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Approximate formulas for the Neutralino Masses in the Nonminimal Supersymmetric Standard Model

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ABSTRACT

We derive a number of approximate analytical formulas for the neutralino masses and neutralino states in the nonminimal supersymmetric standard model containing a Higgs singlet besides the two Higgs doublets of the minimal model. Comparison with the numerical solution for the neutralino masses shows that these formulas serve as an excellent approximation for almost the entire phenomenologically interesting range of parameters.

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I. INTRODUCTION

In supersymmetric theories [1], all particles in the standard model are accompanied by their superpartners. In order to give masses to quarks and leptons, and to cancel triangle gauge anamolies, at least two Higgs doublets $H_1 = (H_1^0, H_1^-)$ and $H_2 = (H_2^+, H_2^0)$, with opposite hypercharge ($Y(H_1) = -1$, $Y(H_2) = +1$), are required in the minimal version of the supersymmetric standard model(MSSM). The fermionic partners of these Higgs bosons mix with the fermionic partners of the gauge bosons to produce two chargino states $\tilde{\chi}_i^{\pm}$, i = 1, 2, and four neutralino states $\tilde{\chi}_i^0$, i = 1, 2, 3, 4 in the MSSM. The neutralino states of the minimal model have been studied in great detail [2-4], because the lightest neutralino state is expected to be the lightest supersymmetric particle(LSP) in supersymmetric theories

In this paper, we make an analysis of the neutralino sector of the nonominimal supersymmetric model (NMSSM) containing two Higgs doublets, H_1 and H_2, and a Higgs singlet chiral superfield N [5], represented by the superpotential [6]

$$W = h_U Q_L U_L^c H_2 + h_D Q_L D_L^c H_1 + h_E L E_L^c H_1 + \lambda H_1 H_2 N - \frac{1}{3} k N^3, \qquad (1.1)$$

where $k \neq 0$ in order to avoid an unacceptable axion in the model. Recently much attention [7,8] has been devoted to the study of the Higgs sector of the nonminimal supersymmetric standard model (NMSSM) (1.1). The reasons for the study of the nonminimal supersymmetric model are twofold. First, the Higgs bilinear term in the superpotential of MSSM can be generated dynamically in the model (1.1), through the trilinear coupling $\lambda H_1 H_2 N$, thereby solving the so called μ problem of the MSSM [9]. Secondly, the minimal supersymmetric standard model makes definite predictions about the spectrum of Higgs bosons and their couplings, including radiative corrections [10]. These predictions about Higgs masses and couplings can be tested experimentally. If these predictions are not borne out, then it would be natural to go to the non-minimal supersymmetric model. In the nonminimal supersymmetric model (1.1), after mixing of Higgs and gauge fermions, there are two chargino $\tilde{\chi}_1^{\pm}$, $\tilde{\chi}_2^{\pm}$, and five neutralino, $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, $\tilde{\chi}_4^0$, $\tilde{\chi}_5^0$, states. The neutralino mass matrix arises from the interaction between gauge and matter multiplets as well as the last two terms in the superpotential (1.1) when the Higgs fields obtain vacuum expectation values. In addition there are supersymmetry breaking gaugino masses, M_1, M_2, M_3 associated with the U(1), SU(2) and SU(3) subgroups of the standard model, respectively. It is a common practice to reduce the parameter freedom by assuming that the three mass scales are equal at some grand unification scale, so that at the electroweak scale the three mass parameters are related [1] through ($M_2 \equiv M$, $M_{\tilde{g}}$ is the gluino mass)

$$M_1 = \frac{3}{5} M' = \tan^2\theta_W \, M, \qquad M_3 = M_{\tilde{g}} = (\alpha_3/\alpha_2)\, M, \tag{1.2}$$

in the standard notation. We shall use these relations in what follows. In the non-minimal model (1.1), because there is an additional gauge singlet fermion $\tilde{N}$, the mass matrix for the neutralinos is a 5 x 5 matrix. We shall choose the following convenient basis for the gaugino-higgsino system of the nonminimal model:

$$\psi_j^0 = (-i\lambda_\gamma, -i\lambda_Z, \psi_H^a, \psi_H^b, \psi_N), \qquad j = 1, 2, 3, 4, 5, \tag{1.3a}$$

where λ_γ and λ_Z are the two component spinors of the photino and zino, respectively, and

$$\psi_H^a = \psi_{H_1}^1 \sin\theta_V - \psi_{H_2}^2 \cos\theta_V, \quad \psi_H^b = \psi_{H_1}^1 \cos\theta_V + \psi_{H_2}^2 \sin\theta_V \tag{1.3b}$$

are the higgsino states, with $\psi_{H_1}^1$, $\psi_{H_2}^2$, ψ_N the two component spinors of the neutral higgsinos $\tilde{H}_1^o$, $\tilde{H}_2^o$ and $\tilde{N}$, respectively, and where [11]

$$\langle H_1^o \rangle = v_1/\sqrt{2}, \quad \langle H_2^o \rangle = v_2/\sqrt{2}, \quad \tan\theta_V = v_1/v_2, \tag{1.3c}$$

The mass term in the Langrangian has the form

$$\mathcal{L}_M = -\frac{1}{2} M_Z \psi_i^0 Y_{ij} \psi_j^0 + \text{h.c.}, \tag{1.4a}$$

where the mass matrix [12]

$$Y = \begin{bmatrix} \Lambda(\alpha \cos^2\theta_W + \sin^2\theta_W) & \Lambda(1-\alpha)\sin\theta_W\cos\theta_W & 0 & 0 & 0 \\ \Lambda(1-\alpha)\sin\theta_W\cos\theta_W & \Lambda(\alpha \sin^2\theta_W + \cos^2\theta_W) & 1 & 0 & 0 \\ 0 & 1 & -\nu \sin 2\theta_V & -\nu \cos 2\theta_V & 0 \\ 0 & 0 & -\nu \cos 2\theta_V & \nu \sin 2\theta_V & \gamma \\ 0 & 0 & 0 & \gamma & -\delta \end{bmatrix}, \tag{1.4b}$$

with

$$\Lambda = \frac{M}{M_Z}, \quad \alpha = \frac{M'}{M}, \quad \nu = \frac{\lambda x}{M_Z}, \quad \gamma = \frac{\lambda\, (v_1^2 + v_2^2)^{1/2}}{\sqrt{2}\, M_Z}, \quad \delta = \frac{2\ kx}{M_Z}, \tag{1.4c}$$

where we have taken out a factor of M_Z so that we deal with dimensionless quantities only. Neglecting CP violation, Y is a real symmetric matrix which can be diagonalized by a 5 x 5 unitary matrix N:

$$N_{im} N_{kn} Y_{mn} = \xi_i \delta_{ik}, \qquad \chi_i^0 = N_{ij} \psi_j^0 \tag{1.5}$$

where $\xi_i = m_i/M_Z$, with m_i being the mass eigenvalue of the neutralino state χ_i^0. Since Y is a real symmetric matrix, we can take N_{im} to be real orthogonal matrix. Some of the mass eigenvalues may be negative. These can be made positive by an appropriate choice of phases in N_{im}, but we shall not do that here. The sign of m_i is related to the CP quantum number of χ_i^0 [13]. The eigenvalues ξ_i of (1.4b) are the solutions of the eigenvalue equation

$$\begin{aligned} &(\xi - \Lambda)(\xi - \Lambda\alpha)[(\xi + \delta)(\xi^2 - \nu^2) - \gamma^2(\xi + \nu \sin 2\theta_V)] \\ &-(\xi - \Lambda\alpha \cos^2\theta_W - \Lambda \sin^2\theta_W)[(\xi + \delta)(\xi - \nu \sin 2\theta_V) - \gamma^2] = 0. \end{aligned} \tag{1.6}$$

Once we obtain the eigenvalues ξ_i, the eigenstates of the neutralino mass matrix can be written as

$$\chi_i^0 = \frac{1}{N_i}\begin{bmatrix} \Lambda(1-\alpha)\sin\theta_W \cos\theta_W[(\nu^2-\xi_i^2)(\xi_i+\nu)+\gamma^2(\xi_i+\nu\sin 2\theta_V)] \\ [\xi_i-\Lambda(\alpha\cos^2\theta_W+\sin^2\theta_W)](\nu^2-\xi_i^2)(\xi_i+\delta)+\gamma^2(\xi_i+\nu\sin 2\theta_V)] \\ [\xi_i-\Lambda(\alpha\cos^2\theta_W+\sin^2\theta_W)][(\nu\sin 2\theta_V-\xi_i)(\xi_i+\delta)+\gamma^2] \\ [\xi_i-\Lambda(\alpha\cos^2\theta_W+\sin^2\theta_W)](\xi_i+\delta)\,\nu\cos 2\theta_V \\ [\xi_i-\Lambda(\alpha\cos^2\theta_W+\sin^2\theta_W)]\,\gamma\,\nu\cos 2\theta_V \end{bmatrix} \qquad (1.7)$$

in the chosen basis. Here N_i is the appropriate normalization factor. The four component Majorana mass eigenstates $\tilde{\chi}_i^0$ of neutralinos are defined as usual in terms of χ_i^0 and $\bar{\chi}_i^0$. The neutralino components given in (1.7) are elements of the transformation matrix N which diagonalizes the mass matrix Y. These will determine the couplings of the neutralino's to the other states in the model.

II. COMPLETE SOLUTIONS

The eigenvalue problem, Eq.(1.6), cannot, in general, be solved analytically. However, for certain special values of the parameters, it reduces to a product of quadratic and linear equations, and can, thus, be solved analytically, as we do in this section. These special cases will serve as a basis for our approximate formulas derived in Section III.

a) $\sin^2\theta_W = 0$, $\sin 2\theta_V = 1$: For these special values of the parameters, the neutralino states are given by

$$\chi_1^0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \chi_2^0 = \begin{bmatrix} 0 \\ \cos\phi \\ \sin\phi \\ 0 \\ 0 \end{bmatrix}, \quad \chi_3^0 = \begin{bmatrix} 0 \\ \sin\phi \\ -\cos\phi \\ 0 \\ 0 \end{bmatrix}, \quad \chi_4^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cos\beta \\ \sin\beta \end{bmatrix}, \quad \chi_5^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sin\beta \\ -\cos\beta \end{bmatrix}, \qquad (2.1)$$

The corresponding neutralino masses are given by

$$\xi_1^0 = \Lambda\alpha,$$

$$\xi_2^0 = \frac{\Lambda - \nu + \sqrt{(\Lambda + \nu)^2 + 4}}{2}, \qquad \xi_3^0 = \frac{\Lambda - \nu - \sqrt{(\Lambda + \nu)^2 + 4}}{2},$$

$$\xi_4^0 = \frac{\nu - \delta - \sqrt{(\nu + \delta)^2 + 4\gamma^2}}{2}, \qquad \xi_5^0 = \frac{\nu - \delta + \sqrt{(\nu + \delta)^2 + 4\gamma^2}}{2} \tag{2.2}$$

with the mixing angles given by

$$\sin\beta = \frac{1}{\sqrt{2}}\left[1 + \frac{\nu + \delta}{\sqrt{(\nu + \delta)^2 + 4\gamma^2}}\right]^{1/2}, \quad \sin\varphi = \frac{1}{\sqrt{2}}\left[1 + \frac{\Lambda + \nu}{\sqrt{(\Lambda + \nu)^2 + 4}}\right]^{1/2} \tag{2.3}$$

b) $\Lambda >> 1$ and/or $\nu >>1$: In this case complete solutions exist only for $\sin 2\theta_V = 1$. This last condition is not required for the analogous situtation in the minimal model. The neutralino states in this limit are

$$\chi_1^0 = \begin{bmatrix} \sin\theta_W \\ \cos\theta_W \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \chi_2^0 = \begin{bmatrix} \cos\theta_W \\ -\sin\theta_W \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \chi_3^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \chi_4^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cos\beta \\ \sin\beta \end{bmatrix}, \quad \chi_5^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sin\beta \\ -\cos\beta \end{bmatrix}, \tag{2.4}$$

with eigenvalues

$$\xi_1^0 = \Lambda, \quad \xi_2^0 = \Lambda\alpha, \quad \xi_3^0 = -\nu, \qquad \xi_{4,5}^0 = \frac{(\nu-\delta) \pm\sqrt{(\nu+\delta)^2+4\gamma^2}}{2}, \tag{2.5}$$

with the mixing angle β same as in (2.3). Note that the states χ_1^0 and χ_2^0 are $\tilde{W}^3$ and $\tilde{B}$ states, respectively, whereas χ_3^0 is a pure doublet Higgsino state.The states χ_4^0 and χ_5^0 are a mixture of doublet and singlet Higgsino states.

c) The limit of $x << v_1, v_2$: This limit is typical of the result that emerges from renormalization group analysis [6] of the model and has been studied for the Higgs sector of the model. In the present case this limit corresponds to taking ν, $\delta \to 0$ in the lowest approximation. Then the mass matrix splits into a 2x2 matrix whose eigenvectors are a mixture of doublet Higgsino and the singlet Higgsino,

$$\chi_4^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, \qquad \chi_5^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}, \tag{2.6}$$

with eigenvalues

$$\xi_{4,5}^0 = \pm\gamma, \tag{2.7}$$

and a 3x3 matrix which cannot be diagonalized analytically. However, in the limit $\sin^2\theta_W=0$, the 3x3 matrix can also be diagonalized analytically with eigenstates and masses given by

$$\chi_1^0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \chi_2^0 = \begin{bmatrix} 0 \\ \cos\phi \\ \sin\phi \\ 0 \\ 0 \end{bmatrix}, \quad \chi_3^0 = \begin{bmatrix} 0 \\ \sin\phi \\ -\cos\phi \\ 0 \\ 0 \end{bmatrix}, \tag{2.8}$$

$$\xi_1^0 = \Lambda\alpha, \qquad \xi_{2,3}^0 = \frac{\Lambda \pm \sqrt{\Lambda^2+4}}{2}, \tag{2.9}$$

$$\sin\phi = \frac{1}{\sqrt{2}} \left[1 + \frac{\Lambda}{\sqrt{\Lambda^2+4}} \right]^{1/2}, \tag{2.10}$$

Here χ_1^0 is a pure photino, whereas $\chi_{2,3}^0$ are mixtures of the zino and the doublet Higgsino.

III. ANALYTICAL FORMULAS AND NUMERICAL RESULTS

Having discussed various special cases where complete analytical solutions are possible, we now discuss several approximation schemes for the neutralino masses which may be of practical value in different domains of the parameter space. The approximation formulas are based on applying perturbation theory to the exact analytical results obtained in Section II. We shall compare these approximate formulas for the neutralino masses with the results obtained by the exact numerical diaognalization of the mass matrix to establish their range of applicability.

Since the number of parameters on which the neutralino mass matrix depends is large, we shall use renormalization group equations as a guiding principle to restrict the parameter space and to motivate specific choice of the parameters for the numerical analysis. The renormalization group equations for the parameters λ and k have infrared fixed points such that if they have values of order 1 or larger at the GUT scale, then at low energies their values will be near the fixed point values [6]:

$$\lambda \sim 0.87, \qquad k \sim 0.63. \tag{3.1}$$

We shall consider the values in (3.1) as a conservative upper limit on the parameters, and use them in our numerical work. The result (3.1), and the first of eqs.(1.2), imply

$$\frac{\nu}{\delta} \sim 0.70, \qquad \gamma \sim 1.40, \tag{3.2}$$

$$\alpha \sim 0.47, \tag{3.3}$$

respectively. Thus, in a renormalization group inspired model we have only three independant parameters describing the neutralino mass matrix which we shall take to be Λ, ν and $\tan\theta_V$. Furthermore, if we assume that there is no explicit or spontaneous CP violation, then one can choose to work in a vacuum state with all three vacuum expectation values real and positive [6], implying a positive $\tan\theta_V$[14]. In order to accomplish this in a renormalization group approach with supersymmetry breaking at GUT scale induced by a universal gaugino mass term $M' = M = M_3 \equiv M_U \neq 0$, with no other soft SUSY breaking terms, M_U must be chosen to be positive. With λ given by (3.1), the effective $\mu(\equiv \lambda x)$ parameter and the gaugino mass parameter M, and hence ν and Λ, are thus both positive, in contrast to the situation that obtains in the minimal model[2]. We shall use this general result to restrict the parameter space in our numerical comparison with our analytical results, although our analytical formulas are valid for any sign of ν and Λ.

III.1. Expansion in $\sin^2\theta_W$ and $\sin 2(\theta_V - \pi/4)$

This approximation scheme, which is based on the analytical solution (a) of section II for $\sin^2\theta_W = 0$ and $\sin 2\theta_V = 1$, is analogous to the corresponding scheme for the minimal model [3]. The expansion is applicable for a large range of Λ, ν and $\tan\theta_V$ values, $\Lambda \leq 10$, $\nu \leq 10$ and $0.1 \leq \tan\theta_V \leq 1$, respectively. The mass matrix Y, Eq.(1.4b), can be written as

$$Y = Y_0 + \Lambda(1-\alpha)\sin^2\theta_W\,\Sigma'_3 + \Lambda(1-\alpha)\sin\theta_W\cos\theta_W\,\Sigma'_1 + 2\nu\sin^2\epsilon\,\Sigma_3 + 2\nu\sin\epsilon\cos\epsilon\,\Sigma_1, \tag{3.4}$$

where Y_0 is the mass matrix for $\sin^2\theta_W = 0$, $\sin 2\theta_V = 1$, and $\epsilon = \theta_V - \pi/4$. The 5x5 matrices Σ_i and Σ'_i are given by

$$\Sigma_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \multicolumn{2}{c}{\sigma_i} & 0 \\ 0 & 0 & & & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \Sigma'_i = \begin{bmatrix} \multicolumn{2}{c}{\sigma_i} & 0 & 0 & 0 \\ & & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{3.5}$$

with σ_i the Pauli matrices. Perturbation theory applied to (3.4) gives

$$\xi_1 = \xi_1^0 + \Lambda(1-\alpha)\sin^2\theta_W + \frac{[\Lambda(1-\alpha)\sin\theta_W\cos\theta_W]^2\cos^2\phi}{\xi_1^0 - \xi_2^0} + \frac{[(\Lambda(1-\alpha)\sin\theta_W\cos\theta_W]^2\sin^2\phi}{\xi_1^0 - \xi_3^0},$$

$$\xi_2 = \xi_2^0 - \Lambda(1-\alpha)\sin^2\theta_W \cos^2\phi + 2\,\nu \sin^2\varepsilon \sin^2\phi$$
$$+ \frac{[\Lambda(1-\alpha)\sin\theta_W \cos\theta_W]^2\cos^2\phi}{\xi_2^0 - \xi_1^0} + \frac{[2\nu \sin\varepsilon \sin\phi \cos\beta]^2}{\xi_2^0 - \xi_4^0}$$
$$+ \frac{[2\nu \sin\varepsilon \sin\phi \sin\beta]^2}{\xi_2^0 - \xi_5^0},$$

$$\xi_3 = \xi_3^0 - \Lambda(1-\alpha)\sin^2\theta_W \sin^2\phi + 2\nu \sin^2\varepsilon \cos^2\phi$$
$$+ \frac{[\Lambda(1-\alpha)\sin\theta_W \cos\theta_W \sin\phi]^2}{\xi_3^0 - \xi_1^0} + \frac{[2\nu \sin\varepsilon \cos\phi \cos\beta]^2}{\xi_3^0 - \xi_4^0}$$
$$+ \frac{[2\nu \sin\varepsilon \cos\phi \sin\beta]^2}{\xi_3^0 - \xi_5^0}, \tag{3.6}$$
$$\xi_4 = \xi_4^0 - 2\nu \sin^2\varepsilon \cos^2\beta + \frac{[2\nu \sin\varepsilon \sin\phi \cos\beta]^2}{\xi_4^0 - \xi_2^0}$$
$$+ \frac{[2\nu \sin\varepsilon \cos\phi \cos\beta]^2}{\xi_4^0 - \xi_3^0},$$

$$\xi_5 = \xi_5^0 - 2\nu \sin^2\varepsilon \sin^2\beta + \frac{[2\nu \sin\varepsilon \sin\phi \sin\beta]^2}{\xi_5^0 - \xi_2^0}$$
$$+ \frac{[2\nu \sin\varepsilon \cos\phi \sin\beta]^2}{\xi_5^0 - \xi_3^0},$$

where ξ_i^0 are the eigenvalues of Y_0 and are given by $\xi_1^0 = \Lambda\alpha$, and ξ_i^0 (i = 2, 3, 4, 5) as in (2.2), with ϕ and β given by (2.3). These eigenvalues are plotted in Fig.1 as a function of ν for $\Lambda = 1.0$ and $\tan\theta_V = 0.4$ ($\sin 2\theta_V \sim 0.7$), together with the exact results for the same set of parameters. It is obvious that (3.6) is an excellent approximation for $\nu \lesssim 10$ for $\tan\theta_V < 1$. This covers almost the entire phenomenologically interesting range of parameters.

III.2 Expansion in ν and $\sin^2\theta_W$

Since ν and δ are related through the renormalization group equation constraint (3.2), this is effectively an expansion in ν, δ and $\sin^2\theta_W$. The limit of small ν and δ is interesting, because it is a result which emerges from a renormalization group analysis of the non-minimal model [6]. If we expand about this limit, we will get approximation formulae for $\nu < 1$ which are valid for all values of $\tan\theta_V$. Starting from the exact solution (2.6) - (2.10) of Section II, and expanding in ν, δ and $\sin^2\theta_W$ we get the eigenvalues

$$\xi_1 = \xi_1^0 + \Lambda(1-\alpha)\sin^2\theta_W + \frac{[\Lambda(1-\alpha)\sin\theta_W\cos\theta_W\cos\phi]^2}{\xi_1^0 - \xi_2^0} + \frac{[\Lambda(1-\alpha)\sin\theta_W\cos\theta_W\sin\phi]^2}{\xi_1^0 - \xi_3^0},$$

$$\xi_2 = \xi_2^0 - \Lambda(1-\alpha)\sin^2\theta_W\cos^2\theta_W + \frac{[(1-\alpha)\sin\theta_W\cos\phi]^2}{\xi_2^0 - \xi_1^0} - \nu\sin 2\theta_V\sin^2\phi + \frac{[\nu\sin 2\theta_V\sin\phi\cos\phi]^2}{\xi_2^0 - \xi_3^0} + \frac{[\nu\cos 2\theta_V\sin\phi]^2}{2(\xi_2^0 - \xi_4^0)}\left[1 + \frac{(\xi_2^0 - \xi_4^0)}{(\xi_2^0 - \xi_5^0)}\right],$$

$$\xi_3 = \xi_3^0 - \Lambda(1-\alpha)\sin^2\theta_W\sin^2\phi - \nu\sin 2\theta_V\cos^2\phi + \frac{[\Lambda(1-\alpha)\sin\theta_W\cos\theta_W\sin\phi]^2}{\xi_3^0 - \xi_1^0} + \frac{[\nu\sin 2\theta_V\cos\phi\sin\phi]^2}{\xi_3^0 - \xi_2^0} + \frac{[\nu\cos 2\theta_V\cos\phi]^2}{2(\xi_3^0 - \xi_4^0)}\left[1 + \frac{(\xi_3^0 - \xi_4^0)}{(\xi_3^0 - \xi_5^0)}\right] \tag{3.7}$$

$$\xi_4 = \xi_4^0 + \frac{\nu\sin 2\theta_V}{2} + \frac{[\nu\sin 2\theta_V]^2}{4(\xi_4^0 - \xi_5^0)} + \frac{[\nu\cos 2\theta_V\sin\phi]^2}{2(\xi_4^0 - \xi_2^0)}$$

$$+\frac{[\nu\cos 2\theta_V\cos\phi]^2}{2(\xi_4^0-\xi_3^0)}-\frac{\delta}{2}+\frac{\delta^2}{4(\xi_4^0-\xi_5^0)},$$

$$\xi_5=\xi_5^0+\frac{\nu\sin 2\theta_V}{2}+\frac{[\nu\sin 2\theta_V]^2}{4(\xi_5^0-\xi_4^0)}+\frac{[\nu\cos 2\theta_V\sin\phi]^2}{2(\xi_5^0-\xi_2^0)}$$

$$+\frac{[\nu\cos 2\theta_V\cos\phi]^2}{2(\xi_5^0-\xi_3^0)}-\frac{\delta}{2}+\frac{\delta^2}{4(\xi_5^0-\xi_4^0)},$$

where ξ_i^0 are the eigenvalues in the limit of ν, δ, $\sin^2\theta_W \to 0$, and are given in (2.7) and (2.9). Note that only ξ_4 and ξ_5 depend on δ. These eigenvalues are plotted in Fig.2, together with the exact results, as a function of ν for $\tan\theta_V = 0.4$ and $\Lambda = 1.0$. From the figure we see that the approximation is of a good quality for $\nu < 1.0$. This approximation is in fact valid for all values of $\tan\theta_V$.

III.3 Expansion in $1/\Lambda$ or $1/\nu$ and $\sin 2(\theta_V-\pi/4)$

We have seen in Section II that complete analytical solutions can be obtained for $\Lambda >> 1$ and/or $\nu >> 1$ with $\sin 2\theta_V = 1$. Taking solution (2.5) as the zeroth approximation, perturbation theory gives,

$$\xi_1=\xi_1^0+\frac{\cos^2\theta_W}{\xi_1^0-\xi_3^0},\qquad \xi_2=\xi_2^0+\frac{\sin^2\theta_W}{\xi_2^0-\xi_3^0},$$

$$\xi_3=\xi_3^0+2\nu\sin^2\varepsilon+\frac{\cos^2\theta_W}{\xi_3^0-\xi_1^0}+\frac{\sin^2\theta_W}{\xi_3^0-\xi_2^0}+\frac{[\nu\sin 2\varepsilon\cos\beta]^2}{\xi_3^0-\xi_4^0}$$

$$+\frac{[\nu\sin 2\varepsilon\sin\beta]^2}{\xi_3^0-\xi_5^0}, \tag{3.8}$$

$$\xi_4=\xi_4^0-2\nu\sin^2\varepsilon\cos^2\beta+\frac{[\nu\sin 2\varepsilon\cos\beta]^2}{\xi_4^0-\xi_3^0}+\frac{[\nu\sin\varepsilon\sin 2\beta]^2}{\xi_4^0-\xi_5^0}$$

$$\xi_5 = \xi_5^0 - 2\,\nu\,\sin^2\epsilon\,\sin^2\beta + \frac{[\nu\,\sin 2\epsilon\,\sin\beta]^2}{\xi_5^0 - \xi_3^0} + \frac{[\nu\,\sin\epsilon\,\sin 2\beta]^2}{\xi_5^0 - \xi_4^0}$$

where ξ_i^0 are the limiting values given in (2.5), and $\epsilon = \theta_V - \pi/4$. The result (3.8) is shown in Fig. 3, where we have plotted the eigenvalues as a function of ν for $\Lambda = 1.0$ and $\tan\theta_V$. We see from the figure that the approximation is fairly good even for low values of Λ when ν is small. The approximation becomes better for larger values of ν for large $\Lambda \geq 2$.

IV. DISCUSSION AND CONCLUDING REMARKS

We have obtained exact analytical formulas for the neutralino masses in NMSSM for some special values of the parameters. Based on these solutions we have build up approximate formulae through a perturbation expansion which cover a wide region of the parameter space relevant for phenomenology, and compared them with exact numerical solution for the neutralino masses. We note that the neutralino states can be obtained, for each of the three cases discussed above, from the general result (1.7). It is important to point out that the approximation formulae (3.6) - (3.8) are valid when the corresponding eigenvalues ξ_i^0 are non-degenerate. In case of degeneracy one must apply degenerate perturbation theory. In our numerical analysis, with the parameter space that we have considered, we have not actually come across a degeneracy. To illustrate this point we consider the solution (a) of Section II. We note that ξ_2^0 and ξ_3^0, and ξ_4^0 and ξ_5^0 are never degenerate for any physical values of the parameters. ξ_1^0 and ξ_2^0 can be degenerate only for negative values of ν, which we have not considered here. Similar remarks apply to the eigenvalues ξ_1^0 and ξ_3^0, and ξ_1^0 and ξ_4^0, etc.

If the future data rules out the minimal supersymmetric model, then in the context of supersymmetry the non-minimal model could be a viable alternative. We have seen that in the context of renormalization group analysis, the effective number of parameters describing the neutralino sector is three, the same as in the minimal model. It will, therefore, be interesting to see whether there are distinctive signatures of the model in the neutralino sector in the context of present and future colliders. This question is under study and will be reported elsewhere [15].

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11 The ratio v_2/v_1 is generally denoted as $\tan\beta$ in the context of minimal supersymmetric standard model. Here we use a different notation because we denote by β one of the mixing angles in a later section.

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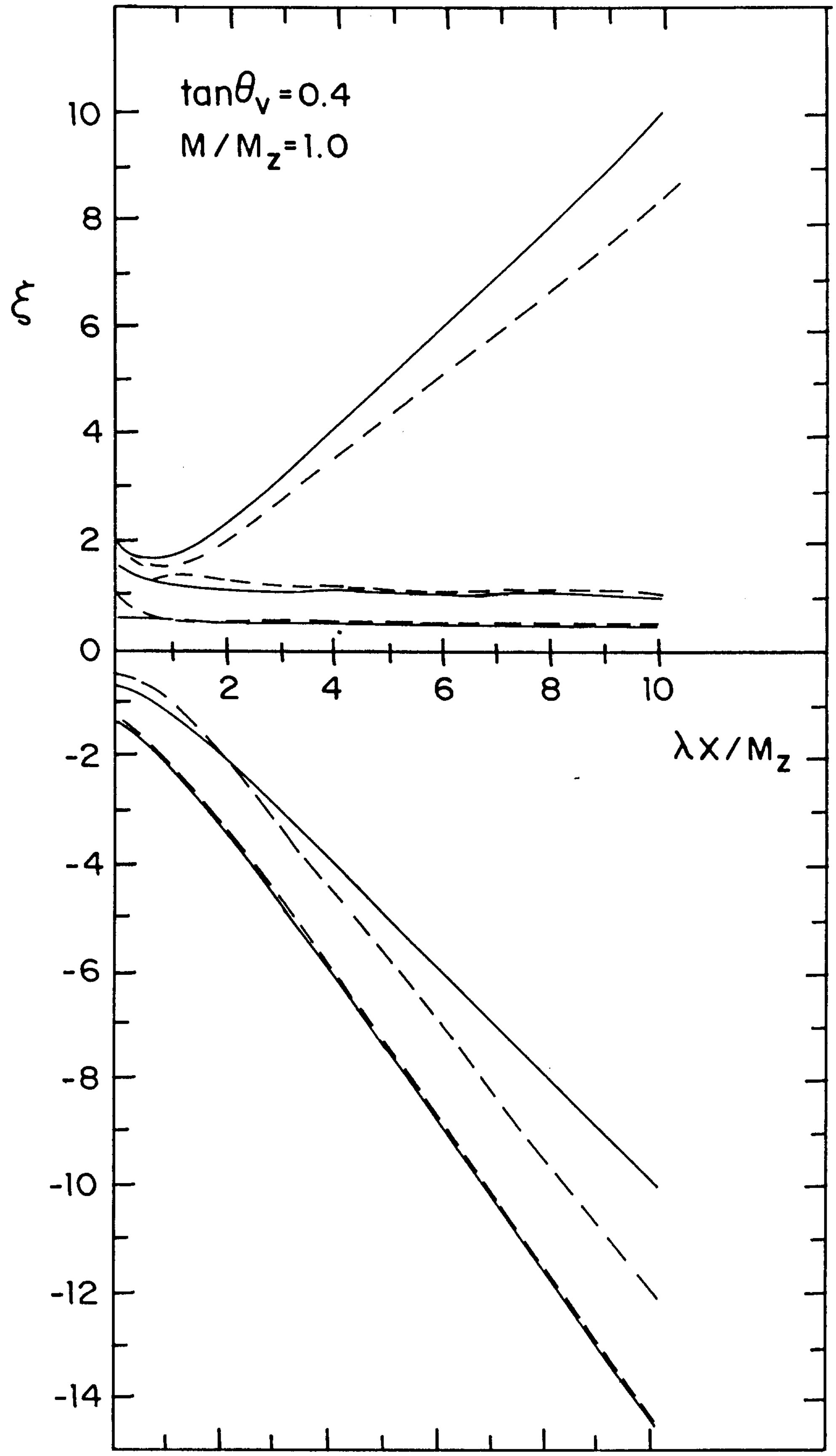
FIGURE CAPTIONS

1. Approximate neutralino masses, result (3.6), as a function of ν for fixed value of Λ = 1.0, $\tan\theta_V = 0.4$, with $\sin^2\theta_W = 0.23$. Solid curves are exact numerical solutions.
2. Neutralino masses as obtained from the approximation formulas (3.7), represented as dashed lines, as a function of ν. Solid lines represent exact solutions. This approximation is valid for all values of $\tan\theta_V$.
3. Approximation formulas (3.8) (dashed lines) for neutralino masses plotted as a function of ν. Solid lines represent exact results. The approximation becomes better for larger values of ν at values of $\Lambda \geq 2$.



$\tan\theta_V = 0.4$
$M/M_Z = 1.0$
ξ
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
2
4
6
8
10
$\lambda X/M_Z$

FIG. 3

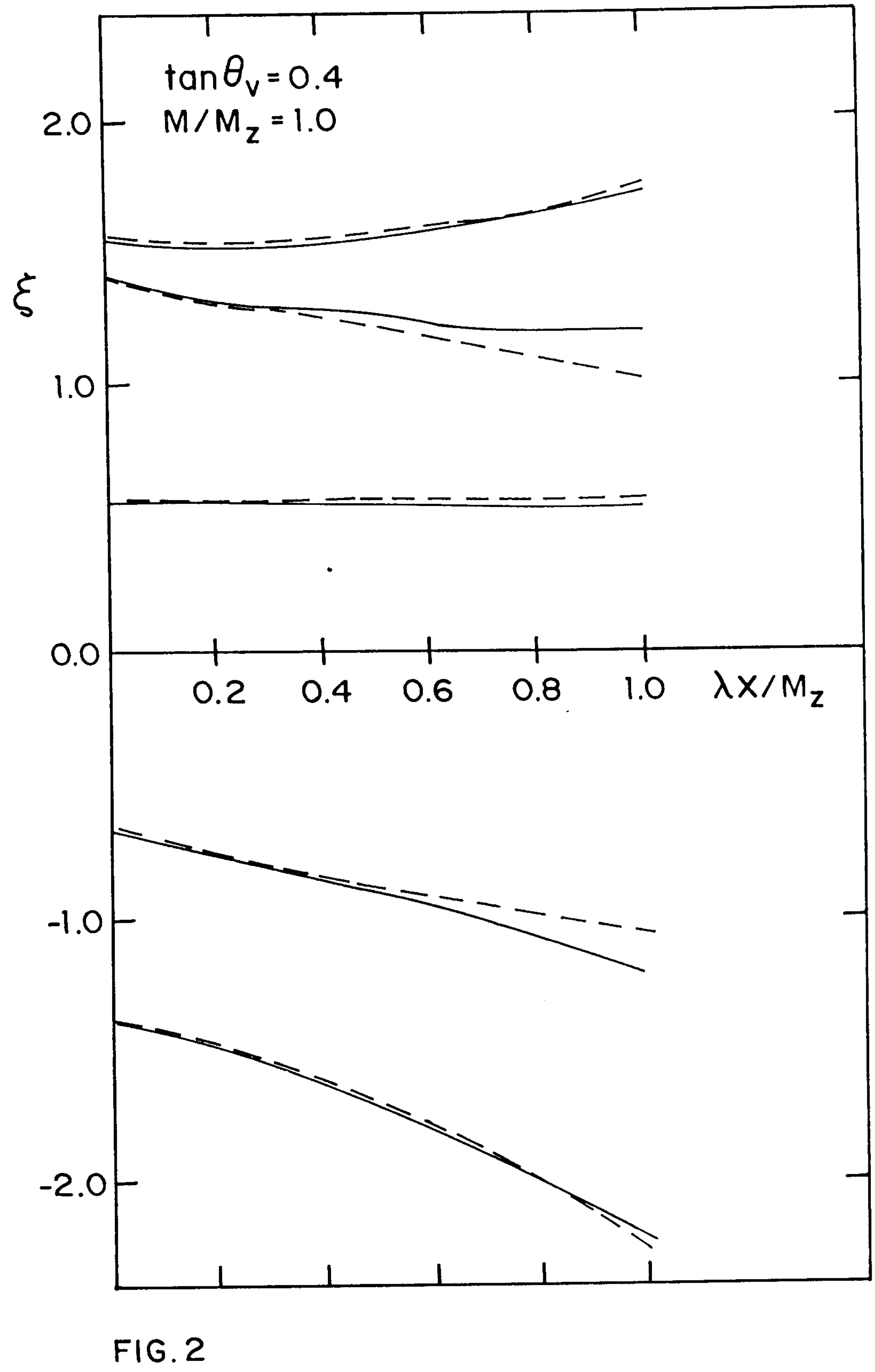


FIG. 2

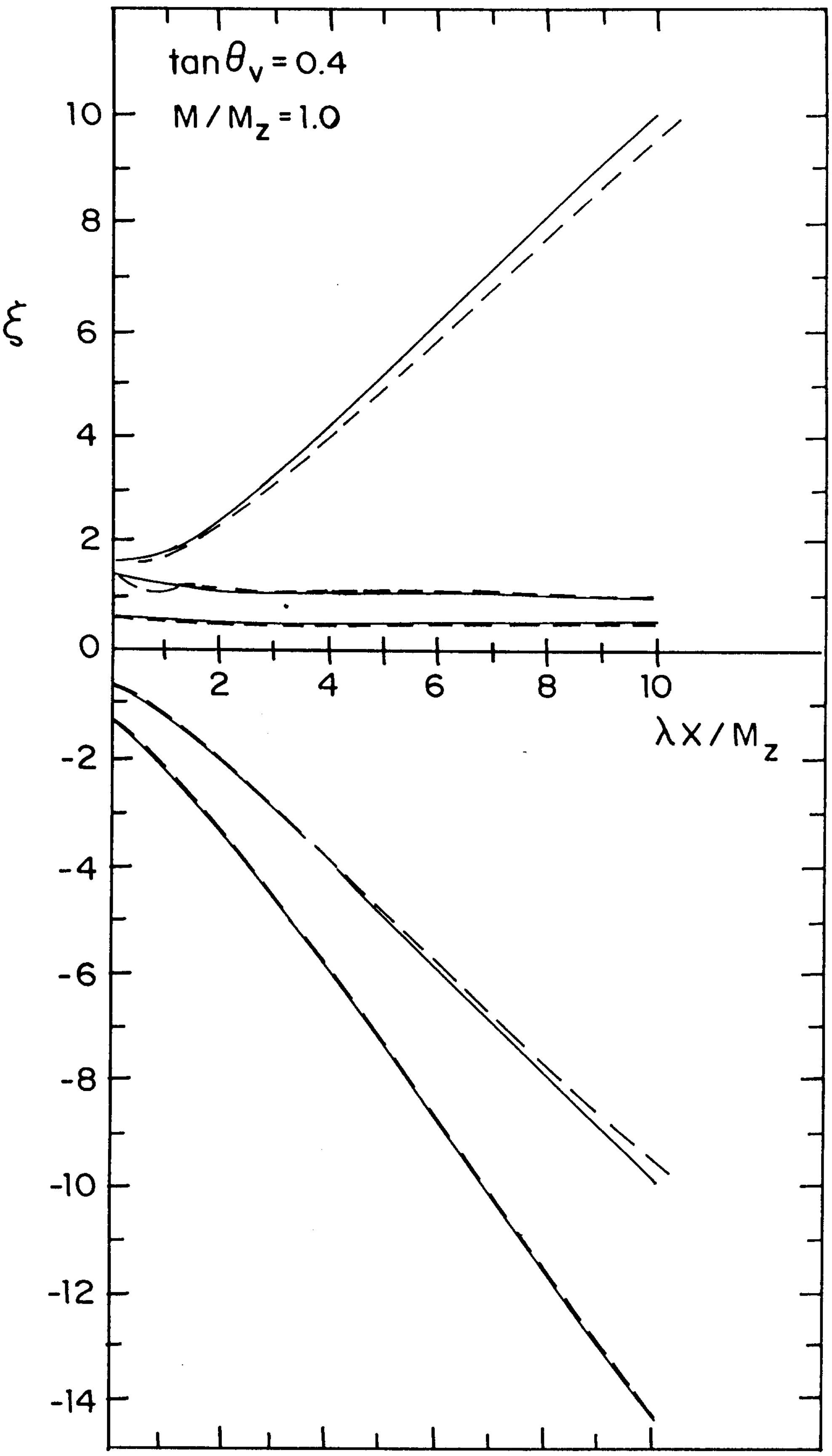


FIG. 1