Applying Revenue Management to the Reverse Supply Chain

Mark Ferguson, Moritz Fleischmann and Gilvan C. Souza

ERIM Report Series *Research in Management*	
ERIM Report Series reference number	ERS-2008-052-LIS
Publication	August 2008
Number of pages	33
Persistent paper URL	http://hdl.handle.net/1765/13211
Email address corresponding author	mfleischmann@rsm.nl
Address	Erasmus Research Institute of Management (ERIM) RSM Erasmus University / Erasmus School of Economics Erasmus Universiteit Rotterdam P.O.Box 1738 3000 DR Rotterdam, The Netherlands Phone: + 31 10 408 1182 Fax: + 31 10 408 9640 Email: info@erim.eur.nl Internet: www.erim.eur.nl

Bibliographic data and classifications of all the ERIM reports are also available on the ERIM website:
www.erim.eur.nl

ERASMUS RESEARCH INSTITUTE OF MANAGEMENT

REPORT SERIES
RESEARCH IN MANAGEMENT

ABSTRACT AND KEYWORDS	
Abstract	We study the disposition decision for product returns in a closed-loop supply chain. Motivated by the asset recovery process at IBM, we consider two disposition alternatives. Returns may be either refurbished for reselling or dismantled for spare parts. Reselling a refurbished unit typically yields higher unit margins. However, demand is uncertain. A common policy in many firms is to rank disposition alternatives by unit margins. We show that a revenue management approach to the disposition decision which explicitly incorporates demand uncertainty can increase profits significantly. We discuss analogies between the disposition problem and the classical airline revenue management problem. We then develop single period and multi-period stochastic optimization models for the disposition problem. Analyzing these models, we show that the optimal allocation balances expected marginal profits across the disposition alternatives. A detailed numerical study reveals that a revenue management approach to the disposition problem significantly outperforms the current practice of focusing exclusively on high-margin options, and we identify conditions under which this improvement is the highest. We also show that the value recovered from the returned products critically depends on the coordination between forward and reverse supply chain decisions.
Free Keywords	Remanufacturing, Revenue Management, Spare Parts Inventory
Availability	The ERIM Report Series is distributed through the following platforms: Academic Repository at Erasmus University (DEAR), DEAR ERIM Series Portal Social Science Research Network (SSRN), SSRN ERIM Series Webpage Research Papers in Economics (REPEC), REPEC ERIM Series Webpage
Classifications	The electronic versions of the papers in the ERIM report Series contain bibliographic metadata by the following classification systems: Library of Congress Classification, (LCC) LCC Webpage Journal of Economic Literature, (JEL), JEL Webpage ACM Computing Classification System CCS Webpage Inspec Classification scheme (ICS), ICS Webpage

Applying Revenue Management to the Reverse Supply Chain

Mark Ferguson

College of Management, Georgia Tech, Atlanta, GA 30307, mark.ferguson@mgt.gatech.edu

Moritz Fleischmann

Rotterdam School of Management, Erasmus University, 3000DR Rotterdam, The Netherlands, mfleischmann@rsm.nl

Gilvan C Souza

Smith School of Business, University of Maryland, College Park, MD 20742, gsouza@umd.edu

Abstract

We study the disposition decision for product returns in a closed-loop supply chain. Motivated by the asset recovery process at IBM, we consider two disposition alternatives. Returns may be either refurbished for reselling or dismantled for spare parts. Reselling a refurbished unit typically yields higher unit margins. However, demand is uncertain. A common policy in many firms is to rank disposition alternatives by unit margins. We show that a revenue management approach to the disposition decision which explicitly incorporates demand uncertainty can increase profits significantly. We discuss analogies between the disposition problem and the classical airline revenue management problem. We then develop single period and multi-period stochastic optimization models for the disposition problem. Analyzing these models, we show that the optimal allocation balances expected marginal profits across the disposition alternatives. A detailed numerical study reveals that a revenue management approach to the disposition problem significantly outperforms the current practice of focusing exclusively on high-margin options, and we identify conditions under which this improvement is the highest. We also show that the value recovered from the returned products critically depends on the coordination between forward and reverse supply chain decisions.

Keywords: Remanufacturing, Revenue Management, Spare Parts Inventory

1 Introduction

Revenue management is concerned with selling the right amount of product or capacity to the right customer at the right time. In this paper, we consider a firm's optimal disposition decision for product returns—remanufacture or dismantle for spare parts—from a revenue management perspective. In our case, the capacity to be allocated is the returned units which can be either remanufactured or dismantled for spare parts. We model this problem in a multi-period, finite horizon setting, which addresses the importance of timing in these decisions.

Our problem is motivated by the asset recovery process at IBM. Information technology products such as personal computers, servers, storage systems, and mainframes coming off of lease are returned to IBM's remanufacturing facility where a disposition decision—remanufacture or dismantle for spare parts—is made. Because the required amount of processing for electronic equipment is much less than in other industries such as automotive engines, we use the terms remanufacturing and refurbishing synonymously in this paper (the term refurbishing is often used for "light" remanufacturing). The remanufacturing operation involves replacing all wearable components, testing, cleaning, and reloading of software to create a standard configuration. Remanufactured units are advertised on IBM's web site (www.ibm.com) for about one month and, if not sold during this time, are salvaged to third-party brokers through an auction. While remanufactured units sold via the web site command attractive margins, unsold units salvaged via the auction can, in most cases, only recover the cost incurred for remanufacturing. Thus, IBM makes a high profit on remanufactured units sold through its regular web site channel, and close to zero profit on remanufactured units that do not sell within the one month window and must be salvaged via the auction.

Instead of remanufacturing a returned unit, IBM can dismantle it to harvest non-wearable parts, such as memory, video cards, and mother boards, which can be used as spare parts for service repairs, or even for selling to customers. The traditional approach for managing spare parts consists of purchasing new parts from the regular supplier as needed. Moreover, suppliers of new parts (e.g., Intel) often have strong incentives to terminate regular production for parts that face slowing demand, such as after the product has been taken out of the market, and may demand a "final buy" from the manufacturer (Cattani and Souza 2003). In that case, the manufacturer often buys a large quantity of parts to last through the period for which it has service contracts. Since demand for spare parts is uncertain during that long period, the firm may run out of inventory and, if the original supplier no longer offers the part, has to procure the part at an alternative supplier (e.g., distributors specializing in hard-to-find parts) at a substantially higher cost. The dismantling decision offers flexibility to meet demand for spare parts in a less expensive way when there are shortages.

When a returned unit arrives at IBM's asset recovery facility, a disposition decision must be made to either remanufacture the unit, to dismantle it for spare parts, or to scrap it for material recycling. In general, remanufacturing and reselling a unit via the web site is more profitable than dismantling the returned unit for parts, which is more profitable than scrapping it. The current practice is to base this decision primarily on unit margins and on product quality. For products that are in a good technical condition, remanufacturing is currently always prioritized over dismantling.

In this paper, we argue that the disposition problem described above is essentially a revenue-management problem. Multiple recovery channels compete for a limited amount of high-quality returned units or cores. In addition to unit margins, the chances of actually selling a unit through a given channel drives this decision. Because prices for remanufactured units sold via the web site are market driven and face restrictions on their range from the new products sales force (to minimize canibalization), IBM faces demand uncertainty— rather than price uncertainty—for units remanufactured. Due to this uncertainty, dismantling for parts can be an attractive alternative, despite its lower unit margins. This option is particularly valuable if the inventory for spare parts is low relative to demand. Dismantling also faces a lower penalty for overproduction because spare parts can be carried in inventory until the end of the service period, which can be many months or even years. The disposition decision has to balance all these factors.

The concept of weighing the opportunity cost of a higher margin, but more uncertain, future demand versus a more certain, but lower margin, present demand is not new. Airlines, hotels, and rental car agencies have been weighing this trade-off for over twenty years using techniques originally referred to as yield management and today, more commonly referred to as revenue management (see Cross 1997 for an entertaining history of the field). The basic idea behind revenue management is to determine how many units of capacity to sell to lower margin customers that are requesting the product today versus how many units to reserve for higher margin customers that may arrive in the future. In the airline industry, customers are typically segmented into leisure and business segments. The leisure segment customers have a lower willingness to pay for a flight but are willing to pay for their seat well in advance of the departure date. The business customers often request seats close to the departure date and are willing to pay a higher price for this added convenience. While this is a major simplification of the actual problem, it captures the airline's main trade-off of how many seats to reserve for future business class customers given a demand of leisure customers that exceeds the seat capacity of the plane. Assuming lower margin customers arrive before higher margin customers, an optimal allocation is to sell to the lower value customers until the marginal opportunity cost (of missing future potential higher margin demand) exceeds the benefit from selling to the lower margin customer. Talluri and van Ryzin (2004) provide an extensive review of the revenue management techniques and algorithms used in practice and/or appearing in the literature.

To link the basic revenue management problem back to IBM's disposition decision regarding a returned unit, consider the spare parts demand as the lower paying customer and the potential sell of a remanufactured unit as a higher paying customer. Each arrival of a returned unit to the asset recovery facility is similar to a low margin customer requesting a seat on an airline. At each returned unit arrival, IBM must decide whether to dismantle the unit for spare parts, if current parts inventory is low, and receive a more certain low margin or to remanufacture the product in hopes of achieving a higher margin in the future by selling it through its web site. If the remanufactured unit does not sell through the web site during the four-week time window, the product is salvaged through an auction at a price roughly equal to the cost incurred for the remanufacturing operation. The additional value added to the product through the remanufacturing process is typically greater than the value obtained from dismantling the unit for spare parts, thus it is more profitable to salvage the remanufactured units that did not sell on the web site via the auction rather than dismantle the remanufactured units for spare parts. In other words, it is only profitable to dismantle the returned unit for parts *before* it undergoes the (costly) remanufacturing process. It is, however, more profitable to dismantle the returned unit, if parts inventory is low, than to remanufacture it and end up not selling it via the web site. Therefore, a remanufactured unit that does not sell via the web site represents the same missed opportunity cost as an empty seat on an airplane (upon departure) that the airline chose not to sell to a low margin customer in hopes that a higher paying customer would buy it instead.

1.1 Contribution to the Literature

The key contribution of this paper is to show how a revenue management approach to the disposition decision can significantly increase profitability. In more specific terms, we make the following contributions:

- We point out that the disposition decision in a closed-loop supply chain has many of the same characteristics as a traditional revenue management problem, even though the operational context of both problems is quite different.

- We develop and solve a stochastic optimization model for the disposition problem and characterize the optimal policy.

- We assess the financial value of the dismantling option. We find that excluding dismantling from the set of dispositions, by always prioritizing the high-margin remanufacturing option, decreases the firm's profit by 20% on average. This shows that an additional disposition option can significantly improve the profitability of a closed-loop supply chain, even if it has lower unit margins.

- Given the complexity of the optimal disposition policy, we investigate the performance of simpler heuristics. We find that allocating returns to remanufacturing and dismantling on the basis of their mean demands (net of inventory) performs reasonably well, relative to the optimal policy, with an average profit deterioration of only 4%. However, we also find that the value of dismantling is critically dependent on the coordination between forward and reverse supply chain decisions. Specifically, we show that the value of dismantling drops by about 60% if the final-buy decision does not appropriately anticipate future streams of returns and their disposition. This reinforces the view that firms should make their operating decisions as an integrated closed-loop supply chain rather than as separate forward and reverse chains.

The rest of the paper is organized as follows: In §2, we position our research in the context of the relevant literature. In §3, we list our key assumptions and notation, present our model and provide some analytic results. In §4, we present numerical results on the value of dismantling from a study based on realistic parameter values. In §5, we summarize our results and conclude with managerial implications. All proofs are provided in Appendix A.

2 Literature Review

Our research draws on two separate streams of literature: revenue management and remanufacturing. In this section, we provide a review of the prominent research in each stream and position our research at the point of their intersection. We begin with an overview of the relevant revenue management literature.

In the past two decades, revenue management has grown into a vast area of research with applications in numerous fields, notably in the service sector. The literature on the use of revenue management in a manufacturing setting has mainly focused on customer segments who are heterogeneous on their willingness to wait for a product and is split between make-to-order (MTO) and make-to-stock (MTS) environments. Research on firms who provide a MTO service has typically focused on capacitated systems where the firm provides discounts based on advance purchase, lead-time, and delivery time flexibility. Other proposed options include some type of admissions control policy where low margin orders may be declined, or quoted a longer lead-time, in anticipation of higher margin orders that may arrive in the future. Most of the work in this area models the manufacturing environment as a capacitated queueing system. A review of this literature can be found in Keskinocak and Tayur (2004). For MTS environments, the majority of the research has focused on inventory management policies that include rationing, where a portion of the inventory is reserved for higher margin customers who may arrive in the future. Topkis (1968), Ha (1997), Deshpande et al. (2003), and Zhao et al. (2005) provide a representative sample of the research in this area. The main difference between these two research streams and our paper is that our focus is on what type of product to produce (harvest for spare parts versus remanufacture for resell as a complete product) while, in the literature streams described above, the physical characteristics of the product is the same for all customers but the customer waiting times differ.

The literature on remanufacturing is part of a broader stream of research on reusable resources, which has been gaining significant momentum over the past decade. The recovery of value from used products and materials has triggered the emergence of 'reverse' supply chains, performing the acquisition and collection of returned goods, their inspection, grading, and disposition, reprocessing operations, such as remanufacturing and recycling, and the remarketing of the processed goods (Guide and Van Wassenhove, 2003). Collectively, these processes link markets that return certain products to new markets for these products. Reverse logistics is concerned with the physical flows through this link (Dekker at al., 2003). Key commercial decisions concern price, quantity, and quality both on the acquisition and on the reselling side (Guide and Van Wassenhove, 2001). Literature on quantitative approaches to these decisions is still relatively scarce. Guide et al. (2003) jointly optimize acquisition and sales prices, taking into account product quality variations. Galbreth and Blackburn (2006) address the interaction between acquisition quantity and quality. Ray et al. (2005) analyze optimal trade-in rebates. On the sales side, several authors have addressed the interaction between new production and remanufacturing, using game-theoretic analyses (e.g. Debo et al. 2005; Ferrer and Swaminathan, 2006; Ferguson and Toktay, 2006).

The disposition decision in the reverse supply chain allocates returned products to an appropriate processing option. In the simplest case, disposition options include a form of recovery, e.g. remanufacturing, and disposal. A richer setting may include several alternative recovery options, such as remanufacturing, harvesting of parts, and material recycling. Most research contributions, as well as business examples, address the disposition decision by means of a relatively long-term priority ranking, typically based on contribution margins. This approach assigns a returned product to the highest ranked option that is technically feasible, given the physical product status (Fleischmann et al., 2004). An exception is Guide et al. (2007), who link the disposition decision to the occupancy rate of the remanufacturing facility; thereby avoiding excessive processing delays.

In this study, we propose that disposition decisions may benefit from a dynamic approach that takes into account short-term variations in commercial opportunities. Two main factors drive our argument, namely demand uncertainty and depreciation. Guide et al. (2006) highlight depreciation as a key issue in closed-loop supply chains and analyze its impact on supply chain design. We address its influence on the short-term disposition decisions. This situation is analogous to classical airline revenue management, where discount decisions depend dynamically on the number of remaining empty seats and on the time until departure. To date, very few articles in the literature apply a revenue management approach to disposition decisions in closed-loop supply chains. The paper that is closes to our analysis is Inderfurth et al. (2001), which investigates the allocation of returned products to multiple alternative reuse options, given stochastic demand and return volumes. The authors analyze the optimal policy structure and calculate optimal control parameters under the assumption of a linear allocation of shortages. We do not make this assumption and instead investigate how to allocate scarce returns—and thereby also shortages—to the different channels so as to maximize profit. Kleber et al. (2002) analyze a related deterministic model and derive optimal disposition rules under time-varying return and demand volumes. In contrast to their analysis, our disposition strategy is driven by expected opportunity costs, rather than by seasonal fluctuations. Finally, we mention two papers that propose price-based revenue management in a closed-loop supply chain. Mitra (2007) determines optimal prices in a stochastic single-period model with multiple reuse options and Gayon and Dallery (2007) compare static and dynamic pricing strategies in a stochastic infinite-horizon model. In contrast to both of these models, our analysis focuses on quantity-based revenue management where the firm's decisions involve the allocation of returned products to alternative reuse options with exogenous prices.

3 Model Formulation and Analysis

3.1 Key Assumptions

Motivated by the context introduced in Section 1, we consider the situation of a firm that receives recoverable products returned from the market. The firm has two options for recovering value from these returns, namely remanufacturing or dismantling for parts. The firm seeks to allocate the returned products to these options so as to maximize expected profits. To model this disposition problem we make a number of assumptions, some of which are specific to our remanufacturing environment, that we discuss below.

Assumption 1 *The arrival of returned units is exogenous to the decision maker.*

There are cases where the decision maker in charge of making the disposition decision can influence the timing and quantity of returned units by offering a higher acquisition price (Guide et al. 2003). In the case of IBM, however, the majority of the returned units arriving at the remanufacturing facility are end-of-lease returns. The average age is three years from the original manufacture of the product. Because the leases are originally written by the sales force for the new products division, the manager in charge of handling product returns has little influence on the volume and timing of the returned units. This scenario is common for companies that lease their products.

Assumption 2 *The prices charged for the remanufactured units are exogenous to the decision maker's problem and constant over the planning horizon.*

Consider again the case at IBM. Because new products are also sold in the remanufactured products's channel (i.e., IBM's web site), the remanufactured units are restricted from being priced below a set percentage of the price for the new products in the same product category. This is done to minimize the cannibalization effect of the remanufactured units on the sales of the new units. There are also brand image concerns that limit IBM from pricing the remanufactured units at a market clearing price. The third party vendors that purchase the unsold (after four weeks on the web site) remanufactured units via the auction agree to sell the units in secondary markets to minimize the cannibalization and negative brand image effects. For these reasons, the prices for remanufactured units made available on the web site are typically set well in advance and are not changed based on the volume or current sales rate of the remanufactured units. Thus, exogenously determined remanufactured prices are a reasonable assumption in our context.

Assumption 3 *Remanufactured units can only be sold during a limited time period. Leftover units at the end of this period are salvaged.*

This assumption is related to the previous one. Given pricing inflexibility, a finite selling period counterbalances product depreciation, which is substantial for electronic products, and clears inventory.

Assumption 4 *Unsatisfied demand for spare parts entails a per-unit penalty cost which is constant over time.*

This assumption is true for the majority of the spare parts. Once a product's demand slows, suppliers typically require a final buy for their components, so as to free valuable capacity for more current (and higher demand) components. The firm thus makes a final buy purchase for each part to meet warranty and contractual repair needs for the remaining life of the product. We consider the firm's situation after this final buy. If the firm runs out of inventory for a part during that period, the firm must procure the part at an alternative, considerably more expensive supplier (e.g., distributors specialized in "hard to find" components). The price of the component from these third-party suppliers are typically constant over time.

Assumption 5 *The per unit cost to remanufacture a returned unit is constant across all units.*

This assumption may seem counter to the claims of other recent work, which states that remanufacturing cost per unit depends on a returned unit's quality (and it is thus not constant across all units). Indeed, at IBM, returned units arrive with various quality levels and thus, the costs associated with bringing the units up to the quality level of the standard configuration needed for resale also vary. What makes this assumption reasonable in our model is the fact that IBM (and other firms in the IT industry) only consider the highest quality returns for remanufacturing. A laptop with a cracked external case, for example, is too costly to remanufacture and is thus screened out after an initial inspection. Therefore, all units considered for remanufacturing have roughly the same quality level and, thus, the same per unit cost. In other industries where remanufacturing is more labor intensive, and there are more wearable mechanical components, this assumption is not likely to hold. Instead, total remanufacturing cost should be convex increasing in the quantity (Ferguson et al. 2008). Our analysis concerns the returns that qualify for remanufacturing in terms of their quality. There may be additional lower quality returns that are too expensive to profitably remanufacture but can still be used for parts harvesting. We consider the demand for parts net of this inflow.

3.2 Single Period Analysis

Table 1: Model Notation

Decision Variables	
Q_r	number of products to be remanufactured
Q_d	number of products to be dismantled
Random Variables	
R	number of products returned $\sim F_R$
D_r	demand for remanufactured products $\sim F_r$
D_d^i	demand for part i from dismantling $\sim F_d^i$, with mean μ^i
Parameters	
a_i	number of parts of type i per returned unit
c_r	unit remanufacturing cost
c_d	unit dismantling cost
p_r	unit sales price for remanufactured product
π^i	penalty of not meeting demand for part i
v	unit salvage value for remanufactured unit not sold

We begin our analysis with a single period model. Table 1 lists our notation. Using the assumptions outlined above, the disposition problem can then be formalized as follows. At the beginning of the period, the firm receives R returns. Of these, the firm decides upon the number of units to be remanufactured Q_r, and the number of units to be dismantled Q_d. The remainder units, $R - Q_r - Q_d$, are scrapped at a cost normalized to zero. Remanufactured returns that are not sold are salvaged at a unit value of v. Demand for part i that is not met is assessed a unit penalty cost π^i, which is the (higher) cost of obtaining the part through an alternative supplier.

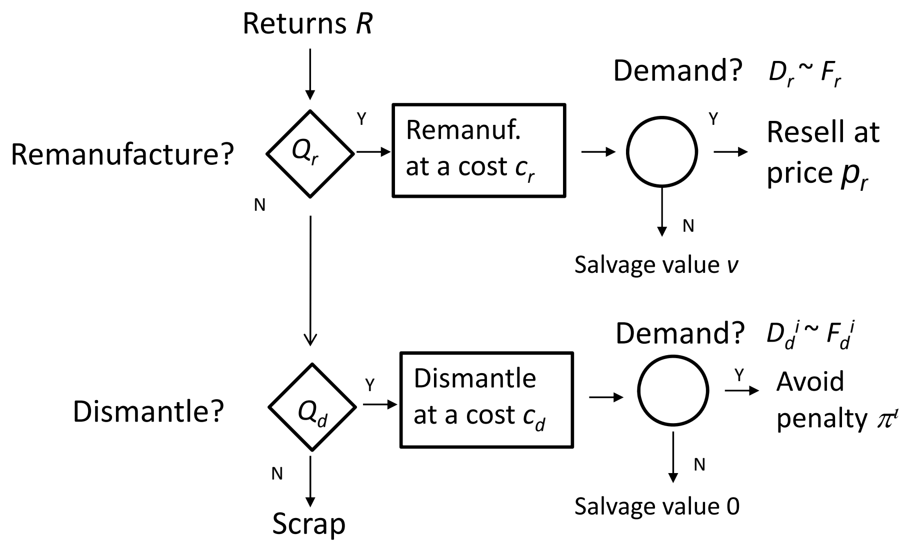

Figure 1: Basic Single Period Model

This setting is illustrated in Figure 1. The firm maximizes its one-period expected profit Π:

$$\max_{Q_r+Q_d\le R} \Pi = E\left[(p_r - v)\min\{D_r, Q_r\} + (v - c_r)Q_r - \sum_i \pi^i(D_d^i - \min\{D_d^i, a_i Q_d\})\right] - c_d Q_d. \quad (1)$$

Considering that $E[\min\{D,Q\}] = Q - \int_0^Q F(u)du$, where $F(\cdot)$ is the cdf of D, (1) becomes:

$$\begin{aligned}\max_{Q_r+Q_d\le R} \Pi &= (p_r - v)\left(Q_r - \int_0^{Q_r} F_r(u)du\right) + (v - c_r)Q_r \\ &\quad -\sum_i \left[\pi^i\mu^i - \pi^i\left(a_i Q_d - \int_0^{a_i Q_d} F_d^i(u)du\right)\right] - c_d Q_d. \end{aligned} \quad (2)$$

The optimal solution of this problem is described in Lemma 1 below:

Lemma 1 *Let*

$$\begin{aligned}\Pi_d(Q) &= \sum_i \pi^i a_i\left(1 - F_d^i(a_i Q)\right) - c_d \\ \Pi_r(Q) &= (p_r - v)\left(1 - F_r(Q)\right) + v - c_r.\end{aligned}$$

Denote by $\tilde{Q}_d$, and $\tilde{Q}_r$ the solutions to $\Pi_d(Q) = 0$ and $\Pi_r(Q) = 0$, respectively. Further, denote by $\hat{Q}_d$, the solution to $\Pi_d(Q) = \Pi_r(R - Q)$. Then, the optimal solution to the disposition problem (2) is

$$(Q_d^*, Q_r^*) = \begin{cases} (\min\{R, \tilde{Q}_d\}, 0) & \textit{if } \Pi_d(R) > \Pi_r(0), \\ (0, \min\{R, \tilde{Q}_r\}) & \textit{if } \Pi_d(0) < \Pi_r(R), \\ (\min\{\tilde{Q}_d, \hat{Q}_d\}, \min\{\tilde{Q}_r, R - \hat{Q}_d\}) & \textit{else }. \end{cases}$$

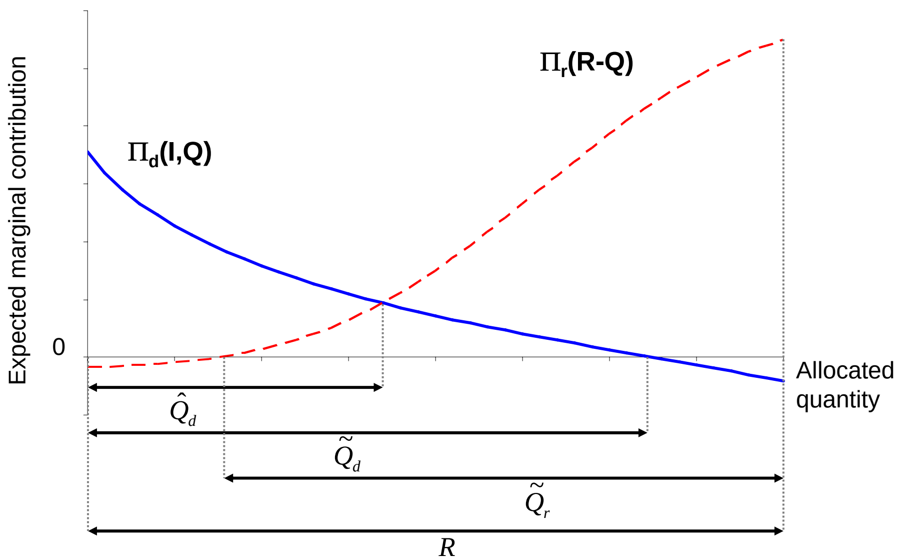

Figure 2: Structure of the Optimal Disposition Policy

In essence, Lemma 1 indicates that if there are enough returns, the firm can satisfy demand for dismantling and remanufacturing at the levels $\tilde{Q}_d$ and $\tilde{Q}_r$, respectively, that set the marginal profit of dismantling and remanufacturing equal to zero. Otherwise, the firm sets the optimal level of dismantling such that the marginal profit of dismantling is equal to the marginal profit of remanufacturing.

Figure 2 illustrates this optimal disposition policy. As in traditional revenue management problems, the optimal decision is determined by the expected marginal contributions, which is intuitive. Unlike in the traditional case however, the optimal decision is not a critical-level policy, in which the quantity available for the low-value channel equals the amount exceeding a certain threshold. This is due to the fact that in the disposition problem, demand in both channels is uncertain.

To illustrate this effect, we consider the special case that demand for parts is deterministic, and equal across parts. Corollary 1 shows that the optimal disposition policy is a critical-level policy in this case.

Corollary 1 *Assume that $D_d^i = d_d$ w.p. 1, for all i. Let $\pi = \sum_i a_i \pi^i$ and assume that $\pi - c_d > 0$. Then the optimal remanufacturing and dismantling quantities satisfy:*

$$(Q_d^*, Q_r^*) = \begin{cases} (R - \min\{R, \check{Q}_r\}, \min\{R, \check{Q}_r\}) & \text{if } R \leq d_d + \check{Q}_r, \\ (d_d, \min\{\tilde{Q}_r, R - d_d\}) & \text{else }, \end{cases}$$

where $\check{Q}_r = F_r^{-1}\left(1 - \frac{c_r - v - \pi + c_d}{p_r - v}\right)$ *and* $\tilde{Q}_r$ *is as defined in Lemma 1.*

As in the general case, the firm determines the optimal amounts of remanufacturing and dismantling by comparing the expected marginal profits of both options. In this deterministic case, however, the marginal profit of dismantling is constant and equal to $\pi - c_d$, for a quantity up to d_d. $\check{Q}_r$ denotes the remanufacturing quantity for which the expected marginal profit equals the marginal dismantling profit. Note that $\check{Q}_r$ is independent of R and is obtained as a newsvendor solution. This is the amount which is protected for the "high-margin customers", i.e. for remanufacturing. Any returns in excess of this quantity are available for dismantling, up to a maximum quantity of d_d. Any remaining returns will again be remanufactured, as long as the expected marginal profit remains positive.

We conclude this section be considering another special case that approximates IBM's situation. At IBM, the salvage value of unsold remanufactured units v is approximately equal to the remanufacturing cost c_r. If $v = c_r$, there exists an optimal solution where returns are either dismantled or remanufactured (i.e., none are scrapped). The reason is because scrapping a returned unit incurs a profit of zero; all units that are remanufactured but not sold via the website also incur a profit of zero if $v = c_r$. Thus, if $v = c_r$, there is no risk in remanufacturing "extra" returns, even for very small probabilities that they will be sold through the website. We formalize this result in Corollary 2 below.

Corollary 2 *If* $v = c_r$*, then there exists an optimal solution to (2) for which* $Q_d^* + Q_r^* = R$.

3.3 Multi-Period Analysis

In many examples, such as at IBM, the period during which a firm offers a repair service, and thus requires spare parts, is much longer than the period during which a returned unit can be sold—typically many months or years versus a few weeks. Firms also typically receive multiple batches of returns during this service period. To capture the dynamics of this situation, we extend our disposition model to a multi-period setting.

The periods are interconnected through the inventory level of spare parts. Any units that are dismantled feed the parts inventory, which is used to meet demand for the parts. If demand for parts exceeds the available inventory, the firm has to procure parts from an expensive backup supplier, as discussed in the single-period analysis. Intuitively, the disposition decision depends on the initial parts inventory level and thereby on the final-buy decision. We therefore include the final-buy decision in our analysis.

We formalize the multi-period disposition problem as follows. For ease of notation we assume that there is only one recoverable part per return, i.e. a_1=1. Our results extend to the more general case but the notation becomes cumbersome. We consider a planning horizon of T periods. Periods are numbered backwards, such that period t indicates there are t periods until the end of the planning horizon. At the beginning of period t, the firm observes I_t, the starting inventory for spare parts, and R_t, the incoming returns in that period. The firm then decides upon the number of returns to remanufacture $Q_{r,t}$ and to dismantle $Q_{d,t}$. Demand for remanufactured units $D_{r,t}$ and for spare parts $D_{d,t}$ is realized. Demand for remanufactured products not met is lost; unsold remanufactured products are salvaged at a unit value v. Demand for spare parts not met from inventory is met through an alternative supplier at a cost π per unit; left over inventory for spare parts is carried over to period $t - 1$ at a holding cost of h per unit. Any left over spare parts at time $t = 0$ have zero salvage value.

At the start of the planning horizon T, the firm makes a final-buy decision, at a cost of c per part, that brings its inventory of spare parts to I_T. The firm aims to maximize expected discounted profits through the planning horizon, given a one-period discount factor α.

This problem can be formulated as a finite-horizon Markov decision process (MDP) as follows.

The state variable is I_t, the actions are $Q_{d,t}$ and $Q_{r,t}$, and the one period profit for $t \geq 1$ is:

$$\begin{aligned}\Pi_t(I_t, Q_{d,t}, Q_{r,t}) &= (p_r - v)\left(Q_{r,t} - \int_0^{Q_{r,t}} F_r(u)du\right) + (v - c_r)Q_{r,t} \\ &\quad -c_d Q_{d,t} - \pi\mu_d + \pi(I_t + Q_{d,t}) - (h+\pi)\int_0^{(I_t+Q_{d,t})} F_d(u)du. \end{aligned} \tag{3}$$

The state transition is defined by $I_{t-1} = (I_t + Q_{d,t} - D_{d,t})^+$. For $I_t \geq 0$, the Bellman recursion of this MDP therefore reads:

$$V_t(I_t) = E_{R_t}\left[\max_{Q_{d,t}+Q_{r,t}\leq R_t}\left(\Pi_t(I_t, Q_{d,t}, Q_{r,t}) + \alpha\int_0^\infty V_{t-1}((I_t + Q_{d,t} - u)^+) f_d(u)du\right)\right], \tag{4}$$

with boundary condition $V_0 \equiv 0$. Recursion (4) provides the disposition decision in each period, given a starting value for inventory. The final buy decision is made at the beginning of period T, and thus

$$V_T^* = \max_{I_T}\{-cI_T + V_T(I_T)\}. \tag{5}$$

The solution of the above MDP has a similar structure and interpretation as in the single-period case. The marginal contributions of both disposition alternatives drive the allocation decision. What is different here is that one also has to take into account the impact on future periods through the resulting inventory level. In addition, the interplay between disposition and final-buy adds another layer to the problem. The essential property that drives the structure of the optimal policy is concavity of the value function in the inventory state variable. We summarize this result in the following theorem

Theorem 1 *For any $t \geq 1$ the MDP defined in (3) and (4) satisfies the following properties:*

(i) $V_t(I)$ *is concave in* I*.*

(ii) $W_t(I_t, Q_{d,t}, Q_{r,t}) := \Pi_t(I_t, Q_{d,t}, Q_{r,t}) + \alpha\int_0^\infty V_{t-1}((I_t + Q_{d,t} - u)^+) f_d(u)du$ *is jointly concave in* $I_t, Q_{d,t}$ *and* $Q_{r,t}$*.*

(iii) Let

$$\Pi_{d,t}(I, Q) = -(h+\pi)F_d(I+Q) + \alpha\int_0^{I+Q} V'_{t-1}(I + Q - u) f_d(u)du - c_d + \pi \tag{6}$$

$$\Pi_r(Q) = (p_r - v)(1 - F_r(Q))) + v - c_r, \tag{7}$$

where V_t' is the first-order derivative of V_t. Denote by $\tilde{Q_{d,t}}$, and $\tilde{Q_{r,t}}$, respectively, the solutions to $\Pi_{d,t}(I_t, \tilde{Q_{d,t}}) = 0$ and $\Pi_r(\tilde{Q_{r,t}}) = 0$. Further, denote by $\hat{Q_{d,t}}$, the solution to $\Pi_{d,t}(I_t, \hat{Q_{d,t}}) = \Pi_r(R - \hat{Q_{d,t}})$. Then, the optimal solution to (4) is

$$(Q^*_{d,t}, Q^*_{r,t}) = \begin{cases} (\min\{R, \tilde{Q_{d,t}}\}, 0) & \text{if } \Pi_{d,t}(I_t, R) > \Pi_r(0), \\ (0, \min\{R, \tilde{Q_{r,t}}\}) & \text{if } \Pi_{d,t}(I_t, 0) < \Pi_r(R), \\ (\min\{\tilde{Q_{d,t}}, \hat{Q_{d,t}}\}, \min\{\tilde{Q_{r,t}}, R - \hat{Q_{d,t}}\}) & \text{else} . \end{cases} \tag{8}$$

The optimal disposition policy has the same structure as in the single-period case. However, the marginal benefit of dismantling (i.e. $\Pi_{d,t}$) now depends on the spare parts inventory level. Note from (6) that an additional unit of inventory shifts the dismantling marginal profit curve in Figure 2 to the left by one unit. This also shifts the intersection point of both marginal profit curves to the left, but not necessarily by a full unit. Therefore, the optimal policy is not a critical-level policy that replenishes the parts inventory up to a certain fixed target level.

Note further that as a consequence of the concavity of V_T, the optimal final-buy quantity I_T^* can be found through a simple myopic search for the maximum.

4 Numerical Results

In this section, we conduct a detailed numerical analysis to assess the performance of our revenue management approach to product disposition and compare it to other approaches found in practice or in the literature. Our objective is (i) to gain further insight into the characteristics of our revenue management approach and the factors that drive it; and (ii) to determine under which conditions our approach significantly outperforms the other policies and when, on the contrary, a simpler heuristic will suffice.

4.1 Experimental Design

We consider a planning horizon with $T = 10$ periods, where each period corresponds to one month. Returns arrive randomly in each period according to a Poisson process with the mean scaled to $\mu_R = 10$ (it may be helpful to think of one unit in this analysis as, say, 1000 units in real life). We consider the simpler case where there is only one recoverable part per return ($a_1 = 1$); this can be thought of as an "aggregate" part. We normalize the remanufactured product's price to $p_r = 1000$. The unit cost of purchasing a spare part at the beginning of the planning horizon through the final buy is $c = 100$, or 10% of the remanufactured product's price; this is a realistic number based on our discussions with IBM. The one-period discount factor is $\alpha = 0.99$, corresponding to an annual cost of capital of 12%; we have experimented with other reasonable discount factors and concluded that they do not impact our results. Finally, we assume $v = c_r$, which is in line with IBM's situation.

We ran a full-factorial experimental design for the remaining parameters of our model. Each factor in the experimental design, a parameter of the model, is explored at three levels: low, medium and high. These levels were chosen based on observed industrial practice as justified below. The total *average* demand for spare parts and remanufactured products is expressed as a fraction $1/k$ of the *average* number of returns per period: $k(\mu_r + \mu_d) = \mu_R$. We choose $k \in \{0.8, 1.0, 1.2\}$, corresponding to cases when the average returns per period are: insufficient to meet the average total demand ($k = 0.8$), equal to average total demand ($k = 1$), and larger than average total demand ($k = 1.2$). Regarding the mix of demand between remanufactured products and spare parts, we assume the demand for spare parts in each period follows a Poisson distribution, with its mean expressed as a fraction of the mean for remanufactured products $\frac{\mu_d}{\mu_r} \in \{0.5, 1, 2\}$; corresponding to reasonable low, medium and high values found in practice. We consider that demand for remanufactured products follows a normal distribution with mean μ_r and a coefficient of variation $CV_r \in \{0.1, 0.4, 0.7\}$, corresponding to low, medium and high levels of demand variability.

Remanufacturing cost per unit relative to price $\frac{c_r}{p_r}$ is set at low, medium and high levels of 0.1, 0.4, and 0.7, respectively. These choices are justified as follows: Agrawal et al. (2008) report values for $\frac{c_r}{p_r}$ in the range 0.05-0.20 for commercial IT equipment; Hauser and Lund (2003) report average values of $\frac{c_r}{p_r}$ in the range 0.45-0.65 for industries where remanufacturing is more labor intensive (and thus remanufacturing is more expensive). Dismantling cost c_d should be lower than c, otherwise dismantling for spare parts is not economically attractive and our problem is not interesting. We therefore varied $\frac{c_d}{c}$ over a wide range between 0.1 and 0.7 to reflect the possible values found in practice. Similarly, the cost of meeting demand for spare parts with the alternative supplier π

should be higher than c, otherwise the problem is not interesting. We thus choose values of $\frac{\pi}{c}$ in the range 1.5-4.5, to reflect a wide range of scenarios. Finally, considering that a period in our study is one month, $\frac{h}{c}$ can be thought of as the monthly holding cost on a percentage basis. We choose values between 0.01 and 0.10 to reflect annual holding costs between 12% and 120%. The high values reflect the additional cost of obsolescence and price decay common in many industries. Thus, the low values are common for products with little value depreciation—around 0.25% per week—such as power tools, whereas the high values correspond to products with very high value decay—around 2% per week—found in some electronic components (Guide et al. 2007).

Our experimental design is summarized in Table 2. There are $3^7 = 2,187$ experimental cells. This experimental setting is used to study several different facets of our problem, as detailed in the next subsections.

Table 2: Experimental Design for Numerical Study ($p_r = 1000$,$c = 100$, $\alpha = 0.99$, $\mu_R = 10$)

Factor description	**Symbol**	**Factor Levels**
Total avg. demand as a fraction of avg. returns	k	0.8, 1.0, 1.2
Mean demand for parts relative to remanuf. products	$\frac{\mu_d}{\mu_r}$	0.5, 1.0, 2.0
CV of demand for remanufactured products	CV_r	0.1, 0.4, 0.7
Relative remanufacturing cost per unit	$\frac{c_r}{p_r}$	0.1, 0.4, 0.7
Relative dismantling cost	$\frac{c_d}{c}$	0.1, 0.4, 0.7
Relative penalty cost for spare parts (alternative vs. regular)	$\frac{\pi}{c}$	1.5, 3.0, 4.5
Holding cost per month (%)	$100\frac{h}{c}$	1, 5, 10

4.2 Revenue-Management Disposition Policy

We first highlight a few characteristics of the optimal disposition policy. The optimal policy allocates, on average, 28% of the returns to dismantling, with a standard deviation of 16%, a minimum of 0.1%, and a 95th percentile of 53%. Since $v = c_r$, the remaining returns are remanufactured. Thus, the share of dismantling is substantial, even though its unit margin $\pi - c_d$ amounts to only 53%, on average, of the remanufacturing margin $p_r - c_r$. This underlines the importance of looking beyond unit margins in the disposition decision. We further elaborate on this point in the next subsection.

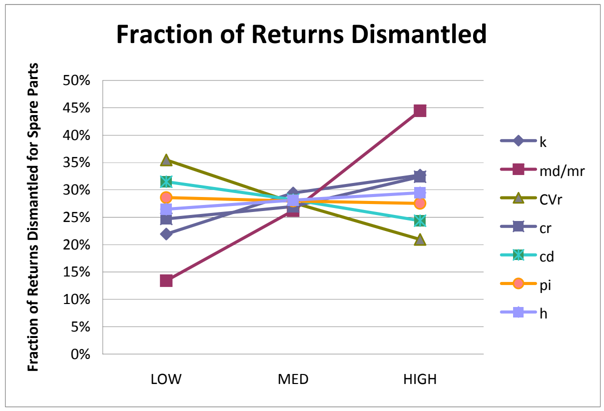

Figure 3: Fraction of Returns Dismantled for Different Levels of Parameter Values

Figure 3 highlights the impact of the various experimental factors on the optimal share of dismantling. For each factor level (e.g., $CV_r = 0.1$), the graph displays the average value of the share of dismantling in the optimal disposition across all respective experimental cells. We observe that the ratio between expected demand for parts and demand for remanufacturing, $\frac{\mu_d}{\mu_r}$, is the main driver. This is not surprising, since one would expect the share of returns allocated to a specific channel to increase with the demand in that channel (ceteris paribus). Among the remaining experimental factors, the CV of demand for remanufacturing appears to have the strongest impact, and its effect on the share of dismantling is negative. With a higher CV of demand, the risk of losing sales in the higher-margin remanufacturing channel increases, which motivates a higher allocation of returns to that channel.

To formalize and quantify these relationships, we performed six single linear regressions. In each regression, the dependent variable is the fraction of returns dismantled, the independent variable is the experimental factor of interest, and there are 729 observations. The value of R^2 in each regression provides a simple quantifiable metric for the impact of each factor (Wagner 1995). We obtained R^2 values of 65% and 14% for $\frac{\mu_d}{\mu_r}$ and CV, respectively. All other R^2 values are smaller than 8%

In addition to the ratio between remanufacturing and dismantling volumes, the fractions of parts demand served through dismantling and the final buy, respectively, also characterizes the optimal policy. The final buy largely determines the demand that is left to fill by dismantling. In our experiments, we found that the optimal policy meets, on average, 54% of demand for spare parts through dismantling, with a standard deviation of 43%, a minimum of 0.2%, and a 95th percentile of 94%. Thus, the fraction of demand covered by dismantling is relatively evenly distributed and the variation is large.

Figure 4 again highlights the impact of the experimental parameters on these outcomes. In addition to the factors identified in Figure 3, the overall volume of returns k appears as an obvious driver—the optimal policy dismantles more when more returns are available. Regression analysis yielded R^2 values of 45%, 22%, and 14% for k, CV_r, and $\frac{\mu_d}{\mu_r}$, respectively, and values smaller than 6% for all remaining factors.

The results in Figure 4 also give an indication of the optimal final-buy quantities in the different scenarios. The demand volume not served through dismantling essentially has to be covered by the final buy. In addition, we also have the emergency supply, but this volume is small, in general.

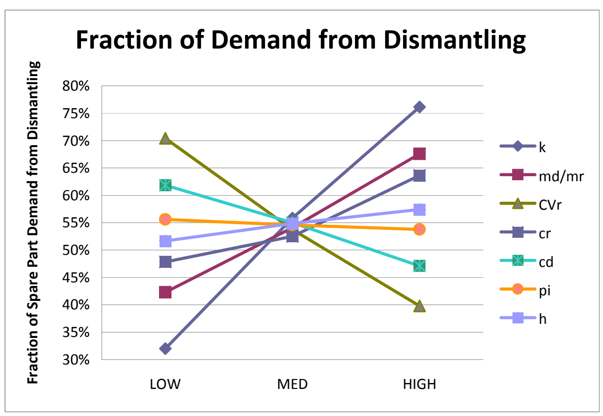

Figure 4: Fraction of Parts Demand Met from Dismantling for Different Levels of Parameter Values

4.3 Value of Dismantling

We now proceed to compare our revenue-management based disposition policy with other policies. As discussed in Sections 1 and 2, a common approach in practice is to rank dispositions by unit margin and to allocate returns depending on their quality condition to the highest-ranked option that is technically feasible. In our case, this would mean remanufacturing all returned units. (Recall that we only consider high-quality returns.) Demand for parts is then met solely with inventory from the final buy plus potential emergency supplies.

We refer to this policy as 'no dismantling' and denote its expected discounted profit through the planning horizon by V_T^{ND}; this value is found by solving the MDP (4)-(5) with $Q_{d,t} = 0$ and $Q_{rt} = R$ for all t (in our experiments, $v = c_r$). Thus, the only decision in the no-dismantling case is the quantity of the final buy I_T^{ND} at time $t = T$. We define the value of dismantling as $\Delta^{ND}(V^*) = 100\% \frac{V_T^* - V_T^{ND}}{V_T^*}$, which is the profit deterioration from not using the dismantling option.

Using the experimental design of Table 2, the mean value of dismantling is 20% (median 10%), with a standard deviation of 30%, a minimum value of 0% and a 95th percentile of 85%. Thus, the value of dismantling is significant, and varies widely across our experimental design. This is in line with the results of Section 4.2: Not only does the optimal policy recommend dismantling a large fraction of the returns for spare parts, it also achieves significant financial benefits by doing so.

The differences between the policies regarding dismantling volumes are also mirrored in the corresponding final-buy quantities. Define $-\Delta^{ND}(I_T)$ as the relative inventory increase at the beginning of planning horizon when moving from the optimal policy to a no-dismantling policy. The median inventory increase is 127%, with a mean value of 290%. This drastically highlights one of the tangible benefits of dismantling, which is a very significant reduction in inventory held for spare parts. Note that the distribution of $\Delta^{ND}(I_T)$ is highly right skewed due to the many cases where the firm carries little or no inventory of spare parts in the optimal policy, due to a relative abundance of returns available for dismantling.

To gain insight into the large variations in the value of dismantling, we again compute averages for each factor level. The results are shown in Figure 5. As in Figure 3, the ratio between μ_d/μ_r turns out to have the strongest impact (R^2=30%). Thus, relatively higher expected demand for parts results in a higher dismantling volume and also in a higher financial contribution of dismantling. In addition, the remanufacturing cost also strongly influences the value of dismantling (R^2=21%). Higher remanufacturing costs reduces the margin of remanufacturing and therefore makes dismantling more attractive.

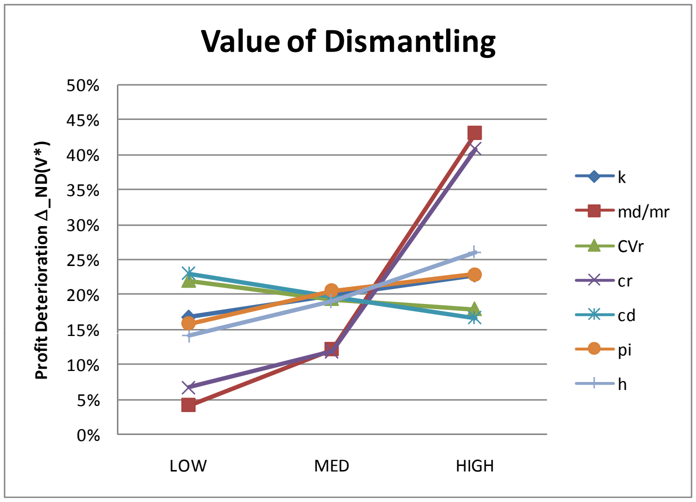

Figure 5: Value of Dismantling for Different Levels of Parameter Values

The remaining experimental factors appear to have a lower impact in Figure 5 (all $R^2 < 3\%$). Qualitatively, we observe that the value of dismantling increases for higher holding cost $\frac{h}{c}$, higher return volume k, higher penalty cost $\frac{\pi}{c}$, lower variability of demand for remanufactured products CV_r, and lower dismantling cost $\frac{c_d}{c}$. These outcomes largely match those regarding the dismantling fraction in Figure 3. It is worth noting the effect of the holding cost rate. A higher value of $\frac{h}{c}$ renders the final buy of spare parts more expensive and therefore makes dismantling a relatively more attractive source for spare parts. While Figure 3 shows that this effect results in only a relatively minor increase of the dismantling volume, Figure 5 shows that the financial value of dismantling increases substantially.

To summarize, ignoring the dismantling option reduces profit by an average of 20% (median value of 10%), and inflates spare parts inventory by a median value of 127%. Further, the average value of dismantling can be significantly higher if the demand for spare parts is high (43%), the remanufacturing cost is high (41%), and the holding cost is high (26%). Note that these outcomes are based on the assumption of riskless remanufacturing (i.e. $v = c_r$). A lower salvage value of remanufactured units is likely to strengthen the importance of dismantling even further.

4.4 Value of Optimal Allocation

We complement the results of the previous section by evaluating another common heuristic. In the case of shortages, many firms allocate available supply to customer orders proportional to their order volumes. This practice has been widely discussed in studies on the bullwhip effect (see e.g. Lee et al., 1997). Inderfurth et al. (2001) apply this linear allocation rule to the disposition decision in a closed-loop supply chain.

We assess the performance of such a linear allocation through a disposition policy that allocates returns to remanufacturing and dismantling proportionally to their respective mean demands, net of inventory. Specifically, given R returns at the beginning of period t, the firm sets $Q_{r,t} = \frac{\mu_r}{\mu_r+\max(\mu_d-I_t/t,0)}R$ and $Q_{d,t} = R - Q_{r,t}$. The second term in the denominator of $Q_{r,t}$ is the expected demand for parts net of inventory. Denote the expected discounted profit through the planning horizon for using this mean demand allocation (MDA) heuristic by V_T^{MDA}. Further denote the *value of optimal allocation* by $\Delta^{MDA}(V^*) = 100\%\frac{V_T^*-V_T^{MDA}}{V_T^*}$, which is the profit deterioration of using the MDA heuristic relative to the optimal policy.

Using the experimental design of Table 2, the average value of optimal allocation $\Delta^{MDA}(V^*)$ is 3.8%, with a median value of 3%, a standard deviation of 3.6%, a minimum value of 0.1% and a 95th percentile of 9.2%. Thus, a simple allocation policy based on mean demands net of inventory performs reasonably well in many cases. This is good news from a managerial perspective, since this allocation rule is much simpler than the optimal allocation, which is based on detailed dynamic optimization, even for the case of a single recoverable part per return.

As for the ND policy, we also consider the effect of the MDA rule on inventories. Define $\Delta^{MDA}(I_T)$ to be the increase of the optimal final buy, and thus of the initial inventory of parts, by using the MDA heuristic vis-a-vis the optimal disposition. The average inventory increase $\Delta^{MDA}(I_T)$ is 44%, and the median increase is 22%, which is a significant value but not nearly as high as in the ND case.

At first sight, it may be surprising that the MDA rule implies higher inventories than the optimal policy. The reason is that the MDA rule does not consider cost effects in its allocation. In the case of low inventories, this may result in expensive out-of-stocks of parts. To prevent this situation, the MDA uses a larger final buy, but at the expense of higher inventories. Consequently, the MDA policy dismantles 16% less returns on average than the optimal policy, with a 95th percentile of 60%. We note, however, that in 21% of the scenarios the MDA policy sets lower inventories and dismantles more than the optimal policy. These are cases where the final buy for the optimal policy is high: i.e. scarce returns, remanufacturing economically more attractive relative to dismantling, and low holding costs.

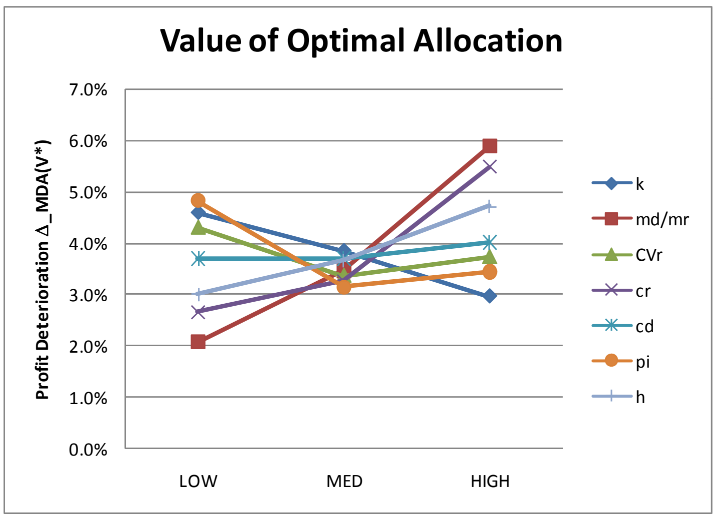

Figure 6: Value of Optimal Allocation for Different Levels of Parameter Values

Similar to the previous sections, we also performed an analysis to gain insights into which factors most influence the value of optimal allocation; the results are shown in Figure 6. The same factors as in the ND case stand out in explaining $\Delta^{MDA}(V^*)$: mean parts demand relative to remanufacturing $\frac{\mu_d}{\mu_r}$ (R^2=19%), remanufacturing cost $\frac{c_r}{c}$ (R^2=10%), and to a less extent holding cost $\frac{h}{c}$ (R^2=4%).

To summarize, a simple linear allocation of returns based on mean demands (net of inventory) performs relatively well, with an optimality gap of 3.8% on average. This supports the practical use of this rule. We note, however, that the performance of the MDA rule is dependent on an appropriate adjustment of the final buy. This turns the disposition problem into an inventory management problem. We address this interplay in more detail in the following subsection.

4.5 Value of Coordination between Final Buy and Dismantling

Our optimal policy assumes the final buy quantity I_T^* takes into account the forecast for future returns, along with an optimal allocation of returns between dismantling and remanufacturing. This assumes a perfect coordination between a purchasing manager, who is responsible for the supply of a particular part, and the reverse supply chain manager, who is responsible for forecasting and processing returns. It has been our experience that in many firms (e.g., IBM, HP, Pitney Bowes) these two managers typically do not belong to the same organizational unit. In particular, final buy decisions for parts are frequently made based on a forecast of demand during the remaining lifetime and service period, without taking into account the potential supply of parts from used units (Cattani and Souza 2003). Thus, the "perfect coordination" assumed by the optimal policy may be unrealistic. We now investigate the *value of coordination*: What is the profit deterioration of having the final buy decision made without consideration of a possible supply of parts from dismantling in the future? The final buy quantity without consideration of future dismantling is I_T^{ND}. Once that decision is made, however, the firm makes the optimal *allocation* decision between dismantling and remanufacturing as returns arrive. Thus, the value of dismantling is given by $\Delta^{NC}(V^*) = 100\% \frac{V_T^*(I_T^*) - V_T^*(I_T^{ND})}{V_T^*(I_T^*)}$.

Using the experimental design of Table 2, the average value of coordination $\Delta^{NC}(V^*)$ is 14.3%, with a median value of 5.7%, a standard deviation of 23%, a minimum value of 0% and a 95th percentile of 67%. Thus, accounting for the dismantling option in the final buy has the potential to significantly improve profits. An analysis similar to the one in Section 4.3 reveals a very similar picture to that of Figure 5, so we omit it for brevity. Thus, the drivers of the value of coordination are the same as those of the value of dismantling: mean part demand relative to remanufacturing demand $\frac{\mu_d}{\mu_R}$, the remanufacturing cost per unit $\frac{c_r}{p_r}$, and to a less extent $\frac{h}{c}$.

To assess these results it is useful to put the value of coordination into another perspective. To this end, we express the value of coordination as the fraction of the dismantling benefit that can be attributed to coordination. This fraction can be expressed as $\tilde{\Delta}^{NC}(V^*) = 100\% \frac{V_T^*(I_T^*) - V_T^*(I_T^{ND})}{V_T^*(I_T^*) - V_T^{ND}(I_T^{ND})}$. The quantity in the numerator is the dismantling benefit attributable to coordination, whereas the denominator represents the total dismantling benefit. The average value of $\tilde{\Delta}^{NC}(V^*)$ is 61%, indicating that 61% of the dismantling benefit can be attributed to coordination between the final buy decision and the processing of returns at the reverse supply chain. Put differently, only 39% of the dismantling benefit can be achieved without coordination between the forward and reverse chains.

An analysis of the factors driving this number is shown in Figure 7; it can be seen that all parameters are roughly equally important in explaining this result, so that the fraction of the dismantling benefit achieved through coordination is high in most scenarios. This result strongly points to the need for coordinating reverse and forward supply chains to achieve maximal profitability.

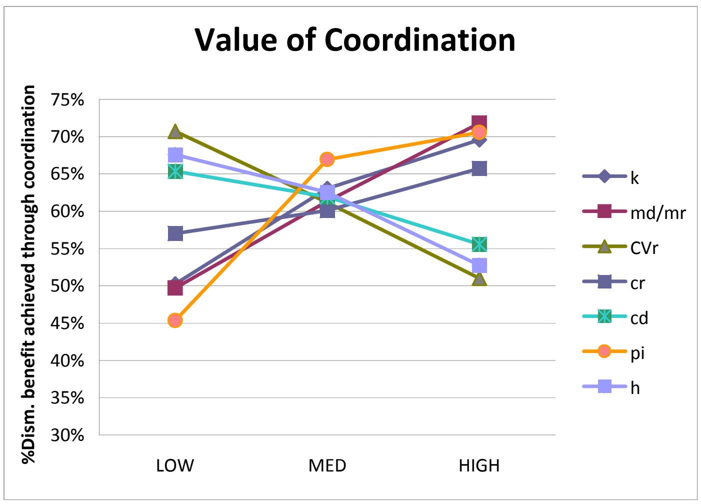

Figure 7: Value of Coordination for Different Levels of Parameter Values

5 Conclusion and Managerial Implications

In this paper, we propose a revenue management perspective to the problem of making optimal disposition decisions for product returns in a closed-loop supply chain. Motivated by a case study of IBM, we consider two disposition alternatives, namely remanufacturing and dismantling for spare parts. We describe how disposition decisions for product returns have typically been made based solely on unit profit margins. We argue that the disposition problem is actually a revenue-management problem and that the optimal decision should balance unit margins and demand uncertainty. The problem resembles the classical airline revenue management problem of allocating seats between the uncertain demand of future higher margin customers with the demand from the lower margin, but more certain, customers. A key difference is that in our scenario, the "seats"—product returns—are uncertain each period, and there are alternative means of meeting demand for the low margin alternative (spare parts), which involve buying a large quantity of parts inventory (final-buy) in advance of the allocation decisions.

We present a single and a multi-period optimization model of the disposition problem. The single period model provides insights into the nature of the allocation decision for a given number of returns. We show that the optimal allocation balances the expected marginal profits of remanufacturing and dismantling. Our multi-period model links the disposition of returns to the final buy decision for spare parts at the beginning of the planning horizon, and takes into account uncertainty in the distribution of returns. We show that the optimal solution has the same structure as the single period model, although its computation is more complex.

We then study, numerically, the expected profit increase of using our revenue management approach versus the current practice of ranking disposition alternatives by unit margins. We show that the value of dismantling is significant with a 20% profit increase on average, despite its lower unit margins which amounts to 53% of the remanufacturing margin, on average. We further show that about 60% of the dismantling benefits can be attributed to the coordination between the forward chain (final-buy decision) and the reverse chain (optimal allocation of returns), quantifying the importance of integrating the decision making of the forward and reverse supply chains. Because of the computational complexity of the optimal policy, we show that in an appropriately coordinated chain, a simple allocation heuristic based on the mean demand rates performs reasonably well, yielding a moderate profit decrease of 4% on average compared to the optimal allocation solution.

Our results have clear implications for the management of closed-loop supply chains. They show that a revenue-management approach to the disposition of returns can significantly enhance profitability. Incorporating additional disposition options can be very valuable, even if they have lower unit margins than current options. This should encourage managers to explore new product recovery alternatives. Harvesting of spare parts can be a particularly attractive option, given the relatively long life cycles of parts. However, careful coordination between the forward and reverse supply chain decisions is indispensable for reaping these benefits. In our experience, few companies to date have reached this level of integration. Our message is that the current reactive approach to the reverse supply chain misses out on significant opportunities.

Our analysis makes a number of assumptions specific to the recovery system considered here. To further the insight into the role of the disposition decision in closed-loop supply chains, extensions to other settings would be valuable. This includes the study of dispositions other than remanufacturing and dismantling. It also includes the incorporation of flexible pricing in the disposition decision. Another very relevant question for future research is how to achieve coordination concerning the disposition of returns when decisions pertaining to the forward and the reverse supply chain are taken by different organizational units.

Acknowledgements

The authors would like to thank the IBM executives for exposing us to this problem.

References

[1] Agrawal, V., M. Ferguson, C. Souza. 2008. Trade-in Rebates for Price Discrimination or Product Recovery. *Working Paper*, Smith School of Business, University of Maryland.

[2] Cattani, K., G. C. Souza. 2003. Good Buy? Delaying End-of-Life Purchases. *European Journal of Operational Research* 146(1) 216-228.

[3] Cross, R. G. 1997. *Revenue Management: Hard Core Tactics for Market Domination.* Broadway Books, New York.

[4] Debo, L.G., L.B. Toktay, L.N. Van Wassenhove. 2005. Market Segmentation and Product Technology Selection for Remanufacturable Products. *Management Science* 51(8) 1193-1205.

[5] Dekker, R., M. Fleischmann, K. Inderfurth, L.N. Van Wassenhove. 2003. *Reverse Logistics: Quantitative Models for Closed-Loop Supply Chains.* Springer Verlag, Berlin.

[6] Deshpande, V., M. Cohen, K. Donohue. 2003. A Threshold Inventory Rationing Policy for Service Differentiated Demand Classes. *Management Science* 49(6) 683-703.

[7] Ferguson, M., B. Toktay. 2006. The effect of competition on recovery strategies. *Production and Operations Management* 15 351-368.

[8] Ferguson, M., V. D. R. Guide Jr., E. Koca, G. C. Souza. 2008. The Value of Quality Grading in Remanufacturing. *Production and Operations Management*, forthcoming.

[9] Ferrer, G. and J. Swaminathan. 2006. Managing New and Remanufactured Products. *Management Science* 52(1), 15-26.

[10] Fleischmann, M., J.M. Bloemhof-Ruwaard, P. Beullens, R. Dekker. 2003. Reverse Logistics Network Design, in *Reverse Logistics: Quantitative Models for Closed-Loop Supply Chains*, R. Dekker, M. Fleischmann, K. Inderfurth, L.N. Van Wassenhove, eds. Springer Verlag, Berlin.

[11] Galbreth, M., J. Blackburn. 2006. Optimal Acquisition and Sorting Policies for Remanufacturing. *Production and Operations Management* 15 384-392.

[12] Gayon, J.-P., Y. Dallery. 2007. Dynamic vs static pricing in a make-to-stock queue with partially controlled production. *OR Spectrum* 29(2) 193205.

[13] Guide Jr., V.D.R., L.N. Van Wassenhove. 2001. Managing Product Returns for Remanufacturing. *Production and Operations Management* 10(2) 142 – 155.

[14] Guide, Jr., V.D.R., L.N. Van Wassenhove. 2003. *Business Aspects of Closed-Loop Supply Chains.* Carnegie Mellon University Press, Pittsburgh, PA.

[15] Guide Jr., V.D.R., R.H Teunter, L.N. Van Wassenhove. 2003. Matching Demand and Supply to Maximize Profits from Remanufacturing. *Manufacturing and Service Operations Management* 5(4) 303 – 316.

[16] Guide Jr., V.D.R., G.C. Souza, L.N. Van Wassenhove, J.D. Blackburn. 2006. Time Value of Commercial Product Returns. *Management Science* 52(8) 1200-1214.

[17] Guide Jr., V. D. R., E. D. Gunes, G. C. Souza, L.N. Van Wassenhove. 2007. The Optimal Disposition Decision for Product Returns. *Operations Management Review*, forthcoming.

[18] Ha, A.Y. 1997. Inventory rationing in a make-to-stock production system with several demand classes and lost sales. *Management Science* 43(8) 1093-1103.

[19] Hauser, W., R. Lund. 2003. *The Remanufacturing Industry: Anatomy of a Giant.* Boston University, Boston, MA.

[20] Inderfurth, K., A.G. de Kok, S.D.P. Flapper. 2001. Product recovery in stochastic remanufacturing systems with multiple reuse options. *European Journal of Operational Research* 133(1) 130-152.

[21] Keskinocak, P., S. Tayur. 2004. Due-Date Management Policies, in *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*, D. Simchi-Levi, S.D. Wu, Z.M. Shen, eds. International Series in Operations Research and Management Science, Kluwer Academic Publishers, Norwell, MA, 485-553.

[22] Kleber, R., Miner, S., G.P. Kiesmuller. 2002. A Continuous Time Inventory Model For a Product Recovery System With Multiple Reuse Options. *International Journal of Production Economics* 79 121-141.

[23] Lee, H.L., V. Padmanabhan, and S. Whang. 1997. Information Distortion in a Supply Chain: The Bullwhip Effect. *Management Science* 43(4) 546-558.

[24] Mitra, S. 2007. Revenue management for remanufactured products. *Omega* 35(5) 553 562.

[25] Ray, S., T. Boyacı, N. Aras. 2005. Optimal Prices and Trade-in Rebates for Durable, Remanufacturable Products. *Manufacturing and Service Operations Management* 7(3) 208 – 228.

[26] Talluri, K. and G.J. van Ryzin. 2004. *The Theory and Practice of Revenue Management.* Kluwer Academic Publishers, Norwell, MA.

[27] Topkis, D. 1968. Optimal Ordering and Rationing Policies in a Nonstationary Dynamic Inventory Model with N Classes. *Management Science* 15(3) 160–176.

[28] Wagner, H.M. 1995. Global Sensitivity Analysis. *Operations Research* 43(6) 948-969.

[29] Zhao, H., V. Deshpande and J.K. Ryan. 2005. Inventory Sharing and Rationing in Decentralized Dealer Networks. *Management Science* 51(4) 531-547.

Appendix A: Proofs

Proof of Lemma 1.

The objective function (2) is jointly concave in Q_r and Q_d because $\frac{\partial^2\Pi}{\partial Q_r^2} = -(p_r - v)f_r(Q_r) \leq 0$, $\frac{\partial^2\Pi}{\partial Q_d^2} = -\sum_i p_d^i a_i^2 f_d^i(a_i Q_d) \leq 0$, and $\frac{\partial^2\Pi}{\partial Q_d \partial Q_r} = 0$; thus the Hessian is negative definite. Denoting by $L(Q_d, Q_r, \lambda_R, \lambda_d, \lambda_r)$ the Lagrangian of this problem, then the KKT conditions $\frac{\partial L}{\partial Q_i} = 0$, $i \in \{r, d\}$; $\lambda_R(R - Q_r - Q_d) = 0$, $\lambda_d Q_d = 0$, $\lambda_r Q_r = 0$ and $\lambda_R, \lambda_d, \lambda_r \geq 0$ are necessary and sufficient for optimality, because the constraint set is a convex set (both variables Q_r and Q_d are bounded by R and 0, and there is only a linear constraint). The Lagrangian for this problem is:

$$\begin{aligned} L(Q_d, Q_r, \lambda_R, \lambda_d, \lambda_r) &= (p_r - v)\left(Q_r - \int_0^{Q_r} F_r(u)du\right) + (v - c_r)Q_r \\ &\quad - \sum_i \pi^i \left(\mu^i - a_i Q_d + \int_0^{a_i Q_d} F_d(u)du\right) - c_d Q_d \\ &\quad + \lambda_R(R - Q_r - Q_d) + \lambda_d Q_d + \lambda_r Q_r. \end{aligned}$$

The first order conditions result in

$$\frac{\partial L}{\partial Q_r} = -\lambda_R + \lambda_r + v - c_r + (p_r - v)\left(1 - F_r(Q_r)\right) \quad = \quad 0 \tag{9}$$

$$\frac{\partial L}{\partial Q_d} = -\lambda_R + \lambda_d - c_d + \sum_i \pi^i a_i \left(1 - F_d^i(a_i Q_d)\right) \quad = \quad 0 \tag{10}$$

Isolating λ_R from (9) and (10), we obtain:

$$\lambda_r + (p_r - v)\left(1 - F_r(Q_r)\right) + v - c_r = \lambda_d + \sum_i \pi^i a_i \left(1 - F_d^i(a_i Q_d)\right) - c_d. \tag{11}$$

We have two cases to consider:

1. The constraint $Q_d + Q_r = R$ is binding. In this case, $Q_r = R - Q_d$, and thus (11) becomes:

$$(p_r - v)\left(1 - F_r(R - Q_d)\right) + v - c_r - \sum_i \pi^i a_i \left(1 - F_d^i(a_i(Q_d))\right) - c_d = \lambda_r - \lambda_d. \tag{12}$$

   If the lefthand side of (12) is strictly positive, then $\lambda_r > 0$ and, by complementary slackness, $Q_r = 0$ and thus $Q_d = R$. Conversely, a strictly negative lefthand side of (12) implies $Q_r = R$ and $Q_d = 0$. Finally, if both Q_d and Q_r are strictly positive, then $\lambda_r = \lambda_d = 0$ and $\hat{Q}_d$ defined in Lemma 1 solves (12). This solution can be found using a simple line-search algorithm.

2. The constraint $Q_d + Q_r = R$ is not binding. In this case, by complementary slackness, $\lambda_R = 0$. For $\lambda_r = 0$, $\tilde{Q}_r$ defined in Lemma 1 solves (9), otherwise $Q_r = 0$. The same argument applies for Q_d.

■

Proof of Corollary 1

The result follows as a special case of Lemma 1. For deterministic and equal demand for parts, we have $\Pi_d(Q) = \pi - c_d$ for $Q \leq d_d$ and 0 otherwise. Therefore $\tilde{Q}_d = d_d > 0$ and $\check{Q}_r = R - \hat{Q}_d$. ■

Proof of Corollary 2

For $v = c_r$ the expected profit (2) is non-decreasing in Q_r. For any solution (Q_d^*, Q_r^*) with $Q_r^* + Q_d^* < R$ the disposition decision $(Q_d^*, R - Q_d^*)$ is therefore also optimal. ■

Proof of Theorem 1

We show Properties (i)-(iii) by induction. For $t = 0$ (i) holds since $V_0 \equiv 0$ by definition. Assume now that (i) holds for $t - 1$. We first show that this implies Properties (ii) and (iii) for t. Subsequently, we show that (i) also holds for t.

We have $\frac{\partial^2 \Pi_t}{\partial Q_{d,t}^2} = \frac{\partial^2 \Pi_t}{\partial I_t^2} = \frac{\partial^2 \Pi_t}{\partial Q_{d,t} \partial I_t} = -(h+\pi) f_{dt}(I_t + Q_{d,t}) \leq 0$, $\frac{\partial^2 \Pi_t}{\partial Q_{r,t}^2} = -(p_r - v) f_{rt} \leq 0$, and $\frac{\partial^2 \Pi_t}{\partial Q_{d,t} Q_{r,t}} = \frac{\partial^2 \Pi_t}{\partial I_t \partial Q_{r,t}} = 0$. Therefore, the current-period expected profit r is jointly concave in $I_t, Q_{d,t}$ and $Q_{r,t}$. The expected discounted future profits are independent of $Q_{r,t}$ and jointly concave in I_t and $Q_{d,t}$ due to the concavity of V_{t-1}. This shows (ii) for t.

The proof of (iii) is identical to the one of Lemma 1, due to the concavity property established in (ii). Note that $\Pi_{dt}(I,Q) = \frac{\partial W_t}{\partial Q_{d,t}}(I,Q,c)$ and $\Pi_r = \frac{\partial W_t}{\partial Q_{r,t}}(I,c,R-Q)$ for any arbitrary value of c.

It remains to be shown that (i) holds for t. It suffices to show that concavity holds for any given value of R_t, which implies that it also holds in expectation. For given R_t the concavity of $\max_{Q_{d,t}+Q_{r,t} \leq R} W_t(I, Q_{d,t}, Q_{r,t})$ in I follows from the concavity of W_t shown in (ii) and from the fact that the set of feasible actions $Q_{d,t}$ and $Q_{r,t}$ is convex and independent of I. ■

Publications in the Report Series Research* in Management

ERIM Research Program: "Business Processes, Logistics and Information Systems"

2008

An Evolutionary Framework for Determining Heterogeneous Strategies in Multi-Agent Marketplaces
Alexander Babanov, Wolfgang Ketter and Maria Gini
ERS-2008-002-LIS
http://hdl.handle.net/1765/10972

Choosing between Auctions and Negotiations in Online B2B Markets for IT Services: The Effect of Prior Relationships and Performance
Uladzimir Radkevitch, Eric van Heck and Otto Koppius
ERS-2008-004-LIS
http://hdl.handle.net/1765/11288

Key Issues in Expansion of End-User Mobile Communication in China
Sunanda Sangwan, Guan Chong and Louis-Francois Pau
ERS-2008-011-LIS
http://hdl.handle.net/1765/11762

Some Comments on the Question Whether Co-Occurrence Data Should Be Normalized
Ludo Waltman and Nees Jan van Eck
ERS-2008-014-LIS
http://hdl.handle.net/1765/11810

Bibliometric Mapping of the Computational Intelligence Field
Nees Jan van Eck and Ludo Waltman
ERS-2008-015-LIS
http://hdl.handle.net/1765/11811

Demand Management Opportunities in E-fulfillment: What Internet Retailers Can Learn from Revenue Management
Niels Agatz, Ann Campbell, Moritz Fleischmann, Jo van Nunen and Martin Savelsbergh
ERS-2008-021-LIS
http://hdl.handle.net/1765/12244

Time Slot Management in Attended Home Delivery
Niels Agatz, Ann Campbell, Moritz Fleischmann and Martin Savelsbergh
ERS-2008-022-LIS
http://hdl.handle.net/1765/12245

Mobile Service Affordability for the Needy, Addiction, and ICT Policy Implications
L-F Pau
ERS-2008-023-LIS
http://hdl.handle.net/1765/12246

CBPRS: A City Based Parking and Routing System
J.L. Boehlé, L.J.M. Rothkrantz and M. van Wezel
ERS-2008-029-LIS
http://hdl.handle.net/1765/12467

Coping with Costly Bid Evaluation in Online Reverse Auctions for IT Services
Uladzimir Radkevitch, Eric van Heck and Otto Koppius
ERS-2008-039-LIS
http://hdl.handle.net/1765/12871

Multi Agent Systems in Logistics: A Literature and State-of-the-art Review
Niels Lang, Hans M. Moonen, F. Jordan Srour and Rob A. Zuidwijk
ERS-2008-043-LIS
http://hdl.handle.net/1765/12902

Generalizing the h- and g-indices
Nees Jan van Eck and Ludo Waltman
ERS-2008-049-LIS
http://hdl.handle.net/1765/13210

Applying Revenue Management to the Reverse Supply Chain
Mark Ferguson, Moritz Fleischmann and Gilvan C. Souza
ERS-2008-052-LIS
http://hdl.handle.net/1765/13211

---

* A complete overview of the ERIM Report Series Research in Management:
https://ep.eur.nl/handle/1765/1

ERIM Research Programs:
LIS Business Processes, Logistics and Information Systems
ORG Organizing for Performance
MKT Marketing
F&A Finance and Accounting
STR Strategy and Entrepreneurship