Approximate Iterative Least Squares Algorithms for GPS Positioning

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***Abstract*—The efficient implementation of positioning algorithms is investigated for Global Positioning System (GPS) and Differential GPS (DGPS). This is particularly important for smart phones with battery limitations. With the help of the information from base stations, Assisted GPS (AGPS) and DGPS can do the positioning more efficiently and more precisely than GPS. In order to do the positioning, the pseudoranges between the receiver and the satellites are required. The most commonly used algorithm for position computation from pseudoranges is non-linear Least Squares (LS) method. Linearization is done to convert the non-linear system of equations into an iterative procedure, which requires the solution of a linear system of equations in each iteration, i.e. linear LS method is applied iteratively. CORDIC-based approximate rotations are used while computing the QR decomposition for solving the LS problem in each iteration. By choosing accuracy of the approximation, e.g. with a chosen number of optimal CORDIC angles per rotation, the LS computation can be simplified. The accuracy of the positioning results is compared for various numbers of required iterations and various approximation accuracies using real GPS data. The results show that very coarse approximations are sufficient for a reasonable positioning accuracy. Therefore, the presented method reduces the computational complexity significantly and is highly suitable for hardware implementation.**

***Keywords*-Global Positioning System (GPS), Differential GPS (DGPS), Least Squares (LS), QR Decomposition (QRD), Coordinate Rotation Digital Computer (CORDIC), approximation methods**

I. INTRODUCTION

Location Based Services (LBS) [1] [2] [3] [4] are wireless 'mobile content' services which are used to provide location-specific information to mobile users moving from location to location. They utilize the ability to make use of the geographical position of the mobile device. Currently, GPS-based techniques [5] [6], network positioning methods [7] as well as other positioning methods [8] [9] are commonly used location estimation methods for LBS.

As a result of the freely available satellite positioning parameters, the GPS system has been widely adopted. Building GPS devices in commercially available cell phones has been achieved by mobile device providers, such that the number of cell phones equipped with GPS functionality has rapidly grown in the last few years.

The next generation of cell phones, usually named smartphones, have integrated lots of functionalities, e.g. big memory, fast processors, mobile network access (GSM, UMTS, LTE), bluetooth, GPS. Therefore, power constraints become more and more important for these devices. In terms of positioning, wireless carriers can communicate location information to the receiver via the cellular network, such that Assisted GPS (AGPS) [10] and DGPS [11] can be used to achieve the positioning faster, at a higher precision and with a lower power consumption than GPS. This improves performance and decreases battery strain. It can be expected that in most application scenarios where a connection from cell phone to the base stations is available, stand-alone GPS will disappear.

In order to do the positioning, an initial set of pseudoranges between the receiver and the satellites is needed. Non-linear LS is the most common method to determine the receiver's position from the pseudoranges. Usually, linearization is done to convert the non-linear problem into an iterative algorithm, which requires the solution of an over-determined system of linear equations in each iteration step itr ($itr = 1,\ 2,\ \cdots,\ itr_{max}$), i.e. linear LS method is applied in each iteration step itr. For solving the linear LS problems in each iteration step an iterative version of the QR decomposition (QRD) [12] is applied in this paper. Instead of annihilating the lower diagonal elements during the QRD, CORDIC-based approximate rotations are used. By choosing the accuracy of the approximation, e.g. by choosing itg ($itg = 1,\ 2,\ \cdots,\ itg_{max}$) optimal CORDIC angles per rotation, the LS computation can be simplified. However, we only obtain an approximate solution to the LS problem, whose accuracy depends on itg. The accuracy of the positioning results of GPS and DGPS methods is compared for varying numbers of iterations of the positioning algorithms and varying numbers of iterations of the iterative QRD using real GPS data. The results show that very coarse approximations are sufficient for obtaining a reasonable position estimate. Therefore, the presented methods reduce the computational complexity and the required power consumption significantly.

The remainder of this paper is organized as follows. GPS and DGPS positioning are introduced in Sec. II resulting in an algorithm, which requires the solution of LS problems in each iteration. For solving these LS problems an iterated version of the QRD is presented in Sec. III using CORDIC-based approximate rotations. The trade-off between iteration of the positioning method and iteration of the iterative QRD is investigated in Sec. IV, where experimental results are given using real GPS data. The paper finishes with a conclusion and an outlook to the future work in Sec. V.

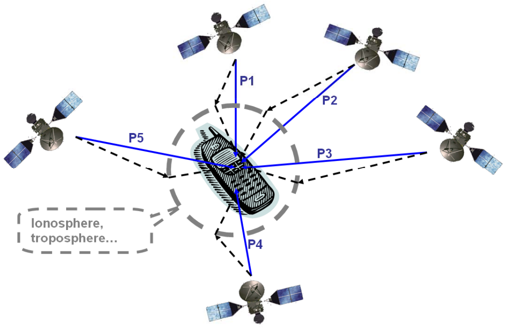

Fig. 1. Pseudoranges: the distance from satellites to GPS receiver

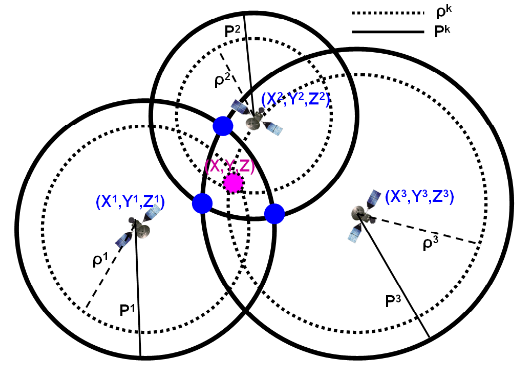

Fig. 2. Observed pseudoranges P^k and geometrical pseudoranges ρ^k

II. GPS Positioning

The whole GPS positioning procedure includes three tasks: acquisition, tracking and positioning [13]. The acquisition searches for satellites and gets their positions. It gives rough estimates of signal parameters. Tracking keeps track of these parameters as the signal properties change over time. After tracking, the navigation data can be extracted and pseudoranges (measured distance from satellites to GPS receiver) can be computed. The final task of the receiver is to compute the user position.

The GPS satellites' arrangement ensures that every point on our planet is in contact with at least six satellites at all times. Each satellite k continuously broadcasts a digital radio signal that includes its position $(X^k,\ Y^k,\ Z^k)$ and its time t^k. Onboard atomic clocks ensure an accurate time to a billionth of a second. The radio signal of a satellite spreads with $c = 3 * 10^8 \frac{m}{s}$ in universe, the velocity of light in vacuum. GPS receivers measure the time delay τ^k of the signal from each satellite k to the receiver, so $\tau^k = t - t^k$, where t is time of receiver. The measurement of t in the receiver is not very accurate (as compared to the satellite time t^k). Furthermore the speed of the radio signal from the satellites is smaller than c because of ionosphere and troposphere. Therefore the measured distance from the satellite to the receiver P^k, measured by $P^k = \tau^k \cdot c$, is a rough distance estimate called 'Pseudorange' (see Fig. 1). The receiver simultaneously collects these measurements from at least four satellites and processes them to solve for position and time measurement error.

A. Observation Equation

The most commonly used algorithm for position computation from pseudoranges is based on the LS method. This method is used to find the receiver position from pseudoranges to four or more satellites.

The basic observation equation for the pseudorange P^k is

$$P^k = \rho^k + c(dt - dt^k) + T^k + \ell^k + e^k. \tag{1}$$

ρ^k is the geometrical range between satellite k and receiver, which can be computed as:

$$\rho^k = \sqrt{(X^k - X)^2 + (Y^k - Y)^2 + (Z^k - Z)^2} \tag{2}$$

where $(X,\ Y,\ Z)$ is the position of receiver (see Fig. 2). dt denotes the receiver clock offset and dt^k is the satellite clock offset. From the ephemerides, which also include information on the satellite clock offset dt^k, the position of the satellite $(X^k,\ Y^k,\ Z^k)$ can be computed. T^k is the tropospheric error and ℓ^k is the ionospheric error. These two errors are computed from a priori models, whose coefficients are part of the broadcast ephemerids. e^k is the observation error of the pseudorange. Therefore, Eq. 1 contains four unknowns X, Y, Z and dt. The error terms are minimized by using the LS method.

(2) is nonlinear with respect to the receiver position $(X,\ Y,\ Z)$, therefore the nonlinear term in (2):

$$f(X,\ Y,\ Z) = \sqrt{(X^k - X)^2 + (Y^k - Y)^2 + (Z^k - Z)^2} \tag{3}$$

is linearized. Starting from an initial position for the receiver $(X_1,\ Y_1,\ Z_1)$, the position estimate is improved iteratively. The center of the Earth $(0,\ 0,\ 0)$ can be chosen as the initial point, if no a-priori-information (as e.g. from AGPS) is available. Let itr be the number of the iterations ($itr = 1,\ 2,\ \cdots,\ itr_{max}$). The increments $\Delta X_{itr},\ \Delta Y_{itr},\ \Delta Z_{itr}$ update the receiver coordinates as follows:

$$\begin{aligned} X_{itr+1} &= X_{itr} + \Delta X_{itr}, \\ Y_{itr+1} &= Y_{itr} + \Delta Y_{itr}, \\ Z_{itr+1} &= Z_{itr} + \Delta Z_{itr}. \end{aligned} \tag{4}$$

The Taylor expansion of $f(X_{itr}+\Delta X_{itr},\ Y_{itr}+\Delta Y_{itr},\ Z_{itr}+\Delta Z_{itr})$ is

$$\begin{aligned} f(X_{itr+1}, Y_{itr+1}, Z_{itr+1}) &= f(X_{itr}, Y_{itr}, Z_{itr}) \\ &+ \frac{\partial f(X_{itr}, Y_{itr}, Z_{itr})}{\partial X_{itr}} \Delta X_{itr} \\ &+ \frac{\partial f(X_{itr}, Y_{itr}, Z_{itr})}{\partial Y_{itr}} \Delta Y_{itr} \\ &+ \frac{\partial f(X_{itr}, Y_{itr}, Z_{itr})}{\partial Z_{itr}} \Delta Z_{itr} \end{aligned} \tag{5}$$

(5) includes only first-order terms, and the partial derivatives are

$$\begin{aligned}\frac{\partial f(X_{itr},Y_{itr},Z_{itr})}{\partial X_{itr}} &= -\frac{X^k - X_{itr}}{\rho^k_{itr}}\\ \frac{\partial f(X_{itr},Y_{itr},Z_{itr})}{\partial Y_{itr}} &= -\frac{Y^k - Y_{itr}}{\rho^k_{itr}}\\ \frac{\partial f(X_{itr},Y_{itr},Z_{itr})}{\partial Z_{itr}} &= -\frac{Z^k - Z_{itr}}{\rho^k_{itr}}\end{aligned}$$

Let $\rho^k_{itr} = \sqrt{(X^k - X_{itr})^2 + (Y^k - Y_{itr})^2 + (Z^k - Z_{itr})^2}$ be the range computed from the satellite position (X^k, Y^k, Z^k) to the approximate receiver position (X_{itr}, Y_{itr}, Z_{itr}), so the first-order linearized observation equation becomes

$$P^k_{itr} = \rho^k_{itr} - \frac{X^k - X_{itr}}{\rho^k_{itr}}\Delta X_{itr} - \frac{Y^k - Y_{itr}}{\rho^k_{itr}}\Delta Y_{itr}$$
$$-\frac{Z^k - Z_{itr}}{\rho^k_{itr}}\Delta Z_{itr} + c(dt_{itr} - dt^k) + T^k_{itr} + \ell^k_{itr} + e^k_{itr}. \quad (6)$$

where dt_{itr} is the estimated clock error for iteration itr at the receiver.

B. Applying Least-Squares Method

(6) can be rewritten as

$$\begin{bmatrix} -\frac{X^k-X_{itr}}{\rho^k_{itr}} & -\frac{Y^k-Y_{itr}}{\rho^k_{itr}} & -\frac{Z^k-Z_{itr}}{\rho^k_{itr}} & +1\end{bmatrix}\begin{bmatrix}\Delta X_{itr}\\ \Delta Y_{itr}\\ \Delta Z_{itr}\\ cdt_{itr}\end{bmatrix}$$
$$= \quad P^k_{itr} - \rho^k_{itr} + cdt^k - T^k_{itr} - \ell^k_{itr} - e^k_{itr}. \quad (7)$$

A unique solution can not be found from a single equation. Therefore $m \geq 4$ satellites are required to form a system of linear equations (usually $m \geq 6$ satellites are available). Let $b^k_{itr} = P^k_{itr} - \rho^k_{itr} + cdt^k - T^k_{itr} - \ell^k_{itr} - e^k_{itr}$ and $\mathbf{b}_{itr} = [b^1_{itr},\ b^2_{itr}, \cdots,\ b^m_{itr}]^T$. Then we obtain the LS problem:

$$min_{\mathbf{x}_{itr}} ||\mathbf{A}_{itr}\mathbf{x}_{itr} - \mathbf{b}_{itr}||_2, \quad (8)$$

where

$$\mathbf{A}_{itr} = \begin{bmatrix} -\frac{X^1-X_{itr}}{\rho^1_{itr}} & -\frac{Y^1-Y_{itr}}{\rho^1_{itr}} & -\frac{Z^1-Z_{itr}}{\rho^1_{itr}} & 1\\ -\frac{X^2-X_{itr}}{\rho^2_{itr}} & -\frac{Y^2-Y_{itr}}{\rho^2_{itr}} & -\frac{Z^2-Z_{itr}}{\rho^2_{itr}} & 1\\ -\frac{X^3-X_{itr}}{\rho^3_{itr}} & -\frac{Y^3-Y_{itr}}{\rho^3_{itr}} & -\frac{Z^3-Z_{itr}}{\rho^3_{itr}} & 1\\ \vdots & \vdots & \vdots & \vdots\\ -\frac{X^m-X_{itr}}{\rho^m_{itr}} & -\frac{Y^m-Y_{itr}}{\rho^m_{itr}} & -\frac{Z^m-Z_{itr}}{\rho^m_{itr}} & 1\end{bmatrix}$$

and $\mathbf{x}_{itr} = [\Delta X_{itr}\ \ \Delta Y_{itr}\ \ \Delta Z_{itr}\ \ cdt_{itr}]^T$. The solution ΔX_{itr}, ΔY_{itr}, ΔZ_{itr} has to be added to the approximate receiver position to get the next approximate position as in (4). These iterations continue until the solution ΔX_{itr}, ΔY_{itr}, ΔZ_{itr} is at meter level.

For solving (8) we need to find $\hat{\mathbf{x}}_{itr}$ which minimizes the length of the error vector $\hat{\mathbf{e}}_{itr} = \mathbf{b}_{itr} - \mathbf{A}_{itr}\hat{\mathbf{x}}_{itr}$ with $||\mathbf{e}_{itr}||^2 = (\mathbf{b}_{itr} - \mathbf{A}_{itr}\mathbf{x}_{itr})^T(\mathbf{b}_{itr} - \mathbf{A}_{itr}\mathbf{x}_{itr})$ the sum of squares of the m separate errors. Minimizing this quadratic gives the normal equations

$$\mathbf{A}^T_{itr}\mathbf{A}_{itr}\hat{\mathbf{x}}_{itr} = \mathbf{A}^T_{itr}\mathbf{b}_{itr} \Rightarrow \hat{\mathbf{x}}_{itr} = (\mathbf{A}^T_{itr}\mathbf{A}_{itr})^{-1}\mathbf{A}^T_{itr}\mathbf{b}_{itr} \quad (9)$$

and the error vector is

$$\hat{\mathbf{e}}_{itr} = \mathbf{b}_{itr} - \mathbf{A}_{itr}\hat{\mathbf{x}}_{itr}. \quad (10)$$

There are various algorithms for solving LS problems [14]. In the subsequent section, we use the QRD by Givens rotations.

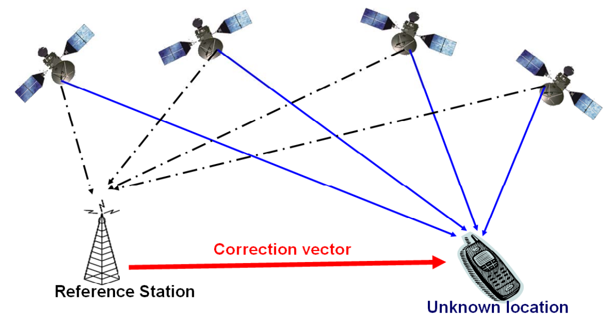

Fig. 3. Differential GPS positioning

C. Differential GPS

Stand-alone GPS will disappear in smartphones, since the wireless carriers can communicate information to the receiver. DGPS is introduced briefly here. It can do the positioning faster, more precisely and with lower power consumption. This improves performance and decreases battery strain. In DGPS method, the accuracy of GPS calculation is increased by simultaneously taking GPS observation at two locations (see Fig. 3), which have identical geometric dilution of precision. Then, the correction of pseudoranges at unknown location is done by using the measured-pseudorange-errors at the known position (reference receiver). The connection from cell phone to the base stations is usually available and the position of base station is known. Base station with known coordinates can be used as a reference receiver. Usually DGPS is applied for obtaining very accurate position estimates. However, if GPS position accuracy is sufficient, we can also apply DGPS in order to simplify the implementation of the receiver (hardware or software). In Sec. IV we will show experimental results, which support this argument. The algorithms used for GPS positioning can also be applied for DGPS. More details can be found in [11] [15].

III. Iterative Solution of LS

In each iteration of the positioning algorithm a LS problem must be computed. Since the pseudoranges are subject to measurement errors and the convergence (number of required iterations) of the algorithm depends on the accuracy of these LS solutions, it is worthwhile to investigate the use of an iterative LS solver and the trade-off between the number of iterations of the positioning method and the number of iterations of the iterative LS solver.

The iterative version of the QR decomposition (QRD) presented in [12] is used for the iterative solution of (8), since it is well suited for hardware implementation, suitable for the adaption to measurement errors (pseudoranges), and yields a solution vector which converges linearly to the exact solution. Here we will briefly review this iterative QRD.

The QR decomposition of a $m \times n$ matrix $\mathbf{A} = \mathbf{Q} \cdot \mathbf{R}$ can be computed by applying a sequence of Givens rotations $\mathbf{G}(\phi_{ij})$ ($\phi_{ij} = \arctan(a_{ij}/a_{jj})$) to the matrix such that the matrix entries below the diagonal of $\mathbf{A}$ are annihilated, i.e. generate $a'_{ij} = 0$ for

$$(i,j) = \{(2,1)(3,1)\dots(m,1)(3,2)\dots(m,2) \\ \dots(n+1,n)\dots(m,n)\}. \tag{11}$$

The resulting matrix is upper triangular and denoted by $\mathbf{R}$. The product of all required Givens rotations forms the orthogonal matrix $\mathbf{Q}^T = \prod_{j=1}^{n} \prod_{i=j+1}^{m} \mathbf{G}(\phi_{ij})$ such that

$$\min_{\mathbf{w}} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2 = \min_{\mathbf{x}} \|\mathbf{R}\mathbf{x} - \mathbf{Q}^T\mathbf{b}\|_2 \tag{12}$$

and the solution $\mathbf{x}$ can be computed by back substitution.

One iteration of the iterative version of the QRD works exactly like the QRD but instead of using exact rotations $\mathbf{G}(\phi_{ij})$ that annihilate a_{ij} ($a'_{ij} = 0$) CORDIC–based approximate rotations are used resulting in $|a'_{ij}| = |d||a_{ij}|$ with $0 \leq |d| < 1$. The CORDIC–based approximate Givens rotations are computed by determining the shift value ℓ, $\ell \in \{0, 1, 2, \dots, w\}$ (w wordlength) corresponding to the CORDIC angle $\phi_{ij}(\ell) = \arctan 2^{-\ell}$ which is closest to the exact rotation angle ϕ_{ij}. Instead of annihilating the matrix elements during the course of the QRD an approximate rotation will only reduce the matrix elements by the maximal factor possible with itg specific CORDIC angles $\phi_{ij}(\ell)$. This CORDIC–based approximate rotation is applied to the QRD for solving LS problems. Since the matrix elements below the diagonal are no longer annihilated the QRD procedure using CORDIC–based approximate rotations must iteratively be applied until the matrix is ultimately upper triangular. One obtains *iterative* versions of the QRD distinguished by itg, which determines the accuracy of the approximation of the rotations.

Defining the lower diagonal quantity for iteration itg:

$$S^{(itg)} = \sum_{j=1}^{n} \sum_{i=j+1}^{m} \left(a_{ij}^{(itg)}\right)^2 \tag{13}$$

the above algorithm guarantees

$$\lim_{itg\to\infty} S^{(itg)} \to 0$$

which is equivalent to

$$\lim_{itg\to\infty} \mathbf{A}^{(itg)} \to \mathbf{R} \;\text{ and }\; \lim_{itg\to\infty} \mathbf{Q}^{T(itg)} \to \mathbf{Q}^T$$

When we apply this method to the LS problems, which must be solved for positioning in (8), very few steps, i.e. $itg \ll w$, of the CORDIC–based approximate rotation are sufficient to obtain similar results as using exact rotations. This yields a significant reduction in computational complexity by itg/w (we will show in the following section for real GPS data that even $itg = 1$ gives reasonable results). Furthermore, this method only requiring shift and add operations is very well suited for hardware implementation.

IV. Experimental results

A. GPS raw data collection

For GPS positioning, a SiGe GN3S Sampler v2 is used to capture the raw GPS data, which are low level signal data (raw intermediate frequency samples) being delivered by the GPS satellite network and processed by the SiGe radio front end [16]. Each GPS satellite continuously broadcasts a navigation message at a rate of 50 bits per second.

For DGPS positioning, raw GPS data are captured at two measurement points with distance about 500 meters simultaneously. At each measurement point, one SiGe GN3S is used. Then, the satellites' positions and the measured pseudoranges can be obtained, which are required for the positioning method.

The obtained raw GPS data at each measurement point includes information of $m = 7$ satellites with matched pseudoranges. 78 raw GPS data records are gained. Afterwards position calculation is done for both GPS and DGPS for various numbers of required iterations itr (see Sec. II) and various approximation accuracies itg (see Sec. III).

B. Experimental results

Before looking at the experimental results, we note, that our GPS raw data are worse than the data from standard GPS receivers. We have also turned off correction algorithms (troposphere correction, ionosphere correction and satellite

TABLE I
MEAN VALUE OF POSITION-ACCURACY RESULTS (IN METER) BY GPS AND DGPS WITH EXACT LS METHOD AND CORDIC-BASED APPROXIMATE ROTATIONS

itr	*itg*	GPS-itr	GPS-itg	DGPS-itr	DGPS-itg
2	1	1241.4	1282.8	1275.2	1368.8
	2		1256.6		1280.6
	3		1246.1		1280.4
	8		1241.4		1275.2
	9		1241.4		1275.2
3	1	38.484	85.756	12.582	19.432
	2		47.744		16.208
	3		39.020		12.605
	8		38.484		12.582
	9		38.484		12.582
4	1	32.472	81.495	8.385	16.762
	2		35.452		11.245
	3		32.985		9.353
	8		32.472		8.385
	9		32.472		8.385
5	1	32.472	81.495	8.385	16.762
	2		35.452		11.245
	3		32.985		9.353
	8		32.472		8.385
	9		32.472		8.385

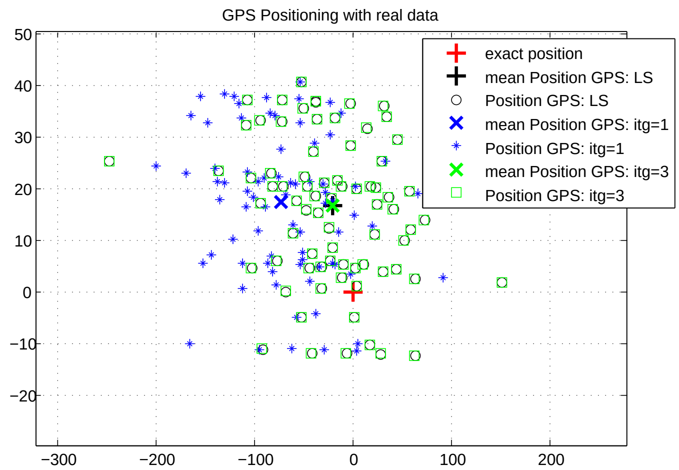

Fig. 4. Positioning results (in meter) with GPS method

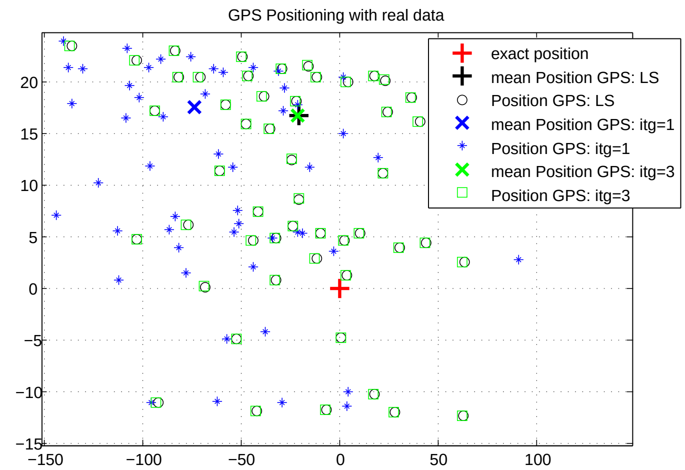

Fig. 5. Enlarged positioning results (in meter) with GPS method

correction). This was done in order to emphasize that even under these circumstances, DGPS can provide an accuracy comparable to standard GPS. Therefore, of course the accuracy of GPS will be much worse than we expect from GPS receivers. However this demonstrates that in future smartphones using DGPS (whenever there is a connection to a base station), it is possible to reduce the requirements significantly (specification of front end, computational effort).

The mean values of the positioning results for the 78 measurements are presented in Tab. I. The calculated positions are compared with the exact positions (known coordinates at the measure points from land surveying office in Bochum Germany) and the accuracy of the positioning results of GPS and DGPS methods for varying itr and itg is compared. The number of required iterations itr is at least two. The second column itg in Tab. I is the approximation accuracy in QRD. Fig. 4 and Fig. 5 show the position estimates of GPS positioning. Fig. 6 and Fig. 7 show these results of DGPS positioning.

1) Results of GPS method: In Tab. I the 3rd column $GPS - itr$ is the accuracy of GPS positioning with exact LS method and the 4th column $GPS - itg$ is the accuracy of GPS positioning using QRD with CORDIC-based approximate rotations for solving LS problems. The results show that with the increasing number of iterations in QRD, the accuracy of $GPS - itg$ also increases. If $itg \geq 8$, the accuracy of the exact LS method is achieved. Especially, if itr is big enough, e.g. $itr \geq 3$, only two iterations in QRD $itg = 2$ is required to compute the position with reasonable positioning accuracy, which means very coarse approximations are sufficient.

The position estimates of 78 measurements are shown in Fig. 4. GPS position with exact LS (black circles), its mean value (black cross), GPS position using iterative QRD with $itg = 1$ (blue stars), its mean value (blue cross), GPS position using iterative QRD with $itg = 3$ (green squares), its mean value (green cross) and the exact position (red cross) are shown in the figure. All the position results are considered as accuracy of position, i.e. the position estimates subtracted by the exact position (so the exact position is set to $(0,\ 0)$).

Fig. 5 is an enlarged part of Fig. 4. It is obvious to notice that GPS position using QRD with $itg = 3$ (the green squares) are more closer to GPS position with exact LS (black circles) than GPS position using QRD with $itg = 1$ (blue stars). If the number of iterations in QRD increases, i.e itg increases, the position results will become more and more similar to the results of exact LS. The mean value of $itg = 3$ is almost the same as the mean value of exact LS (green cross is almost at the same position as black cross in Fig. 5).

2) Results of DGPS method: In Tab. I the 5th column $DGPS - itr$ is the accuracy of DGPS positioning with exact LS method and the 6th column $DGPS - itg$ is the accuracy of DGPS positioning using QRD with CORDIC-based approximate rotations for solving LS problems. The results show that with the increasing number of iterations in QRD, the accuracy of $DGPS - itg$ also increases. If $itg \geq 8$, the accuracy of the exact LS method is achieved. Especially, if itr is big enough, e.g. $itr \geq 3$, only one iteration in QRD $itg = 1$ is required to compute the position with reasonable positioning accuracy, which means very coarse approximations are again sufficient in DGPS positioning.

Fig. 7 is an enlarged part of Fig. 6. It is also obvious to notice that DGPS position using QRD with $itg = 3$ (the green squares) are closer to DGPS position using exact LS (black circles) than DGPS position using QRD with $itg = 1$ (blue stars). If the number of iterations itg in QRD increases, the position results will become more and more similar to the results of exact LS. The mean value of $itg = 3$ is almost the same as the mean value of exact LS (green cross is almost at the same position as black cross in Fig. 7).

Finally, we notice that even for low-cost front ends, turned-off correction procedures and small itg, the accuracy of DGPS is about 10m (i.e. what we usually expect from high-end GPS receivers). Therefore the presented $DGPS - itg$ method is suitable for LBS using very small itg. Furthermore it is even

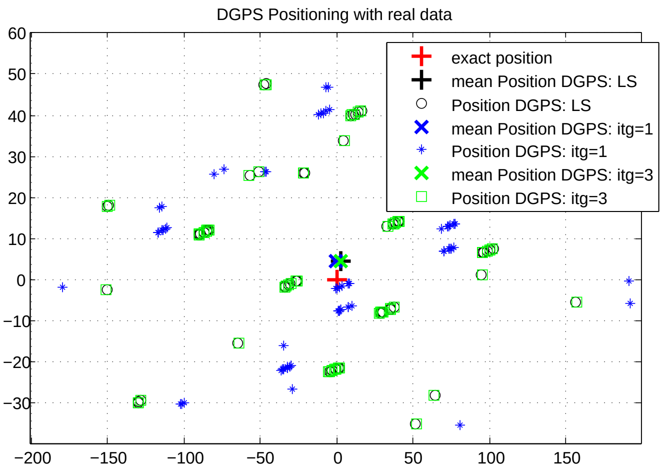

Fig. 6. Positioning results (in meter) with DGPS method

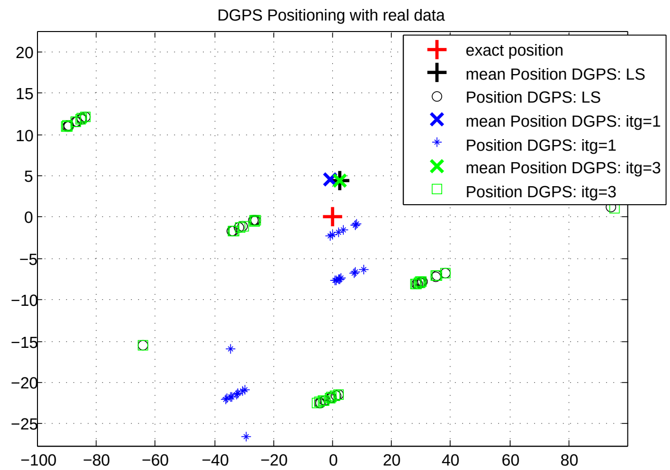

Fig. 7. Enlarged positioning results (in meter) with DGPS method

possible to adapt the positioning accuracy by choosing itr and itg.

V. Conclusion

An iterative LS approach and an iterative version of the QRD using CORDIC-based approximate rotation are applied to position computation. The accuracy of the positioning results of GPS and DGPS methods is compared for various numbers of required iterations itr and various approximation accuracies itg of CORDIC-based approximate rotations by using real GPS data. It is shown that a significant reduction concerning computational complexity and hardware requirements can be obtained. Furthermore, this method only requiring shift and add operations is very well suited for hardware implementation.

The presented method is very efficient for the implementation of the standard triangulation method based on non-linear LS. In future work we apply this idea to *recursive* computations of the position estimates using Orthogonal DGPS [17] and Kalman Filter based recursive GPS algorithms [18]. In the first case no iterative LS method is required for positioning which leads to further reduction of the computational effort and the power consumption. In the second case using the square root version of the Kalman filter [19] QRD can be applied and the required approximation accuracy (number of itg) can be investigated to obtain desired position accuracy.

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