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Applying an integrated fuzzy gray MCDM approach: A case study on mineral processing plant site selection

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ABSTRACT

Accurate site selection of a processing plant could result in decreasing total mining costs. The site selection problem could be solved by multi-criteria decision-making (MCDM) methods. This paper introduces a new approach by integrating fuzzy AHP and gray MCDM methods to solve all decision-making problems. The approach is applied in case of a copper mine area. The critical criteria are considered as adjacency to the crusher, adjacency to tailing dam, adjacency to a power source, distance from blasting sources, sufficient land availability, and safety against floods. After studying the mine map, six feasible alternatives are prioritized using the integrated approach. The results indicated that sites A, B, and E take the first three ranks. The separate results of fuzzy AHP and gray MCDM confirm that alternatives A and B have the first two ranks. Moreover, field investigations approved the results obtained by the applied approach.

Keywords : *Fuzzy theory, gray theory, site selection, open pit mining*

1. Introduction

Selecting the optimal location for a plant is an important and fundamental factor to companies success in all engineering disciplines It also has a significant impact on decreasing total costs and increasing resources efficiency [1]. Knowing the importance of deciding on this issue, decision makers should select a site not only suitable for present conditions but also sufficiently flexible in case of future changes if necessary [2]. Similar to other engineering disciplines, in mining projects, the optimal selection of a mineral processing plant location is an important issue. Since a processing plant would be used during the mine life, selecting an appropriate location could greatly reduce the costs and ensure success of the process. Site selection process for a mineral processing plant consists of considering potential inconsistent criteria and suggesting multiple feasible alternatives. Therefore, as a problem it could be defined and solved by multi-criteria decision-making (MCDM) process.

Different techniques based on the MCDM process have been already introduced to solve decision-making problems. Simple additive weighting (SAW) [3], analytic hierarchy process (AHP) [4], analytic network process (ANP) [5], elimination and choice expressing reality (ELECTRE) [6], technique for order preference by similarity to ideal solution (TOPSIS) [7], Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [8], the preference ranking organization method for enrichment evaluation (PROMETEE) [9], decision making trial and evaluation laboratory (DEMATEL) [10], and data envelopment analysis (DEA) [11] are some of common MCDM techniques. So far, different MCDM techniques in certain and fuzzy environments have been used in order to site selection by many authors.

Ataei used fuzzy AHP to select the suitable site of alumina cement plant in East Azerbaijan province,Iran [12]. *Yang* applied fuzzy TOPSIS for locating an industrial unit [13]. *Yavuz* incorporated a multi-attribute decision-making model in fuzzy environment and AHP to select the optimal location of a dimension stone plant and applied it to case studies in Turkey [1]. *Safari et al.* employed AHP to select a mineral processing plant site in Phase I of Sangan iron ore mine [14]. Furthermore, they utilized fuzzy TOPSIS to select the appropriate site for a mineral processing plant in case of Sangan mine Phase II [15]. *Choudhary and Shankar* used fuzzy TOPSIS AHP to select a thermal power plant site in India [16]. *Mousavi et al.* utilized an integrated Delphi AHP PROMETHEE method to find a plant site [17]. *Sriniketha et al.* utilized AHP and PROMETHEE II to select an optimal location of the facilities [2]. *Asakereh et al.* used a fuzzy AHP in geographical information system (GIS) software environment to select the most suitable sites for solar energy farms in Iran [18]. *Azizi et al.* used a combination of ANP and DEMATEL in GIS software environment to select the location of a wind power plant in northwestern Iran [19]. *Sozen et al.* developed a DEA-based model combined with TOPSIS to find the optimal location of solar plants in various regions of Turkey [20]. *Singh* developed an extent fuzzy AHP based approach to select the best geographical locations of facilities under a real time process [21]. *Sindhu et al.* utilized fuzzy TOPSIS and AHP to select the suitable location of a solar farm in India [22]. *Lee et al.* developed an MCDM model by use of VIKOR, fuzzy ANP, and the interpretive structural modeling to select the optimal location of the photovoltaic solar plant and applied it to a case study in Taiwan [23]. *Bakhtavar et al.* represented a mathematical model by use of fuzzy-weights and integer goal programming to select the optimal location of a dimension stone processing plant for adjacent small-scale dimension stone quarries [24].

As the literature review suggests, the employed and developed methods in plant site selection could be classified as two categories: industrial and mineral processing plant site selection methods. Very limited case studies have been conducted in processing plant site selection. Therefore, regarding to the importance of this issue further studies are required using more reliable approaches, especially under uncertain environment based on fuzzy and gray theories. It is often difficult to assign a precise number during an MCDM process. In this cases, to determine the relative importance of criteria experts and decision-makers could easily use fuzzy numbers rather than precise

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numbers [25].

In this study, a different approach is developed under uncertain conditions using fuzzy and gray theories considering the importance and qualitative ratings of criteria as linguistic variables. The new approach is to improve the results of fuzzy AHP and gray MCDM. Gray MCDM is a simple and powerful tool to solve MCDM problems. It also has advantages over the fuzzy approach, and its application for the processing plant site selection has not been reported yet. Moreover, this study considers more secure distance of the selected site from blasting points and distance from crusher as new criteria for selecting a processing plant site.

2. The introduction of the developed approach

2.1. Uncertainty in decision-making

Some alternatives are analyzed in MCDM problems. In addition to alternatives, a decision-maker should examine several criteria in case of each alternative. In decision-making problems, the preference information related to alternatives and criteria is a part of judgment process. Usually there is a level of uncertainty in real world, and there are always uncertain conditions in various stages of studying a problem. In some cases, the judgments are ambiguous. There is always a kind of uncertain data in MCDM problems. In such problems, decision-makers express their evaluations by the linguistic variables [26].

Uncertain conditions could be described using fuzzy and gray approaches. In many decision-making problems, all or a part of the data are uncertain, in form of fuzzy. The fuzzy theory was first introduced in 1965 by Lotfi Zadeh as he published his first paper entitled the fuzzy sets [27]. This paper was the beginning of a new approach in mathematics which developed in other sciences later. The fuzzy theory is used for solving problems in uncertain conditions. It can describe many vague concepts and variables in scientific forms to provide a context for reasoning and decision-making.

Deng introduced gray theory by classifying all systems into three categories of white, black, and gray [28]. The white parts include fully known information, the black sections include unknown information and systems, and the gray parts consist of partial unknown information and uncertainty for a system [28].

2.2. Fuzzy AHP

In Fuzzy AHP, the opinions of experts are used along with the uncertainties in their evaluations and decisions. According to Chang's method [29], fuzzy AHP steps to achieve the most desirable alternative are as follows:

- Drawing a hierarchical diagram
- Defining fuzzy numbers for pairwise comparisons
- Creating the pairwise comparison matrix (A) using fuzzy numbers according to the following matrix [29]:

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \dots & \tilde{a}_{2n} \\ . & . & . & . \\ . & . & . & . \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & 1 \end{bmatrix}$$

- Calculating S_i (fuzzy synthetic degree value) for each row of pairwise comparison matrix. S_i is a triangular fuzzy number calculated through Eq. 1 [29].

$$S_i = \sum_{j=1}^{m} M_{gi}^{j} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} \tag{1}$$

where, *i* and *j* are The number of rows and columns, respectively

M_{gi}^{j}, Triangular fuzzy numbers of pairwise comparison matrix

$\sum_{j=1}^{m} M_{\mathrm{gi}}^{j}$, $\sum_{i=1}^{n} \sum_{j=1}^{m} M_{\mathrm{gi}}^{j}$, and $\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{\mathrm{gi}}^{j}\right]^{-1}$ could be calculated through Eq. 2-4, respectively.

$$\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \tag{2}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{i=1}^{m} l_i, \sum_{i=1}^{m} m_i, \sum_{i=1}^{m} u_i \right) \tag{3}$$

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{m} u_i}, \frac{1}{\sum_{i=1}^{m} m_i}, \frac{1}{\sum_{i=1}^{m} l_i} \right) \tag{4}$$

- Calculating the degree of possibility of S_i

In general, if $M_1 = (l_1.m_1.u_1)$ and $M_2 = (l_2.m_2.u_2)$ are two fuzzy triangular numbers, then the degree of possibility of $M_1 \geq M_2$ is defined as following [29]:

$$V(M_1 \geq M_2) = hgt(M_1 \cap M_2) = \mu_{M_2}(d) = \begin{cases} 1 & if\ m_2 \geq m_1 \\ 0 & if\ l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} & otherwise \end{cases} \tag{5}$$

The degree of possibility for a triangular fuzzy number to be greater than *k* triangular fuzzy numbers could be determined using Eq. 6 [29]:

$$V(M \geq M_1, M_2, \dots, M_k) = V[(M \geq M_1) and (M \geq M_2) and \dots and (M \geq M_k)] \tag{6}$$
$$= Min V(M \geq M_i), \quad i = 1,2,3,\dots,k$$

- Calculating the weight of criteria and alternatives in the pairwise comparison matrix through Eq. 7. The normalized weighting vector is given by Eq. 8 [29].

$$d'(A_i) = Min V(S_i \geq S_k) \quad k = 1,2,\dots,n, \quad k \neq i \tag{7}$$

$$W' = (d'(A_1), d'(A_2), \dots, d'(A_n))^T \quad A_i = (i = 1,2,\dots,n) \tag{8}$$

- Calculating the final weighting vector through normalizing the calculated weighting vector in the previous step as Eq. 9 [29].

$$W = (d(A_1), d(A_2), \dots, d(A_n))^T \tag{9}$$

Based on the weight of each alternative, the alternative which have the greatest weight would be selected as the best alternative [29].

2.3. Gray MCDM approach

2.3.1. Gray numbers

A gray number is defined as a number which has an unknown value; however, its domain and the lower and upper bounds are known [26]. Usually, a gray number is shown as $G|_{\underline{a}}^{\bar{a}}$. A gray number which has lower and upper bounds is defined as interval gray number [26]. Main mathematical operations on two interval gray numbers and the constant (a) are defined as follows [30]:

$$\otimes \mathrm{G}_1 + \otimes \mathrm{G}_2 = \left[\underline{\mathrm{G}_1} + \underline{\mathrm{G}_2}. \bar{\mathrm{G}}_1 + \bar{\mathrm{G}}_2\right] \tag{10}$$

$$\otimes \mathrm{G}_1 - \otimes \mathrm{G}_2 = \left[\underline{\mathrm{G}_1} - \underline{\mathrm{G}_2}. \bar{\mathrm{G}}_1 - \bar{\mathrm{G}}_2\right] \tag{11}$$

$$\mathrm{a} \times \otimes \mathrm{G}_1 = \left[\mathrm{a} \times \underline{\mathrm{G}_1}. \mathrm{a} \times \bar{\mathrm{G}}_1\right] \tag{12}$$

$$\otimes \mathrm{G}_1 \div \otimes \mathrm{G}_2 = \left[\underline{\mathrm{G}_1}. \bar{\mathrm{G}}_1\right] \times \left[\frac{1}{\underline{\mathrm{G}_2}}. \frac{1}{\bar{\mathrm{G}}_2}\right] \tag{13}$$

$$\otimes \mathrm{G}_1 \times \otimes \mathrm{G}_2 = \left[\min\left(\underline{\mathrm{G}_1\mathrm{G}_2}. \underline{\mathrm{G}_1}\bar{\mathrm{G}}_2. \bar{\mathrm{G}}_1\underline{\mathrm{G}_2}. \bar{\mathrm{G}}_1\bar{\mathrm{G}}_2\right), \max\left(\underline{\mathrm{G}_1\mathrm{G}_2}. \underline{\mathrm{G}_1}\bar{\mathrm{G}}_2. \bar{\mathrm{G}}_1\underline{\mathrm{G}_2}. \bar{\mathrm{G}}_1\bar{\mathrm{G}}_2\right)\right] \tag{14}$$

The length of a gray number is calculated through Eq. 15.

$$L(\otimes G) = \bar{G} - \underline{G} \tag{15}$$

For two gray numbers $\otimes G_1$, $\otimes G_2$, the gray possibility degree is defined as Eq. 16 [26].

$$(\otimes G_1 \leq \otimes G_2) = \frac{max\left((0.L^*) - max\left(0.\bar{G}_1 - \underline{G_2}\right)\right)}{L^*}; L^* = L_2 - L_1 \tag{16}$$

Remark as follows [31]:

if $\underline{G_1} = \underline{G_2}$, $\bar{G}_1 = \bar{G}_2$, then $\otimes G_1 = \otimes G$ and $P\{\otimes G_1 \leq \otimes G\} = 0.5$

if $\underline{G_2} > \bar{G}_1$, then $\otimes G_2 > \otimes G$ and $P\{\otimes G_1 \leq \otimes G\} = 1$

if $\bar{G}_2 < \underline{G_1}$, then $\otimes G_2 < \otimes G$ and $P\{\otimes G_1 \leq \otimes G\} = 0$

if $P\{\otimes G_1 \leq \otimes G\} > 0.5$, then also $\otimes G_2 > \otimes G$
if $P\{\otimes G_1 \leq \otimes G\} < 0.5$, then also $\otimes G_2 < \otimes G$

2.3.2. The steps of gray MCDM

The main steps to decide on the gray procedure are introduced as follows [31, 32]:

The weight of each criterion is determined according to decision makers judgment using linguistic variables. The linguistic variables with interval gray numbers are shown in Table 1.

Table 1. The weight of criteria determination scale [31, 32].

Scale	$\otimes W$
Very low	[0,0.1]
Low	[0.1,0.3]
Medium low	[0.3,0.4]
Medium	[0.4,0.5]
Medium high	[0.5,0.6]
High	[0.6,0.9]
Very high	[0.9,1]

The status of each alternative is determined regarding each criterion according to decision maker's judgment using the linguistic variables as given in Table 2.

Table 2. Evaluation scale of alternatives [31, 32].

Scale	$\otimes W$
Very poor	[0,1]
Poor	[1,3]
Medium poor	[3,4]
Fair	[4,5]
Medium good	[5,6]
Good	[6,9]
Very good	[9,10]

As a result, the following gray decision matrix is provided. It includes gray numbers ($\otimes G$) based on their equivalent linguistic variables from Table 2.

$$D = \begin{bmatrix} \otimes G_{11} & \otimes G_{12} & \dots & \otimes G_{1n} \\ \otimes G_{21} & \otimes G_{22} & \dots & \otimes G_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \otimes G_{m1} & \otimes G_{m2} & \dots & \otimes G_{mn} \end{bmatrix}$$

- Gray decision matrix is normalized as follows:

$$D^* = \begin{bmatrix} \otimes G^{*11} & \otimes G^{*12} & \dots & \otimes G^{*1n} \\ \otimes G^{*21} & \otimes G^{*22} & \dots & \otimes G^{*2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \otimes G^{*m1} & \otimes G^{*m2} & \dots & \otimes G^{*mn} \end{bmatrix}$$

Eq. 17 and 19 are used to normalize positive and negative criteria, respectively.

$$\otimes G_{ij}^{*} = \left[\frac{\underline{a}_{ij}}{G_j^{max}} . \frac{\overline{a}_{ij}}{G_j^{max}}\right] \tag{17}$$

$$\otimes G_j^{max} = max_{1 \leq i \leq m}\{\underline{a}_{ij}\} \tag{18}$$

$$\otimes G_{ij}^{*} = \left[\frac{G_j^{min}}{\overline{a}_{ij}} . \frac{G_j^{min}}{\underline{a}_{ij}}\right] \tag{19}$$

$$\otimes G_j^{min} = min_{1 \leq i \leq m}\{\underline{a}_{ij}\} \tag{20}$$

- The weighted normal gray decision matrix is provided as follows:

$$N = \begin{bmatrix} \otimes N_{11} & \otimes N_{12} & \dots & \otimes N_{1n} \\ \otimes N_{21} & \otimes N_{22} & \dots & \otimes N_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \otimes N_{m1} & \otimes N_{m2} & \dots & \otimes N_{mn} \end{bmatrix}$$

$$\otimes N_{ij} = \otimes G_{ij}^{*} \times \otimes W_j \tag{21}$$

- Eq. 22 to 24 provide the positive ideal to rank alternatives.

$$V^{max} = \{\otimes G_1^{max}. \otimes G_2^{max}. \dots . \otimes G_n^{max}\} \tag{22}$$

$$V^{\max} = \left\{ \left[\max_{1 \leq i \leq m} \alpha_{i1}, \max_{1 \leq i \leq m} \beta_{i1} \right], \left[\max_{1 \leq i \leq m} \alpha_{i2}, \max_{1 \leq i \leq m} \beta_{i2} \right], \dots, \left[\max_{1 \leq i \leq m} \alpha_{in}, \max_{1 \leq i \leq m} \beta_{in} \right] \right\} \tag{23}$$

$$\otimes N_{ij} = [\alpha_{ij}. \beta_{ij}] \tag{24}$$

- Eq. 25 determines the gray possibility degree for each alternative.

$$P\{V_i \leq V^{\max}\} = \frac{1}{n} \sum_{j=1}^{n} P\{\otimes N_{ij} \leq G_j^{\max}\} \tag{25}$$

- Alternatives are ranked. In this case, a better rank is indicated by the smaller gray possibility degree.

2.4. Developed approach

A new hybrid approach is developed based on fuzzy AHP and gray MCDM concepts to rank alternatives. It adjusts the ranking of alternatives through the following steps:

- Prioritizing alternatives using fuzzy AHP as described earlier
- Ranking the alternatives using gray MCDM
- Determining the integrated value of each alternative through Eq. 26

$$WH = WG - WF \tag{26}$$

where,

WH is Integrated value of each alternative;

WG is Gray possibility degree of each alternative;

WF is Fuzzy AHP weight of each alternative.

3. Results and discussion

3.1. Case study

The Sungun copper mine is investigated as an example for applicability of the proposed approach in details. The mine, situated near Iran borders with the Republic of Azerbaijan and Armenia. It is located at a distance of 100 km, northeast of Tabriz city and 25 km north of Varzeghan city on 46 ° 43′ longitude and 38 ° 42′ latitude. The geological reserve of the mine was estimated as approximately 796 million tons. Annual production was planned to be 7 million tons for the first seven years of mine life. It is 14 million tons for the next years, according to the mine plan. The mineral processing plant was designed to produce 150000 and 300000 tons of concentration annually, during Phases 1 and 2, respectively.

3.2. Alternatives and criteria

The required data are collected and evaluated based on first step of site selection process. For this purpose, a general map of the mine area (Fig. 1), including topography, pit limits, the main roads, infrastructures, and blasting levels is initially imported into the AutoCAD software. Then, a mesh network is defined using meshes with approximately 8200 m^2 area equal to the required area for Sungun processing plant construction. The target is to identify the alternatives better, and to measure the distance of each alternative from the mine infrastructures, as well.

The meshes given in Fig. 1 consider the potential alternatives for the mineral processing plant site. Many alternatives (meshes) must be eliminated from the mesh network because of their undesirable location. For this purpose, a filtering process is carried out as following. The alternatives placed entirely or partially within the ultimate pit limits are initially eliminated. Afterwards, the alternatives that located at very long distances from the crusher site and the exit point of the mine are eliminated as the inappropriate and infeasible cases. Furthermore, the alternatives that located on map borders and main roads of the mine are disregarded. A safe distance is considered from the valleys and cliffs located in the mine area; hence, the alternatives on the safe distance are also rejected. Finally, as illustrated in Fig. 1, only six alternatives (A, B, C, D, E, and F) remain for consideration during the decision-making process using proposed approach. Since a processing plant might be constructed near a mine, where there are the mineralization zones, locating it on a probable reserve is inevitable. To avoid this problem, it is of uttermost importance to provide a map that consists of alteration and mineralization trends and considers the detailed exploration results and ultimate pit limits.

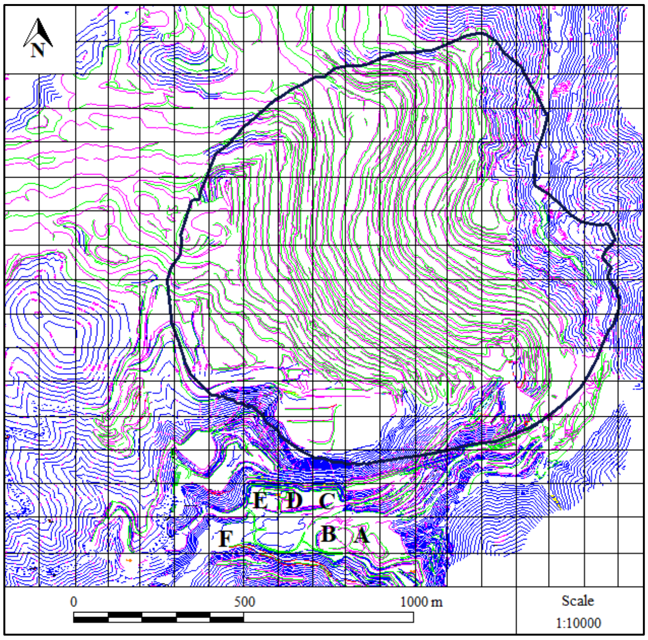

Fig. 1. Feasible alternatives for selecting the most appropriate processing plant site.

After investigating the various criteria concerned in literature of processing plant site selection and considering the specific conditions of Sungun mine, six important criteria summarized in Table 3 are deemed.

Table 3. The most critical criteria.

Criteria	Description
C_1	Adjacency (approaching) to crusher
C_2	Adjacency to tailing dam
C_3	Adjacency to power source
C_4	The availability of sufficient land
C_5	Safety against floods
C_6	Distance from bench-blasting sources

So far, in similar studies, the criterion of distance from mine has been considered [12, 14, 15, 24]. In this study, the criterion of distance from crusher is regarded because the main volume of the extracted ore is directly hauled to crusher before entering to the processing plant. Ground vibration due to large scale blasting is iterative and dominate phenomenon that could damage the processing plant of Sungun mine. Therefore, the criterion of distance from the blasting sources is taken into consideration. In the Sungun area, the six alternatives are located at lower levels, concerning the highest level of the Sungun pit. In this case, the distance of the nearest blasting level to the plant site within the critical depth of 90 meters is measured and investigated for the safety concerns. It is obvious that the alternative with longer distance from blasting sources is safer than other alternatives.

3.3. Applying the developed approach

Usually all or a part of data in MCDM problems are uncertain. Therefore, they could be stated through fuzzy numbers. During the first step of proposed approach, only the fourth and fifth criteria, introducing the availability of sufficient land and safety against floods, respectively, are considered fuzzy numbers. Other criteria in this step have a certain nature. In second step, all data are considered using linguistic variables based on uncertain gray numbers. To rank alternatives applying the proposed approach, alternatives are initially prioritized by fuzzy AHP method, and their weights are determined. Then, alternatives are prioritized employing gray MCDM based on their gray possibility degrees. Finally, the most appropriate alternative is selected through the third step of the integrated approach.

3.3.1. The first step (fuzzy AHP) of the developed approach

The analytical hierarchy diagram for selecting a processing plant location at Sungun mine is shown in Fig. 2.

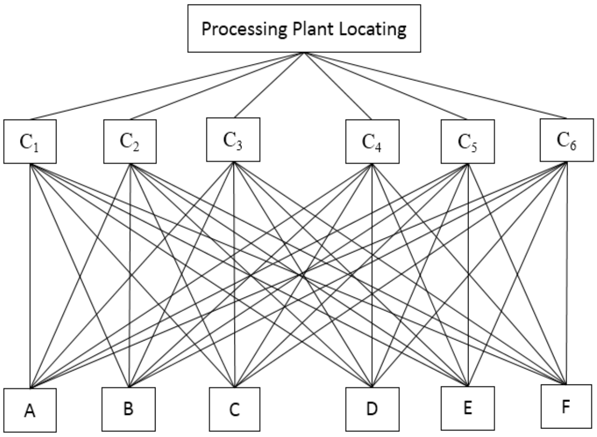

Fig. 2. Analytical hierarchy diagram to locate a processing plant.

Linguistic variables could be expressed in crisp and triangular fuzzy numbers for pairwise comparison. There are many fuzzy scales to express the linguistic variables. The fuzzy triangular scale given in Table 4 is used in this study. A group of four experts (consists of two mining engineering academic professors and two experts from Sungun mine) discussed their ideas using the concept of Delphi technique and the scale presented in Table 4 to make a common decision in pairwise comparison matrix form, given in Table 5.

Table 4. Triangular fuzzy pairwise comparison based on a linguistic scale [33].

Crisp value	Linguistic scale	Fuzzy number
1	Equal importance	(1, 1, 1) if diagonal; (1, 1, 3) otherwise
2	Intermediate value	(1, 2, 4)
3	Moderate dominance	(1, 3, 5)
4	Intermediate value	(2, 4, 6)
5	Strong dominance	(3, 5, 7)
6	Intermediate value	(4, 6, 8)
7	Demonstrated dominance	(5, 7, 9)
8	Intermediate value	(6, 8, 9)
9	Absolute dominance	(7, 9, 9)

Table 5. Pairwise comparison matrix of criteria for locating a processing plant.

	C_1	C_2	C_3	C_4	C_5	C_6	Weight
C_1	(1,1,1)	(1,2,4)	(1,2,4)	(1/3,1,1)	(1,1,3)	(1/4,1/2,1)	0.1923
C_2	(1/4,1/2,1)	(1,1,1)	(1,1,1)	(1/5,1/3,1)	(1/3,1,1)	(1/6,1/4,1/2)	0.079
C_3	(1/4,1/2,1)	(1,1,1)	(1,1,1)	(1/5,1/3,1)	(1/3,1,1)	(1/6,1/4,1/2)	0.079
C_4	(1,1,3)	(1,3,5)	(1,3,5)	(1,1,1)	(1,2,4)	(1/3,1,1)	0.2344
C_5	(1/3,1,1)	(1,1,3)	(1,1,3)	(1/4,1/2,1)	(1,1,1)	(1/5,1/3,1)	0.1463
C_6	(1,2,4)	(2,4,6)	(2,4,6)	(1,1,3)	(1,3,5)	(1,1,1)	0.27

The pairwise comparison matrix of alternatives by considering the availability of sufficient land is given in Table 6. Note that the sufficient land availability criterion is regarded to focus on alternatives which take enough area for constructing the processing plant, proper bed rock, suitable topography with low costs for leveling, and the minimum environmental restrictions. The pairwise comparison matrix of alternatives considering “safety against floods” is given in Table 7.

Table 6. Pairwise comparisons of alternatives considering C_4.

	A	B	C	D	E	F	Weight
A	(1,1,1)	(1,1,3)	(1,3,5)	(2,4,6)	(1,3,5)	(4,6,8)	0.284
B	(1/3,1,1)	(1,1,1)	(1,2,4)	(1,3,5)	(1,2,4)	(3,5,7)	0.252
C	(1/5,1/3 ,1)	(1/4,1/2,1)	(1,1,1)	(1,1,3)	(1,1,1)	(1,3,5)	0.16
D	(1/6,1/4,1/2)	(1/5,1/3 ,1)	(1/3,1,1)	(1,1,1)	(1/3,1,1)	(1,2,4)	0.1153
E	(1/5,1/3 ,1)	(1/4,1/2,1)	(1,1,1)	(1,1,3)	(1,1,1)	(1,3,5)	0.16
F	(1/8,1/6,1/4)	(1/7,1/5,1/3)	(1/5,1/3 ,1)	(1/4,1/2,1)	(1/5,1/3 ,1)	(1,1,1)	0.0284

Table 7. Pairwise comparisons of alternatives considering C_5.

	A	B	C	D	E	F	Weight
A	(1,1,1)	(1,2,4)	(5,7,9)	(3,5,7)	(4,6,8)	(4,6,8)	0.504
B	(1/4,1/2,1)	(1,1,1)	(3,5,7)	(1,3,5)	(2,4,6)	(2,4,6)	0.3754
C	(1/9,1/7,1/5)	(1/7,1/5,1/3)	(1,1,1)	(1/4,1/2,1)	(1/3,1,1)	(1/3,1,1)	0
D	(1/7,1/5,1/3)	(1/5,1/3,1)	(1,2,4)	(1,1,1)	(1,1,3)	(1,1,3)	0.1207
E	(1/8,1/6,1/4)	(1/6,1/4,1/2)	(1,1,3)	(1/3,1,1)	(1,1,1)	(1,1,1)	0
F	(1/8,1/6,1/4)	(1/6,1/4,1/2)	(1,1,3)	(1/3,1,1)	(1,1,1)	(1,1,1)	0

Four criteria are numerical and do not require a pairwise comparison. The distances measured from the mine map are shown in Table 8. Table 9 summarizes relative weight of each alternative.

Table 8. Distances of the alternatives from the quantitative criteria (m).

	C_1	C_2	C_3	C_6
A	1113.6	272.2	962.5	970.1
B	1225.3	424	827.9	1013.4
C	1182.8	506.5	732.9	814.7
D	1341.1	648.9	594.1	915.9
E	1531.1	806.7	428.1	1042.3
F	1719.9	943.2	483.7	1288.7

Table 9. Calculated weights for each quantitative criterion by considering each alternative.

	C_1	C_2	C_3	C_6
A	0.1982	0.31	0.1072	0.16047
B	0.1763	0.199	0.1245	0.16764
C	0.1871	0.1666	0.14062	0.135
D	0.16505	0.13	0.1735	0.1515
E	0.1446	0.105	0.2408	0.1724
F	0.1287	0.09	0.2131	0.2132

Table 10 summarizes alternatives ranking using fuzzy AHP method. According to the results given in Table 10, alternatives A and B, which include very high scores in comparison with other alternatives, take the first and second ranks, respectively.

Table 10. Alternatives ranking based on the first step of the integrated approach.

Alternatives	A	B	C	D	E	F
Weight	0.2547	0.2187	0.1342	0.1413	0.1392	0.1129
Rank	1	2	5	3	4	6

3.3.2. The second step (gray MCDM) of the developed approach

To determine the weight of each criterion the decision-maker's opinions are expressed by linguistic variables in seven ranking scales from very low to very high. Afterwards, the linguistic variables turn into gray numbers [34]. As a result, the weight of each criterion is obtained as given in Table 11.

Table 11. The most critical criteria.

Criteria	Weight
C_1: Adjacency to the crusher	[0.5-0.6]
C_2: Adjacency to tailing dam	[0.3-0.4]
C_3: Adjacency to power source	[0.3-0.4]
C_4: The availability of sufficient land	[0.9-1]
C_5: Safety against floods	[0.4-0.5]
C_6: Distance from bench-blasting sources	[0.6-0.9]

Next, gray decision matrix is formed as presented in Table 12. Gray numbers in the table represent the linguistic variables with a range from very poor to very good.

Table 12. The gray decision matrix.

	C_1	C_2	C_3	C_4	C_5	C_6
A	[9-10]	[9-10]	[1-3]	[4-5]	[9-10]	[9-10]
B	[6-9]	[6-9]	[3-4]	[5-6]	[6-9]	[9-10]
C	[6-9]	[5-6]	[4-5]	[0-1]	[1-3]	[5-6]
D	[5-6]	[4-5]	[5-6]	[3-4]	[4-5]	[4-5]
E	[4-5]	[3-4]	[9-10]	[6-9]	[3-4]	[5-6]
F	[1-3]	[1-3]	[6-9]	[9-10]	[3-4]	[3-4]

Afterwards, the gray decision matrix is normalized and weighted (Table 13).

Table 13. Normalized and weighted decision matrix based on the gray numbers.

	C_1	C_2	C_3	C_4	C_5	C_6
A	[0.45-0.6]	[0.27-0.4]	[0.12-0.3]	[0.36-0.5]	[0.36-0.5]	[0.54-0.9]
B	[0.3-0.54]	[0.18-0.36]	[0.09-0.16]	[0.45-0.6]	[0.24-0.45]	[0.54-0.9]
C	[0.3-0.54]	[0.15-0.24]	[0.12-0.2]	[0-0.1]	[0.04-0.15]	[0.3-0.54]
D	[0.25-0.36]	[0.12-0.2]	[0.15-0.24]	[0.27-0.4]	[0.16-0.25]	[0.24-0.45]
E	[0.2-0.3]	[0.09-0.16]	[0.27-0.4]	[0.54-0.9]	[0.12-0.2]	[0.3-0.54]
F	[0.05-0.18]	[0.03-0.12]	[0.18-0.36]	[0.81-1]	[0.12-0.2]	[0.18-0.36]

Based on the fifth step of the gray algorithm, the positive ideal is defined as follows:

$$V^{\max} = \{[0.45 - 0.6].[0.17 - 0.4].[0.27 - 0.4].[0.81 - 1].[0.36 - 0.5].[0.54 - 0.9]\}$$

Based on gray possibility degree equation described in the sixth step of the gray algorithm, all alternatives are compared with the ideal alternative resulted from step 5. Finally, the alternatives are ranked as shown in Table 14.

Table 14. Ranking of the alternatives based on gray possibility degree.

Alternatives	A	B	C	D	E	F
Gray possibility degree	0.666	0.787	0.9615	1	0.8894	0.8683
Rank	1	2	5	6	4	3

As a result, alternatives A and B are ranked first and second, respectively.

3.3.3. Prioritizing the alternatives through the developed approach

As explained in previous sections, according to the integrated approach, alternatives are initially prioritized using fuzzy AHP and gray MCDM methods. Afterwards, by subtracting the gray possibility degree and weight of fuzzy AHP for each alternative, combined weight is achieved using Eq. 26. The integrated scores of alternatives are summarized in Table 15.

Table 15. Ranking of the alternatives based on gray possibility degree.

Alternatives	A	B	C	D	E	F
Combined rating	0.4113	0.5683	0.8273	0.8587	0.7502	0.7554
Rank	1	2	5	6	3	4

Based on results of the integrated approach, sites A and B are of the first and second rank, respectively.

3.3.4 Validation

An approach is proposed by integrating fuzzy AHP and gray MCDM methods to decide in general and in particular on the most appropriate processing plant site at the Sungun copper mine. The decision is made using the integrated approach based on an expert team idea from fuzzy AHP and gray MCDM steps. The results obtained from both fuzzy AHP and gray MCDM methods indicated that alternatives A and B are the first and second choices. Although this denotes that both methods confirmed each other in case of first two priorities (choices), there are some differences in prioritizing the other alternatives. The order of other alternatives based on fuzzy AHP is D, E, C, and F; whereas, gray MCDM ranks them as F, E, C, and D. The main difference between the results of the two methods is the priority of the third and sixth alternatives. The investors proposed alternative F as an ideal plant site, which has the longest distance from blasting sources, after concerning the predicted impacts of the blast-induced ground vibration and the probable damages. Although the investors accepted high costs of site F for leveling the ground, the management team believed that it also imposed high transportation costs over the plant life due to its long distance from the crusher and tailing dam.

The integrated approach is proposed to adjust the differences between fuzzy AHP and gray MCDM methods. After prioritizing the alternatives using the integrated approach, the order of the alternatives is A, B, E, F, C, and D. The fourth rank for Alternative F is closed by the field investigations, as well as the opinion of the mine management team. As shown on meshed area map, all six possessed sites have potential to build a processing plant, and they are accurately selected as the alternatives.

4. Conclusion

Knowing the importance of deciding on plant site selection problems, decision makers should select a site not only suitable for present conditions but also sufficiently flexible in case of future changes if necessary. Different MCDM techniques in certain and fuzzy states have been introduced and developed for selecting the suitable location for units. There is always a kind of uncertain data in MCDM problems, and evaluations could be made by use of fuzzy and gray methods employing the linguistic variables. The approach developed in this paper aimed to adjust differences between the results of fuzzy AHP and gray MCDM in all situations where a decision should be made, especially in case of processing plant site selection. Based on the specific conditions of Sungun mine, six criteria have been utilized for assessing six alternatives. According to common pairwise comparison on the criteria by a group of four experts, distance from bench-blasting sources (weight = 0.27) has been known as the most important criterion during the first step of developed approach. Fuzzy AHP ranked first three alternatives as A (weight = 0.2547), B (weight = 0.2187), and D (weight = 0.1413); whereas, gray MCDM prioritized them as A (degree = 0.666), B (degree = 0.787), and F (degree = 0.8683). The integrated approach indicated the order of A, B, and E by subtracting the gray possibility degree and the weight of fuzzy AHP. Field investigations indicated the accuracy of the results and applicability of the developed approach. A suggestion for future studies is to consider the impacts of the mineralization zones around pit limits on processing plant site selection. In this case, a map consisting of alteration and mineralization trends is required along with detailed exploration results and ultimate pit limits. The sufficient land availability criterion focuses on enough area availability for constructing a processing plant, proper bedrock, suitable topography with low costs for leveling, and minimum environmental restrictions. It is recommended to considering them as separate criteria in case of the processing plant site selection.

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