Approximability and Hardness in Multi-Objective Optimization

Christian Glaßer Christian Reitwießner Heinz Schmitz
Maximilian Witek

We study the approximability and the hardness of combinatorial multi-objective NP optimization problems (multi-objective problems, for short). Our contributions are:

- We define and compare several solution notions that capture reasonable algorithmic tasks for computing optimal solutions.
- These solution notions induce corresponding NP-hardness notions for which we prove implication and separation results.
- We define approximative solution notions and investigate in which cases polynomial-time solvability translates from one to another notion. Moreover, for problems where all objectives have to be minimized, approximability results translate from single-objective to multi-objective optimization such that the relative error degrades only by a constant factor. Such translations are not possible for problems where all objectives have to be maximized (unless P = NP).

As a consequence we see that in contrast to single-objective problems (where the solution notions coincide), the situation is more subtle for multiple objectives. So it is important to exactly specify the NP-hardness notion when discussing the complexity of multi-objective problems.