*Transactions in GIS*, 2008, 12(1): 5–29

Review Article

# Approaches to Semantic Similarity Measurement for Geo-Spatial Data: A Survey

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**Abstract**
Semantic similarity is central for the functioning of semantically enabled processing of geospatial data. It is used to measure the degree of potential semantic interoperability between data or different geographic information systems (GIS). Similarity is essential for dealing with vague data queries, vague concepts or natural language and is the basis for semantic information retrieval and integration. The choice of similarity measurement influences strongly the conceptual design and the functionality of a GIS. The goal of this article is to provide a survey presentation on theories of semantic similarity measurement and review how these approaches – originally developed as psychological models to explain human similarity judgment – can be used in geographic information science. According to their knowledge representation and notion of similarity we classify existing similarity measures in geometric, feature, network, alignment and transformational models. The article reviews each of these models and outlines its notion of similarity and metric properties. Afterwards, we evaluate the semantic similarity models with respect to the requirements for semantic similarity measurement between geospatial data. The article concludes by comparing the similarity measures and giving general advice how to choose an appropriate semantic similarity measure. Advantages and disadvantages point to their suitability for different tasks.

## 1 Introduction

Psychologists consider similarity judgement as probably the most central construct in human cognition (Medin et al. 1993, Goldstone 1994, Gentner and Markman 1995,

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Goldstone and Son 2005). Humans use similarity for storing and retrieving information, to compare new situations to similar experiences in the past; also category learning and concept formation hinges crucially on similarity. While computers can process the binary decision of equivalence or non-equivalence of two things very fast and precisely, it poses a complex and non-trivial problem to compute similarity. Also in geographic information science (GIScience), similarity plays a major role in many applications such as spatial decision support systems, data mining or pattern recognition. Current solutions developed from classical artificial intelligence and machine learning hardly include psychological findings about human similarity judgement. Although similarity has long been the subject of investigation in psychology to understand cognitive processes, there exists no common theory on semantic similarity measurement. This article provides the reader with a survey presentation on theories for semantic similarity measurement and reviews how these approaches – originally developed as psychological models to explain human similarity judgment – can be formalized and used in GIScience. Since the choice of an adequate similarity measure influences strongly the functionality of geographic information systems (GIS), the results of this analysis will help to revise the conceptual design of a GIS and include cognitively plausible similarity measures.

This survey starts with a short introduction to the field of semantic similarity (section 2.1) and outlines the peculiarities of geospatial objects and concepts (section 2.2). From there we derive the requirements which form the basis for our evaluation of the similarity measures. Sections 3 to 7 each explain a model for semantic similarity measurement: the geometric model, the feature model, the network model, the alignment model and the transformational model. Each section about a similarity model is organized as follows: after introducing the main idea of the model, we describe the knowledge representation and the applicable similarity measures. We focus on the notion of similarity and discuss the metric or non-metric properties of the measure. By describing the representational model separately from the similarity measure we can reveal distinct underlying notions of similarity for the same representational model. Moreover not all measures are necessarily as expressive as the representational model and they may account only for a subset of aspects covered by the representational model. Afterwards we present prominent representatives of each model. A discussion of the applicability of the similarity model for semantic similarity measurement in GIS concludes each section. Section 8 reviews the approaches and gives general advice for selecting one similarity measure. Advantages and disadvantages point to their suitability for different tasks. The final section (#9) summarizes the approaches and outlines how they influence each other.

## 2 Semantic Similarity Measurement in GIS

### 2.1 Notion of Semantic Similarity

Two major notions of similarity are found in existing semantic similarity measures: commonalities and differences or the semantic distance (Figure 1).

*Commonalities and differences* between two representations of concepts are taken as one indicator for similarity: the more commonalities and the less differences, the higher is the similarity. While some similarity assessments base their measurement on an unstructured comparison, other representations allow for a structured comparison: the common element must play an analogous role in the representation to increase the

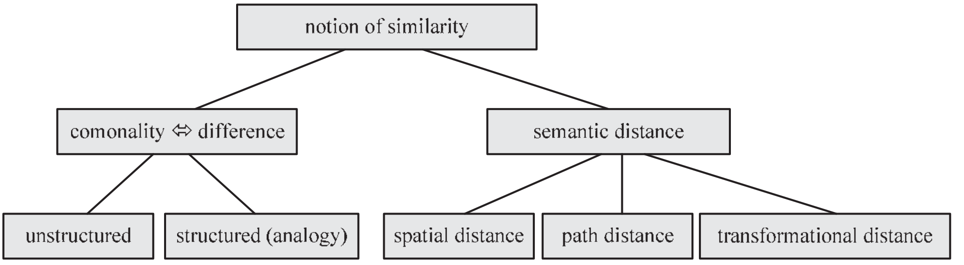


**Figure 1** Different notions of similarity

commonalties between two concepts. To apply *semantic distance* as a notion for semantic similarity, all concepts must be represented in a common framework with some specified metric. Some similarity measures use a multi-dimensional space as framework and the Euclidian or city-block metric for distance measurement. Semantic distance in a tree or network structure is defined by the length of the shortest path between nodes. The transformational distance is common to use for representations based on sets of transformations which can be composed and executed one after another. The distance is measured via the number of transformations or their complexity.

### *2.2 Geospatial Objects and Concepts*

In the context of semantic similarity we distinguish two constituents of a conceptualization: objects and concepts. Following the definition by Sloman et al. (1998, p. 192) a "concept is an idea that characterizes a set or category of objects", i.e. a geospatial concept describes the idea that characterizes a geographic feature type. A geospatial object refers to the single geographic feature. Similarity measures must be able to deal with objects *and* concepts to measure the interoperability between geospatial data sources.

The semantics of geospatial objects and concepts is complex and has some special characteristics: they are typically described by properties such as shape, size and location. In addition, relations, in particular spatial relations, play a major part in the semantic description. Therefore we review all similarity measures with respect to their ability to deal with geospatial objects and concepts and their ability to include properties and spatial relations in the semantic description.

## 3 Geometric Model

The following five sections each present one model for semantic similarity measurement. As a running example we use the geospatial concept 'floodplain': a floodplain is a low lying, flat area of land which is periodically waterlogged due to flooding. Floodplains are located next to rivers. Geometric models were first used in psychology to exploit the analogy to space for measuring similarity (Attneave 1950; Torgerson 1958, 1965): concepts are modelled within a multi-dimensional space and their spatial distance indicates the semantic similarity.

### *3.1 Knowledge Representation*

Geometric models are based on the notion of multi-dimensional vector spaces. Each dimension is used to describe properties of objects and concepts. The value of a property is separated from its quality and shown as a value on a dimension. This separation enables the comparison of two properties of the same quality, e.g. tiny is more similar to small than to huge. Most geometric models focus on modelling only objects. Due to their singular character they can be represented by vectors specifying one point in the vector space. In section 3.3 we present conceptual spaces, a geometric model which accounts also for concepts.

Many parallels can be drawn between geometric models and multi-dimensional scaling (MDS) (Shepard 1987, Nosofsky 1992), but there exist also some fundamental differences: MDS uses as input subjects' judgments about pair-wise similarities and determines the number of dimensions. The goal is to reach the maximum correlation between the subjects' similarity judgments and the corresponding distances in the multi-dimensional space with a minimum number of dimensions. Geometric models have a given set of dimensions and determine the values to describe each object. Similarity can be determined via calculation of the spatial distance.

### *3.2 Similarity Measure*

*Semantic distance in analogy to spatial distance.* Geometric models use the analogy of semantic distance to spatial distance: similarity is measured as a function of the spatial distance. The most commonly used similarity measures are the Minkowski distance measures (equation 1):

$$d_{ij} = \left[ \sum_{k=1}^{n} \left| x_{ik} - x_{jk} \right|^{r} \right]^{1/r} \tag{1}$$

where $n$ is the number of dimensions, $x_{ik}$ is the value of dimension $k$ for stimulus $i$ and $x_{jk}$ is the value of dimension $k$ for stimulus $j$. The Minkowski metric is a generic formula: $r = 1$ results in the city-block distance and $r = 2$ in the Euclidian distance (Suppes et al. 1989). Similarity is a linear (sometimes also exponentially) decaying function of the Minkowski distance $d_{ij}$ (Attneave 1950, Melara et al. 1992); the absolute identification confusability – often taken as an indirect measure for similarity – is an exponentially decaying function of distance $d_{ij}$ (Shepard 1958a, b).

*Metric properties of geometric similarity.* A core property and at the same time the most heavily criticized property of geometric similarity models are their metric assumptions. The vector space and therefore also the similarity has to meet the three metric axioms:

*Minimality.* This axiom (equation 2) states that, if the spatial distance between two concepts is zero, then the concepts are the same. It follows from this axiom that the maximum similarity exists between a concept and itself.

$$d(i, j) = 0 \Rightarrow i = j \wedge d(i, j) \geq 0 \tag{2}$$

Based on several experiments – cognition of Morse code (Rothkopf 1957) or cognition of rectangles varying in size and reflectance (Attneave 1950) – critics of metric similarity claim that the axiom of minimality does not hold for similarity. Other studies state as well that self-proximities of concepts differ by taking the response time as a measure of similarity: the same-different judgment takes longer for complex concepts than for simple concepts (Takane and Sergent 1983). According to Johannesson (2002) these results cannot simply be transferred to similarity measurement: all experiments demonstrating the absence of the minimality axiom deal with confusability or response time, but none with direct similarity judgements.

*Symmetry.* This axiom (equation 3) states that the distance and therefore also the semantic similarity from one concept to another is the same as vice versa.

$$d(i, j) = d(j, i) \tag{3}$$

Like Tversky in his experiment of similarity between countries – North Korea is more similar to Red China than vice versa (Tversky 1977) – many other researchers from different domains showed that symmetry between concepts does not always hold. He states that similarity or dissimilarity is judged depending on the prominence or relative salience of concepts. Although geometric models in general assume symmetry, there exist geometric models which take asymmetry of similarity into account, for instance Krumhansl's distance density model (Krumhansl 1978) or extensions to Gärdenfors' conceptual spaces such as the Relative Prominence Model (Johannesson 2000).

*Triangle Inequality.* The axiom (equation 4) states that the distance between two concepts is always smaller than or equal to the distance between both concepts via a third concept.

$$d(i, j) + d(j, k) \geq d(i, k) \tag{4}$$

It is trivial to come up with examples for similarity judgments that violate the triangle inequality, e.g. the famous example from James (1892/1961): a lamp is similar to the moon and the moon is similar to a soccer ball; but a lamp is not similar to a soccer ball. While the first comparison in this example emphasizes the aspect of providing light and the second focuses on the shape dimension, it is unclear which dimensions are used in the soccer ball – lamp comparison. These similarity judgements are based on different saliencies of properties, which depend on the context. Tversky and Gati (1978) showed the systematic violation of the triangle inequality when the similarity of two concepts $A$ and $B$ was measured on one and the similarity of concept $B$ to $C$ was measured on a different dimension. Therefore similarity on different dimensions is not transitive. Krumhansl (1978) and Johannesson (2000) proposed extensions to geometric models to take into account effects of context changes in the similarity judgement.

Here we would like to emphasize that the triangle inequality as shown in Equation (4) refers only to semantic distances. Many authors (such as Tversky 1977) infer from this axiom that triangle inequality holds also for similarity, i.e. from the equation about the distances $d1 + d2 \geq d3$ it is inferred that the similarities $s1 + s2 \leq s3$. Mathematically, this conclusion is not correct.[1]

*Requirements and assumptions.* Like all property-based similarity measures the geometric model has some assumptions underlying the dimensions:

1. Independence of representational element: geometric models in general assume properties to be independent. Independent properties are modelled via orthogonal

dimensions. Dependent properties can be modelled by dimensions with angles smaller than 90°, e.g. Raubal (2004).
2. Solvability: the set of properties used to describe a concept must be sufficiently rich and representative for the conceptualization. A property-set that does not reflect the human conceptualization can obviously not provide good similarity results.
3. Comparability of representational elements: the equivalence of intervals on dimensions must be preserved across dimensions by normalising the dimensions. To make dimensions comparable and represent measurands in the same relative unit, we propose to use standardization methods from statistics such as the z-transformation.
4. Complexity of representation: by adding more information to a description the semantic distance can only increase and similarity decreases. Therefore geometric models are suited only for comparison of concepts with an identical number of dimensions. Alternatively, one may add an additional factor which computes the similarity depending on the number of dimensions as done in Schwering and Raubal (2005a, b).

### *3.3 Representatives for Geometric Models*

The most famous representative for geometric models are *conceptual spaces* introduced by Gärdenfors (2000). Conceptual spaces represent information at a conceptual level and are formed by a set of quality dimensions. The dimensions are closely connected to qualities perceivable by the human sensory system. For example, the colour domain is formed through the integral dimensions hue, saturation and brightness. Objects are represented as a point in a conceptual space and concepts are modelled as n-dimensional regions.

Krumhansl (1978) proposed the *distance density model* as a geometric similarity measure. It focuses only on objects and calculates the similarity as a function of the spatial distance and the density of stimulus points.[2] By including the density of the stimulus points, Krumhansl takes into account that the similarity also depends on the granularity of objects, i.e. similarity is less in sub-regions with dense stimulus points than with low density. This similarity measure meets the concerns about minimality and takes into account that the confusion probability is higher in regions with high density of stimulus points.

### *3.4 Evaluation for Similarity Measurement between Geospatial Data*

*Objects and concepts.* The representational model of conceptual spaces is developed to model objects and concepts likewise. However, the similarity measure can only measure the semantic distance between points in the conceptual space. The semantic distance between concepts is estimated by reducing the concept to a single exemplar. Gärdenfors proposes to use the prototypical instance of each concept and measure the semantic similarity between prototypical instances in place of the whole concept. Prototypes are assumed to be "central points in the categories they represent" (Gärdenfors 2000, p. 97). Distances can also be determined by calculating the average distances between a limited number of exemplars, by determining the distances between convex hulls of each concept or by dimension-wise distances (Schwering and Raubal 2005a).

*Properties and spatial relations.* Conceptual spaces represent properties on quality dimensions, but do not support semantic relations between concepts. Schwering and Raubal (2005b) proposed an extension to conceptual spaces to model spatial relations as compound dimensions. However, these "compound dimensions" suffer from the problem that the relational structure is hidden in the dimension label and cannot be used for similarity judgement.

## 4 Feature Model

Both geometric and feature models use properties to describe concepts. While properties in geometric models are dimensions with ordered values, properties in feature models are Boolean: features either hold or do not hold for a concept. Two concepts having the same feature are similar in some respect. Similarity measures of feature models underlie the assumption that similarity of concepts increases the more common and the less distinct features these concepts have. Using feature models to compare concepts and measure their similarity became popular with Tversky's famous critique of metric similarity (Tversky 1977).

### *4.1 Knowledge Representation*

The feature model is based on a set-theoretic knowledge representation: concepts are represented via an unstructured set of features that hold for the specific concept.

Features correspond to components, concrete or abstract properties of the concept. A wetland (Figure 2) may be described by the feature 'low vegetation' reflecting the component vegetation of a wetland, 'flat' for the relief of the wetland, 'often water-logged' for its hydrologic features and 'area' for its shape. Like dimensions, features can represent nominal, ordinal, interval and ratio scaled variables. Features may refer to two types of dimensions: additive and substitutive dimensions (Tversky and Gati 1982). Features depending on additive dimensions may be added to the feature set regardless of the other features in the set. However, features based on substitutive dimensions are not independent from other features. An example for a substitutive dimension would be the dimension surface relief: an object can be described as flat or steep, but it cannot have the feature flat and steep at the same time.[3] Substitutive dimensions are therefore collections of features and any object can have only one feature of the collection (Markman 1999).

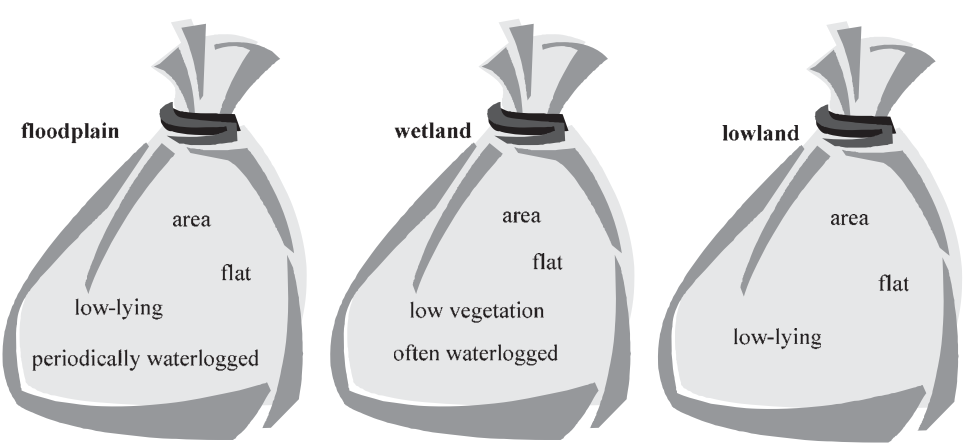


**Figure 2** The concepts 'floodplain', 'wetland' and 'lowland' are modelled via unstructured sets of features, which hold for something to be an extension of the concepts

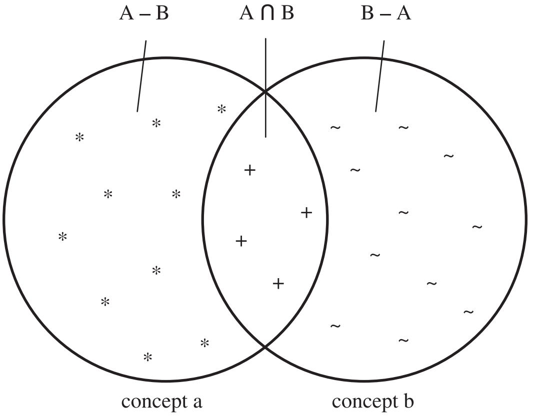


**Figure 3** The matching feature similarity measures the similarity by applying set-theoretic operations (*A* indicates the set of features of concept *a* and *B* those of concept *b*)

### *4.2 Similarity Measure*

*Feature-matching model.* The feature-matching model accounts for the fact that similarity is not necessarily metric. Concepts are represented as collections of features. By representing each concept with a different set of features, elementary set operations can be applied to estimate similarities and differences.

Tversky and colleagues (Tversky 1977, Tversky and Gati 1978, Sattath and Tversky 1987) propose a set-theoretic similarity measure expressing the similarity between concepts *a* and *b* as a function of their common and distinct features (equation 5, illustrated in Figure 3).

$$s(a, b) = F(A \cap B, A - B, B - A) \tag{5}$$

Intersections or subtractions of feature sets are based only on entire feature matches. While geometric similarity accounts for intra-dimensional similarity, feature similarity cannot measure partial matches, e.g. the waterlogged feature 'periodically waterlogged' is as distinct from 'sometimes waterlogged' as from 'always waterlogged'. Feature models allow for representing ordinal and cardinal features, but the similarity measure does not account for their ordering.

*Non-metric properties of feature-based similarity*. The feature matching model consists of three components: the distinct features of *A* to *B*, the distinct features of *B* to *A* and the common features of *A* and *B*. Depending on the weighting of the components in the similarity function *F* (equation 5) this function is non-metric. Tversky was probably the strongest advocate of non-metric similarity. He proved empirically that the three metric axioms do not hold in human similarity assessment and stated that "minimality is somewhat problematic, symmetry is apparently false and the triangle inequality is hardly compelling" (Tversky 1977, p. 329).

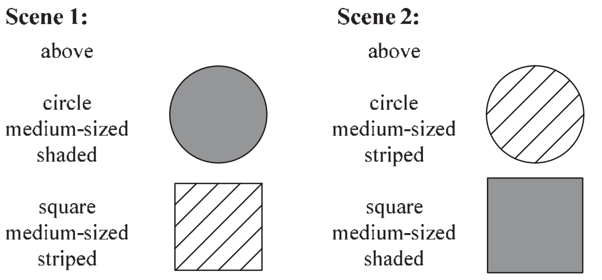


**Figure 4** Two spatial scenes are described by a set of features. The similarity between these scenes depends on the correct alignment of these features (Gentner and Markman 1995, p. 114)

*Requirements and assumptions.* As the geometric model, the feature model has some assumptions to provide reliable similarity measurements (Tversky 1977):

1. Independence of representational elements: the degree to which each feature (or a pair or set of features) shared by two concepts affects similarity must not be dependent on any other shared feature (or pair or set of features). An example illustrates the problem (Figure 4): Comparing only the features of the circles lead to two common versus one distinct feature. The degree of similarity changes, if the features of the squares are included.
2. Solvability: the feature-set must be sufficiently rich and representative. Like the other approaches, the feature model measures similarity between representations. If a representation of a concept is inadequate, i.e. it contains a non-representative or incomplete set of features, the resulting similarity is necessarily also inadequate.
3. Invariance of representational elements: the intervals between features are assumed to be equivalent across factors.

### *4.3 Representatives of the Feature Model*

The most famous feature model is the *contrast and ratio models* by Tversky, which represent similarity function *F* as a weighted difference of common and distinctive feature sets (contrast model, equation 6) or as its normalised version (ratio model, equation 7).

$$S(a, b) = \theta * f(A \cap B) + \alpha * f(A - B) + \beta * f(B - A) \tag{6}$$

The scale $f$ can either be a simple function determining the cardinality of the set or a functional reflection of the salience or prominence of the various features.

$$S(a, b) = \frac{f(A \cap B)}{f(A \cap B) + \alpha * f(A - B) + \beta * f(B - A)} \tag{7}$$

Both models reflect Tversky's assertion that similarity assessment is directional and asymmetric. Tversky's non-metric feature-matching models are probably the most commonly used measure for similarity among prototype theorists (Laurence and Margolis 1999).

The *Matching-Distance Similarity Measure (MDSM)* by Rodriguez et al. (1999) and Rodriquez and Egenhofer (2003, 2004) is a feature model developed for similarity measurement of geospatial concepts. It is based on the ratio model, extends it by introducing different kinds of features and applies it to concepts.

### *4.4 Evaluation for Similarity Measurement between Geospatial Data*

*Objects and concepts.* Tversky applied the feature-matching model only to objects. Each object is allowed to have no more than one property from a substitutional property set. Rodriguez and colleagues ease this restriction: in MDSM they allow also for mutually exclusive properties.

*Properties and spatial relations.* The feature-matching model is unable to relate two objects in a structured way. It can express structure only by compound features such as 'nextToRiver'. However, since feature models cannot detect any partial feature matches, no relational similarity is detected between 'nextToWaterbody' and 'nextToRiver'. A relational representation of these features – e.g. nextTo(floodplain, waterbody) and nextTo(floodplain, river) – could detect similarity via an alignment of arguments.

## 5 Network Model

Network models are graph-based and use semantic networks for knowledge representation. Semantic networks have their roots in psychology, where they were used to model the human semantic memory.

### *5.1 Knowledge Representation*

The term semantic network encompasses a family of graph-based representations. Semantic networks are composed of labelled nodes and edges. Nodes represent units of knowledge, e.g. objects, concepts or properties. Edges link nodes with each other and represent the relations between them explicitly. However, the graph notation itself isn't the decisive point – a semantic network needs a standard terminology and standard semantics especially for relations between concepts (Luger 2001). Although their representational model always has the same structure, network models still differ quite a lot: depending on the implementation, some network models allow only for taxonomic relations, others for hyponomic and partonomic and again others include any kinds of hierarchic and association relations. Some semantic networks restrict the direction of the relations and assign weights to model their importance.

### *5.2 Similarity Measure*

Like geometric models, network models measure similarity based on the notion of distance. Most approaches use graph-theoretic algorithms such as the shortest path algorithm and weighted path length or information theoretic measures for the calculation. Graph-theoretic similarity measures in semantic networks are metric, if the distance between concepts is measured regardless of the direction of the arches. Distance algorithms that take the direction into account compute non-metric similarity: the similarity is asymmetric, but the triangle inequality holds and self similarity equals always zero. Information theoretic similarity measures are also non-metric.

*Requirements and assumptions.* The following requirements and assumptions underlie the network model:

1. Solvability: the relations between concepts must be sufficiently rich and representative. The similarity of two concepts can only be measured, if a path between them exists. Similarity between concepts not being connected via any relation cannot be computed.
2. Comparability of representational elements: graph-theoretic similarity measures assume that each relation is relevant for the similarity judgment and has the same influence on similarity. The distance algorithm valuates each relation equally. Similarity measures based on information content differ exactly in this point.

### *5.3 Representatives of the Network Model*

Rada et al. (1989) proposed a similarity measure called DISTANCE to compute the conceptual distance which measures similarity between two concepts within the same network. DISTANCE was designed for semantic networks with taxonomic relations (later extended to association relations). It measures the distance between two nodes or sets of nodes and is defined by the "average minimum path length over all pair-wise combinations of nodes between two subsets of nodes" (Rada et al. 1989, p. 17). Rada et al. believe that human similarity judgement is metric and that the asymmetry of similarity between concepts is not derived "from the asymmetry of similarity [. . .] but from the existence of another asymmetric relationship between concepts" (Rada et al. 1989, p. 19) such as fuzzy category-membership.

The feature-based similarity in *MDSM* (section 4.3) includes relations between concepts via a semantic neighbourhood.

Resnik (1995, 1999) proposes a semantic similarity measure of concepts based on the notion of information content. She uses a taxonomy with multiple inheritance as the representational model. Existing approaches such as DISTANCE assume that all relations represent uniform distances. Yet, in real taxonomies the distances covered by one single relation vary a lot. Resnik's information content-based semantic similarity measure overcomes these shortcomings: The probability of a concept increased the more concepts it subsumes, i.e. the higher it is within the ontology. The information content of a concept can be computed from its probability. By analogy to information theory (Ross 1998), Resnik defines the information content of a concept as the negative logarithm of its probability. The similarity of two concepts $c_1$ and $c_2$ is the maximal information content of all concepts subsuming $c_1$ and $c_2$. Figure 5 illustrates a small example.

The information content-based similarity measure is symmetric and transitive. Thus, in contrast to distance by Rada et al., the minimality axiom does not hold for Resnik's similarity measure: the similarity from a concept to itself is the negative logarithm of its information content. Only the single concept on top of the hierarchy reaches the self-similarity of one.

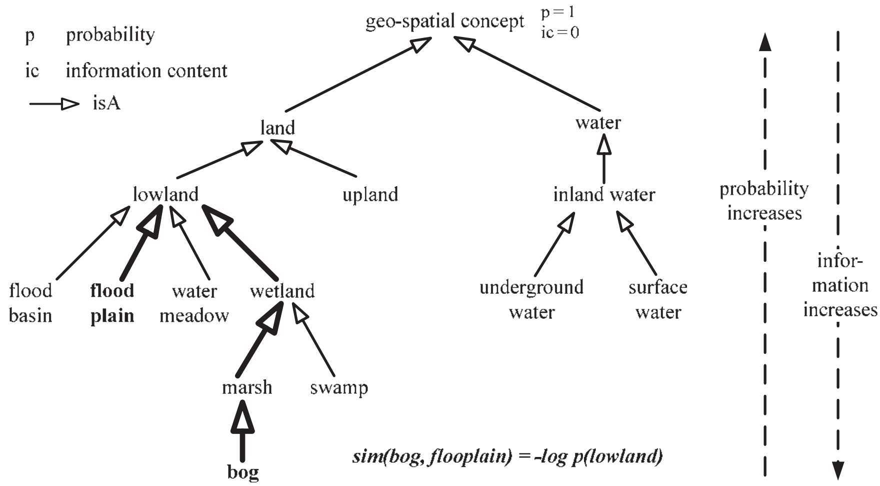


**Figure 5** The probability and the information content depend on the concept's position in the taxonomy

### *5.4 Evaluation for Similarity Measurement between Geospatial Data*

*Objects and concepts*. The network model was intended to model only concepts, but all similarity measures support semantic similarity assessment between objects as well.

*Properties and spatial relations*. The strength of the network approach is the representation of relations between concepts: hierarchic (in Resnik's approach) and hierarchic and partonomic (in Rada's approach and in MDSM) with a possible extension to spatial relations (Schwering 2004). Pure network models do not describe concepts any further (e.g. by features or dimensions) though for the similarity measurement network models can be combined with the feature model like Rodriguez and colleagues did in the semantic neighbourhood.

## 6 Alignment Model

The idea of using alignment of elements for similarity measurement arose from the studies about structural alignment and mapping in analogy by Gentner and Markman (1994, 1995, 1997). Like the feature model, the alignment model uses commonalities and differences as notion of similarity, but includes the relational structure in which properties or relations are found.

### *6.1 Knowledge Representation*

Alignment models represent knowledge in a structured way by adopting Gentner's structural alignment framework. Objects are represented by their properties incorporated into a system of relations that hold between them. Central to this representation are alignment relations which indicate a structural analogy of two representational elements – properties or relations – belonging to two different objects. Alignment relations must be structurally consistent and systematic. Structurally consistent relations follow two constraints: each element corresponds to at most one element (one-to-one mapping) and the corresponding arguments of each pair of matching relations also match (parallel connectivity). The one-to-one mapping is difficult to apply in similarity measurement between concepts (section 6.4). The alignment is called systematic, if there exists a deep interconnected structure of matching features and relations.

### *6.2 Similarity Measure*

Similarity measurement of alignment models examines the commonalities between relational structures: while geometric and feature models search only for *matching* elements, alignment models also account for whether these matching elements *align* or not.

Figure 6 shows two scenes where a car (a lorry) is towing a ship (a car). The lorry towing the car in the left picture is placed in correspondence to the car towing the ship in the right picture. These are alignable differences. The moon in the left and the right picture are aligned as well – an alignable match. The birds in the left picture do not have any correspondence in the right picture and are therefore a non-alignable difference. The cars in both scenes are similar according to their properties, but they play different roles in the relational system – one is towing, the other one being towed. This situation – also called cross mapping – is a non-alignable match. These different types of matches and mismatches have different influences on the final similarity value: alignable matches increase the similarity more than non-alignable matches.

*Requirements and assumptions*. The following assumptions underlie the alignment model:

1. Independence of representational elements: the alignment model assumes that each element (or pair or set of elements) used for the description of a scene is independent.
2. Homogeneous structure: the advantages of the alignment model only become evident when both compared objects have a homogeneous structure. The alignment rules require a uniform structure to work automatically.
3. Solvability: the set of elements must be sufficiently rich and representative.
4. Comparability of representational elements: the alignment model assumes that each element has the same influence on the (dis)-similarity.

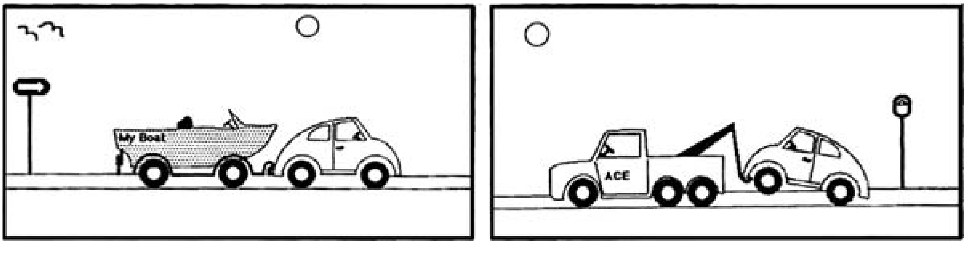


**Figure 6** These scenes show two towing scenes, which contain alignable and non-alignable matches as well as alignable and non-alignable differences (Gentner and Markman 1995, p. 123)

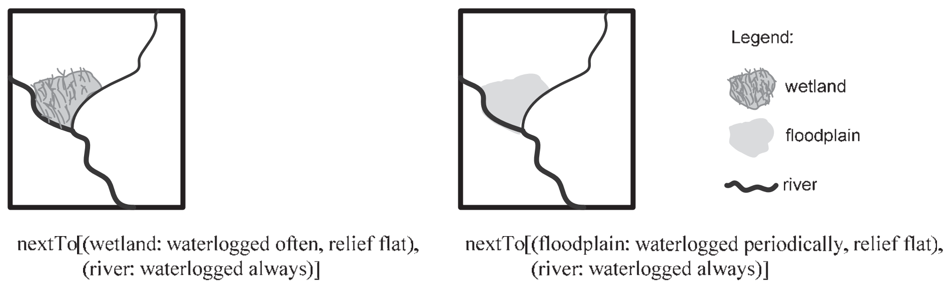


**Figure 7** The alignment model proposed by Goldstone describes spatial scenes in a structured way with roles, components and features

### *6.3 Representative for the Alignment Model*

Goldstone (1994) and Goldstone and Medin (1994) introduced the *Similarity, Interactive Activation, and Mapping (SIAM) model*, an alignment model for similarity measurement between spatial scenes. Similarity is measured in an iterative learning process based on neural networks.

SIAM describes spatial scenes by roles, components of the scene and features: roles are binary relations with components as arguments, which themselves contain feature slots filled in with particular values. Figure 7 shows a spatial scene and its description. Relations describe hierarchical or propositional representations of components, such as the spatial relation in this example. SIAM computes all possible alignments of features and relations and iteratively revises inconsistent alignments until it is consistent and similarity can be determined (Goldstone 1994).

### *6.4 Evaluation for Similarity Measurement between Geospatial Data*

*Objects and concepts*. SIAM was developed to measure similarity between objects (spatial scenes). The rule of one-to-one mapping is difficult to apply to similarity between concepts, if they are described at different levels of granularity. This may entail that elements describing one concept may correspond to more fine-granular elements of the other concept. The base concept might be a general concept 'flooding area' and the target concept is a more complex concept 'floodplain' having a part 'water meadow' (Figure 8). Both 'floodplain' and 'water meadow' map to the single concept 'flooding area'. Alignment models consider this as inconsistent.

Structural commonalities are certainly important to human similarity judgments, but SIAM's determination of alignments is probably too strict to cope with this complex task of similarity measurement between concepts.

*Properties and spatial relations*. SIAM supports hierarchical relations as well as propositional representations between objects (e.g. spatial relations). Properties are also included in the similarity measure: the properties of each object are divided into feature slots and feature values.

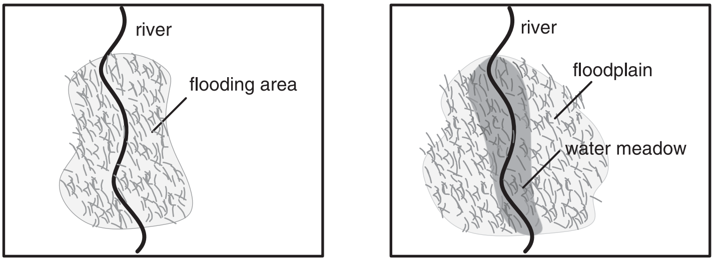


**Figure 8** In these two spatial scenes the flooding area in the left picture aligns to floodplain and water meadow, a part of the floodplain, in the right picture

## 7 Transformational Model

All similarity measures presented so far describe the concepts according to their properties and relations and measure similarity based on the comparison of these descriptions. Transformational models compute similarity in a different way: they define transformations required to distort one concept into another and similarity is defined in terms of the number of transformations needed to make concepts transformationally equal.

### *7.1 Representational Model*

In transformational models concepts are described according to the transformations needed to make one concept equal to another concept. The representational model comprises a set of transformations that can be applied to modify concepts. However, this set of transformations depends greatly on the nature of the concepts: The frequently used perceptual transformations are not sufficient to describe the relation of conceptual stimuli. Although in principle transformations are not restricted to being perceptual, the identification of transformations to modify meaning is not easy. The Rutherford analogy between the solar system and an atom illustrates the problem: "A miniaturization transformation could be applied to the solar system. However, this single transformation is not nearly sufficient; a nucleus is not simply a small sun" (Goldstone and Son 2005, p. 27). To apply transformational models to conceptual information a set of constrained transformations must be defined. It is much more tenable to define such a set for perceptual stimuli than for conceptual.

### *7.2 Similarity Measure*

Transformational models use the number of transformations needed to make one concept identical to another as the basis for the similarity calculation. The similarity is assumed to decrease monotonically when the number of transformations increases (Imai 1977, Wiener-Ehrlich 1980). Hahn et al. (2003) proposed to use the Kolmogorov complexity as the mathematical foundation to include the complexity of transformations in the similarity calculation.

*Metric properties of the transformational model.* The transformational model is asymmetric, but the metric axioms minimality and triangle inequality hold (Hahn and Chater 1997): the similarity between identical concepts is maximal, because no transformation is required to transform concepts into themselves (minimality axiom). The transformations required to distort concept *A* to *B* concatenated with the transformations required for distorting *B* to *C* are more or equal to the transformations required for directly transforming *A* to *C* (triangle inequality). Hahn and Chater (1997) argue that similarity in transformational models is asymmetric, because inverse transformations do not necessarily have the same complexity. For example, transforming a complex concept 'China' with a lot of detailed information into a simple concept 'North Korea' with little information simply requires the deletion of the additional information in 'China' and the transformation of the remaining information. For the transformation the other way around – from 'North Korea' to 'China' – all the detailed information about 'China' must be built up.

*Requirements and assumptions.* Several assumptions are made for the transformational model to provide reliable similarity measurements:

1. Solvability: the set of transformations must be sufficiently rich, i.e. it must comprise at least all transformations necessary to make each of the regarded concepts transformationally equal to another.
2. Comparability of representational elements: to use the number of transformations as a measure for similarity, the degree to which each transformation affects the similarity must be equal, i.e. all transformations must be of the same complexity. The Kolmogorov complexity theory (Vitanyi and Li 1997) makes two transformations with different complexity comparable.
3. Complexity of representation: the transformation of a simple concept into another concept being equally simple requires only a simple transformation, while the transformation of two complex concepts described in minute detail is naturally more complex. Hahn and Chater (1997) propose to solve this problem by measuring the similarity depending on the complexity of the concepts themselves.

## *7.3 Representatives of the Transformational Model*

Transformational models were applied to perceptual stimuli, usually symbol chains such as alphabetic strings (Wiener-Ehrlich et al. 1980), chains of filled and unfilled circles (Imai 1977) or geometric complexes (Hahn et al. 2003). The transformations focus on the perceptual attributes only. Operations such as mirror, reverse, add symbols are used to modify the order of symbol chains. Operations modifying the geometric arrangement are rotation, reflection, translation and dilation (Goldstone and Son 2005). Recently Hahn et al. (2003) proposed a transformational model based on representational distortion, a theoretical framework for similarity judgments. This model represents concepts with computer programs to generate them. The inputs of this transformational model are representations of two concepts *A* and *B*. The computer program *P* to distort or transform the representation of concept *A* in the representation of concept *B* is determined. The complexity of *P* is used to measure the similarity: According to Kolmogorov complexity theory, the complexity of a representation is the length of the shortest computer program that can generate that representation (Vitanyi and Li 1997).

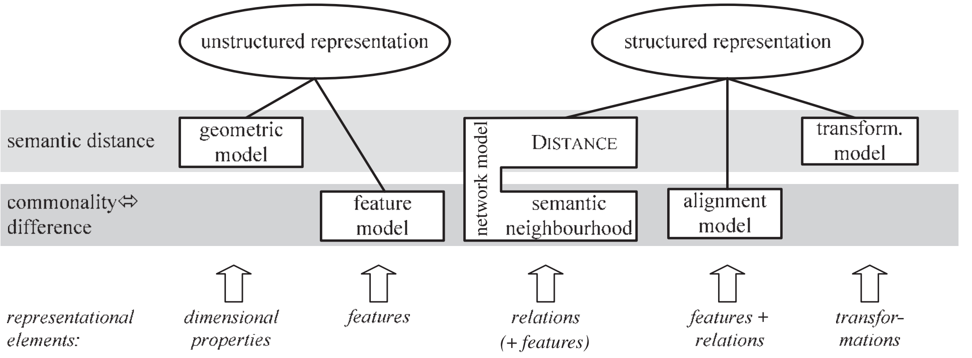


**Figure 9** Semantic similarity measures are based on different notions of similarity

### *7.4 Evaluation for Similarity Measurement between Geospatial Data*

*Objects and concepts.* So far, the transformational model was only applied to relatively simple objects and perceptual similarity. In general it can be applied to geospatial concepts and conceptual similarity, but this requires a framework of conceptual transformations. This framework – if general enough this would lead to a semantic reference system (Kuhn 2003, 2005) – is the core component of the similarity assessment.

*Properties and spatial relations.* The transformational model uses transformations for the semantic comparison of objects or concepts. These transformations can refer to the properties or relations of the respective concept: for instance transform the waterlogged value from periodically to often or delete the nextTo relation of concept 'floodplain' to represent the concept 'wetland'. The effort needed to distort one property or relational structure into another must be defined within the reference framework.

## 8 Review of Existing Semantic Similarity Measurements

### *8.1 Review and Comparison*

Each of the five similarity measures has a different mathematical foundation, uses different knowledge representation and notion of similarity. Figure 9 gives an overview of the semantic similarity measures presented in this survey, Table 1 contrasts their differences with respect to the knowledge representation and Table 2 with respect to the notion of similarity.

Geometric-, feature- and alignment models describe concepts directly, while network- and transformational models use an indirect way by describing a concept's relations or transformations to other concepts. Like feature models, alignment models use features for the semantic description. But the alignment model separates features into attribute-values pairs (feature slots and feature values) which are ordered like values on dimensions. Network- and alignment models both use relations, but different kinds: relations in network models are hierarchic or associative relations only. Alignment models use hierarchic and associative relations to describe the role of components

**Table 1** Comparing the representational models of various similarity measures[1]

| Representational elements<br>Representational structure | Geometric Model | Feature Model | Network Model | Alignment Model | Transformational Model |
|---|---|---|---|---|---|
| **Mathematical model** | | | | | |
| – Multidimensional space | X | | | | |
| – Set theory | | X | | | |
| – Graph theory | | | X | | |
| – Neural network | | | | X | |
| – Transformations | | | | | X |
| **Representational elements** | | | | | |
| – Dimensions (direct description) | X | | | (X) | |
| – Feature (direct description) | | X | | X | |
| – Relation (indirect description via other objects/concepts) | | | X | X | |
| – Transformation (indirect description) | | | | | X |
| **Representation structure** | | | | | |
| – Structured representation | | | X | X | (X) |
| – Unstructured representation | X | X | | | X |

[1] The X indicates that the criterion holds for the model. The (X) indicates that the criterion holds only for some extensions of this model.

**Table 2** Comparing the similarity measures of various similarity approaches

| Notion of similarity<br>Type of match<br>Degree of match | Geometric Model | Feature Model | Network Model | Alignment Model | Transformational Model |
|---|---|---|---|---|---|
| **Applicable to** | | | | | |
| – Objects | X | X | X | X | X |
| – Concepts | (X) | (X) | X | | X |
| **Notion of similarity** | | | | | |
| – Semantic distance as dissimilarity | X | | X | | X |
| – Commonalities versus differences | | X | X | X | |
| – Structural analogy | | | | X | |
| **Type of match** | | | | | |
| – No distinction in alignable/ non-alignable match | | X | | | |
| – Only alignable match | X | | | | |
| – Distinction in alignable/ non-alignable match | | | | X | |
| **Degree of match** | | | | | |
| – Various degrees [0, 1] | X | | | X | |
| – Match versus non-match (0, 1) | | X | | | |

within one object and alignment relations to align analogous features or components of different objects. Transformations can also be seen as an extended kind of alignment relations: they state not only that there exists *a* relation, but specify also how these concepts correspond.

Structured knowledge representations are important for similarity measurement, because they "make explicit the relations between elements in a situation, and allow complex representation to be constructed through the combination of simpler elements" (Markman 1999, p. 124). The geometric and the feature model are both unstructured representation, because neither can structure concepts into parts and describe the parts: the geometric model describes concepts only by dimensional properties, but it relates properties on one dimension to each other and therefore provides more structure than the feature model. Network models can relate parts to wholes and various concepts to each other and therefore are classified under structured representations. The alignment model uses features and relations for the description: objects are structured in their parts, described by features and related to other elements. Therefore, the alignment model allows for a structured representation. Alignment relations specify analogous elements of the two objects, between which the similarity is measured. Alignment relations enable the similarity measure to account for structural analogy. The transformational model is a structured representation: a concept represented by a computer program can be described at a high level of complexity and the transformations may be arbitrarily complex as well.

Geometric-, feature-, alignment- and transformational models were originally developed for similarity between objects (Table 2). Gärdenfors proposed conceptual spaces, a geometric model to compare concepts as well. Rodriguez et al. extended the feature model for similarity measurement between concepts. Hahn et al. proposed to use computer programs for the description, which can handle complex concepts. Network models are based on semantic networks that describe concepts or objects and therefore measure similarity between both.

Geometric-, network- and transformational models all use distance as a notion of similarity, but due to their different representational models distance is measured differently: geometric models measure the spatial distance, network models measure the distance in graphs and transformational models take the number or complexity of transformations as distance. These distances are interpreted as the semantic distance of stimuli or the dissimilarity. The distance must be transformed to a similarity value using either a linear or exponentially decaying function as proposed in the literature. We assign Resnik's similarity measure as well to the group of semantic distance-based similarity measures: the algorithm computes the common superconcept with the highest information content. The resulting similarity is comparable with the shortest path length computed by DISTANCE extended by length of the hierarchic links. The concept probability can be interpreted as the semantic distance and the function transforming probability into information content is analogous to the transformation of semantic distance into similarity. Feature models balance the commonalities and differences. Alignment models compute similarity according to the degree of matching features (similar to the feature model), but consider as well the structural analogy between scenes. Feature and alignment models both compute directly a similarity value.

Alignment models distinguish between different kinds of matches: matches that can be aligned increase the similarity more than those that cannot be aligned. Geometric models compare concepts in the same multi-dimensional space, therefore only aligned dimensions exist. Both models allow for a different degree of similarity, typically normalised between 0 and 1. Feature models do not align features before matching them. They distinguish only matching or non-matching features. Network and transformational models do not match relations or transformations: they simply compute the distance from concept $A$ to $B$.

### *8.2 Choosing a Similarity Measure*

The choice of a similarity measure for a certain application is a complex process. In many cases the requirements already restrict the set of possible approaches: For instance, to compare semantic descriptions containing only properties and neglecting relations requires the geometric or the feature model. If you want to compare geographic features in the context of a spatial scene you can use SIAM; however, SIAM cannot be used to compare geographic feature types semantically. The metric properties also determine the suitability of a similarity measure: similarity in a retrieval task is directed (from the query to the data source elements) and therefore an asymmetric implementation of similarity makes sense. In an integration task, a symmetric similarity measure is probably more adequate, because joining two data sources is a symmetric and non-directed task. The comparison given in the above section can be used to match requirements with the characteristics of the semantic similarity measures.

In general however, there is no best practise to choose a similarity measure. This is also because the human similarity judgment process is not always the same. The way humans perceive similarity is influenced by context and experience (Goldstone and Son 2005):

- The similarity between two concepts depends on other concepts in consideration when judging the similarity. In human subject tests for example, the similarity judgment is affected by which objects were presented on previous trials. Similarity between two concepts decreases when the number of similar concepts increases. Krumhansl's distance density model takes this effect into account.
- The similarity between concepts depends on the context which is considered to play a major role in similarity measurement (Roth and Shoben 1983, Wendell 1994). The geometric and the feature model take context effects into account by assigning different weighting factors to dimensions or features. The importance of properties may be perceived differently depending on the task. In a categorization task humans tend to group individuals together that agree on one or several properties. All individuals in a category are similar "with respect to" a particular property (Goodman 1972, Medin et al. 1993). This particular property is considered very important and significant for the categorization. But the importance of properties also depends on the concepts themselves. A feature shared by all concepts of the group in consideration has no diagnostic significance when judging the similarity between pairs of concepts. The salience of properties depends on the other concepts in this group. Tversky's feature-matching model and Johannesson's relative prominence model include weighting factors to reflect the importance of a property.
- The way of judging similarity depends also on the observer. In an experiment Suzuki et al. (1992) asked experts and non-experts to solve the Tower of Hanoi problem and judge the similarity of the various states and the goal. While the experts with a computer science background judge the similarity based on the number of required moves to distort the puzzle into the goal, non-experts base their judgments on shared superficial features (Suzuki et al. 1992). The notion of similarity of the transformational model applies very well to the expert's way of measuring similarity, while the feature model is more appropriate to represent the non-experts similarity judgment process.

For every domain and application task it must be examined which approach satisfies the restrictions and can be applied. The suitability of the way of describing concepts – directly by properties, indirectly by relations or transformations to other concepts or by a combination of both – and the notion of similarity must be checked in every individual case.

## 9 Summary and Outlook

The different approaches for semantic similarity measurement have all their origin in psychology and reflect how humans compare concepts and judge similarity. Still the methods of assessing similarity differ greatly. They use different theories on how humans perceive and structure knowledge about concepts – occupying a region in a vector space or as a set of features – and on how they define a notion of similarity for their model. Similarity in feature models and network models are built on different theories of semantic memory. Alignment models are geared to analogical reasoning and assume that similarity is a high-level cognitive process. Finally, transformational models assume that humans base their similarity judgments on the transformational distance needed to make two concepts transformationally equivalent.

Figure 10 summarizes all five approaches with their various variants by illustrating their development and how they influenced each other. MDS was the first model used

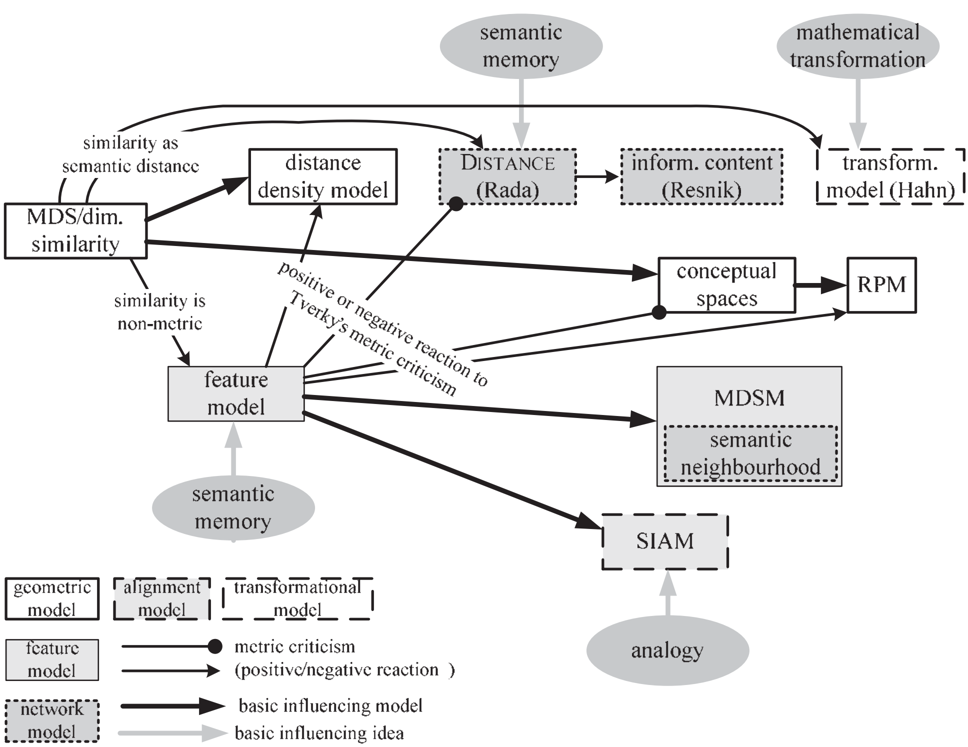


**Figure 10** The development of semantic similarity measures

to analyze human similarity judgment. Its metric properties were heavily criticized in the 1970s and led to the first non-metric similarity measure: the feature-matching model by Tversky. Starting from Tversky's critique of metric similarity, a number of different approaches developed, which take asymmetric properties into account: Krumhansl's distance density model and Johannesson's RPM are asymmetric variants of the geometric model. Goldstone developed SIAM, which seizes the non-metric similarity and combines it with structural analogy. Rada denies Tversky's criticism and argues for metric similarity. He developed the algorithm DISTANCE for network models. Resnik's information content-based similarity model includes different link distances. The early geometric models and Tversky's feature-matching model only accounted for individuals. MDSM extends Tversky's model to account for concepts. Gärdenfors' conceptual spaces are a geometric model for representations of objects and concepts: they account for prototype effects by the way concepts are modelled. Conceptual spaces provide a natural way of measuring similarity via the spatial distance of objects and concepts. Transformational models picked up the notion of semantic distance, but applied it to a framework of transformations.

Ongoing research about semantic similarity between geospatial data aims at overcoming the shortcomings of similarity measures by combining different approaches to semantic similarity measurement. For example, the Hybrid Model (Schwering 2005) integrates the geometric model, namely Conceptual Spaces by Gärdenfors, with semantic networks. This allows using properties as well as relations for the semantic description. The similarity measure applies the notion of semantic distance like the geometric and the network model, but also includes structural commonalities like the alignment model. A combination of various approaches often increases the expressiveness and accounts better for the complexity of human similarity judgement.

Logic-based representations based on description logics are widely accepted in the semantic web. Reasoning mechanisms such as subsumption reasoning is used to identify matching concepts from different ontologies (Lutz and Klien 2006). Current research investigates the usage of similarity in formal ontologies to compare ontologies (e.g. Janowicz 2006a, b; Janowicz et al. 2007). Such approaches are applicable to logic-based ontologies that build the basis for the geospatial semantic web. However, they do not focus on the explanation of human similarity measurement as the psychologically motivated approaches presented in this survey do.

## Acknowledgements

This work has been supported by the UK Ordnance Survey through the project "GeoSemantics".

## Notes

1 Let similarity s be the decaying function $s = 1/(1 + d)$. From $d1 + d2 \geq d3$ with $d1 = d2 = d3 = 1$ follows $s1 = s2 = s3 = 1/2$ and $s1 + s2 > s3$.
2 Stimulus points are the vectors representing objects in a multidimensional space.
3 This does not necessarily hold for concepts, though.

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