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Applied supersymmetry and supergravity

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Abstract

In this review I discuss in some detail the structure and physical consequences of global and local supersymmetric (SUSY) gauge theories. Section 1 contains motivations for SUSY theories, whilst §§ 2 and 3 explain what supersymmetry is, and what are its physical properties. The observable consequences of SUSY at low energies and superhigh energies are discussed in §§ 4 and 5. The physical structure of simple ($N=1$) local SUSY ($\equiv$ supergravity) is given in § 6, whilst § 7 contains the physics of simple supergravity both at superhigh as well as at low energies. The experimental evidence (?) for supersymmetry is analysed in § 8, whilst § 9 contains the conclusions. Amazingly enough, we find that gravitational effects, as contained in supergravity theories, may play a rather fundamental role at all energy scales. This strong interrelation between gravity and particle physics is unprecedented.

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1. Motivation(s) for supersymmetry

Unification of all elementary particle forces has been the holy grail of theoretical physics. The first realistic step towards this end has been the highly successful unification between electromagnetic and weak interactions, now called electroweak interactions. The obvious (and natural) next step is then the amalgamation of electroweak and strong interactions, justifiably called grand unified theories (GUTS). (For reviews on GUTS see e.g. Ellis 1984, Nanopoulos 1980, Langacker 1981.) The qualitative successes (e.g. charge quantisation, equality of different coupling constants and equality of certain quark–lepton masses at superhigh energies, natural understanding of quark and lepton quantum numbers, etc), as well as the quantitative successes (e.g. disparity of coupling constants and of quark–lepton masses at low energies, determination of the electroweak mixing angle (θ_{EW}), possible limit on the number of flavours, virtually massless neutrinos, etc) are rather well known (Ellis 1984, Nanopoulos 1980, Langacker 1981). It is also well known that the grand unification scale M_{X} is rather large

$$M_{\mathrm{X}} \sim 10^{15}\,\mathrm{GeV}. \tag{1.1}$$

GUTS contain baryon and lepton number violating interactions, and the presently observed proton stability ($\tau_{\mathrm{p}} > 10^{31}$–$10^{32}$ yr) immediately puts a lower bound on M_{X} which more or less is saturated by (1.1). The existence of two scales, the electroweak scale ($M_{\mathrm{W}} \sim 100\,\mathrm{GeV}$) and the GUT scale M_{X}, so different

$$\frac{M_{\mathrm{W}}}{M_{\mathrm{X}}} \leqslant 10^{-13} \tag{1.2}$$

creates a fundamental problem for GUTS.

The gauge hierarchy problem: how is it possible to keep these two scales separate, incommunicado? In ordinary field theories, even if we fix the parameters at the classical level to satisfy (1.2), quantum corrections will undo this relationship and we will have to adjust it at each order in perturbation theory. This does not sound right if we claim to have a natural theory. The heart of the problem is the existence of scalar fields. The only way to break a gauge theory and keep renormalisability is through *spontaneous breaking* (SB). The simplest way to achieve SB is to allow certain scalar fields to have a vacuum expectation value (VEV). In GUTS we need two sets of scalars, one set to break the big group G which supposedly contains all interactions down to $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ at M_{X}, and thus the VEV of these fields $V \sim M_{\mathrm{X}}$, and another set to break $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ down to $\mathrm{SU}(3) \times \mathrm{U}(1)_{\mathrm{EM}}$, and thus they should get VEVS $v \sim M_{\mathrm{W}}$.

Since the masses of these Higgs fields are proportional to their VEVS, we end up with light Higgs of masses $\mathrm{O}(M_{\mathrm{W}})$ and heavy Higgs of masses $\mathrm{O}(M_{\mathrm{X}})$. Again this can be arranged at the classical level. But, since light and heavy Higgs are both coupled to gauge bosons, fermions and scalars, already at the 1-loop level there is a communication between the light and heavy Higgs, through the above mentioned fields running around the loop. That immediately creates corrections to light scalar masses $\mathrm{O}(M_{\mathrm{X}})$, while only corrections at most $\mathrm{O}(M_{\mathrm{W}})$ are allowed, i.e., end of the gauge hierarchy!

This happens because there is no symmetry able to keep scalars massless (or virtually massless: $M_W \ll M_X$), in contrast to gauge or chiral symmetries which keep gauge bosons or fermions massless.

One way out would be to abandon completely the use of scalar fields as a means of SB. Then we have to attempt dynamical SB. This approach has been tried (technicolour, extended technicolour, ...) but led to fatal flaws, so it became evident that technicolour has created more heat than light (Farhi and Susskind 1981). So back to scalars again. One may now try a different way. Is there any possibility that when one adds up all the 1-loop corrections to light scalar masses, they practically vanish, becoming at most of order M_W? We may exploit the fact that, thanks to the 'spin-statistics theorem and all that', boson and fermion loops differ by an all-over minus sign. Then if suitable relations exist between fermion and boson masses on the one hand and gauge, Yukawa and scalar self-coupling constants on the other, the hope of cancellation between different 1-loop diagrams may be realised. Well, this is a very neat way to discover supersymmetry (SUSY) (Gol'fand and Likhtman 1971, Volkov and Akulov 1973, Wess and Zumino 1974a; for reviews, see Fayet and Ferrara 1977, Wess and Bagger 1983). Actually, one may rigorously prove that the only way to get the desired 1-loop cancellation is through supersymmetry (Vetman 1981, Inami *et al* 1983, Deshpande *et al* 1981). As we will see later, a remarkable property of supersymmetry is that if the cancellation occurs at the 1-loop level, then it occurs automatically to all orders in perturbation theory. The technical aspect of the gauge hierarchy problems then has been solved.

Since exact supersymmetry implies equal fermion boson masses, which is not seen experimentally, supersymmetry has to be broken. The corrections to the light Higgs masses will be $O(M_{LESB})$ and they should better not exceed $O(M_W)$, otherwise the gauge hierarchy problem strikes back again, thus

$$M_{LESB} \leqslant O(M_W) \tag{1.3}$$

where M_{LESB} refers to the low energy supersymmetry breaking, i.e. the SUSY breaking that the low energy world suffers. Talking about hierarchies, another serious problem, this time concerning quantum chromodynamics (QCD), naturally comes to mind. Non-perturbative effects in QCD have the disturbing feature of adding a term:

$$\epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} \theta \tag{1.4}$$

where $F^a_{\mu\nu}$ is the gluon field strength and θ is a new parameter. There is a contribution from the non-perturbative term (1.4) to d_n, the dipole electric moment of the neutron (DEMON)

$$d_n \simeq 10^{-16} \theta \, e \text{ cm} \tag{1.5(a)}$$

which imposes a severe upper bound on θ

$$\theta \leqslant O(10^{-9}) \tag{1.5(b)}$$

when the present experimental upper bound (Altarev *et al* 1981, Dress *et al* 1977, Ramsey 1978) $d_n < O(10^{-25} \, e \text{ cm})$ on DEMON is used. This is the strong *CP*-hierarchy problem. Again supersymmetry solves the technical aspect of this problem (Ellis *et al* 1982a). Starting with $\theta = 0$, one proves (Ellis *et al* 1982a) that in spontaneously broken SUSY type theories, θ naturally lies below the limit posed by (1.5). The same type of miraculous cancellations, as mentioned above, occur again. Supersymmetric theories are very well behaved; they respect hierarchies.

We have by now enough physical motivation to have a close look at the structure of supersymmetric theories.

2. Supersymmetry (SUSY)

All kinds of symmetries that one normally uses in particle physics, global (like isospin, eightfold way, ...) or gauge (SU(3)×SU(2)×U(1), SU(5), O(10), E_6, ...), always transform fermions to fermions and bosons to bosons. Supersymmetry, or fermion-boson symmetry, tries to bypass this prejudice and aims to be a theory in which a fermion-boson transformation will also be possible. Indeed, such theories have been constructed and are in full accord with all the standard laws of quantum field theory (Gol'fand and Likhtman 1971, Volkov and Akulov 1973, Wess and Zumino 1974; for reviews see Fayet and Ferrara 1977, Wess and Bagger 1983). In its simplest form, one has to extend the usual Poincaré algebra of the generators of space-time rotations and translations, $M^{\mu\nu}$ and P^{μ}, to contain a self-conjugate (Majorana) spin $\frac{1}{2}$ generator, Q_α, which turns boson fields to fermion fields and vice versa. Schematically,

$$Q_\alpha|\text{boson}\rangle = |\text{fermion}\rangle. \tag{2.1}$$

$\alpha = 1, 2, 3, 4$ is the spinor index.

They satisfy the following (anti)commutation rules:

$$[Q_\alpha, M^{\mu\nu}] = \mathrm{i}(\sigma^{\mu\nu}Q)_\alpha \tag{2.2}$$

$$[Q_\alpha, P_\mu] = 0 \tag{2.3}$$

$$\{Q_\alpha, \bar{Q}_\beta\} = -2(\gamma_\mu)_{\alpha\beta}P^\mu \tag{2.4}$$

in which $\sigma^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$ and $\bar{Q} = Q^{\mathrm{T}}\gamma^0$.

The Q_α are four Hermitian operators and we use Majorana representation for Dirac matrices. Because of the spinorial character of the generators Q_α, the extended Poincaré algebra, called supersymmetry algebra, involves both commutation and anticommutation relations. It is not an ordinary Lie algebra, but what the mathematicians call a graded Lie algebra (GLA). The spinorial generator Q_α is a grading representation of the Poincaré Lie algebra. Supersymmetry extends this algebra in a rather *non-trivial way*; one can associate in irreducible representations a *finite* number of bosons and fermions. This fact is extremely important if one wants to construct conventional renormalisable quantum field theories invariant under supersymmetry and satisfying the usual Wightman axioms. Graded Lie algebras play perhaps a unique role in particle physics, because they realise truly relativistic spin-containing symmetries in which particles of different spin belong to the same supermultiplet. It is remarkable that by making the spinorial generators transform as some representation of an internal symmetry group, the resulting algebra provides also a fusion between space-time and internal symmetry *overcoming* previous no-go theorems (Coleman and Mandula 1967). Irreducible multiplets combine in this case fermions and bosons with different internal quantum numbers.

The physical meaning of the (anti)commutation rules (2.2), (2.3) and (2.4) is rather apparent. Equation (2.2) simply states that Q transforms as a spinor, while equation (2.3) states that the spinor charges are conserved and are translation invariant. Presumably the most important is equation (2.4), and it suggests the terminology that the supersymmetry charges are the 'square root of translations'. Clearly, SUSY involves the structure of space-time and it is this connection that is fully developed in local supersymmetry or supergravity (van Nieuwenhuizen 1981).

It is very easy to show that SUSY implies a relation between particles of different spin. Apply the spinor charge Q_α to a particle state $|P, S\rangle$ of definite momentum and helicity:

$$Q_\alpha |P, S\rangle = a|P, S+\tfrac{1}{2}\rangle + b|P, S-\tfrac{1}{2}\rangle. \tag{2.5}$$

Because of (2.3), the RHS is a superposition of particles of the same momentum and energy—thus, *the same mass.* Furthermore, addition of angular momentum implies that these particles have helicities $S \pm \frac{1}{2}$. Supersymmetry transformations connect states which differ by $\frac{1}{2}$ unit of spin. These states fill in the so-called supermultiplets. They therefore relate bosons to fermions. One may easily generalise the (anti)commutation rules (2.2), (2.3) and (2.4) to involve more than one spinorial charge, say Q_α^N, $N = 1, 2, \ldots, 8$. Then we talk about N-extended supersymmetry in contrast to $N = 1$ or simple supersymmetry. Since here we are not going to be involved with $N > 1$ supersymmetries we stick to the above given (anti)commutation rules.

When trying to mix SUSY with gauge theories we had better keep in mind certain general features and constraints that more or less come out from first principles.

1. In global supersymmetry the supercharges (Q_α) always commute with gauge symmetries; the commutator, if not zero, would be a supersymmetry transformation depending on the infinite number of parameters of the gauge symmetry, so would have to be a local SUSY. Thus,

$$[Q_\alpha^N, G] = 0. \tag{2.6}$$

There are two immediate consequences of this fact:

(i) all members of a supermultiplet have the same internal quantum numbers. This means that

(ii) only $N = 1$ (global) SUSY makes phenomenological sense, since $N \geqslant 2$ (global) supersymmetric theories always yield fermions in real representations of $SU(3) \times SU(2) \times U(1)$ in sharp contrast with what we observe experimentally at low energies (parity violation exists both in charged and neutral currents). For example in $N = 2$ global supersymmetry there are two types of supermultiplets containing spin $\frac{1}{2}$ fermions: $(\frac{1}{2}, 0, -\frac{1}{2})$ and $(1, \frac{1}{2}, 0)$. Both of them provide vector-like theories. The $(\frac{1}{2}, 0, -\frac{1}{2})$ multiplet relates fermions of helicity $\frac{1}{2}$ and $-\frac{1}{2}$ which would have to have the same quantum numbers. The $(1, \frac{1}{2}, 0)$ multiplet relates fermions of helicity $\frac{1}{2}$ to massless bosons of helicity 1. But massless bosons of helicity 1 are always gauge bosons, transforming in the adjoint representation, which is real. So whether we consider the $(1, \frac{1}{2}, 0)$ or the $(\frac{1}{2}, 0, -\frac{1}{2})$ multiplet, the fermions in $N = 2$ (or $N > 2$) global SUSY transform in a real representation of the gauge group: helicity $\frac{1}{2}$ and helicity $-\frac{1}{2}$ transform equivalently. This observation justifies our attitude of not considering very seriously any $N \geqslant 2$ global SUSY theories.

2. An immediate consequence of equation (2.4) is that in global SUSY theories with SB the Hamiltonian H is the sum of the squares of the supersymmetry charges

$$H = \sum_{\alpha=1}^{4} Q_\alpha^2. \tag{2.7}$$

Since H is the sum of squares of Hermitian operators, the energy of any state is positive or zero.

Clearly, if there exists a SUSY invariant state, that is a state annihilated by Q_α, then it is automatically the true vacuum state since it has zero energy, and any state that is not invariant under supersymmetry has positive energy. Thus, in contrast with ordinary gauge theories, if a SUSY state exists, it is the ground state and SUSY is not SB. Only if there does not exist a state invariant under SUSY, SUSY is SB. In this case the ground state energy is positive. Obviously, it is far more difficult to achieve SUSY SB than to achieve gauge symmetry SB. The supersymmetric state would have to be ostracised from the physical Hilbert space.

In global SUSY theories, it is impossible to spontaneously break an N-extended SUSY to an $N'(N > N' \geq 1)$ SUSY, because all Q_α^N satisfy separately equation (2.7) and if one breaks down some N, then all of them break down as well. No stepwise extended SUSY SB in global supersymmetric theories is possible. Things are different though in supergravity.

We are ready now to move to the construction of $N=1$ SUSY models.

3. Physical properties of supersymmetry

From our previous discussion of the general features of supersymmetry theories, we recall the fact that only $N=1$ global SUSY theories make phenomenological sense. Thus each supermultiplet contains only two kinds of particles, with identical internal quantum numbers but with a 'spin-shift' of $\frac{1}{2}$ unit.

Let us take an arbitrary gauge group, with gauge mesons A_μ^a (spin 1) and fermionic partners λ^a (spin $\frac{1}{2}$) called gauginos, belonging to the adjoint representation of the gauge group. They consist of the vector multiplet. The gauginos have to have spin $\frac{1}{2}$ and not $\frac{3}{2}$ because of renormalisability. In addition, we may introduce left-handed fermions ψ_L^i in an arbitrary multiplet of the gauge group. They form supersymmetry multiplets

$$\varphi_i \equiv \begin{bmatrix} \psi_i^L \\ s_i \end{bmatrix} \tag{3.1}$$

with complex scalar bosons s_i. They consist of the chiral multiplets. The right-handed fermion fields are the complex conjugates of the left-handed fields, $\psi_{jR} = (\psi_L^j)^*$ and their SUSY partners are the complex conjugates s_j^* of the s^i. We have to use scalar fields and not spin-1 fields to complete the 'fermion' multiplet because of renormalisability; the only allowed spin-1 bosons are gauge bosons, but then the fermions should belong to the adjoint representation of the gauge group, in contradiction to what we see experimentally. It is very interesting that renormalisability plus 'observation' define the superpartners uniquely. The superpartners of the observed fermions (quarks, leptons) are called sfermions (squarks, sleptons) and have spin 0, while the superpartners of Higgs are called higgsinos and have spin $\frac{1}{2}$. We will see below why in $N=1$ global SUSY, the usual Higgs fields cannot be identified with the superpartners of the observed quarks and leptons.

In addition, we introduce a function $f(\varphi_i)$, which is known as the 'superpotential'. f must be an analytic function of the φ_i, i.e. a function of φ_i but not of their complex conjugates φ_j^*. For a renormalisable theory f should be at most cubic in the φ_i, otherwise f is restricted only by gauge invariance. The general form of f is $f(\varphi_i) = a_i\varphi^i + a_{ij}\varphi^i\varphi^j + b_{ijk}\varphi^i\varphi^j\varphi^k$, where a_i, a_{ij}, b_{ijk} are gauge covariant tensors.

Notice that in SUSY theories the usual Higgs fields cannot be identified with the superpartners of the observed quarks and leptons (sfermions). In ordinary gauge theories we can use a Higgs doublet (H_2) to give masses to up quarks, while its charge conjugate (H_2^C) can provide masses to charged leptons and down quarks. In SUSY theories, since the superpotential f is a function only of the φ_i and not of the φ_j^*, H_2 and H_2^C should be chosen to be completely unrelated, different, fields. Thus, even if we identified the ($s\nu$, se) doublet with H_2^C, we are missing H_2, i.e. we are left with massless up quarks (u, c, t)! Furthermore, in a GUT theory, quarks and leptons are sitting in the same multiplet; thus by identifying sleptons with 'weak' Higgs fields, we have to interpret squarks as colour Higgs fields, which is catastrophic, since these colour Higgs fields will cause protons to decay instantly. For example, in SU(5), the Higgs doublet H_2 is sitting in the same multiplet, a 5-plet of SU(5), with a coloured Higgs triplet, H_3. The colour Higgs mediates proton decay and since it is coupled to quarks and leptons with the normal Yukawa couplings, it had better be superheavy ($\geqslant 10^{10}$ GeV), otherwise matter will disintegrate instantly! Thus, if we identified s-ups or s-downs with coloured Higgs fields, we are in big trouble since s-ups and s-downs cannot weigh much more than M_W ($\ll 10^{10}$ GeV) if we want to solve the gauge hierarchy problem. Finally, the existence of a pair of Higgs supermultiplets of *opposite helicities* is crucial in cancelling the Adler-Bell-Jackiw anomalies of the higgsino sector.

It is an unfortunate fact of supersymmetric life that no known particle can be the spartner of any other particle. In $N=1$ SUSY, all particles and their spartners must have identical $SU(3)_C \times SU(2)_L \times U(1)_Y \times$ global baryon and lepton quantum numbers. No known particles fit into the appropriate pairs, so we must invent a doubling of the elementary particle spectrum as seen in table 1.

Table 1. Spectrum of supersymmetric particles.

Particle	Spin	Sparticle	Spin
quark $q_{L,R}$	$\frac{1}{2}$	squark $\tilde{q}_{L,R}$	0, 0
lepton $l_{L,R}$	$\frac{1}{2}$	slepton $\tilde{l}_{L,R}$	0, 0
photon γ	1	photino $\tilde{\gamma}$	$\frac{1}{2}$
gluon g	1	gluino $\tilde{g}$	$\frac{1}{2}$
W	1	wino $\tilde{W}$	$\frac{1}{2}$
Z	1	zino $\tilde{Z}$	$\frac{1}{2}$
Higgs H	0	shiggs $\tilde{H}$	$\frac{1}{2}$

Gauge theories including the above mentioned supermultiplets embody the following relations between couplings (Fayet and Ferrara 1977):

$$g\bar{f}_{L\text{ or }R}\gamma_\mu f_{L\text{ or }R}G^\mu \rightarrow \sqrt{2}g(\bar{f}_{L\text{ or }R}\tilde{G}_{L\text{ or }R})\tilde{f}_{L\text{ or }R} + (\text{HC}) \tag{3.2}$$

$$P \equiv \lambda(f_L f'_L)H \rightarrow \lambda[(f_L\tilde{H}_L)\tilde{f}'_L + \tilde{f}_L(f'_L\tilde{H}_L)] \tag{3.3}$$

where the $f_{L,R}$ are left- (right-)handed fermions, G^μ are gauge bosons, $\tilde{G}$ are gauginos, the products of two fermions are denoted by

$$(f_L f'_L) \equiv \epsilon^{\alpha\beta} f_\alpha f'_\beta \tag{3.4}$$

and the $\tilde{f}$ ($\tilde{H}$) are 'sfermion' ('shiggs') partners of conventional fermions and Higgs bosons.

It is convenient to introduce the concept of superspace, which possesses anticommuting (Grassmannian) coordinates θ_α as well as the conventional space-time coordinates x_μ. Supermultiplets can then be represented by superfields which can be expressed as power series in the superspace coordinates θ_α, which are of finite order because they anticommute. Chiral superfields φ are spin-0 fields of the form

$$\varphi(x) = s(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) \tag{3.5}$$

whereas vector superfields V are written as

$$V = -\theta\sigma^\mu\theta V_\mu(x) + (-\mathrm{i}\tilde{\theta}\tilde{\theta}\theta\lambda(x) + \text{HC}) + \tfrac{1}{2}\theta\theta\tilde{\theta}\tilde{\theta}[D(x) - \mathrm{i}\,\partial_\mu V^\mu(x)]. \tag{3.6}$$

The so-called auxiliary fields F and D can be eliminated using the equations of motion, leaving us with physical degrees of freedom corresponding just to the supermultiplet structures. The interactions of the supermultiplets are derived from a supersymmetric action which can be written schematically as:

$$A = \int [\bar{\varphi}\varphi\, \mathrm{e}^{2V} + f(\varphi) + VV] \tag{3.7}$$

where the integral over space and superspace denotes a $\int \mathrm{d}^4x\, \mathrm{d}^4\theta$ for the first term, which is a kinetic term for the chiral supermultiplet $\varphi \equiv (s, \psi)$ and an $\int \mathrm{d}^4x\, \mathrm{d}^4\theta$ for the last term, which is a kinetic term for the gauge supermultiplets $(V, \tilde{V})$. The middle term in (3.7) is a $\int \mathrm{d}^4x\, \mathrm{d}^2\theta$ which gives Yukawa interactions, fermion masses and multiple scalar interactions. The object $f(\varphi)$ is a cubic polynomial, introduced above

$$f(\varphi) = a_{ij}\varphi_i\varphi_j + b_{ijk}\varphi_i\varphi_j\varphi_k \tag{3.8}$$

called the superpotential. Fermion interactions are obtained from $f(\varphi)$ (3.8) by removing two φ and putting in their spin-$\frac{1}{2}$ ψ components, while taking the scalar components s of any remaining φ:

$$(\psi_i\psi_j)\left.\frac{\partial^2 f}{\partial\varphi_i\,\partial\varphi_j}\right|_{\varphi_k = s_k} = a_{ij}(\psi_i\psi_j) + b_{ijk}s_k(\psi_i\psi_j). \tag{3.9}$$

The first term on the right-hand side of (3.9) is a fermion mass term, while the second term is a conventional Yukawa interaction. The multiscalar interactions obtained from $f(\varphi)$ are

$$\sum_i |F_i|^2 = \sum_i \left|\frac{\partial f}{\partial\varphi_i}\right|^2\Bigg|_{\varphi_i = s_i} = |a_{ij}s_j + b_{ijk}s_js_k|^2. \tag{3.10}$$

We easily derive from (3.8) a $(\text{mass})^2$ matrix:

$$(m_s^2)_{ik} = a_{ik}a_{jk}^* = (m_\psi m_\psi^*)_{ik} \tag{3.11}$$

which we see to be identical with the fermion $(\text{mass})^2$ matrix derived from (3.9). Thus, fermion and boson masses are identical, as we would expect from exact SUSY (2.1). Gauge interactions also give quartic scalar interactions (Fayet and Ferrara 1977)

$$\tfrac{1}{2}\sum_a g_a^2\, |s^* T^a s|^2. \tag{3.12}$$

Thus, the full scalar potential is given by

$$V(s_i, s_i^*) = \sum_i \left|\frac{\partial f}{\partial s_i}\right|^2 + \tfrac{1}{2}\sum_a g_a^2 |(s^*, T^a s)|^2 \tag{3.13}$$

where the second sum runs over all generators a of the gauge group, the g_a are the gauge coupling constants and T^a are the generators of the gauge group acting on the representation of the group furnished by the s_i. If the gauge group is not semi-simple but contains U(1) generators, say Y_l with charge g_{Yl}, then its contribution to (3.13) becomes

$$\tfrac{1}{2}g_{Y_l}^2|(s^*, Y_l s)+\xi_l|^2 \tag{3.14}$$

where ξ_l are arbitrary constants of mass dimension 2. Usually one defines

$$F_i \equiv \frac{\partial f}{\partial s_i} \tag{3.15}$$

and

$$D_a \equiv (s^*, T^a s)+\delta_{aY_l}\cdot\xi_l \; (T^{Y_l}\equiv Y_l) \tag{3.16}$$

and thus

$$V(s_i, s_i^*)=\sum_i |F_i|^2+\tfrac{1}{2}\sum_a g_a^2|D_a|^2. \tag{3.17}$$

In the classical approximation, the zero point energy of the fields may be neglected, and the energy of the ground state just equals the minimum of the potential $V(s_i, s_i^*)$. SUSY is unbroken if $V=0$ for $s_i=\langle s_i\rangle$. As V is a sum of squares, $V=0$ if each term separately vanishes. Thus the condition for unbroken SUSY is

$$F_i \equiv \frac{\partial f}{\partial s_i}=0 \tag{3.18}$$

for each field s_i, and

$$D_a \equiv (s^*, T^a s)+\delta_{aY_l}\cdot\xi_l=0 \tag{3.19}$$

for every generator T^a of the gauge group.

If (3.18) and (3.19) have a simultaneous solution, SUSY is unbroken at the tree level. Otherwise, SUSY is spontaneously broken. It is not difficult to show that a necessary condition to have spontaneous SUSY breaking at the tree level in a supersymmetric gauge theory is to have one of the following two conditions satisfied.

(i) The group G should contain at least a neutral field X with linear terms in the superpotential f. This is called F-type breaking because (3.18) cannot be satisfied.

(ii) The group G should contain at least an Abelian factor U(1) with a non-vanishing ξ in (3.16). This is called D-type breaking because (3.19) cannot be satisfied.

Similarly to the SB of global continuous symmetry where there are Goldstone bosons, in the case of SB of global SUSY a Goldstone fermion, called a goldstino, should be present. In general, up to a normalisation factor the goldstino is given by:

$$\psi_g=\tfrac{1}{2}g_a D_a\lambda^a+\frac{\partial f}{\partial\varphi_i}\psi_i. \tag{3.20}$$

Let us define the coupling M_S^2 of the supercurrent $S_{\mu\alpha}(Q_\alpha=\int d^3x\, S_{0\alpha})$ to the goldstino by:

$$\langle 0|S_{\mu\alpha}|\psi_\beta\rangle=M_S^2(\gamma_\mu)_{\alpha\beta}. \tag{3.21}$$

There is a simple and fundamental relation between the value of the potential at the minimum (vacuum energy) V_0 and M_S^2:

$$V_0=(M_S^2)^2=M_S^4. \tag{3.22}$$

The physical meaning of M_S is apparent: M_S^2 is the 'order parameter' of supersymmetry. A value of M_S different from zero implies that supersymmetry is spontaneously broken. In particular, if we denote by ϵ the coupling of the goldstino to a supermultiplet, then there is a mass splitting between the boson and the fermion inside the supermultiplet:

$$m_B^2 - m_F^2 = M_S^2 \epsilon. \tag{3.23}$$

The form of the mass splitting (3.23) is very suggestive. If one wishes, one may arrange things in such a way that, by making the coupling of the goldstino to certain supermultiplets small ($\epsilon \ll 1$), these supermultiplets suffer mass splittings, much smaller than the primordial SUSY breaking scale M_S. This simple mechanism, the *SUSY decoupling* mechanism (SUDEC), has been discovered only recently (Barbieri *et al* 1982a, b).

Its importance in constructing realistic SUSY models is difficult to overestimate (Barbieri *et al* 1982a, b). The reason is very simple. As we have seen before (see (1.3)), because of the gauge hierarchy problem, M_{LESB}, the mass splitting that the low energy world supermultiplets suffer, has to be of order M_W. Then if we identify M_S with M_{LESB}, à la Fayet (1980), no realistic model can be constructed (Barbieri *et al* 1982a, b, Farrar and Weinberg 1983). All Fayet type models (Fayet 1980) ($M_S \sim M_{LESB}$) suffer either from Adler–Bell–Jackiw type anomalies or/and flavour changing neutral currents or/and other pathologies related to the standard established low energy phenomenology (Barbieri *et al* 1982a, b, Farrar and Weinberg 1983). It seems that we definitely need

$$M_S \gg M_{LESB} \tag{3.24}$$

which, in turn, means that necessarily the SUDEC mechanism (Barbieri *et al* 1982a, b) has to be employed ($\epsilon \ll 1$). Actually, thanks to the 'magic' properties of SUSY theories (non-renormalisation theorems), one can prove (Barbieri *et al* 1982a, b, Polchinsky and Susskind 1982) that (3.24) persists to all orders in perturbation theory, i.e. it is stable against large radiative corrections. Indeed, realistic models have already been constructed (Barbieri *et al* 1982a, b) where one usually finds that

$$M_S^2 \simeq M_W M_{\text{Planck}} \tag{3.25}$$

implying that, by using (1.3) and (3.23),

$$\epsilon_{\text{LOW}} \approx \frac{M_W}{M_{\text{Pl}}}. \tag{3.26}$$

Things become very interesting because local SUSY or supergravity (van Nieuwenhuizen 1981) cannot be neglected any more (Barbieri *et al* 1982c). In local SUSY the goldstino becomes the missing longitudinal components of the spin $\frac{3}{2}$ gravitino, the gauge fermion of local supersymmetry (the superpartner of the spin 2 graviton) through the superhiggs effect (Volkov and Soroka 1973, Deser and Zumino 1977). In analogy with ordinary gauge theories in which the gauge boson mass is given by

$$M \sim g\langle\varphi\rangle \tag{3.27}$$

where g is the gauge coupling constant and $\langle\varphi\rangle$ is the VEV of the scalar field causing the breaking, i.e. the order parameter of the gauge symmetry, the mass of the gravitino is given by (Deser and Zumino 1977)

$$m_{3/2} \sim (G_{\text{Newton}})^{1/2} M_S^2 = \frac{M_S^2}{M_{\text{Pl}}} \tag{3.28}$$

in terms of the gravitational coupling constant G_N ($\equiv 1/M_{Pl}^2$) and M_S^2, the order parameter of supersymmetry. One then finds (Cremmer *et al* 1978a, 1979, 1982, 1983a, Ellis and Nanopoulos 1982b) extra contributions to the RHS of (3.23) proportional to $m_{3/2}^2$. By putting together (3.25) and (3.28) we find that

$$m_{3/2} \sim M_W \tag{3.29}$$

implying that the extra gravitational contributions to (3.23) proportional to $m_{3/2}^2$ cannot be neglected any more (Barbieri *et al* 1982c), as they are of the same order of magnitude as the non-gravitational ones. One may even suspect that it is possible to create the whole M_{LESB} through gravitational effects. This possibility will be explored further in § 6. There is another way of breaking global supersymmetry: *the soft way*, i.e. interactions of dimensions less than four which do not lead to quadratic divergences (Girardello and Grisaru 1982). For example, scalar and gaugino mass terms belong to the list of allowed soft SUSY breaking terms. The ultraviolet properties of the theory, e.g. softening of divergences, etc, hold true in soft breaking as they do in SB (Girardello and Grisaru 1982), so soft breaking may be employed in physical applications as good as SB. Amazingly enough, recent progress has shown that it is almost impossible (Barbieri *et al* 1982a, b, Farrar and Weinberg 1983) to construct appealing SUSY models with spontaneous global SUSY breaking, while realistic SUSY models have appeared (Ellis 1984, Nilles 1984) satisfying all possible phenomenological constraints, with a very definite pattern of soft SUSY breaking emerging from the spontaneous breakdown of local supersymmetry. Gravity seems to play a rather fundamental role here, as will be discussed later.

One of the central features of supersymmetric theories is that, thanks to their fermion–boson symmetry, there are a lot of cancellations between 'badly' behaving graphs. This amounts to a much better behaved field theory in the ultraviolet, compared with the ordinary field theories, which make them very attractive. Actually, recently it has been proven that the $N = 4$ SUSY Yang–Mills (YM) field theory in $d = 4$ space-time dimensions is *finite* (Mandelstam 1983a, b, Howe *et al* 1984)!

The remarkable property of SUSY gauge theories which has recently made them so interesting is their non-renormalisation theorem (Ferrara *et al* 1974, Ferrara and Piquet 1975, Grisaru *et al* 1979, Iliopoulos and Zumino 1974, Wess and Zumino 1974b). Since all loop diagrams have a $\int d^4\theta$ form, there is no intrinsic renormalisation of the superpotential $f(\varphi)$, but only wavefunction renormalisations of the chiral supermultiplets:

$$\varphi_i \to Z_{ij}\varphi_j \tag{3.30}$$

and renormalisations of the gauge couplings g_α. There is no intrinsic renormalisation of the Yukawa coupling parameters a_{ij}, b_{ijk} in $f(\varphi)$ (3.8). This means that we can 'set and forget' them: any small (or vanishing) superpotential term remains small (or vanishing) in all orders of perturbation theory.

'*Set it and forget it.*' It is this property that solves (Kaul 1982, Maiani 1979, Witten 1981) the technical aspect of the gauge hierarchy and strong *CP*-hierarchy problems (Ellis *et al* 1982a) that we discussed in the beginning as motivation for SUSY theories. By the same token, one can prove that if SUSY is unbroken at the tree level, it remains unbroken to all orders in perturbation theory (Ferrara *et al* 1974, Ferrara and Piquet 1975, Grisaru *et al* 1979, Iliopoulos and Zumino 1974, Wess and Zumino 1974b). It sounds like a magic world full of miracles. But still, it is true!

4. 'Low energy' physics and SUSY

Indeed, if SUSY provides the solution (Kaul 1982, Maiani 1979, Witten 1981) to the gauge hierarchy problem, then the phenomenological implications are tremendous. Firmly, each one of the 'standard' particles of the low energy world (quarks, leptons, gauge bosons, Higgs) should have their superpartners in a mass range at most not far above M_W. This fact makes the situation very exciting because the hope exists that these particles will be discovered in the not very far future. Present experimental limits from PETRA and PEP put the mass of any new charged particles approximately above 20 GeV, which puts a lower bound on the mass of charged SUSY particles. I will not discuss here how to find SUSY particles since this has been discussed in lengthy detail elsewhere (for an exhaustive review see Nanopoulos and Savoy-Navarro 1984).

Here I will discuss the constraints that well established phenomenological facts like absence of flavour-changing neutral currents (FCNC), absence of strong *CP*-violation, $g-2$, etc, impose on SUSY models. Clearly, the introduction of new particles, which are coupled to the low energy world with ordinary gauge or Yukawa couplings and of mass not far above M_W, sound like trouble. For example, analogously to the gauge boson-fermion-fermion coupling there is a gaugino-sfermion-fermion coupling of comparable strength (see equation (3.2)). Such kinds of couplings contribute to all kinds of rare processes, like the real and imaginary part of K_L-K_S, $K_L\to\mu\mu$, $\mu\to e\gamma$, etc.

We saw before that exact SUSY would require degenerate spin 0 and spin $\frac{1}{2}$ particles:

$$m_0^2=m_{1/2}^2. \tag{4.1}$$

Since all squarks and charged sleptons must have masses $\geqslant O(20)$ GeV, it must be that SUSY is broken. A first approach to describing SUSY breaking might be to introduce soft SUSY breaking, i.e. interactions of dimension less than four which do not lead to quadratic divergences (Girardello and Grisaru 1982). These could include the desirable

$$L_{\rm SUSY}\ni -m_{\tilde q}^2|\tilde q|^2-m_{\tilde l}^2|\tilde l|^2-m_{\tilde V}(\tilde V\tilde V) \tag{4.2}$$

with $m_{\tilde q}^2$ and $m_{\tilde l}^2$ being general matrices in both flavour and helicity spaces. (Recall that since quarks and charged leptons have both left- and right-handed components q_L, q_R, l_L, l_R, there must exist all the corresponding spartners $\tilde q_L$, $\tilde q_R$, $\tilde l_L$, $\tilde l_R$.) We must be careful with our choice of $m_{\tilde q}^2$ and $m_{\tilde l}^2$ so as to avoid problems with flavour-changing neutral interactions (Ellis and Nanopoulos 1982a). Recall first the usual formalism (Ellis *et al* 1976) for the Cabibbo-Kobayashi-Maskawa (CKM) mixing for quarks: a general mass matrix m_q is diagonalised to m_q^D by unitary transformations $U_{L,R}^q$ on the quark fields:

$$m_q\bar q_R q_L=m_q^D\bar q_R^D q_L^D\colon\; m_q^D=U_R^q m_q U_L^{q\dagger};\; q_{L,R}^D=U_{L,R}^q q_{L,R}. \tag{4.3}$$

When we make the transformations (4.3) the neutral gauge boson interactions remain flavour-diagonal:

$$\bar q_{L,R}\gamma^\mu q_{L,R}G_\mu^0=\bar q_{L,R}^D(U_{L,R}^{q\dagger}U_{L,R}^q)\gamma^\mu q_{L,R}^D G_\mu^0=\bar q_{L,R}^D\gamma^\mu q_{L,R}^D G_\mu^0. \tag{4.4}$$

However, the left-handed charged currents acquire non-trivial CKM angles because the unitary rotations U_L^u, U_L^d are in general different:

$$\begin{aligned}\bar u_L\gamma^\mu d_L W_\mu^\dagger&=\bar u_L^D(U_L^{u\dagger}U_L^d)\gamma^\mu d_L W_\mu^\dagger\\&=\bar u_L^D U_{\rm CKM}\gamma^\mu d_L W_\mu^\dagger\colon\; U_{\rm CKM}=U_L^{u\dagger}U_L^d.\end{aligned} \tag{4.5}$$

Consider now what could happen to the corresponding neutral gaugino interactions:

$$\tilde{\bar{q}}_{\mathrm{L,R}}(q_{\mathrm{L,R}}\tilde{G}^0_{\mathrm{L,R}}) = \tilde{\bar{q}}^{\mathrm{D}}_{\mathrm{L,R}}(\tilde{U}^{q\dagger}_{\mathrm{L,R}}U^q_{\mathrm{L,R}})(q^{\mathrm{D}}_{\mathrm{L,R}}\tilde{G}^0_{\mathrm{L,R}}) \tag{4.6}$$

where the new unitary rotations $\tilde{U}^q_{\mathrm{L,R}}$ diagonalise $m_{\tilde{q}}$ to $m^{\mathrm{D}}_{\tilde{q}}$:

$$(m^{\mathrm{D}}_{\tilde{q}})^2 = U^q m^2_{\tilde{q}} U^{q\dagger}\colon\ q^{\mathrm{D}}_{\mathrm{L,R}} = U^q_{\mathrm{L,R}} q_{\mathrm{L,R}}. \tag{4.7}$$

The interaction (4.6) would contain flavour-non-diagonal neutral gaugino couplings unless we arrange that

$$\tilde{U}^q_{\mathrm{L,R}} = U^q_{\mathrm{L,R}} \tag{4.8}$$

which in turn implies that the SUSY CKM angles should be identical to the conventional CKM angles:

$$\tilde{U}_{\mathrm{CKM}} \equiv \tilde{U}^{u\dagger}_{\mathrm{L}}\tilde{U}^d_{\mathrm{L}} = U^{u\dagger}_{\mathrm{L}}U^d_{\mathrm{L}} = U_{\mathrm{CKM}}. \tag{4.9}$$

You might wonder how important it is to ensure the absence of flavour-changing neutral gaugino interactions. If they were present, the $\tilde{\gamma}$ and $\tilde{g}$ SUSY box diagrams of figure 1 would make disastrously large contributions to the $\mathrm{K}_1-\mathrm{K}_2$ mass difference (Barbieri and Gatto 1982, Campbell 1983, Ellis and Nanopoulos 1982a, Inami and Lim 1982). Furthermore, if the photino $\tilde{\gamma}$ were light enough to be produced in $\mathrm{K}^\pm \to \pi^\pm\tilde{\gamma}\tilde{\gamma}$ decay, the rate would greatly exceed the experimental upper limit on $\mathrm{K}^\pm \to \pi^\pm +$nothing observed, unless $\tilde{U}^q_{\mathrm{L,R}} = U^q_{\mathrm{L,R}}$ (Suzuki 1982). Of course, it is not necessarily the case that $m_{\tilde{\gamma}} < (m_{\mathrm{K}} - m_\pi)/2$, so we may not need to worry about $\mathrm{K}^\pm \to \pi^\pm\tilde{\gamma}\tilde{\gamma}$ decay, but the problem with $\mathrm{K}_1-\mathrm{K}_2$ box diagrams exists for all $\tilde{\gamma}$ or $\tilde{g}$ with masses O(100) GeV. To avoid this problem by ensuring the equality (4.8) of $\tilde{U}$ and U matrices, we must demand that the $m^2_{\tilde{q}}$ matrix be a simple function of the quark mass matrix m_q, presumably a quadratic function:

$$m^2_{\tilde{q}} = \tilde{m}^2 \mathbf{1} + C_1 m_q \tilde{m} + C_2 m^2_q. \tag{4.10}$$

By making the ansatz (4.10) we also avoid any problem (Barbieri and Gatto 1982, Campbell 1983, Ellis and Nanopoulos 1982a, Inami and Lim 1982, Suzuki 1982) with $\tilde{\mathrm{W}}^\pm$ box diagrams. Equation (4.10) guarantees that

$$m^2_{\tilde{\mathrm{c}}} - m^2_{\tilde{\mathrm{u}}} = \mathrm{O}(1)(m^2_{\mathrm{c}} - m^2_{\mathrm{u}}) \tag{4.11}$$

if the $C_i = \mathrm{O}(1)$, and, therefore, the $\tilde{\mathrm{W}}$ box diagrams will be suppressed by a super-GIM mechanism to

$$\mathrm{O}\left(\frac{\alpha G_{\mathrm{F}}\sin^2\theta_{\mathrm{C}}}{4\pi' s}\right)\left(\frac{m^2_{\tilde{\mathrm{c}}} - m^2_{\tilde{\mathrm{u}}}}{m^2_{\tilde{\mathrm{W}},\tilde{q}}}\right) = \mathrm{O}\left(\frac{\alpha G_{\mathrm{F}}\sin^2\theta_{\mathrm{C}}}{4\pi' s}\right)\left(\frac{m^2_{\mathrm{c}} - m^2_{\mathrm{u}}}{m^2_{\tilde{\mathrm{W}},\tilde{q}}}\right) \tag{4.12}$$

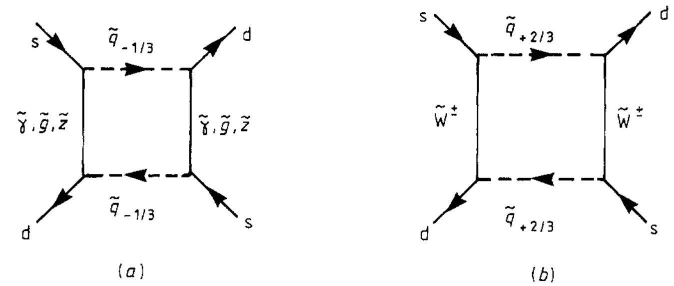

Figure 1. (*a*) Neutral gaugino and (*b*) charged gaugino SUSY box diagrams contributing to the K_1–K_2 mass difference (Barbieri and Gatto 1982, Campbell 1983, Ellis and Nanopoulos 1982a, Inami and Lim 1982).

which is of the same order as the $W^{\pm}$ box diagrams if $m_{\tilde{W}}$ and $m_{\tilde{q}}=O(m_W)$. Our problem is then to derive the desirable form (4.10) in a natural way. Lo and behold, mass matrices of the form (4.10) do occur in most SB SUSY models of F or D type or in softly broken SUSY models where the soft breaking is provided from supergravity effects, as will be discussed later.

Similar results are obtained from the analysis of $g-2$ (Grifols and Méndez 1982, Ellis *et al* 1982c, Barbieri and Maiani 1982) or of the absence of strong *CP*-violation (Ellis *et al* 1982a). Once again, we find that *F*- or *D*-type SB SUSY models, or supergravity induced softly broken SUSY models, easily satisfy very stringent types of constraints (Grifols and Méndez 1982, Ellis *et al* 1982c, Barbieri and Maiani 1982). It is remarkable that realistic SUSY models satisfy automatically severe low energy phenomenological constraints (Barbieri and Gatto 1982, Barbieri and Maiani 1982, Campbell 1983, Ellis *et al* 1976, 1982c, Ellis and Nanopoulos 1982a, Grifols and Méndez 1982, Inami and Lim 1982, Suzuki 1982), which have been the nemesis of other alternatives like technicolour models (Farhi and Susskind 1981).

Being very happy with the low energy front, let us move now to the GUT front.

5. Supersymmetric GUTS

The main reason for supersymmetrising grand unified theories is of course the solution (Kaul 1982, Maiani 1979, Witten 1981) of the cumbersome gauge hierarchy problem. We have seen that a proliferation of the 'low energy' particle spectrum is then necessarily unavoidable. Every 'known' particle, fermion, Higgs boson or gauge boson should have its corresponding superpartner with characteristic mass differences of order $O(M_W)$. Additional problems to the ones discussed in the previous section appear. The new 'low energy' degrees of freedom will definitely modify the standard programme of grand unification and, in general, there is the danger that the whole programme will be mucked up. It is remarkable that in SUSY GUTS the standard success of ordinary GUTS remains more or less intact. So let us see how the unification programme changes. Our SUSY GUT should contain at least the supersymmetrised $SU(3)\times SU(2)\times U(1)$ model. This piece of information is enough to give a kind of general analysis. It is clear from the beginning that the unification point is going to be raised. The new 'light' degrees of freedom involve fermions and scalars; thus their contribution to the various β functions has the effect of delaying the change of the various coupling constants with energy. Notably, the strong coupling constants fall down with energy much smoother than before and so it will take 'longer' for the different coupling constants to 'meet'. At the same time, one expects a larger unification coupling constant. More precisely, in 'minimal' type SUSY GUTS (Dimopoulos and Georgi 1981, Sakai 1981) one finds, for the coefficients of the SU(3), SU(2) and U(1) β functions,

$$\begin{aligned}\beta_3&=9-f\\ \beta_2&=6-f-h/2\\ \beta_1&=-f-3h/10\end{aligned} \tag{5.1}$$

where f represents the number of flavours ($f\geqslant 6$) and h stands for the number of 'light' Higgs doublets ($h\geqslant 2$).

Concerning the coupling constants we get, using equation (5.1),

$$\begin{aligned}
\frac{1}{\alpha_3(m)} &= \frac{1}{\alpha_{SG}} - \frac{1}{2\pi}(9-f)\ln\left(\frac{M_{SX}}{m}\right) \\
\frac{1}{\alpha_2(m)} &= \frac{1}{\alpha_{SG}} - \frac{1}{2\pi}\left(6-f-\frac{h}{2}\right)\ln\left(\frac{M_{SX}}{m}\right) \\
\frac{1}{\alpha_1(m)} &= \frac{1}{\alpha_{SG}} - \frac{1}{2\pi}\left(-f-\frac{3h}{10}\right)\ln\left(\frac{M_{SX}}{m}\right)
\end{aligned} \tag{5.2}$$

where as usual $\alpha_i \equiv g_i^2/4\pi$ $(i=1,2,3)$, α_{SG} is the SUSY GUT unification fine structure constant and M_{SX} is the SUSY GUT unification mass; m is a 'low energy' mass scale larger than or equal to $\sim O(M_W)$. We can recast equations (5.2) in a more useful form:

$$\ln\frac{M_{SX}}{M_W} = \frac{2\pi}{18+h}\left(\frac{1}{\alpha(M_W)} - \frac{8}{3}\frac{1}{\alpha_3(M_W)}\right) \tag{5.3}$$

$$\sin^2\theta_{EW}(M_W) = \frac{(3+h/2)+(10-h/3)\alpha(M_W)/\alpha_3(M_W)}{18+h} \tag{5.4}$$

and

$$\frac{1}{\alpha_{SG}} = \frac{(9-f)/\alpha(M_W) - [6-(8f/3)-h]/\alpha_3(M_W)}{18+h} \tag{5.5}$$

where, for simplicity, we have identified the supersymmetry breaking scale (M_{LESB}) with M_W.

Using equations (5.3)-(5.5), and taking into account higher order corrections, we get (Einhorn and Jones 1982, Ellis *et al* 1982e)

$$M_{SX} \simeq \begin{cases} 6\times 10^{16}\Lambda_{\overline{MS}} & \text{for } h=2 \\ 3\times 10^{15}\Lambda_{\overline{MS}} & \text{for } h=4 \end{cases} \tag{5.6}$$

where the present favourable value of $\Lambda_{\overline{MS}}$ (the QCD scale parameter evaluated in the modified minimal subtraction scheme with four flavours) is between 100 and 200 MeV. The electroweak angle is calculated to be (Einhorn and Jones 1982, Ellis *et al* 1982e):

$$\sin^2\theta_{EW}(M_W)\Big|_{\text{theor.}} = \begin{cases} 0.236\pm 0.003 & \text{for } h=2 \\ 0.259\pm 0.003 & \text{for } h=4 \end{cases} \tag{5.7}$$

while $\alpha_{SG} \simeq 1/24$ to $1/25$ for six flavours and two light Higgs doublets. The present measured value of $\sin^2\theta_{EW}(M_W)$ is (Ellis 1984, Nanopoulos 1980, Langacker 1981):

$$\sin^2\theta_{EW}(M_W)|_{\text{exper.}} = 0.215\pm 0.010\pm 0.007. \tag{5.8}$$

We move next to the m_b/m_τ ratio in SUSY GUTS. Here we find (Einhorn and Jones 1982, Ellis *et al* 1982e):

$$\left(\frac{m_b}{m_\tau}\right)_{\text{SUSY}}\left(\frac{m_b}{m_\tau}\right)^{-1}_{\text{ord.}} = \left(\frac{\alpha_3(M_W)}{\alpha_{SG}}\right)^{8/9}\left(\frac{\alpha_3(M_W)}{\alpha_G}\right)^{-4/7} \tag{5.9}$$

and substituting $\alpha_{SG} \simeq 1/24$, $\alpha_G \simeq 1/41$ and $\alpha_3(M_W) \simeq 0.12$, we get

$$\left(\frac{m_b}{m_\tau}\right)_{\text{SUSY}}\left(\frac{m_b}{m_\tau}\right)^{-1}_{\text{ord.}} \simeq 1 \tag{5.10}$$

and thus by recalling that $(m_b/m_\tau)_{\text{ord.}} \simeq 2.8$–$2.9$, which literally coincides with its experimentally measured value, we declare that (5.10) is a very successful relation. We find this 'coincidence' remarkable. The situation is rather clear. As was expected the unification scale moves upwards and the unification coupling constant increases, as does the electroweak angle always compared with the ordinary GUTS results (Ellis 1984, Nanopoulos 1980, Langacker 1981). The m_b/m_τ remains unchanged, a surprise at least to me! Concerning the value of $\sin^2\theta_{\text{EW}}$, it seems to be a bit high for the case of two light Higgs doublets compared with the experimental value (5.8). On the other hand, the increase of the grand unification scale by a factor of O(10) with respect to ordinary GUTS suppresses the conventional (gauge-boson mediated) proton decay mode, $p \to e^+\pi^0$, by a factor of $O(10^4)$ compared with the ordinary GUTS value, thus evading any conflict with the present experimental lower bound (Ellis 1984, Nanopoulos 1980, Langacker 1981). However, the show is not over! It has been remarked (Weinberg 1982a, Sakai and Yanagida 1982) that in a large class of SUSY grand unified theories, if there are no preventing symmetries, there are loop diagrams that may cause rapid proton decay. For example, by 'dressing up' diagrams of the form shown in figure 2, where sf and $\tilde{H}_{SX}$ represent the SUSY partners of 'light' fermions (f) and 'superheavy' coloured Higgs triplets respectively, one may get 'looping' proton decay (figure 3)

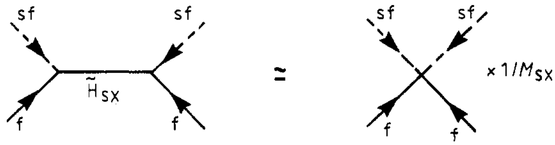

Figure 2. Dimension five operators contributing to proton decay.

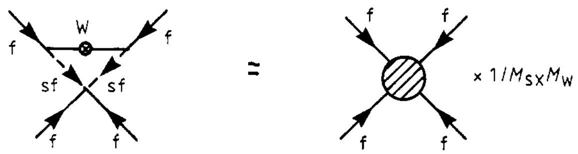

Figure 3. 'Looping' proton decay through dimension five operators.

where, again, $\tilde{W}$ stands for the SUSY partners of the charged weak bosons. The bizarre thing here is that $\tau_p \propto M_{SX}^2 M_W^2$ and not $\tau_p \propto M_{SX}^4$. One may then naively think that these kinds of SUSY theories are dead because they cause a too rapid proton decay (Weinberg 1982a, Sakai and Yanagida 1982). A more careful analysis showed though that things are different (Ellis *et al* 1982e). Indeed, we have found that in such theories the proton lifetime can easily be 10^{31} yr or a bit longer (not much longer, though), and with the very 'peculiar' characteristic decay mode $\bar{\nu}_\tau K^+$ (Ellis *et al* 1982e). The appearance of 'strange' particles in the final decay products of the nucleon should not sound strange. As is apparent from figures 2 and 3, 'looping' nucleon decay involves Yukawa couplings; thus the nucleon will prefer to decay predominantly to the heaviest energetically allowed quark, i.e. the strange quark! Thus in the so-called softly broken SUSY GUTS (without 'preventing' symmetries) we find (Ellis *et al* 1982e)

$$
\begin{gathered}
\tau_N \simeq O(10^{31\pm2})\ \text{yr} \\
\tau(N \to \bar{\nu}_\tau K) \gg \Gamma(N \to \bar{\nu}_\mu K) \gg \Gamma(N \to \bar{\nu}\pi, \mu^+ K) \gg \Gamma(N \to \mu^+\pi) \\
\gg \Gamma(N \to e^+ K) \gg \Gamma(N \to e^+\pi).
\end{gathered} \tag{5.11}
$$

But the surprises are not over. Very recently we have found (Nanopoulos *et al* 1982, Nanopoulos and Tamvakis 1982a, Srednicki 1982a, b) that SUSY GUTs may solve naturally the monopole problem. In doing so, though, we may upset the standard solution of the baryon asymmetry problem. One way to reconcile this puzzle and keep both solutions intact (Nanopoulos *et al* 1982, Nanopoulos and Tamvakis 1982a, Srednicki 1982a, b) is the existence of 'light' superheavy triplets, i.e. $M_{H_3} \sim 10^{10}$ GeV. Further details on this interesting possibility will be given in § 7. Actually we find (Nanopoulos and Tamvakis 1982a, b, c, Ellis *et al* 1984a) in this case $\sin^2 \theta_{EW} \simeq 0.220$, much closer to the experimental value given by equation (5.8) than in other SUSY GUTS (see equation (5.7)). But it is well known (Ellis *et al* 1979a) that such higgsons mediate proton decay with lifetime $\sim O(10^{31\pm 2})$ yr, and we find that in SUSY GUTs the decay modes are given by (Nanopoulos and Tamvakis 1982a, b, c, Ellis *et al* 1984a):

$$\begin{aligned} &\tau_N \simeq O(10^{31\pm2})\ \mathrm{yr} \\ &\Gamma(\bar{\nu}_\mu K^+, \mu^+ K^0) : \Gamma(\bar{\nu}_e K^+, e^+ K^0, \mu^+\pi^0) : \Gamma(e^+\pi^0, \bar{\nu}_e \pi^+) \\ &\quad \simeq 1 : \sin^2\theta_C : \sin^4\theta_C \qquad (\theta_C \simeq \text{Cabibbo angle}). \end{aligned} \tag{5.12}$$

All these predictions have to be contrasted with the ordinary GUT predictions (Ellis 1984, Nanopoulos 1980, Langacker 1981):

$$\begin{aligned} &\tau_N \simeq 10^{29\pm2}\ \mathrm{yr} \\ &B(N \to e^+ \text{ non-strange}, \bar{\nu}_e \text{ non-strange}, \mu^+ \text{ or } \bar{\nu}_\mu \text{ strange}) : \\ &B(N \to e^+ \text{ strange}, \bar{\nu}_e \text{ strange}, \mu^+ \text{ or } \bar{\nu}_\mu \text{ non-strange}) \\ &\quad = 1 : \sin^2\theta_C . \end{aligned} \tag{5.13}$$

The contrast between equations (5.11), (5.12) and (5.13) is rather dramatic. Apart from the case where the protons decay in the 'conventional' way (equation (5.13)) but with $\tau_p \propto M_{SX}^4$ and M_{SX} as given by equation (5.6) (which will make life very, very difficult, if not impossible), all other possibilities are very interesting and hopefully not impossible to test experimentally. It should be emphasised once more that *proton decay* in *SUSY GUTs* at an *observable rate* always involves *strange particles* (K, ...) in the *final state.* This striking difference in the proton decay modes between SUSY and ordinary GUTs is maintained also in supergravity models, as we shall see later. Experiment will tell us!

One may wonder if there are at all realistic SUSY models encompassing all different phenomenological constraints previously mentioned. Indeed, realistic SUSY model building is not an easy task. However, any effort is worthwhile since SUSY models are left as the only candidates for a physical description of the world, at least up to energies of the Planck scale M_{Pl}. Any high standard(s) SUSY model should satisfy the following *two SUSY golden rules.*

(1) It should provide naturally an acceptable form of SUSY breaking such that

$$\underset{\substack{\uparrow \\ \text{experiment}}}{O((20\ \mathrm{GeV})^2)} \leq m_B^2 - m_F^2 \equiv \tilde{m}^2 \leq \underset{\substack{\uparrow \\ \text{gauge hierarchy}}}{O(M_W^2)} \tag{5.14}$$

where $\tilde{m}^2$ is a typical boson–fermion mass splitting of a supermultiplet. The sparticle mass spectrum should be such that not only all types of low energy constraints are satisfied (e.g. equation (4.10)) but in addition some possible potential problems of the standard model should find a satisfactory resolution.

(2) It should provide a complete solution to the three-fold gauge (scale) hierarchy problem: create, stabilise and *dynamically* explain the scale hierarchy. All (small) mass scales should be determined *dynamically* in terms of one fundamental one, the super-Planck scale $M \equiv M_{Pl}/\sqrt{(8\pi)} \simeq 2.4\times 10^{18}$ GeV:

$$\frac{M_W}{M} \simeq \frac{\tilde{m}}{M} \simeq O(10^{-16}). \tag{5.15}$$

Surprisingly enough, such *no-scale models* (Ellis *et al* 1984b, c, g, h, i, j) have recently been constructed. Since their construction involves a lot of very interesting physics, it is worth discussing the different steps that lead (uniquely?) to them. Furthermore, it should be recalled that we would like to understand in a natural, satisfactory way:

(i) How to separate at the tree level the masses of the Higgs doublet and its GUT partner the coloured Higgs triplet.

(ii) How to incorporate gravitational interactions, etc.

(iii) The absence of the cosmological constant.

A possible answer to all these questions may potentially be found in the framework of local SUSY theories or supergravity (SUGAR), where we move next (for a review, see van Nieuwenhuizen 1981). It should be stressed that the move to supergravity theories is not only for aesthetic reasons, but is entailed by the structure of realistic SUSY models, as discussed above (see the remarks after equation (3.29)).

6. Physical structure of simple ($N = 1$) supergravity

We are then led to consider local SUSY gauge theories (Cremmer *et al* 1978a, 1979, 1982, 1983a). The effective theory below the Planck scale must be $N = 1$ supergravity (Ellis *et al* 1983e). The restriction to $N = 1$ follows from the apparent left-right asymmetry of the 'known' gauge interactions. Since we are dealing with local SUSY, the breaking of SUSY *must be* spontaneous, *not* explicit, if Lorentz invariance or unitarity are not to be violated. It is remarkable that the effective theory below M_{Pl} has been determined (Ellis *et al* 1983e) to be a spontaneously broken $N = 1$ local SUSY gauge theory (Cremmer *et al* 1978a, 1979, 1982, 1983a).

We start with a reminder of the structure of $N = 1$ supergravity actions containing gauge and matter fields (if not explicitly stated, we use natural units $k^2 \equiv 8\pi G_N = (8\pi/M_{Pl}^2) \equiv 1/M^2 = 1$):

$$A = \int d^4x\, d^4\theta\, E\{\Phi(\varphi, \bar{\varphi}e^{2V}) + \mathrm{Re}\,[R^{-1}g(\varphi)] + \mathrm{Re}\,[R^{-1}f_{ab}(\varphi)\, W_\alpha^a \epsilon^{\alpha\beta} W_\beta^b]\} \tag{6.1}$$

where E is the superspace determinant, Φ is an arbitrary real function of the chiral superfields φ and their complex conjugates $\bar{\varphi}$, V is the gauge vector supermultiplet, R is the chiral scalar curvature superfield, g is the chiral superpotential, f_{ab} is another chiral function of the chiral superfields φ, and W_α^a is a gauge-covariant chiral superfield containing the gauge field strength. In addition to all the obvious general coordinate transformations, local supersymmetry and gauge invariance, the action (6.1) is also invariant under the transformations (Cremmer *et al* 1978a, 1979, 1982, 1983a):

$$J \equiv 3 \ln(-\tfrac{1}{3}\Phi) \to J + K(\varphi) + K^*(\bar{\varphi}) \qquad g(\varphi) \to e^{K(\varphi)} g(\varphi) \tag{6.2}$$

where $K(\varphi)$ is any analytic function of φ. These transformations can be related to a description of the chiral superfields φ as coordinates on a Kähler manifold with Kähler potential G, defined by

$$G \equiv J - \ln(\tfrac{1}{4}|g|^2) \tag{6.3}$$

and the transformations (6.2) are known as Kähler gauge transformations (Zumino 1979).

The general couplings of chiral and vector multiplets (3.1) to $N=1$ supergravity is specified by two functions of the complex scalar fields φ_i contained in the chiral multiplets (Cremmer *et al* 1978a, 1979, 1982, 1983a). An analytic function $f_{ab}(\varphi) = f_{ba}(\varphi)$, related to the YM part of the Lagrangian, gives for the kinetic terms of the gauge fields

$$\frac{-1}{4}(\mathrm{Re}\, f_{ab}) F^a_{\mu\nu} F^{b\mu\nu} + \frac{\mathrm{i}}{4}(\mathrm{Im}\, f_{ab}) F^a_{\mu\nu} \tilde{F}^{b\mu\nu} \tag{6.4}$$

(a, b are indices of the adjoint representation of the gauge group G). Then, a real gauge invariant function $G(\varphi, \varphi^*)$, the Kähler potential (6.3), defines the scalar kinetic terms, given by

$$G^i_j (\partial_\mu \varphi^j)(\partial^\mu \varphi^*_i) \tag{6.5}$$

in the notation

$$G_i \equiv \frac{\partial G}{\partial \varphi^i}, \qquad G^i \equiv \frac{\partial G}{\partial \varphi^*_i}, \qquad G^i_j \equiv \frac{\partial^2 G}{\partial \varphi^*_i \, \partial \varphi^j}.$$

The kinetic terms have a form characteristic of supersymmetric non-linear σ models. The scalar fields φ_i in $N=1$ SUGAR span a Kähler manifold with G^i_j as its metric (Zumino 1979). Clearly, the functions $G(\varphi, \varphi^*)$ and $f_{ab}(\varphi)$ largely determine the physics of the $N=1$ SUGAR YM theories (Cremmer *et al* 1978a, 1979, 1982, 1983a). Indeed, the scalar potential V has two terms (Cremmer *et al* 1978a, 1979, 1982, 1983a, Barbieri *et al* 1982c, Arnowitt *et al* 1982, 1983a, b, Bagger 1983, Bagger and Witten 1982a, b):

$$V = V_c + V_g. \tag{6.6}$$

The gauge potential V_g reads

$$V_g = \tfrac{1}{2}(\mathrm{Re}\, f^{-1}_{ab}) D^a D^b \tag{6.7}$$

where the real functions D^a are

$$D^a = g_a G_j (T^a)^j_i \varphi^i \tag{6.8}$$

(g_a is the gauge coupling constant associated with the normalised generator T^a). The 'chiral' potential is

$$V_c = \exp\{G[G_i G^j (G)^{-1j}_i - 3]\}. \tag{6.9}$$

It is apparent from the potential (6.9) that, unlike the global case, spontaneously broken SUSY does not imply $\langle V\rangle > 0$. This is fortunate since one can now obtain spontaneous breakdown of local SUSY (the super-Higgs effect) in Minkowski space ($\langle V\rangle = 0$). The theory also contains a gravitino mass term, $m_{3/2}\bar{\psi}_{\mu L}\sigma^{\mu\nu}\psi_{\nu L}$, with

$$m_{3/2} = \langle \exp(G/2)\rangle. \tag{6.10}$$

The most naive choice for the functions G and f_{ab} would be the ones that provide canonical scalar and vector kinetic terms,

$$G^j_i = \delta^j_i \qquad f_{ab} = \delta_{ab} \tag{6.11}$$

corresponding to a flat Kähler manifold. In such cases, one writes

$$G = \varphi_i \varphi^{i*} + \ln |f(\varphi)|^2 \tag{6.12}$$

where $f(\varphi)$ stands for the gauge invariant superpotential. The 'minimal' choice of G and f_{ab} in (6.11) has some rather unpleasant consequences. The cosmological constant $\langle V\rangle = \Lambda$ is zero due to some unbearable fine-tuning of the parameters in G. Furthermore, scalar boson masses are proportional (Ellis and Nanopoulos 1982b) to the gravitino mass (6.10), as they are given by the curvature of the potential (6.9) at the minimum. In this case, the gravitino mass is essentially a free parameter and because of its relation (Ellis and Nanopoulos 1982b) to scalar boson masses or equivalently to $\tilde{m}$ (5.14), it has to be chosen *by hand* $O(M_W)$. *Dynamical* determination of $\tilde{m}$ is excluded, thus violating the second SUSY golden rule.

There is, however, a very elegant way to circumvent these two unsatisfactory points: there exist non-trivial Kähler potentials for which the chiral potential V_c is identically zero. Supersymmetry is, however, *broken*. Vacuum expectation values are not determined by the classical theory, ditto for the gravitino mass. The cosmological constant is *naturally* zero (Cremmer *et al* 1983b, Chang *et al* 1983). As we shall see later, radiative corrections are then used (Ellis *et al* 1984b, c, g, h, i, j) to determine the various scales of gauge symmetry breaking, which in general will be closely related to the gravitino mass. Eureka! This is what we are aiming at to satisfy the second SUSY golden rule. In principle, it is sufficient to require zero chiral potential only in the direction of a gauge singlet complex scalar z field, the Polonyi field. In such a case, we may rewrite (6.10) as (Cremmer *et al* 1983b):

$$V_c = 9\exp(\tfrac{4}{3}G)G^{-1}_{zz^*}\frac{\partial^2}{\partial z\,\partial z^*}\exp(-\tfrac{1}{3}G) \tag{6.13}$$

and

$$V_c \equiv 0 \qquad \text{implies} \qquad \frac{\partial^2}{\partial z\,\partial z^*}\exp(-\tfrac{1}{3}G) = 0$$

with solution (Cremmer *et al* 1983b)

$$G = -3\ln(z + z^*). \tag{6.14}$$

The scalar kinetic term $G_{zz^*}(\partial_\mu z)(\partial_\mu z^*)$ is never canonical, and the gravitino mass is (Cremmer *et al* 1983b)

$$m_{3/2} = \langle (z + z^*)^{-3/2}\rangle. \tag{6.15}$$

$m_{3/2}$ is *undetermined* but non-zero since

$$\langle G_{zz^*}\rangle = 3(m_{3/2})^{4/3} \neq 0. \tag{6.16}$$

$V_c \equiv 0$ entails a very particular geometry of the Kähler manifold. The Kähler curvature

$$R_{zz^*}\left(\equiv \frac{\partial^2}{\partial z\, \partial z^*} \ln G_{zz^*}\right)$$

is given by (Cremmer *et al* 1983b):

$$R_{zz^*} = \tfrac{2}{3} G_{zz^*} \tag{6.17}$$

or R ($\equiv R_{zz^*}/G_{zz^*}$) $= \frac{2}{3}$. Equation (6.17) means that the Kähler manifold is an Einstein space (maximally symmetric space), i.e. that the scalar field z is a coordinate of the coset space SU(1, 1)/U(1) (Cremmer *et al* 1983b, Ellis *et al* 1984g). The non-compact global SU(1, 1) invariance can be checked explicitly in the whole Lagrangian apart from the gravitino mass term (Ellis *et al* 1984g, Ferrara and van Proeyen 1984, Derendinger and Ferrara 1984). It is very interesting to notice (Cremmer *et al* 1983b) that the $N = 1$ Lagrangian for one chiral multiplet with vanishing potential corresponds, up to the gravitino mass term, to a particular truncation of $N = 4$ supergravity, which is known (Cremmer *et al* 1978b) to possess an SU(1, 1) non-compact global symmetry. Vanishing chiral potentials for an arbitrary number n of chiral multiplets also exist (Cremmer *et al* 1983b, Ellis *et al* 1984b, c, g, h, i, j, Ferrara and van Proeyen 1984, Derendinger and Ferrara 1984). In one case (Ellis *et al* 1984i), the scalar fields ϕ_i, $i = 1, 2, \ldots, n$, are coordinates of an SU(n, 1)/SU(n)$\times$U(1) coset space, which is an Einstein space with curvature given by (6.17) but with $(n+1)$ replacing 2 in the numerator.

Clearly, in both cases $n = 1$ or $n > 1$ the 'flatness' of the potential implies massless scalar bosons, neglecting radiative corrections. There is some kind of 'curvature conservation' between the Kähler manifold spanned from the scalar fields of the chiral multiplets and the chiral potential V_c, as schematically represented by figure 4.

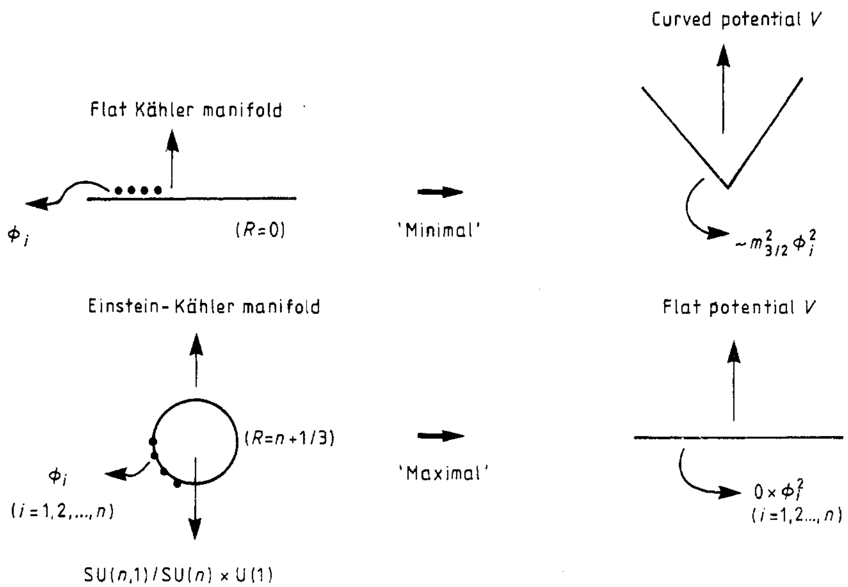

Figure 4. Relation between the form of the Kähler manifold and the induced scalar (chiral) potential.

It should be stressed that higher than four ungauged extended supergravities also exhibit invariances under non-compact global groups which contain SU(1, 1) as a subgroup (Cremmer and Julia 1979):

$$N=5: \mathrm{SU}(5,1); \qquad N=6: \mathrm{SO}^*(12); \qquad N=7,8: \mathrm{E}_{7,7}. \tag{6.18}$$

In addition, extended $N \geqslant 4$ and gauged $N \geqslant 2$ supergravities do inevitably contain (Cremmer *et al* 1978b, Cremmer and Julia 1979) non-minimal kinetic terms for the gauge smultiplets ($f_{ab} \neq \delta_{ab}$ in equation (6.4)). As a result, tree level gaugino masses are introduced, of the form (Cremmer *et al* 1978a, 1979, 1982, 1983a):

$$m_{\tilde{V}} = m_{3/2} \frac{\langle 0|f'|0\rangle}{\langle 0|f|0\rangle} \frac{G'}{G} \tag{6.19}$$

for $f_{ab} = \delta_{ab} f(\phi_i, z)$, and with primes indicating z differentiation. The usefulness of this remark will become apparent shortly. After this unavoidable digression into the 'esoterics' of $N=1$ SUGAR, the road is now open to physical applications.

$N=1$ supergravity YM field theories seem to be the only available framework for physics description below the Planck scale M_{Pl}. Alas, these theories are not renormalisable! This fact should not bother us, since we are going to use the $N=1$ SUGAR framework anyway as an effective theory to describe physics for scales below M_{Pl}.

Indeed, Ellis, Tamvakis and myself (Ellis *et al* 1983e) have suggested interpreting equation (6.1) as an effective action suitable for describing particle interactions at energies $\ll M_{\mathrm{Pl}}$ just as chiral $\mathrm{SU}(N) \times \mathrm{SU}(N)$ Lagrangians were suitable for describing hadronic interactions at energies $\ll 1$ GeV. In much the same way as we know that physics gets complicated at $E = 1$ GeV, with many new hadronic degrees of freedom having masses of this order, we also expect many new 'elementary particles' to exist with masses $\mathrm{O}(M_{\mathrm{Pl}})$. It may well be that all the known light 'elementary particles', as well as these heavy ones, are actually composite, and that at energies $\gg M_{\mathrm{Pl}}$ a simple preonic picture will emerge, analogously to the economical description of high-energy hadronic interactions in terms of quarks and gluons. It may even be that these preonic constituents are themselves ingredients in an extended supergravity theory (Cremmer and Julia 1979, Ellis *et al* 1980). But let us ignore these speculations for the moment and return to our pedestrian phenomenological interpretation of the action (6.1).

The well known rules of phenomenological Lagrangians (Callan *et al* 1968, Coleman *et al* 1968, Weinberg 1968) are that one should write down all possible interactions consistent with the conjectured symmetries (e.g. chiral $\mathrm{SU}(2) \times \mathrm{SU}(2)$), and only place absolute belief in predictions which are independent of the general form of the Lagrangian (e.g. $\pi\pi$ scattering lengths). These are the reliable results which could also be obtained using current algebra arguments. It does not make sense to calculate strong interaction radiative corrections (read: supergravity loop corrections) to these unimpeachable predictions: these are ambiguous until we know what happens at the 1 GeV scale (read: M_{Pl}), and our ignorance can be subsumed in the general form of the phenomenological Lagrangian, in which any and all possible terms are present *a priori* (read: non-trivial J, non-polynomial g and f_{ab}). On the other hand, non-strong interaction radiative corrections can often be computed meaningfully (e.g. the $\pi^+ - \pi^0$ mass difference, large numbers of pseudo-Goldstone boson masses in extended technicolour theories). Similarly, it makes sense to compute matter interaction (gauge, Yukawa, Higgs) corrections to the tree level predictions of the effective action (6.1).

Since the supergravity action is non-renormalisable, and since both the Φ and f_{ab} terms in the action (6.1) have a $\int \mathrm{d}^4\theta$ form, we expect general variants of them to be generated by loop corrections. Presumably, radiative corrections maintain the essential geometry of the Kähler manifold (Zumino 1979). Therefore, we expect loop corrections to fall into the class of Kähler gauge transformations (6.2). The only analogous transformation allowed in a conventional renormalisable theory is $K=$ constant, corresponding to a wavefunction renormalisation. In our case, more general gauge functions $K(\varphi)$ might appear.

Thus it seems that we have enough justification to use the $N=1$ SUGAR framework as an effective theory to describe physics below M, according to the general scheme

$$\mathscr{L}(N=1\ \text{SUGAR}) \xrightarrow[E<M_{\text{Pl}}]{} \mathscr{L}(N=1\ \text{SUSY}) + \mathscr{L}_{\text{SOFT}} \tag{6.20}$$

where $\mathscr{L}_{\text{SOFT}}$ stands for a highly constrained set of soft SUSY breaking terms. The passage described by (6.20) is carried out by making a choice for G (6.3) and f_{ab} (6.4) such that:

(1) Supersymmetry is spontaneously broken;
(2) Certain fields associated with this breaking (z) decouple (*hidden* sector);
(3) Certain fields become superheavy (X);
(4) Remaining fields (φ_i) are to be observed in low energy theory (*observable* sector). After shifting all fields by their VEVs, and discarding terms involving decoupled (z) or superheavy fields (X), $\mathscr{L}_{\text{SOFT}}$ is obtained

$$\begin{aligned}\mathscr{L}_{\text{SOFT}} = m^2 \sum_i |\varphi_i|^2 + \left(m \sum_n (A-3+n) f_n + \text{HC} \right) \\ -\tfrac{1}{2}(m_{\tilde g}\tilde g \tilde g + m_{\tilde W}\tilde W \tilde W + m_{\tilde B}\tilde B \tilde B + \text{HC})\end{aligned} \tag{6.21}$$

while more general forms (Hall *et al* 1983, Soni and Weldon 1983) are certainly possible but irrelevant to our discussion here. A is a model-dependent parameter (Nilles *et al* 1982) of O(1) and f_n is the nth term in the superpotential $f(\varphi_i) = \Sigma_n f_n = \Sigma_n C_n \varphi_i^n$ with $n=1,2,3,\ldots$, not necessarily terminating at 3, because non-renormalisable terms are allowed in these effective theories. If we imagine that the low energy theory is embedded in a GUT model at some GUT scale below the Planck mass, then all gaugino masses are equal at the GUT scale, so that only one single parameter M_0 is needed (M_0 *in principle* may be of $\mathrm{O}(M_{\mathrm{W}})$):

$$m_{\tilde V}(M_{\mathrm{X}}) = M_0 \tag{6.22}$$

while at lower energies $m_{\tilde V}$ evolves in a manner identical for the gauge couplings:

$$\frac{m_{\tilde V}(\mu)}{M_0} = \frac{\alpha_\alpha(\mu)}{\alpha_{\mathrm{G}}}, \qquad \alpha = 1,2,3 \tag{6.23}$$

with α_α, α_{G} the usual SU(3), SU(2), U(1), GUT fine-structure constants. The mass parameters m and $m_{\tilde V}$ in (6.21) depend on the form of G and f_{ab}. We may distinguish three interesting cases (see also figure 4):

(i) '*Minimal*' (6.11) (Barbieri *et al* 1982d, Nilles *et al* 1982)
(All scalar fields z; φ_i satisfy (6.12)):

$$\left\{\begin{matrix} m = m_{3/2} \\ m_{\tilde V} = 0 \end{matrix}\right. \rightarrow \tilde m = \mathrm{O}(m_{3/2}) \tag{6.24(a)}$$

(ii) '*Mini-maxi*' (Ellis *et al* 1984g, j)
(All scalar fields φ_i satisfy (6.12), while the z (Polonyi) field satisfies (6.14)):

$$\begin{cases} m = m_{3/2} \neq 0 \ (6.15) \\ \quad\downarrow \\ \begin{array}{l}\text{undetermined at}\\ \text{tree level}\end{array} \rightarrow \tilde{m} = \mathrm{O}(m_{3/2}) \\ m_{\tilde{V}} = \begin{cases} 0 \\ \text{or} \\ \text{given by (6.19)} \end{cases} \end{cases} \tag{6.24(b)}$$

(iii) '*Maximal*' (Ellis *et al* 1984b, c, h, i)
(All scalar fields z, φ_i satisfy (6.14)-like G):

$$\begin{cases} m = 0; \qquad A = 0 \\ m_{\tilde{V}} \neq 0 \ (6.19) \\ \quad\downarrow \\ \begin{array}{l}\text{undetermined at}\\ \text{tree level}\end{array} \rightarrow \tilde{m} = \mathrm{O}(m_{\tilde{V}}). \end{cases} \tag{6.24(c)}$$

It is interesting to notice that in the 'maximal' case (iii), the emerging low energy theory (Ellis *et al* 1984i) (*observable* sector) is globally supersymmetric ($m = 0$, $A = 0$) and so all the burden of the necessary global SUSY breaking shifts unavoidably to the gaugino mass $m_{\tilde{V}}$ (Ellis *et al* 1984b, c, h, i). If the gaugino mass is non-zero (5.14), then radiative corrections will generate non-zero scalar masses. Thus, we expect the gaugino mass to be $\mathrm{O}(M_{\mathrm{W}})$, but the gravitino mass could *a priori* be very different (Ellis *et al* 1984b, c, h). This possible *decoupling* (Ellis *et al* 1984b, c, h) of the local SUSY breaking parameter ($m_{3/2}$) and the global SUSY breaking parameter ($\tilde{m}$) has some very interesting particle physics and cosmological implications to be discussed later. In order to cover all cases, $\tilde{m}$ will generically stand for either $m_{3/2}$ or $m_{\tilde{V}}$, both originating from supergravity and providing the 'seed' for global SUSY breaking. Clearly, the form of $\mathscr{L}_{\mathrm{SOFT}}$ as given by (6.21) is simple enough.

In the physics applications which follow, we shall make extensive use of two main characteristics of the general framework discussed above. First, since we are dealing with an effective theory (the $N = 1$ SUGAR action is non-renormalisable), the superpotential g is not any longer necessarily constrained by renormalisability to be at most cubic, but it may contain any higher powers, suitably scaled, by inverse powers of M_{Pl}, the natural cut-off of the theory (Ellis *et al* 1983e). Secondly, because of the non-renormalisation theorems (Ferrara *et al* 1974, Ferrara and Piquet 1975, Iliopoulos and Zumino 1974, Grisaru *et al* 1979, Wess and Zumino 1974b) of SUSY ('set it and forget it' principle), we may set, as we wish, certain parameters equal to zero, even if no symmetry implies that—a very different situation from ordinary gauge theories. Here, no apologies are needed. As explained in detail before, most of the physics is contained in the 'observable' sector superpotential $f(\varphi_i)$. Here we shall assume that, in one way or another, the 'hidden' sector has played its role, as discussed previously, and we shall concentrate on the form of $f(\varphi_i)$. We follow the natural (cosmic) evolution of things starting at energies below M_{Pl} and 'coming down' to M_{W}. So we distinguish physics around the GUT scale (M_{X}) and physics around the electroweak (EW) scale (M_{W}).

All physics from M_{Pl} down to (and including) low energies should emerge from such a programme. We will show next that this is indeed possible.

7. Physics with simple ($N = 1$) supergravity

7.1. *Physics around the* GUT *scale* (M_{X})

The superhigh energy regime ($\sim 10^{16}$ GeV) is the theorists' paradise. There is a lot of freedom in building models, even though the constraints both from particle physics and cosmology become tighter and tighter. For definiteness, simplicity and out of habit, we shall take as our prototype GUT an SU(5) type model (Ellis 1984, Langacker 1981, Nanopoulos 1980). All GUT physics information will be contained in f_{GUT}, the GUT part of the 'observable sector' superpotential. There is no consensus about the definite form of this superpotential, but it should unavoidably contain a piece (f_{I}) that breaks SU(5) down to SU(3) × SU(2) × U(1) and, if possible, a piece (f_{II}) providing some explanation about the tree-level gauge hierarchy problem, so we write:

$$f_{\mathrm{GUT}} = f_{\mathrm{I}} + f_{\mathrm{II}}. \tag{7.1}$$

For example, we may take (Nanopolous *et al* 1983c):

$$f_{\mathrm{I}} = \frac{a_1}{M} X^4 + \frac{a_2}{M^2} X^2 \operatorname{Tr}(\Sigma^3) \tag{7.2}$$

and (Kounnas *et al* 1983d):

$$f_{\mathrm{II}} = \bar{\theta} H\left(\lambda_1 \frac{\Sigma^2}{M} + \lambda_2 \frac{\Sigma^3}{M^2} + \dots\right) + \bar{H}\theta\left(\lambda'_1 \frac{\Sigma^2}{M} + \lambda'_2 \frac{\Sigma^3}{M^2} + \dots\right) + M_\theta \bar{\theta}\theta \tag{7.3}$$

where $X = \mathbf{1}$, $\Sigma = \mathbf{24}$, $\overset{(-)}{\theta} = \overset{(-)}{50}$, $\overset{(-)}{H} = \overset{(-)}{5}$ are chiral superfields of SU(5). The Higgs fields H and $\bar{H}$ couple to quark and lepton fields in the usual way. All components of θ and $\bar{\theta}$ have a bare mass M (which is taken to be of order M_{X} or larger), and so remain heavy after SU(5) breaks to SU(3) × SU(2) × U(1). After minimising the potential, obtained by plunging into (6.12) and (6.9) the sum of f_{I} and f_{II} as given by (7.2) and (7.3), we get zero VEV for $\overset{(-)}{H}$ and $\overset{(-)}{\theta}$ but non-zero ones for:

$$\begin{aligned} \langle X \rangle &= \left(\frac{m_{3/2}}{M}\right)^{3/8} M \\ \langle \Sigma \rangle &= \left(\frac{m_{3/2}}{M}\right)^{1/4} M. \end{aligned} \tag{7.4}$$

Furthermore, we find (Nanopoulos *et al* 1983c) that the SU(3) × SU(2) × U(1) symmetric minimum is the *lowest one* for *all values* of a_1 and a_2, with a value:

$$V_{\mathrm{eff}} \simeq -\left(\frac{m_{3/2}}{M}\right)^{5/2} M^4. \tag{7.5}$$

What do these results mean? First, since the VEV of Σ sets the scale of SU(5) breaking, we find that the GUT scale M_{X} satisfies (Nanopoulos *et al* 1983c):

$$M_{\mathrm{X}}^4 \simeq \mathrm{O}(m_{3/2} M^3) \tag{7.6}$$

which is a highly successful relation. Using as an input the ratio $(M_X/M) \sim 10^{-2}$–10^{-4}, we obtain that $m_{3/2} \sim O(100\,\text{GeV})$! More generally, relations of the form $M_X^{2p-2} \simeq O(m_{3/2}M^{2p-3})$ with $p \geq 3$, are also possible (Nanopoulos *et al* 1983c) by suitably modifying the exponents in (7.2). The supergravity hierarchy problem has been solved in a rather simple way.

Secondly, the $SU(3) \times SU(2) \times U(1)$ symmetric minimum is lower in energy density than the SU(5) symmetric minimum $X = \Sigma = 0$ by an amount $(m_{3/2}/M)^{5/2}M^4$. Thirdly, the barrier between these two minima is never larger than $(m_{3/2}/M)^{5/2}M^4$, the same as the splitting between the states. Why this is so can be seen by noting that if we replace X by its VEV (7.4) in (7.2), the effective renormalisable self-coupling of Σ is $10^{-12}\,\text{Tr}\,(\Sigma^3)$. Thus we have generated (Nanopoulos *et al* 1983c) a small renormalisable coupling for Σ from our starting point of only non-renormalisable interactions among X and Σ. This small coupling suppresses the barrier between the SU(5) and the $SU(3) \times SU(2) \times U(1)$ phases. The consequences of this suppression for supercosmology (Nanopoulos *et al* 1982, Nanopoulos and Tamvakis 1982a, Srednicki 1982a, b) are difficult to overestimate. Simply, it now makes possible the transition from the SU(5) to the $SU(3) \times SU(2) \times U(1)$ phase at temperatures $T \sim 10^{10}\,\text{GeV}$, which was previously blocked, since the barrier between the two phases was of the order of $(M_X)^4$. Incidentally, in this picture, the number density of GUT monopoles is naturally suppressed below its present experimental upper bound (Nanopoulos *et al* 1982, Nanopoulos and Tamvakis 1982a, Srednicki 1982a, b).

It should be clear that the basic result—small renormalisable couplings arising from non-renormalisable ones suppressed only by inverse powers of M—is quite general and does not depend on the detailed form of the superpotential (f_{I}) (Nanopoulos *et al* 1983c). The main characteristics of these types of models (Nanopoulos *et al* 1983c, Kounnas *et al* 1983b, c) are that they provide relations of the type (7.6); they make possible 'delayed' SU(5) to $SU(3) \times SU(2) \times U(1)$ phase transitions at $T \sim 10^{10}\,\text{GeV}$, and they contain more 'light' particles than the ones in the minimal SUSY $SU(3) \times SU(2) \times U(1)$ model. This last fact may sound dangerous when calculating M_X, $\sin^2\theta_{\text{EW}}$ and m_b/m_τ, since in general an arbitrary increase of 'light' stuff gives an out-of-hand increase (Ellis *et al* 1982e, Einhorn and Jones 1982, Nanopoulos and Ross 1982) and thus experimentally unacceptable values for the above mentioned quantities. A more careful analysis (Kounnas *et al* 1983a, 1984) of these cosmological acceptable models (CAM) shows that they make predictions as successful (for $\sin^2\theta_{\text{EW}}$, $m_b/m_\tau, \ldots$) as *at least* the ones (Einhorn and Jones 1982, Ellis *et al* 1982e) of the phenomenologically acceptable minimal type models (MIM). For a detailed thorough phenomenological analysis of CAMs, see Kounnas *et al* (1983a, 1984).

Next, we discuss (Kounnas *et al* 1983d) physics related to f_{II} as given in (7.3). The VEV of Σ not only breaks SU(5) to $SU(3) \times SU(2) \times U(1)$ but also provides a mass term which mixes the colour triplets in H and $\bar{H}$ with those in θ and $\bar{\theta}$. However, there is no weak doublet in the **50**, and so the *weak doublets* in H and $\bar{H}$ *remain massless*. The colour triplets will have a mass matrix (Kounnas *et al* 1983d):

$$\begin{pmatrix} 0 & \dfrac{M_X^2}{M} \\ \dfrac{M_X^2}{M} & M_\theta \end{pmatrix} \tag{7.7}$$

where M_θ should be of order M_X or larger $(\leq M_{\text{Pl}})$, to avoid having particles from θ

and $\bar{\theta}$ influencing the renormalisation group equations at scales below M_X (or even M_{Pl}). The eigenvalues of this mass matrix are $O(M_\theta)$ and $O(M_X^4/M_\theta M^2)$; this latter eigenvalue is about 10^{10} GeV for $M_X \sim O(10^{16}$ GeV) and $M_\theta \sim O(M_{Pl})$. In this case, the Higgs colour triplet can be used to generate (Nanopoulos *et al* 1982, 1983c, Nanopoulos and Tamvakis 1982a, Srednicki 1982a, b) the baryon number of the universe after the SU(5) to SU(3) × SU(2) × U(1) transition which, as discussed earlier, occurs at temperatures $T \sim 10^{10}$ GeV in CAMS (Kounnas *et al* 1983a, 1984). It is remarkable that $O(10^{10}$ GeV) is the lower bound (Ellis *et al* 1979a) allowed for colour triplet Higgs masses from present limits (Ellis 1984, Langacker 1981, Nanopoulos 1980) on proton decay ($\tau_p > 10^{31-32}$ yr). If indeed there are 10^{10} GeV Higgs triplets, then protons should decay predominantly to $\bar{\nu}_\mu$K, μK with a lifetime $\sim O(10^{31-32}$ yr), as discussed before (see equation (5.12)). Of course in this case one has to control the menace of dimension 5 operators mediating proton decay (see figures 2 and 3), because they lead to a catastrophically short proton lifetime (Nanopoulos and Tamvakis 1982a, b, c, Ellis *et al* 1984a).

The role of supergravity in this natural explanation (Kounnas *et al* 1983d) of the Higgs triplet-doublet mass splitting (≡ tree-level gauge hierarchy problem) is fundamental, in several aspects. The same kind of explanation had been suggested before in the framework of renormalisable global SUSY GUTS, where Σ^2 in (7.3) was replaced by a **75** of SU(5) and higher than two powers of Σ were absent (Masiero *et al* 1982, Grinstein 1982). Unfortunately, the use of **75** drastically conflicts with cosmological scenarios (Nanopoulos and Tamvakis 1982a, Nanopoulos *et al* 1982, 1983c, Srednicki 1982a, b, Kounnas *et al* 1983b, c) based on SUSY GUTS. The barrier between the SU(5) and SU(3) × SU(2) × U(1) phases is impossible to overcome *unless* most of the **75** is very light ($\sim M_W$). But then all hell breaks loose. A light **75** makes the gauge coupling in the SU(5) phase decrease at lower energies so there is no phase transition at all (Nanopoulos and Tamvakis 1982a, Nanopoulos *et al* 1982, 1983c, Srednicki 1982a, b). Furthermore, the presence of these new light particles in the SU(3) × SU(2) × U(1) phase changes the renormalisation group equations, and prevents perturbative unification. On the contrary, in SUGAR theories, since we may use non-renormalisable terms, we may replace the fundamental **75** by an 'effective' **75** contained in Σ^2. Unlike a light **75**, a light **24** neither makes the SU(5) gauge coupling decrease at energies below M_X, nor upsets perturbative unification. The previously mentioned cosmological scenarios of Nanopoulos *et al* (1982, 1983c), Nanopoulos and Tamvakis (1982a), Srednicki (1982a, b) and Kounnas *et al* (1983b, c) can proceed without modification. In addition, SUSY non-renormalisation theorems (Ferrara *et al* 1974, Ferrara and Piquet 1975, Iliopoulos and Zumino 1974, Grisaru *et al* 1979, Wess and Zumino 1974b) ensure the stability of the triplet-doublet splitting to all orders in perturbation theory. Since the only modifications of the theory are at the GUT scale M_X, it seems that we have got (Kounnas *et al* 1983d) a harmless and elegant solution of the tree level, and for that matter, to all orders in perturbation theory, gauge hierarchy problem.

SUGAR models give good physics at the GUT scale—unique, cosmologically acceptable breaking of SU(5) to SU(3) × SU(2) × U(1), with an explanation of the smallness of the gravitino mass (Nanopoulos *et al* 1983c) ((7.6)-like relations), and a natural explanation (Kounnas *et al* 1983d) of the Higgs triplet-doublet splitting, cosmologically fitted and general enough. We believe that even if the very specific form of f_I in (7.2) may change, then f_{II} as given by (7.3) (or its obvious generalisation to other GUT models) will always be a useful part of the f_{GUT}.

After finding plausible explanations for the SUGAR hierarchy problem (gravitino mass $\sim O(100$ GeV)), the tree level and higher-order gauge hierarchy problem (triplet-doublet Higgs splitting), it is time to explain the last gauge hierarchy problem (1.2), i.e. why is $M_W/M_X \leqslant O(10^{-13})$? This problem brings us naturally to our next subject.

7.2. *Physics around the electroweak scale* (M_W)

Although there is no consensus on the best way to incorporate grand unification in SUGAR models, a *unique minimal low energy model* has recently emerged (Ellis *et al* 1983c, e, Alvarez-Gaumé *et al* 1983, Ibanez and Lopez 1983). In this model, the physics of the TeV scale is described by an effective $SU(3)\times SU(2)\times U(1)$ gauge theory, in which the breaking of weak interaction gauge symmetry is induced by renormalisation group scaling of the Higgs $(\text{mass})^2$ operators (Ellis *et al* 1983e). Much of the attractiveness of this model stems from the fact that no gauge symmetries or fields beyond those required in any low energy SUSY theory are included. Sometimes, it may happen, as is the case of cosmologically acceptable models (CAMS) (Kounnas *et al* 1983a, 1984), that there are GUT relics which are light ($\sim M_W$), but they do not seem to play any fundamental role at low energies, so we may neglect them in our present discussion. Furthermore, adding random chiral superfields to the low energy theory may be problematic. For example, the presence of a gauge singlet superfield coupled to the Higgs doublets and added to trigger $SU(2)\times U(1)$ breaking (Barbieri *et al* 1982d, Nilles *et al* 1982, Hall *et al* 1983, Soni and Weldon 1983), usually (but not always (Ferrara *et al* 1983)) destroys (Nilles *et al* 1983, Lahanas 1983) any hope of understanding the gauge hierarchy problem; the reason being (Nilles *et al* 1983, Lahanas 1983) that in a GUT theory, the gauge singlet couples not only to the Higgs doublets but also to their associate, superheavy colour triplets. Then we have to try hard (Ferrara *et al* 1983) to avoid $(M_W M)^{1/2} \simeq 10^{10}$ Higgs doublet masses, generated by (Nilles *et al* 1983, Lahanas 1983) one-loop effects involving colour triplets. Something smells fishy.

We focus then on the standard low energy $SU(3)\times SU(2)\times U(1)$ gauge group, containing three generations of quarks and leptons, along with two Higgs doublets, as chiral superfields. The low energy effective superpotential (f_{LES}) of the model consists only of the usual Yukawa couplings of quark and lepton superfields to the Higgs superfields, along, *in general*, with a mass term coupling the two Higgs doublets, H_1 and H_2. Explicitly, in a standard notation:

$$f_{LES} = h_{ij} U_i^c Q_j H_2 + \tilde{h}_{ij} D_i^c Q_j H_1 + f_{ij} L_i E_j^c H_1 + m_4 H_1 H_2 \tag{7.8}$$

where a summation over generation indices (i, j) is understood and $Q(U^c)$ denote generically quark doublets (charge $-\frac{2}{3}$ antiquark singlet) superfields, while $L(E^c)$ refers to lepton doublets (charge $+1$ antilepton singlet) superfields. With the exceptions of the top quark (h_t) Yukawa coupling and the mass parameter m_4, which *in principle* may be of order $O(m_{3/2})$, all other parameters appearing in (7.8) contribute to the masses of the observed quarks and leptons and are known to be small. Neglecting these small couplings, the effective low energy potential (V_{LEP}) can be written as (see (6.9) and (6.21)):

$$\begin{aligned} V_{LEP} = \sum_{i=1}^{3} & (m_{L_i}^2|L_i|^2 + m_{E_i}^2|E_i^c|^2 + m_{Q_i}^2|Q_i|^2 + m_{U_i}^2|U_i^c|^2 + m_{D_i}^2|D_i^c|^2) \\ & + m_1^2|H_1|^2 + m_2^2|H_2|^2 + Ah_t m_{3/2}(U_3^c Q_3 H_2 + \text{HC}) \\ & + Bm_{3/2} m_4(H_1 H_2 + \text{HC}) + h_t m_4(H_1^+ Q_3 U_3^c + \text{HC}) \\ & + h_t^2(|Q_3|^2|U_3^c|^2 + |Q_3|^2|H_2|^2 + |U_3^c|^2|H_2|^2) + \text{`}D\text{ terms'}. \end{aligned} \tag{7.9}$$

The effective parameters appearing in (7.9) take, at large scales ($\sim M_X$), the values:

$$\begin{aligned} &m_1^2(M_X) = m_2^2(M_X) = m_{3/2}^2 + m_4^2(M_X) \\ &m_{Q_i}^2(M_X) = m_{U_i}^2(M_X) = m_{D_i}^2(M_X) = m_{L_i}^2(M_X) = m_{E_i}^2(M_X) = m_{3/2}^2 \\ &A(M_X) = A; \qquad B(M_X) = A - 1; \qquad (i = 1, 2, 3) \end{aligned} \tag{7.10}$$

as dictated by (6.21). It should be stressed once more that the boundary conditions (7.10) are exact, if we only neglect corrections at the Planck scale, ignore the scaling of parameters from M to M_X, and pay no attention to corrections (Hall *et al* 1983, Soni and Weldon 1983) at the GUT scale. All these effects are expected to be small and it is assumed that they do not seriously disturb (7.10) and the picture hereafter.

It is apparent from (7.9) that SUGAR models can easily succeed in giving weak interaction scale masses ($m_{3/2} \sim M_W$) to squarks, sleptons and gauginos (see (6.21)). Alas, SUGAR models also give large positive (mass)^2 to the Higgs doublets, thus making the breaking of SU(2)$\times$U(1) difficult. One way to overcome this difficulty is the introduction (Barbieri *et al* 1982d, Nilles *et al* 1982, Hall *et al* 1983, Soni and Weldon 1983) of a gauge singlet coupled to H_1 and H_2, but, as mentioned above, with disastrous effects (Nilles *et al* 1983, Lahanas 1983) for the gauge hierarchy. A particularly simple solution to the SU(2)$\times$U(1) breaking relies upon the fact that the boundary conditions (7.10) need be satisfied only at M_X (or M), and that large renormalisation group scaling effects can produce a negative value for m_H^2 at low energies (Ellis *et al* 1983e). The full set of renormalisation group equations for the parameters in V_{LEP} (6.21) has been written elsewhere (Inoue *et al* 1982). Here we concentrate on the most interesting equation, the one for the mass-squared of the Higgs (m_2^2), which gives mass to the top quark:

$$\mu\frac{\partial}{\partial\mu}\begin{bmatrix} m_2^2 \\ m_{U_3}^2 \\ m_{Q_3}^2 \end{bmatrix} = \frac{h_t^2}{8\pi^2}\begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} m_2^2 \\ m_{U_3}^2 \\ m_{Q_3}^2 \end{bmatrix} + \frac{|A|^2 h_t^2 m_{3/2}^2}{8\pi^2}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - \frac{h_t^2 m_4^2}{8\pi^2}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - \frac{8\alpha_3}{3\pi} m_{\tilde{g}}^2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \tag{7.11}$$

where we have neglected gauge couplings other than the 'coloured' one, α_3 ($\equiv g_3^2/4\pi$), $m_{\tilde{g}}$ is the gluino mass (see (6.21) and (6.23)), and Yukawa couplings other than h_t, for the top, have been dropped. The physics content of (7.11) is apparent. Since μ is decreasing (we come from high energies down to low energies), the sign of the first two terms in (7.11) is such as to make all m_2^2, $m_{U_3}^2$, $m_{Q_3}^2$ smaller at low energy with the decrease of m_2^2 becoming more pronounced because of the 3:2:1 weighting. On the other hand, the sign of the last two terms in (7.11) is such as to make $m_{U_3}^2$ and $m_{Q_3}^2$ (the squark masses) larger at low energy, but have no direct effect on m_2^2 (notice the 'zeros' in the corresponding matrices in (7.11)). Indirectly though, the net effect on m_2^2 of the last two terms in (7.11) is to enhance further its decrease at low energies, by increasing $m_{U_3}^2$ and $m_{Q_3}^2$, which then drive down m_2^2 via the first two terms of (7.11). This is exactly what we are after! We want large ($\sim M_W^2$) and positive squarks and slepton (masses)^2, *but* negative Higgs (mass)^2 to trigger SU(2)$\times$U(1) breaking. The ways of obtaining negative Higgs (mass)^2 now become clear (see (7.11)). We have to use either a large top Yukawa coupling (h_t), *or* large A, *or* large m_4, *or* a fourth generation to provide large Yukawa couplings, *or* some suitable physically plausible combination of the above possibilities. There are pros and cons for every one of the above situations. In the case of large h_t, a lower bound on the mass of the top quark is set (Alvarez-Gaumé *et al* 1983, Ellis *et al* 1983c, e, Ibanez and Lopez 1983):

$$m_t > O(60\,\mathrm{GeV}) \tag{7.12}$$

which some people may find uncomfortable. We may avoid a large h_t by moving it into the large A (>3) regime (Ellis *et al* 1983c, Claudson *et al* 1983). The price though is high. The phenomenologically acceptable vacuum becomes unstable against tunnelling into a vacuum in which all gauge symmetries, including colour and electromagnetism, are broken. We must (Ellis *et al* 1983c, Claudson *et al* 1983) then arrange things in such a way that the lifetime for this vacuum decay process is greater than the age of the universe. Some people, not without reason, may find this possibility dreadful. We may avoid large h_t and/or large A by using non-vanishing m_4 ($\sim m_{3/2}$) where a rather satisfactory picture then emerges (Kounnas *et al* 1983a, 1984). Some people may object here to the basic assumption of large m_4 ($\sim m_{3/2}$), since in the case of natural triplet-doublet Higgs splitting-type models (Kounnas *et al* 1983d) (see (7.3)), m_4 has a tendency to be small, if not zero, even though other sources of m_4 may be available.

Finally, we come to the possibility of a fourth generation which, suitably weighted, may help us to avoid large h_t, A or m_4. The problem here is that low energy phenomenology (evolution of coupling constants, $m_b/m_\tau, \ldots$) (Kounnas *et al* 1983a, 1984) as well as firm cosmological results like nucleosynthesis (especially ^{4}He abundance) (Olive *et al* 1981), may suffer almost unacceptable modifications. Furthermore, one has to watch out for the mass of the fourth generation charged lepton, since it is going to behave like m_2^2 in (7.11), and thus $m_{L_4}^2$ and $m_{E_4}^2$ may easily go negative, breaking electromagnetic gauge invariance.

Whatever mechanism (if any) turns out to be correct, it is rather remarkable that in SUGAR-type models, there is a simple explanation of the *breaking* of $SU(2)\times U(1)$ and of the *non-breaking* of $SU(3)\times U(1)_{EM}$. Furthermore, for the first time, we have a simple explanation of why $M_W \lll M_X$ (or M), i.e. a *simple solution* of the cumbersome *gauge hierarchy problem.* Starting with a positive Higgs (mass)^2, of order $m_{3/2}^2$ at M_X, and noticing that (see (7.11)) the evolution with μ^2 of the Higgs (mass)^2 is very slow (logarithmic), it is not surprising that we have to come down a long way in the energy scale, before the Higgs (mass)^2 turns negative and is thus able to trigger $SU(2)\times U(1)$ breaking. Another very amazing fact is that the values of the parameters of the low energy world seem to cooperate with us. Since quarks are feeling strong interactions (7.11) tells us that quarks may enjoy large masses (Yukawa couplings) without making squark (masses)^2 negative, because of the last term ($\sim\alpha_3$), which easily balances off large Yukawa couplings, without any sweat. On the other hand, since leptons are not feeling strong interactions, the balance-off between the weak gauge couplings and large Yukawa couplings becomes extremely delicate and could be problematic. How nice that for all three generations, leptons and down quarks weigh less than 5 GeV and *especially* for the third generation that the top quark (t) is heavier than the bottom quark (b). An inverse situation would be disastrous, because, in any reasonable GUT, a very heavy b quark would mean a very heavy τ lepton, thus making electromagnetic gauge invariance tremble in such SUGAR-type schemes. I will not go any further into the esoterics of this type of $SU(2)\times U(1)$ breaking model, since a rather thorough and detailed exposé of these types of theories and of their phenomenological consequences is now available (Kounnas *et al* 1983a, 1984). It should be stressed that things are now very constrained, as we see from table 2, taken from Kounnas *et al* (1983a, 1984) where the *whole* low energy spectrum is worked out in terms of very few parameters,

Table 2. Particle spectrum (Kounnas *et al* 1983a, 1984). Physical mass spectrum of the cosmologically acceptable model (CAM) and the minimal model (MIM) corresponding to the same gravitino mass $m_{3/2} \simeq$ 15 GeV for top quark masses equal to 25 GeV, 35 GeV and 50 GeV respectively. ξ denotes the ratio of the gaugino to the gravitino mass at M_X. All masses are in GeV units. The light neutral Higgs gets its mass via radiative corrections.

		CAM	MIM	CAM	MIM	CAM	MIM
$A(M_X)$		3	3	2.8	2.8	2.0	1.6
$m_{3/2}$		15	15	15	15	15	15
ξ		2.8	2.2	3.2	3.1	3.5	1.9
$m_4(M_X)$		15	17	16	18	11	7
Top		25	25	35	35	50	50
All families	$(\text{sleptons})_L$	29	27	32	36	35	25
	$(\text{sleptons})_R$	21	20	22	23	23	19
1st and 2nd families	$(\text{squark})_L$	58	77	66	108	72	67
	$(\text{squark})_R$	54	74	61	104	67	65
		54	74	60	103	66	65
3rd family	$(\text{sbottom})_L$	58	76	64	106	68	66
	$(\text{sbottom})_R$	54	74	60	104	66	65
	$(\text{stop})_L$	81	96	95	132	112	106
	$(\text{stop})_R$	26	54	23	78	21	37
	Charged Higgs	96	93	95	94	88	83
	Neutral Higgs	106	104	105	105	100	95
		3	3	4.4	5.3	6	5
	'Axion'	51	46	49	48	35	19
	Gluinos	42	84	47	118	52	72
	Photino	11	4.6	9.4	7.3	4	3
	HW, WH-inos	89	87	87	94	84	90
		78	82	79	79	82	75
	HZ, HZ-inos	99	108	99	116	101	106
		92	85	91	80	88	83
	Axino	26	23	24	24	17	9

$m_{3/2}$, $A(M_X)$, $\xi \equiv M_0/m_{3/2}$ (see (6.22)) and $m_4(M_X)$. Eventually, with more theoretical insight, we hope to determine even these very few parameters, thus predicting *uniquely* the low energy spectrum. For example, we have already discussed ways of determining $m_{3/2}$ (see (7.6)), while some people may favour $A(M_X) = 3$ as a natural solution (Cremmer *et al* 1983b) to the absence of the cosmological constant problem, etc. Among other interesting things contained in the table, the existence of a very light ($\sim$(3–6) GeV) neutral Higgs, with the usual Yukawa couplings to matter, should not escape our attention. Since such a particle is a common feature of a large class of models (Kounnas *et al* 1983a, 1984, Ellis *et al* 1983c), a search in the $\Upsilon \to H^0 + \gamma$ channel, which is expected to be a few per cent of the $\Upsilon \to \mu^+\mu^-$ decay, may turn out to be very fruitful.

7.3. *No-scale models*

As already mentioned above, the complete solution of the gauge hierarchy problem demands both scales, M_W and $m_{3/2}$, to be explained. We saw before how by using non-renormalisable interactions, e.g. equation (7.2), we can relate the gravitino mass to the M_X, e.g. equation (7.6), working only at the tree level. Some people may find this approach a bit *ad hoc* and it may be very much dependent on the specific form of the non-renormalisable terms, which deviates slightly from the standard philosophy of effective Lagrangians: put more weight on results emerging from the general symmetries and not from the very specific form of the effective Lagrangian. Well, here is a résumé of our new, more ambitious approach (Ellis *et al* 1984b, c, g, h, i, j).

An interesting possibility in this framework is the determination of the weak interaction scale by dimensional transmutation (Kounnas *et al* 1983a, 1984, Ellis *et al* 1983c). The multiplicative renormalisation (7.11) of the soft SUSY breaking mass parameters means that the renormalisation scale μ_0 at which m_H^2 goes negative is *independent* of the magnitude of $\tilde{m}^2$. The value of μ_0 is determined by the logarithmic rate of evolution specified by the renormalisation group equations (7.11). Hence

$$\mu_0 = M \exp\left(-\frac{\mathrm{O}(1)}{\alpha_\mathrm{t}}\right). \tag{7.13}$$

Once $m_H^2 < 0$, it is possible to have weak gauge symmetry breaking and $M_\mathrm{W} = \mathrm{O}(\mu_0)$, implying through (7.13) the highly desirable relation

$$\frac{M_\mathrm{W}}{M} \simeq \exp\left(-\frac{\mathrm{O}(1)}{\alpha_\mathrm{t}}\right) \simeq 10^{-16}. \tag{7.14}$$

The *dynamical determination of* M_W has been *realised* (Kounnas *et al* 1983a, 1984, Ellis *et al* 1983c). The first half (5.15) of the second SUSY golden rule has been satisfied. It should be emphasised that we have tacitly assumed $\tilde{m} < \mu_0$, otherwise the renormalisation group equations (RGE) will be frozen at some renormalisation scale $\mu > \mu_0$, implying $m_H^2 > 0$ and thus no $\mathrm{SU(2)_L \times U(1)}$ breaking. But who determines $\tilde{m}$? In the 'minimal' case (6.24(*a*)), $\tilde{m}$ is put in by hand and that is no good. In the 'mini-maxi' or 'maximal' cases (6.24(*b*) and (*c*)) $\tilde{m}$ is undetermined at the tree level and we should use non-gravitational standard model radiative corrections to determine it, thus finally realising the no-scale model dream (Ellis *et al* 1984b, c, g, h, i, j).

To explore the basic mechanism (Ellis *et al* 1984j) for this trick, we consider the usual low energy Higgs potential (7.9) of the SUSY standard model, in an idealised limit where the mixing between the two light Higgs doublets is neglected

$$V = \left(\frac{g_2^2 + g'^2}{8}\right)(|H_1|^2 - |H_2|^2)^2 + m_1^2|H_1|^2 + m_2^2|H_2|^2. \tag{7.15}$$

We denote by H_2 the Higgs field coupled to the t quark. Radiative corrections (7.11) drive $m_2^2 < m_1^2$ and when $m_2^2 < 0$ weak $\mathrm{SU(2)_L \times U(1)}_Y$ gauge symmetry is spontaneously broken and the vacuum energy can become negative:

$$V_{\text{min.}} = -\frac{2|m_2^2|^2}{(g_2^2 + g'^2)} \propto (-1)\tilde{m}^4. \tag{7.16}$$

In writing equation (7.16), we have recalled that since the Higgs mass is multiplicatively renormalised (7.11) for $\mu > \mathrm{O}(\tilde{m})$, the Higgs mass is always $\propto \tilde{m}$, and hence $V_{\text{min.}} \propto |m_2^2|^2 \propto \tilde{m}^4$. The negative coefficient in equation (7.16) means that increasing $|m_2^2|$ and hence $\tilde{m}^2$ is energetically preferred, at least for small values of $\tilde{m}$. However, if $\tilde{m}$ gets to be larger than the scale μ_0 at which m_2^2 falls through zero, then the evolution of m_2 with the renormalisation scale μ will become truncated at $\mu = \mathrm{O}(\tilde{m}_2) > \mu_0$, m_2^2 will never become negative, and the potential will always be positive semidefinite. The general form of $V_{\text{min.}}(\tilde{m})$ is therefore as shown in figure 5.

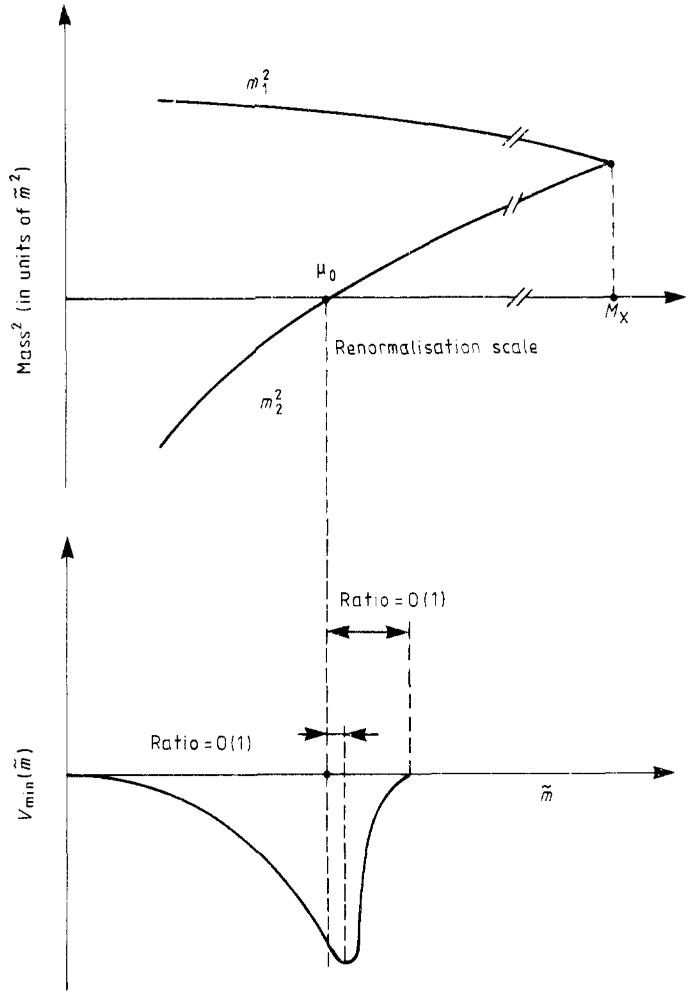

Figure 5. Sketch of the variation of the SUSY-breaking mass parameters m_1^2, m_2^2 with the renormalisation scale μ. The Higgs (mass)^2 $m_2^2=0$ at a scale μ_0, which determines the dynamically preferred value of $\tilde{m}$, as seen in the bottom half of the figure.

It decreases from zero to negative values as $\tilde{m}$ increases from zero, but then rises to zero again for some $\tilde{m}=O(\mu_0)$. (The precise value depends on the choice of renormalisation scheme, but physical parameters such as particle masses do not depend on this choice.) It is apparent from figure 6 that there must (Ellis *et al* 1984g, j) be a dynamically preferred value of $\tilde{m}$ in the range $(0, \mu_0)$. Indeed, one finds (Ellis *et al* 1984g, j):

$$V_{\text{min.}}(\tilde{m}) = -c^2\tilde{m}^4 \ln^2 \frac{\tilde{m}^2}{d(\mu_0^2)} \tag{7.17}$$

with c and d calculable parameters. Minimising the potential (7.17) with respect to $\tilde{m}$, one finds a minimum at

$$\tilde{m} \simeq O(\mu_0). \tag{7.18}$$

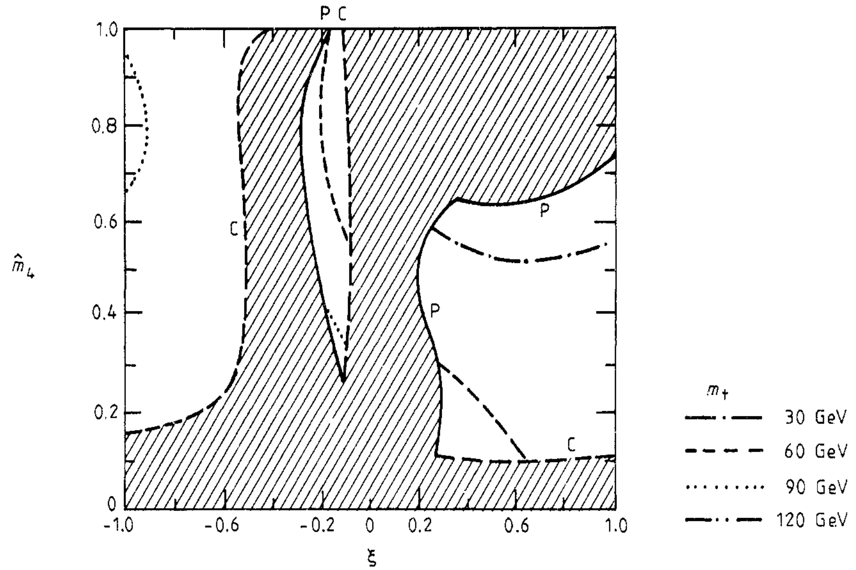

Figure 6. Values of m_t (in GeV) for general values of the Higgs mixing parameter $\hat{m}_4$, $\xi = m_{\tilde{V}}/m_{3/2}$ and $A = 3$. Our present vacuum is unstable in the allowed region on the right of the figure. The shaded domain indicates values of $\hat{m}_4$ and ξ disallowed because of the absence of a charged sparticle with mass less than 20 GeV (P), and/or because of an excessive cosmological density of the lightest neutral sparticle (C).

Remember that μ_0 is a dimensional transmutation scale (7.13), so that the preferred value of $\tilde{m}$ is also fixed by dimensional transmutation, leading through equations (7.14) and (7.18) to the golden relation (5.15),

$$\frac{M_W}{M} \simeq \frac{\tilde{m}}{M} \simeq \exp\left(-\frac{O(1)}{\alpha_t}\right) \simeq 10^{-16}. \tag{7.19}$$

This simultaneous and dynamical determination (Ellis *et al* 1984b, c, g, h, i, j) of M_W and $\tilde{m}$ combined with the non-renormalisation theorems (Ferrara *et al* 1974, Ferrara and Piquet 1975, Iliopoulos and Zumino 1974, Grisaru *et al* 1979, Wess and Zumino 1974b) of global SUSY (stabilisation) and with the classic 'missing partner' mechanism (6.21) (Kounnas *et al* 1983d, Masiero *et al* 1982, Grinstein 1982) (natural tree-level GUT Higgs–electroweak Higgs splitting), *completely* solves the gauge (scale) hierarchy problem.

No-scale SUSY standard models (Ellis *et al* 1984g, j) contain three adjustable parameters: possible non-zero gaugino masses $\xi \equiv m_{\tilde{V}}/m_{3/2}$, non-zero H_1H_2 mixing characterised by a mixing parameter m_4 ($\hat{m}_4 \equiv m_4/m_{3/2}$) and the A parameter (6.21) (Nilles *et al* 1982). As seen in figure 6, we find (Ellis *et al* 1984g, j) domains of ξ and $\hat{m}_4$ which give phenomenologically acceptable models for which all charged sparticles have masses above 20 GeV (denoted by P), and the cosmological density of the lightest stable neutral sparticle is less than 2×10^{-29} g cm^{-3} (denoted by C).

No-scale SUSY GUT models have also been constructed (Ellis *et al* 1984i). In most cases, it is *obligatory* (Ellis *et al* 1984i) to use the 'maximal' alternative (6.24(*c*)) because global SUSY ($m = 0$, $A = 0$), at the GUT scale M_X is badly needed in order to get rid of the highly *un*desirable huge GUT vacuum energy. That is good news, since the decoupling entailed in principle between $m_{3/2}$ and $\tilde{m}$ is very welcome. There are two very interesting physical implications of this gravitino mass liberation movement

(Ellis *et al* 1984b, c, h), one cosmological and the other hierarchical. As is well known (Pagels and Primack 1982, Weinberg 1982b), standard cosmological constraints (critical energy density, primordial nucleosynthesis, etc) *exclude* gravitino masses in the region 1 keV to 10^4 GeV, thus excluding $m_{3/2} \simeq \mathrm{O}(M_{\mathrm{W}})$. Inflation can save (Ellis *et al* 1982d) the gravitino by diluting its original number density, but gravitino regeneration processes (Krauss 1983, Nanopoulos *et al* 1983a) put a rather low upper bound (Ellis *et al* 1984f, Krauss 1983, Nanopoulos *et al* 1983a, Khlopov and Linde 1984) on the reheating temperature $T_{\mathrm{R}} \leqslant \mathrm{O}(10^{10}\,\mathrm{GeV})$. This is rather unfortunate, because then 'standard' baryosynthesis (Olive 1984) is in conflict with long-lived protons (Ellis *et al* 1984a, Kim *et al* 1984). This is the gravitino problem. It seems that we need either superlight (<1 keV) or (super)heavy ($>10^4$ GeV) gravitinos. No-scale models encompassing both possibilities have been constructed (Ellis *et al* 1984b, c, h), thus resolving the gravitino problem in a Gordian-knot way by 'cutting' the $m_{3/2} \sim \mathrm{O}(M_{\mathrm{W}})$ relation. Actually, the superlight gravitino case has a very interesting hierarchical implication (Ellis *et al* 1984b, c): it completely solves the strong *CP* problem (5.1). One simply notices from (6.4) that non-minimal gauge boson kinetic terms, necessarily needed for non-vanishing gaugino masses (6.19), give rise to $\theta_{\mathrm{QCD}} \sim \langle \mathrm{Im}\, f_{ab} / \mathrm{Re}\, f_{ab} \rangle$. Then, since there are *two* dynamical degrees of freedom (Re z, Im z), it is evident that these will determine dynamically two physical parameters. Since the gravitino is decoupled, we may naturally choose $m_{\tilde{V}}$ and θ_{QCD} as the two dynamically determined quantities. While standard model radiative corrections fix the gaugino mass $m_{\tilde{V}}$, the only non-trivial dynamical dependence on θ_{QCD} comes from non-perturbative QCD effects which favour $\theta_{\mathrm{QCD}} = 0$. The dynamical freedom accorded to us by the Im f_{ab} field enables us to 'relax' to $\theta_{\mathrm{QCD}} = 0$, thus *completely* solving (Ellis *et al* 1984b, c) the strong *CP* problem. In this case, in a class of no-scale models (Ellis *et al* 1984b, c), the gravitino mass is determined to be

$$m_{3/2} = \mathrm{O}\left(\frac{m_{\tilde{V}}^{p}}{M^{p-1}}\right) \ll m_{\tilde{V}} \tag{7.20}$$

with $1 < p < 2$, in order to avoid too singular couplings to ordinary matter (Fayet 1980).

Turning now to baryon decay, an interaction of the form (Ellis *et al* 1983e):

$$f \ni \frac{\lambda}{M} \bar{F}TTT \tag{7.21}$$

where $\bar{F}$ is a $\bar{\mathbf{5}}$ of matter (quark+lepton) chiral superfields in SU(5), T is a $\mathbf{10}$ of matter superfields and λ is some generic Yukawa coupling, could replace the Higgs exchange in the Weinberg-Sakai-Yanagida (Weinberg 1982a, Sakai and Yanagida 1982) loop diagram for baryon decay. The magnitude of the diagram with (7.21) relative to the conventional Higgs diagram (see figure 3) is:

$$\left[\frac{\lambda}{M}\right] \Big/ \left[\frac{\lambda^2}{M_{H_3}}\right] \simeq \left[\frac{M_{H_3}}{\lambda M}\right]. \tag{7.22}$$

The ratio (7.22) could easily be >1, making a non-renormalisable superpotential interaction the dominant contribution to proton decay. A careful analysis of SUGAR-induced baryon decay shows (Ellis *et al* 1983b), surprisingly enough, that the expected hierarchy of decay modes is similar to that (Ellis *et al* 1982e, 1984a, Nanopoulos *et al* 1982, Nanopoulos and Tamvakis 1982a, b, c, Sakai and Yanagida 1982, Srednicki 1982a, b, Weinberg 1982a) coming from conventional minimal SUSY GUTs as given by (5.11) and (5.12). One might have wrongly expected that no hard and fast predictions could be made about gravitationally induced baryon decay modes. Anyway, this mechanism could give observable baryon decay even if the GUT mass $M_X \simeq M$.

Incidentally, similar terms like (7.21) have been considered (Nanopoulos and Srednicki 1983a) in efforts to explain the 'lightness' of the first two generations of quarks and leptons. One replaces (Nanopoulos and Srednicki 1983a) direct Yukawa couplings for the first two generations with (very schematically)

$$f \ni \frac{\tilde{\lambda}_2'}{M} \bar{H}\Sigma T_2 \bar{F}_2 + \frac{\tilde{\lambda}_2'}{M} H T_2 T_2 \Sigma + \frac{\tilde{\lambda}_1'}{M^2} \bar{H}\Sigma^2 T_1 \bar{F}_1 + \frac{\tilde{\lambda}_1'}{M^2} H T_1 T_1 \Sigma^2 + \ldots \qquad (7.23)$$

which not only repairs (Ellis *et al* 1983e, Nanopoulos and Srednicki 1983a) wrong relations like $m_d(M_X) \simeq m_e(M_X)$, very difficult to correct (Ibanez 1982, Masiero *et al* 1983) in conventional SUSY GUTs, *but* also provides reasonable masses for the first two generations. Indeed, it follows from (7.23) that the second generation is getting masses $(M_X/M)M_W \sim (0.1\text{-}1\,\text{GeV})$, while the first generation masses are $(M_X/M)^2 M_W \sim (1\text{-}10\,\text{MeV})$, exactly what was ordered. It is amazing that in SUGAR models, by increasing M_X ($\sim 10^{16}$ GeV), relative to its ordinary GUT value (10^{14} GeV), and by decreasing M_{Pl}, what is relevant is the super-Planck scale M ($\sim 10^{18}$ GeV), the highly desired ratio $(M_X/M) \sim 10^{-2}$, which appears naturally. It seems now, for the first time, that gravitational interactions may be responsible for the masses of at least the first two generations. Once more, non-renormalisable interactions contained in SUGAR models provide a simple solution (Nanopoulos and Srednicki 1983a) to another hierarchy problem, the *fermion mass hierarchy problem*.

I hope I have convinced you by now that (no-scale) phenomenological supergravity models are worth considering and I proceed next to discuss experimental evidence for (or against) them.

8. Experimental evidence (?) for supersymmetry

Low energy supersymmetric models have a very rich structure that makes them experimentally vulnerable at accessible (present or very near future) energies, if SUSY indeed solves the gauge hierarchy problem (see equation (5.14)). Since several reports exist, covering, in rather lengthy detail, the numerous experimental consequences of SUSY (Nanopoulos and Savoy-Navarro 1984, Haber and Kane 1984b) I shall limit my discussion here to the relation between SUSY and the new experimental results mentioned in the beginning.

Let me start with *direct evidence*, i.e. with SUSY particle production. As I emphasised before (see (3.2) and (3.3)), SUSY particles in general can only be produced in pairs. This means that among the decay products of every sparticle, there must be another sparticle, and hence the lightest sparticle must be stable. The lightest sparticle is probably neutral (Ellis *et al* 1984d) and not strongly interacting and the most likely candidate may be the photino $\tilde{\gamma}$. Thus, *a characteristic signature* for SUSY could be (Nanopoulos and Savoy-Navarro 1984, Haber and Kane 1984b) *missing energy-momentum* carried away by weakly interacting photinos, for example, from gluino or squark-pair production (Kane and Leveille 1982, Harrison and Llewellyn Smith 1982, 1983):

$$\mathrm{p} + \bar{\mathrm{p}} \to \tilde{g}\tilde{g} + \mathrm{X} \qquad \tilde{q}\tilde{q} + \mathrm{X}$$
$$\tilde{g} \to q q \tilde{\gamma} \qquad \tilde{q} \to q \tilde{\gamma} \qquad (8.1)$$

or from supersymmetric W decay

$$\begin{array}{l} p+\bar{p} \longrightarrow W+X \\ \qquad\qquad \hookrightarrow \tilde{W}+\tilde{\gamma} \\ \qquad\qquad\qquad \hookrightarrow q\bar{q}\tilde{\gamma};\ l\nu\tilde{\gamma}. \end{array} \tag{8.2}$$

It is highly exciting that the observed UA1 and UA2 'Zen' (UA1 1984b) or 'Wen' (UA2 1984) events, with their characteristic large missing energy-momentum, may be naturally due to production and decay of SUSY particles. Indeed, detailed studies (Ellis and Kowalski 1984a, b, Reya and Roy 1984a, b, Haber and Kane 1984a, Barger *et al* 1984, Allan *et al* 1984) have shown that the UA1 'Zen' (monojet) events (UA1 1984b) could be due to either squark or gluino production (and decay), according to (8.1), if either $m_{\tilde{q}}$ or $m_{\tilde{g}} = O(40\ \mathrm{GeV})$; lower masses down to O(30 GeV) cannot yet be firmly (?) excluded. As is apparent from figure 7, for $m_{\tilde{g}} \leq O(40\ \mathrm{GeV})$ and when all experimental cuts are taken into account, the total cross section for $\tilde{g}\tilde{g}$ production is dominated by the one-jet final state (Ellis and Kowalski 1984a, b). Similar results hold true for

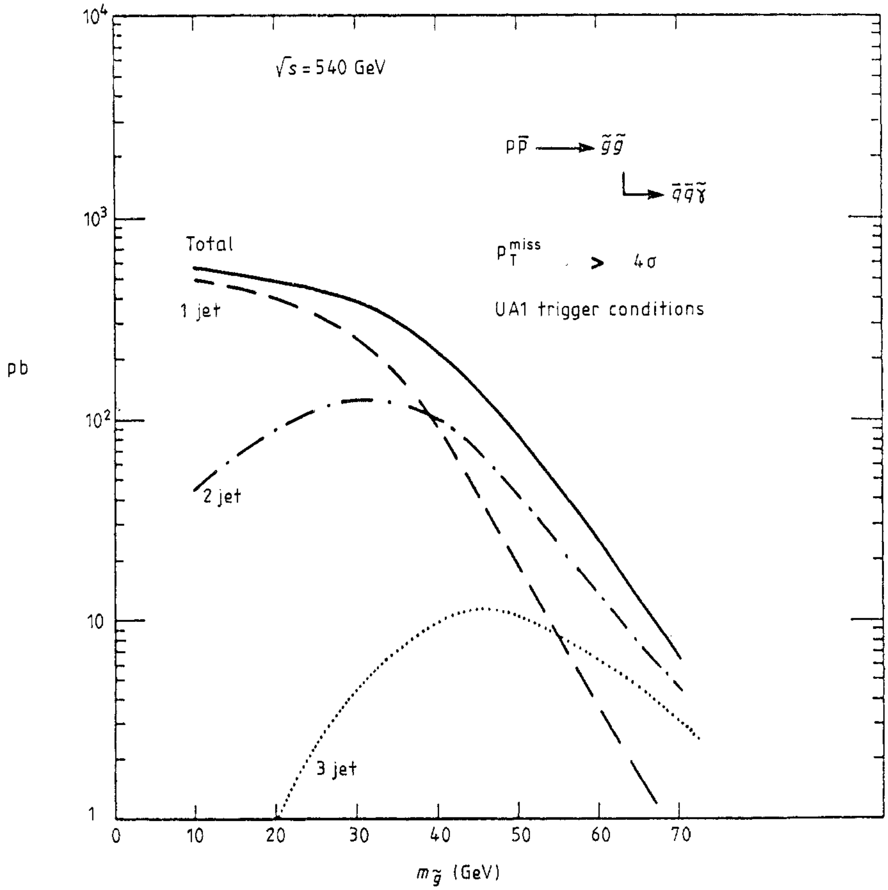

Figure 7. The total and topological cross sections for $\tilde{g}\tilde{g}$ production followed by $\tilde{q} \rightarrow q\bar{q}\gamma$ decay giving one-, two- and three-jet final states with $p_T^{miss.} > 4$, fulfilling the UA1 trigger requirements (from Ellis and Kowalski 1984a, b).

the squark case. So, despite the naive expectations from (8.1) that $\tilde{g}\tilde{g}$ or $\tilde{q}\tilde{q}$ production should give four-jet or two-jet final states respectively, surprisingly enough for $m_{\tilde{g}}$ or $m_{\tilde{q}} \lesssim O(40\,\text{GeV})$ one-jet final states dominate, in accordance with experiment (UA1 1984b).

Are then the observed events with large $p_T^{\text{miss.}}$ due to the production of either $\tilde{g}$ or $\tilde{q}$ with mass O(40 GeV)? Both are possible interpretations of the UA1 data (UA1 1984b), but neither can be confirmed or refuted until more data are accumulated. Nevertheless, the squark interpretation has been favoured (Ellis and Kowalski 1984a, b) on two grounds: (i) the hardness of the observed missing $p_\perp$ spectrum, which is more naturally explained by two-body $\tilde{q} \to q + \tilde{\gamma}$ decays and (ii) the thinness of the observed monojets, which disfavours $\tilde{g} \to q + \bar{q} + \tilde{\gamma}$ decay which yields monojets with invariant masses up to O(20 GeV) and an average of O(10 GeV), while the squark decay yields monojet invariant masses O(2 GeV) consistent with the observed ones. Other more contrived explanations are still possible. For example, if $\tilde{g}$ and $\tilde{\gamma}$ are both very light and approximately degenerate (~3 GeV) to ensure long enough $\tilde{g}$ lifetime, then $qg \to \tilde{q}\tilde{g} \to q(\tilde{\gamma}\tilde{g})$ with $m_{\tilde{q}} \sim 100$ GeV may also explain (Barger *et al* 1984) the 'monojet' events. Concerning the 'photon' UA1 event(s) (UA1 1984b), it may possibly be a monojet with a large collimated electromagnetic component containing one or more π^0 or η whose charged multiplicity fluctuated down to zero. This event actually contains some soft charged tracks which are nearby in angle and could perhaps be associated in the 'monojet'. Another source of photons could be (Tracas and Vlassopoulos 1984) $\tilde{q}q \to \tilde{\gamma}\gamma$, assuming $m_{\tilde{q}} \lesssim E_T^{\text{trigger}}$ such that the SUSY content of the proton is plausibly excited, but in this case, one should take care of the jet coming from the decay of the left-over spectator $(\bar{\tilde{q}})$. Putting everything together, it does not look unreasonable to me to pursue the interpretation (Ellis and Kowalski 1984a, b) of the UA1 monojet data as squarks with $m_{\tilde{q}} \simeq 40$ GeV, and infer a lower limit $m_{\tilde{g}} \gtrsim 40$ GeV on the gluino mass. Furthermore, such an assumption is not in contradiction with possible explanations of the UA2 'Wen' events (UA2 1984). For example (Haber and Kane 1984a), $p\bar{p} \to \tilde{W}\tilde{g}$ or $\tilde{W}\tilde{q}$ production, followed by $\tilde{W} \to e\nu\tilde{\gamma}$ and $\tilde{g} \to qq\tilde{\gamma}$ or $\tilde{q} \to q\tilde{\gamma}$, would produce 'Wen' events (e+jet+large amounts of $p_T^{\text{miss.}}$). It has been argued (Haber and Kane 1984a) that with $m_{\tilde{q}} \simeq (40\text{–}60)$ GeV, $m_{\tilde{g}} \simeq (70\text{–}100)$ GeV and $m_{\tilde{W}} \simeq (35\text{–}40)$ GeV, one may probably get suitable (?) rates. In addition, such an explanation is consistent with the observation that in the three UA2 'Wen' events, the missing 'ν' vector is consistently larger than the p_T of the observed electron. They should on average be equal in $W \to e\nu$ decay, but could easily be different in SUSY, where the 'ν' is actually a combination of one ν and two photinos $\tilde{\gamma}$.

If we indeed buy the squark explanation ($m_{\tilde{q}} = 40$ GeV; $m_{\tilde{g}} \gtrsim O(40\,\text{GeV})$) of the Zen–Wen events, then its phenomenological implications are rather dramatic. In a large class of phenomenological supergravity models discussed before (Kounnas *et al* 1983a, 1984, Ellis *et al* 1983c, 1984b, c, g, h, i, j), one may write down (Kounnas *et al* 1983a, 1984) convenient approximate formulae for physical squark and slepton masses at relevant renormalisation scales $\mu = O(M_W)$:

$$m_{\tilde{q}}^2 \simeq m^2(1 + 7.6\xi^2), \qquad \tilde{q} = \tilde{u}, \tilde{d}, \tilde{s}, \tilde{c}, \tilde{b} \tag{8.3}$$

and

$$\begin{aligned} m_{\tilde{l}_L}^2 &\simeq m^2(1 + 0.15\xi^2) \\ m_{\tilde{l}_R}^2 &\simeq m^2(1 + 0.5\xi^2) \end{aligned} \tag{8.4}$$

where m and ξm ($\equiv m_{\tilde{V}}(M)$) are respectively the scalar boson and gaugino masses at $\mu = M_X$ or M (m is commonly, but not always necessarily, identified with the gravitino mass). The corresponding formulae for gaugino masses take the simple form (see also (6.23))

$$m_{\tilde{g}} = \frac{\alpha_3}{\alpha_G} m_{\tilde{V}}(m) \qquad m_{\tilde{\gamma}} = \frac{\alpha_1}{\alpha_G} m_{\tilde{V}}(m) \tag{8.5}$$

implying

$$\frac{m_{\tilde{\gamma}}}{m_{\tilde{g}}} = \frac{3}{8} \frac{\alpha}{\alpha_3} \frac{1}{\sin^2 \theta_W} \tag{8.6}$$

where it has been assumed that, thanks to grand unification, all gaugino masses are equal at M_X or M, not necessarily an unavoidable assumption (Ellis *et al* 1984i). Clearly enough, as it has been repeatedly emphasised in the literature (Kounnas *et al* 1983a, 1984, Ellis *et al* 1983c, 1984b, c, g, h, i, j) for some time now, the supergravity sparticle mass spectrum is rather tight. For example, assuming $m_{\tilde{q}} = 40$ GeV, $m_{\tilde{g}} \geq$ O(40 GeV) and $m_{\tilde{l}_{L,R}} \geq 20$ GeV, it is trivial to show (Ellis and Sher 1984) that the set of equations (8.3)-(8.6) imply:

$$\begin{aligned} 20\,\text{GeV} \leq m_{\tilde{l}_{L,R}} \leq 30\,\text{GeV} \\ 5\,\text{GeV} \leq m_{\tilde{\gamma}} \leq 10\,\text{GeV} \end{aligned} \tag{8.7}$$

and

$$40\,\text{GeV} \leq m_{\tilde{g}} \leq 60\,\text{GeV}$$

a rather 'light' and easily experimentally accessible spectrum. In addition, while SUSY in general entails the existence of at least two Higgs doublets, dimensional transmutation (Kounnas *et al* 1983a, 1984) or no-scale type (Ellis *et al* 1984b, c, g, h, i, j) supergravity models ask for the existence of a 'light' (sometimes (Kounnas *et al* 1983a, 1984) ≤ 10 GeV) neutral Higgs boson. Actually, if indeed (8.7) holds, then the O(10 GeV) upper bound is certainly quite firm (Kounnas *et al* 1983a, 1984, Ellis and Sher 1984). The suggestion was then made (Kounnas *et al* 1983a, 1984), some time ago, that $\Upsilon \to H^0 + \gamma$ is an excellent place (Wilczek 1977, Ellis *et al* 1979b) to look for such a 'non-standard' light Higgs. If indeed (8.7) holds true, then news (good or bad) should come very soon from almost everywhere: $e^+e^- \to \tilde{l}^+\tilde{l}^-$; $e^+e^- \to (\tilde{\gamma}\tilde{\gamma})\gamma$ (Ellis and Hagelin 1983, Fayet 1982) (one event has already been reported (Prepost 1984)), $p\bar{p} \to (\tilde{g}\tilde{g}$ or $\tilde{q}\tilde{g}) + X$, $p\bar{p} \to (W^+ \to \tilde{l} + \tilde{\nu}) + X$ or $p\bar{p} \to (Z^0 \to \tilde{l}\tilde{l}, \tilde{\nu}\tilde{\nu}) + X$ and $\Upsilon \to H^0 + \gamma$. Wait and see!

There are other very interesting features concerning low energy phenomenology stemming from the general form of V_{LEP} (see (7.9) and (7.10)) in SUGAR models. Very tight constraints coming from natural suppression of flavour-changing neutral currents (FCNC) (Barbieri and Gatto 1982, Campbell 1983, Ellis and Nanopoulos 1982a, Inami and Lim 1982), absence of large corrections to $(g-2)$ (Barbieri and Maiani 1982, Ellis *et al* 1982c, Grifols and Méndez 1982) and ρ ($\equiv (M_W/M_Z \cos\theta_{EW})^2$) (Alvarez-Gaumé *et al* 1983, Barbieri and Maiani 1983, Lim *et al* 1983) as well as to θ_{QCD} (Ellis *et al* 1982a), which have been the nemesis of SUSY models with arbitrary and explicit soft SUSY breaking, are satisfied in SUGAR models. The highly constrained set of soft SUSY operators (6.21) in SUGAR models fits the bill (Ellis *et al* 1983e, Alvarez-Gaumé *et al* 1983). Concerning FCNC, (7.10) guarantees the super-GIM mechanism, since the mass matrices for the quarks and leptons are diagonalised by the same transformation that renders the mass matrices for their scalar partners and gluino couplings generation diagonal. Despite the fact that this property does not survive, in general, after renormalisation, it has been shown (Lahanas and Nanopoulos 1983, Donoghue *et al* 1983) that these effects are controllable. Furthermore, the Buras stringent upper bound (Buras 1981) on the top quark mass ($<O(40\,\mathrm{GeV})$), coming from kaon phenomenology (K_L-K_S and $K_L\to\mu^+\mu^-$ systems), is avoided (Lahanas and Nanopoulos 1983) in SUGAR models. There are a lot of cancellations between ordinary and SUSY contributions in K processes (Inami and Lim 1982), such that the top quark mass may be stretched up to 100 GeV without problem (Lahanas and Nanopoulos 1983). That sounds very satisfactory, especially for SUGAR models (Ellis *et al* 1983c, Alvarez-Gaumé *et al* 1983, Ibanez and Lopez 1983) that do need a large top quark mass for $SU(2)\times U(1)$ breaking. It looks like a self-service situation. Similar comments apply in the case of $(g-2)_\mu$ or ρ, where it has been shown that SUGAR model contributions are acceptable (Kounnas *et al* 1983a, 1984, Alvarez-Gaumé *et al* 1983, Barbieri and Maiani 1983, Lim *et al* 1983). Typical values for SUGAR contributions are (Kounnas *et al* 1983a, 1984, Kosower *et al* 1984) $|\Delta(g-2)_\mu|\leqslant(3\times10^{-9})$ and (Alvarez-Gaumé *et al* 1983, Barbieri and Maiani 1983, Lim *et al* 1983) $\Delta\rho\leqslant0.01$, which compare favourably with the present experimental upper bounds of (4×10^{-8}) and (0.03) respectively, but are large enough to be interesting. Better experimental bounds, especially on $\Delta\rho$, could be revealing.

9. Conclusions

We have shown that gravitational effects, as contained in SUGAR theories, cannot be neglected any longer in the regime of particle physics. On the contrary, it may be that supergravitational effects are really responsible: *for* the SU(5) breaking at M_X with an automatic triplet-doublet Higgs splitting; *for* the $SU(2)\times U(1)$ breaking (and $SU(3)\times U(1)_{EM}$ *non-breaking*) at M_W, naturally exquisitely smaller than M_X; *for* the 'constrained' soft SUSY breaking at $\tilde{m}$, hierarchically smaller than M in a natural way; *for* definite, at present experimentally acceptable departures from the 'standard' low energy phenomenology (like the DEMON, or $\Delta\rho$, with values below, but not far from, their present experimental upper bounds, or the existence of very light ($<O(10\,\mathrm{GeV})$) neutral Higgs bosons), as well as a rather well defined low-energy SUSY spectrum; *for* observable baryon decay even if $M_X\simeq M$; and *for* the light fermion masses of the first two generations. Incidentally, it has been argued (Ginsparg *et al* 1983) recently that the observed (Fernandez *et al* 1983, Lockyer *et al* 1983) long b-quark lifetime and a 'light' top quark (UA1 1984a) (~40 GeV) may not fit together in the conventional Kobayashi-Maskawa six-quark *CP*-violation model, as they give too small *CP*-violation. It has even been suggested (Ginsparg *et al* 1983) that a 'light' top quark will be a signal for a fourth generation in the standard model. Well, this is not the case in a SUSY-KM model (Gérard *et al* 1984a, b). We have found (Gérard *et al* 1984a, b) that long τ_b and a 'light' top quark provide enough *CP*-violation, if we supersymmetrise the standard KM model. For us (Gérard *et al* 1984a, b) a long-lived b-quark and 'light' top maybe is a signal for supersymmetry but not necessarily for a fourth generation (Ginsparg *et al* 1983). Furthermore, supergravity theories may provide, for the first time, a problem-free cosmological scenario (Ellis *et al* 1983d, Nanopoulos *et al* 1983a, b, Gelmini *et al* 1983a, b, Enqvist and Nanopoulos 1984a, b; for reviews see

Olive 1984, Nanopoulos 1983), from primordial inflation through GUT phase transitions to baryon and nucleosynthesis, ostracising troublesome particles such as GUT monopoles, gravitinos (Ellis *et al* 1982d, 1984f, Nanopoulos *et al* 1983a, Krauss 1983, Khlopov and Linde 1984), Polonyi fields (Nanopoulos and Srednicki 1983b) or other SUSY relics (Ellis *et al* 1984d).

Putting the whole thing together, it becomes apparent that spontaneously broken $N=1$ local SUSY gauge theories, with their prosperous and appropriate structure, may well serve as an effective theory describing all physics from $M_{\rm Pl}$ down to (and including) low energies, with well defined and rich experimental consequences. What's next then? Well, we really have to understand where this highly successful theory comes from. There are reasons to believe (Cremmer and Julia 1979, Ellis *et al* 1980) that $N=8$ extended supergravity, suitably broken down to $N=1$ supergravity (Barbieri *et al* 1981, Ellis *et al* 1982b), may provide the fundamental theory. But this next move asks for a deep understanding of physics at Planck energies, which is as exciting as it is difficult, taking into account that even quantum mechanics may need modification (Hawking 1982, Ellis *et al* 1984e), if quantum gravitational effects have to be considered seriously. It may even be needed to consider extended fundamental objects like superstrings (Green 1985).

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