Then engineering judgment must determine how well such data represent the population of interest and how much one can rely on the information.

Most statistical work assumes that the data are obtained by simple random sampling from the population of interest. Such sampling gives each possible set of n units the same chance of being the chosen sample; random numbers should be used to ensure random selection. In practice, other statistical sampling methods are sometimes used, the most common methods being stratified sampling and two-stage sampling. Data analyses must take into account the sampling method. This book assumes throughout that simple random sampling is used. Some samples are taken haphazardly, that is, without probability sampling. Such samples may be quite misleading.

In practice, measurements must be meaningful and correct. Also, one needs to avoid blunders in handling data. Bad data can be unknowingly processed by computers and by hand.

Failure and exposure. Failure must be precisely defined in practice. For dealings between producers and consumers, it is essential that the definition of a failure be agreed upon in advance to minimize disputes. For many products, failure is catastrophic, and it is clear when failure occurs. For some products, performance slowly degrades, and there is no clear end of life. One can then define that a failure occurs when performance degrades below a specified value. Of course, one can analyze data according to each of a number of definitions of failure. One must decide whether time is calendar time or operating hours or some other measure of exposure, for example, the number of start-ups, miles traveled, energy output, cycles of operation, etc. Also, one must decide whether to measure time of exposure starting at time of manufacture, time of installation, or whatever. Engineers define failure and exposure.

Types of data. The proper analysis of data depends on the type of data. The following paragraphs describe the common types of life data from life tests and actual service.

Most nonlife data are **complete**; that is, the value of each sample unit is observed. Such life data consist of the time to failure of each sample unit. Figure 3.1*a* depicts a complete sample. Chapters 3, 6, and 10 treat such data. Much life data are incomplete. That is, the exact failure times of some units are unknown, and there is only partial information on their failure times. Examples follow.

Sometimes when life data are analyzed, some units are unfailed, and their failure times are known only to be beyond their present running times. Such data are said to be **censored on the right.** Unfailed units are called run-outs, survivors, removals, and suspended units. Similarly, a failure time known only to be before a certain time is said to be **censored on the left.** If all unfailed units have a common running time and all failure times are earlier, the data are said to be **singly censored** on the right. Singly censored data arise when units are started on test together and the data are analyzed before all units fail. Such data are singly **time censored** if the censoring time is fixed; then the number of failures in that fixed time is random. Figure 3.1*b* depicts such a sample. Time censored data are also called **Type I censored.** Data are singly **failure censored** if the test is stopped when a specified number of failures occurs, the time to that fixed number of failures being random. Figure 3.1*c* depicts such a sample. Time censoring is more common in practice; failure censoring is more common in the literature, as it is mathematically more tractable. Chapters 3, 7, and 11 treat singly censored data.

Much data censored on the right have differing running times intermixed with the failure times. Such data are called **multiply censored** (also progressively, hyper-, and arbitrarily censored). Figure 3.1*d* depicts such a sample. Multiply censored data usually come from the field, because units go into service at different times and have different running times when the data are recorded. Such data may be time censored (running times differ from failure times, as shown in Figure 3.1*d*) or failure censored (running times equal failure times, as shown in Figure 3.1*e*). Chapters 4, 8, and 12 treat such data.

A mix of **competing failure modes** occurs when sample units fail from different causes. Figure 3.1*f* depicts such a sample. Data on a particular

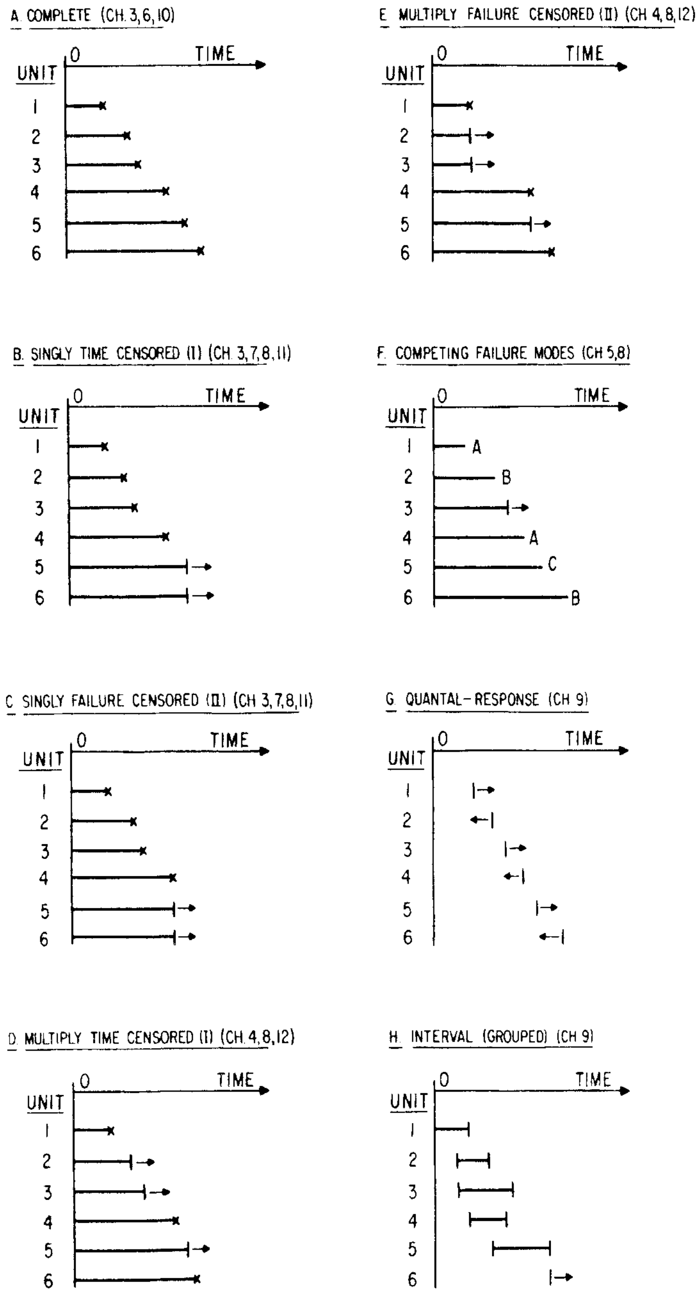

Figure 3.1. Types of data (failure time ×, running time ↦, failure occurred earlier ↤)

failure mode consist of the failure times of units failing by that mode. Such data for a mode are multiply censored. Chapter 5 treats such data in detail; Chapter 8 does so briefly.

Sometimes one knows only whether the failure time of a unit is before or after a certain time. Each observation is either censored on the right or else on the left. Such life data arise if each unit is inspected once to see if it has already failed or not. These data are **quantal-response** data, also called sensitivity, probit, and all-or-nothing response data. Figure 3.1*g* depicts such a sample. Chapter 9 treats such data.

When units are inspected for failure more than once, one knows only that a unit failed in an interval between inspections. So-called **interval** or grouped data are depicted in Figure 3.1*h*. Such data can also contain right and left censored observations. Chapter 9 treats such data.

Data may also consist of a mixture of the above types of data.

Analyses of such censored and interval data have much the same purposes as analyses of complete data, for example, estimation of model parameters and prediction of future observations.

Nature of data analysis. This section briefly describes the nature of data analysis. It advises how to define a statistical problem, select a mathematical model, fit the model to data, and interpret the results.

The solution of a real problem involving data analysis has seven basic steps.

1. Clearly state the real problem and the purpose of the data analysis. In particular, specify the numerical information needed in order to draw conclusions and make decisions.

2. Formulate the problem in terms of a model.

3. Plan both collection and analyses of data that will yield the desired numerical information.

4. Obtain appropriate data for estimating the parameters of the model.

5. Fit the model to the data, and obtain the needed information from the fitted model.

6. Check the validity of the model and data. As needed, change the model, omit or add data, and redo steps 5 and 6.

7. Interpret the information provided by the fitted model to provide a basis for drawing conclusions and making decisions for the real problem.

This book gives methods for steps 5 and 6. The other steps involve the judgment of engineers, managers, scientists, etc. Each of the steps is discussed below, but full understanding of these steps comes only with experience. Data analysis is an iterative process, and one usually subjects a data set to many analyses to gain insight. Thus, many examples in this book involve different analyses of the same set of data.

1. A clear statement of a real problem and the purpose of a data analysis is half of the solution. Having that, one can usually specify the numerical information needed to draw practical conclusions and make decisions. Of course, an analysis provides no decisions—only numerical information for people who make them. If one has difficulty specifying the numerical information needed, the following may help. Imagine that any desired amount of data is available (say, the entire population), and then decide what values calculated from the data would be useful. Statistical analysis estimates such values from limited sample data. If such thinking does not clarify the problem, one does not understand the problem. Sometimes there is a place for exploratory data analyses that do not have clear purposes but that may reveal useful information. Data plots are particularly useful for such analyses.

2. To state a problem in terms of a model, one chooses a statistical distribution for performance. Often the model is a simple and obvious one, widely used in practice: for example, a lognormal distribution for time to insulation failure. When a suitable model is not obvious, display the data various ways, say, on different probability papers. Such plots often suggest a suitable model. Indeed, a plot often reveals needed information and can serve as a model itself. Another approach is to use a very general model that is likely to include a suitable one as a special case. After fitting the general model to the data, one often sees which special case is suitable. Still another approach is to try various models and select the one that best fits the data. The chosen model should, of course, provide the desired information. Examples of these approaches appear in later chapters.

3. Ideally a tentative model is chosen before the data are collected, and the data are collected so that the model parameters can be estimated from the data. Sometimes when data are collected before a model and data analyses are determined, it may not be possible to fit a desired model, and a less realistic model must be used.

4. Practical aspects of data collection and handling need much forethought and care. For instance, data may not be collected from the population of interest; for example, data may be from appliances on service contract (self-selection) rather than from the entire population. Many companies go to great expense collecting test and field data but end up with inadequate data owing to lack of forethought.

5. To fit a chosen model to the data, one has a variety of methods. This step is straightforward; it involves using methods described in this book.

Much of the labor can (and often must) be performed by computer programs.

6. Of course, one can mechanically fit an unsuitable model just as readily as a suitable one. An unsuitable model may yield information leading to wrong conclusions and decisions. Before using information from a fitted model, one should check the validity of the model and the data. Such checks usually employ graphical displays that allow one to examine the model and the data for consistency with each other. The model may also be checked against new data. Often different models fit a set of data within the range of the data. However, they can give very different results outside that range.

7. Interpretation of results from the fitted model is easy when the above steps are done properly, as practical conclusions and decisions are usually apparent. A possible difficulty is that the information may not be accurate or conclusive enough for practical purposes. Then more data for the analysis is needed or one must be content with less reliable information. Also, most models and data are inaccurate to some degree. So the uncertainty in any estimate or prediction is greater than is indicated by the corresponding confidence or prediction interval.

Data analysis methods. Some specific data analysis methods are discussed below—estimates, confidence intervals, hypothesis tests, and predictions. These methods are treated in detail in later chapters.

Estimates and confidence intervals. Using sample data, the book provides estimates and confidence intervals for the parameters of a model. The estimates approximate the true parameter values. By their width, confidence intervals for parameters indicate the uncertainty in estimates. If an interval is too wide for practical purposes, a larger sample may yield one with the desired width. Chapters 6 through 9 provide such analytical estimates and confidence limits and examples.

Hypothesis tests. Chapters 10, 11, and 12 provide statistical tests of hypotheses about model parameters. A statistical test compares sample data with a hypothesis about the model. A common hypothesis is that a parameter equals a specified value; for example, the hypothesis that a Weibull shape parameter equals unity implies that the distribution is exponential. Another common hypothesis is that corresponding parameters of two or more populations are equal; for example, the standard two-sample t-test compares two population means for equality. If there is a statistically significant difference between the data and the hypothesized model, then there is convincing evidence that the hypothesis is false. Otherwise, the hypothesis is a satisfactory working assumption. Also, a test of fit or a test for outliers may result in rejection of the model or data.

Predictions. Most statistical methods are concerned with population parameters (including percentiles and reliabilities) when a population is so large that it can be regarded as infinite. One then uses estimates, confidence intervals, and hypothesis tests for **parameters** (or **constants**). However, in many business and engineering problems, the data can be regarded as a sample from a theoretical distribution. Then one usually wishes to predict the **random values** in a **future** sample from the same distribution. For example, one may wish to predict the random warranty costs for the coming year or the random number of product failures in the coming quarter, using past data. Then one wants a **prediction** for the future random value and a **prediction interval** that encloses that future random value with high probability. Many prediction problems go unrecognized and are incorrectly treated with methods for population parameters. Methods for such problems are now being developed, and this book brings together many of them. Chapters 6 through 8 include prediction methods and examples.

Practical and statistical significance. Confidence intervals indicate how (im)precise estimates are and reflect the inherent scatter in the data. Hypothesis tests indicate whether observed differences are statistically significant; that is, whether a difference between a sample of data and a hypothesized model (or whether the difference between a number of samples) is large relative to the inherent random scatter in the data. Statistical significance means that the observed differences are large enough to be convincing. In contrast, practical significance depends on the true differences in the actual populations; they must be big enough to be important in practice. Although results of an analysis may be important in practice, one should not rely on them unless they are also statistically significant. Statistical significance assures that results are real rather than mere random sampling variation.

A confidence interval for such differences is often easier to interpret than a statistical hypothesis test. The interval width allows one to judge whether the results are accurate enough to identify true differences that are important in practice. Chapters 10, 11, and 12 give examples of such confidence intervals and their application to comparisons.

Numerical calculations. Numerical examples are generally calculated with care. That is, extra figures are used in intermediate calculations. This good practice helps assure that answers are accurate to the final number of figures shown. For most practical purposes, two or three significant final figures suffice. A reasonable practice is to give estimates and confidence