

Approximate R -symmetries and the μ term

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ABSTRACT: We discuss the role of approximate $U(1)_R$ symmetries for the understanding of hierarchies in Nature. Such symmetries may explain a suppressed expectation value of the superpotential and provide us with a solution to the MSSM μ problem. We present various examples in field theory and string-derived models.

KEYWORDS: Supersymmetric Standard Model, Superstring Vacua, Global Symmetries

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1 Introduction

Some of the most fundamental questions in theoretical physics are related to the observation of huge hierarchies in Nature. According to the traditional criteria [1], such hierarchies are “natural” if they are a consequence of an approximate symmetry. Low-energy supersymmetry provides an explanation for the stability, but not the origin, of the hierarchy between the electroweak scale and the GUT or Planck scale. To also explain its origin one should find a reason why the scale of supersymmetry breakdown is so much smaller than the fundamental scale. In Minkowski vacua, with gravity-mediated supersymmetry breaking, superpartner masses are of the order of the gravitino mass $m_{3/2}$. This in turn is essentially given by the vacuum expectation value (VEV) of the superpotential \mathcal{W} . For supersymmetry to stabilize the electroweak hierarchy, $\langle \mathcal{W} \rangle$ should be of the order of a TeV, or about 10^{-15} in Planck units.

A conventional strategy to obtain vacuum solutions with small supersymmetry breaking (and cosmological constant zero) in a given model is to identify field configurations in which $\langle \mathcal{W} \rangle$ completely vanishes at the perturbative level. A suppressed $\langle \mathcal{W} \rangle$ and supersymmetry breaking should then be obtained from non-perturbative effects. This has the appealing feature that the scale of supersymmetry breakdown is set by dimensional transmutation [2], and is thus naturally small.

Recently it has been argued that the hierarchy between the fundamental and the supersymmetry breaking scale might also be due to an approximate $U(1)_R$ symmetry. Approximate $U(1)_R$ symmetries appear naturally in the low-energy effective field theories of heterotic string compactifications on orbifolds. They may explain a highly suppressed $\langle \mathcal{W} \rangle$ in settings in which the typical VEVs of the relevant fields are only somewhat below the fundamental scale [3].

In any case, even if hierarchically small soft masses are explained, the minimal supersymmetric extension of the standard model, the MSSM, still suffers from the so-called μ problem. That is, the coefficient μ of the Higgs bilinear coupling $\mu H_u H_d$ has to be of the order of the soft masses as well, in order to give rise to a reasonable phenomenology. There are various proposals to solve this problem [4, 5].

The main point of this study is to show that approximate R -symmetries have the potential to solve both these problems simultaneously. A $U(1)_R$ symmetry, under which $H_u H_d$ is neutral, forbids both the μ term and any non-trivial $\langle \mathcal{W} \rangle$. If $U(1)_R$ is merely an approximate symmetry, $\langle \mathcal{W} \rangle$ can be viewed as an order parameter for $U(1)_R$ breaking, setting consequently also the size of the μ term. Hence, a μ term of the order of the gravitino mass $m_{3/2}$ can be explained by an approximate R -symmetry. We explore this relation both in the context of field theory and string-derived MSSM models.

This paper is organized as follows. In section 2 we recollect some basic facts on supersymmetric ground states and explain how an (approximate) R -symmetry allows us to control the VEV of the superpotential. In section 3 we discuss how an R -symmetry can relate the μ term to $m_{3/2}$ in field-theoretic examples with generic superpotentials. Section 4 is devoted to a discussion of solutions of the μ problem in string-derived models. Section 5 contains a collection of example models in which approximate R -symmetries lead to suppressed superpotential VEVs. Finally, section 6 contains our conclusions.

2 Supersymmetric ground states

2.1 Trivial vs. non-trivial field configurations

Consider a 4D $\mathcal{N} = 1$ supergravity theory with chiral superfields Φ_i ($1 \leq i \leq M$), which could, for instance, arise as the low-energy effective field theory governing the massless spectrum of some string compactification. The scalar potential is given by

$$V = e^K (D_i \mathcal{W} D_{\bar{j}} \overline{\mathcal{W}} K^{i\bar{j}} - 3|\mathcal{W}|^2) . \tag{2.1}$$

Here we ignore possible contributions of D -terms and follow the usual notation conventions: K is the Kähler potential, $D_i = \partial_i + (\partial_i K)$ is the Kähler-covariant derivative with respect

to the superfield Φ_i , and $K^{i\bar{j}}$ is the inverse Kähler metric. We work in units where the Planck scale is set to unity, $M_{\text{P}} = 1$.

To account for supersymmetry breaking and an (almost) vanishing cosmological constant, one should find systems in which both terms in the parentheses in equation (2.1) are nonzero and (almost) cancel each other. Furthermore, in order to solve the gauge hierarchy problem of the standard model (SM), one is lead to consider settings in which $\langle \mathcal{W} \rangle \sim 10^{-15}$, such that the gravitino mass is at the TeV scale.

In string-derived models, calculating the full scalar potential is usually impossible for practical purposes. Even in the relatively simple setting of orbifold compactifications, computing higher-order superpotential coefficients tends to be fairly cumbersome, and the number of fields entering the scalar potential (2.1) is typically of the order of dozens or hundreds. For these reasons it is usually not feasible to analytically minimize the full scalar potential. However, often one is only interested in the low-energy physics appearing from the expansion about some vacuum. A good strategy is therefore to make an ansatz for a vacuum configuration, defined by a set of fields with non-vanishing VEVs, with the help of string selection rules and phenomenological considerations. One should then show a posteriori that this choice is self-consistent. The properties of the model which one may deduce from this ansatz should, of course, not depend too sensitively on the specific values of the coefficients.

In the traditional approach, one would seek configurations in which $D_i \mathcal{W}$ and \mathcal{W} vanish at the perturbative level, leading to a supersymmetric Minkowski vacuum. To be more specific, let us briefly discuss an example. In [6] a heterotic orbifold model (with the precise chiral spectrum of the MSSM) was discussed, in which the potential for SM singlets from the second and fourth twisted sectors is F - and D -flat to all orders in perturbation theory. This is because any allowed superpotential coupling involves at least two other fields, such that \mathcal{W} as well as $\partial_a \mathcal{W}$ vanish (where a runs over the SM singlets from the second and fourth twisted sectors). Hence the scalar potential (2.1) equals zero. That is, the system that emerges by setting all other fields to zero has, at the perturbative level, a supersymmetric Minkowski ground state, but also a huge moduli space.

Let us now look instead at systems where all fields enter a non-trivial superpotential. These emerge, for instance, if one switches on other fields in the above model. An immediate objection against such systems is that, in order to obtain a small gravitino mass, one has to satisfy

$$D_i \mathcal{W} \ll 1 \quad \text{and} \quad \langle \mathcal{W} \rangle \ll 1 \tag{2.2}$$

simultaneously, which constitutes $M + 1$ constraints for M fields Φ_i . In other words, one might expect that for solutions of $D_i \mathcal{W} \ll 1$, which can be chosen such that the D -terms vanish (cf. [7]), one would always obtain $\langle \mathcal{W} \rangle$ of order one. However, while this argument certainly applies for completely generic superpotentials, it is not true in the presence of an approximate $U(1)_R$ symmetry. In this case, one obtains [3]

$$\langle \mathcal{W} \rangle \sim \langle \Phi \rangle^N, \tag{2.3}$$

where $\langle \Phi \rangle$ denotes the typical VEV scale of the Φ_i fields and N is the order at which the approximate $U(1)_R$ is explicitly broken in \mathcal{W} . If $\langle \Phi \rangle$ is slightly suppressed against the

fundamental scale and N is sufficiently large, $\langle \mathcal{W} \rangle$ can be hierarchically small and induce a gravitino mass in the TeV range. In other words, a mild hierarchy between $\langle \Phi \rangle$ and the fundamental scale is power-law enhanced in the presence of an approximate R -symmetry, very similarly to the well-known Froggatt-Nielsen scheme [8]. In such settings one often finds that all Φ_i fields attain masses, with the would-be R -axion gaining a mass of the order $\mathcal{M}/\langle \Phi \rangle^2$. In particular, one obtains point-like vacuum configurations, i.e. vacua with dimension zero moduli space, where a hierarchically small gravitino mass is explained by an approximate, perturbative symmetry (rather than by non-perturbative effects).

2.2 Supersymmetric Minkowski vacua as a consequence of $U(1)_R$

Let us review the derivation of equation (2.3) in a way which slightly differs from the one in [3]. Consider first a globally supersymmetric model of chiral superfields which has an exact $U(1)_R$ symmetry. Then, in a supersymmetric vacuum, the expectation value of the superpotential vanishes: since the R -charges r_i in each term of \mathcal{W} add up to 2, \mathcal{W} is a weighted homogeneous polynomial in the fields satisfying

$$2\mathcal{W} = \sum_i r_i \Phi_i \partial_i \mathcal{W} . \tag{2.4}$$

In supersymmetric vacua the $\partial_i \mathcal{W}$ vanish, and therefore $\langle \mathcal{W} \rangle = 0$.

For a model with an exact $U(1)_R$ symmetry, a critical point of the superpotential necessarily also represents a supersymmetric vacuum in supergravity: $\partial_i \mathcal{W} = 0$ and $\mathcal{W} = 0$ imply $D_i \mathcal{W} = 0$. This is irrespective of whether or not the corresponding field configuration represents a physical vacuum of some globally supersymmetric effective theory, obtained by decoupling Planck-scale physics. (In fact, in concrete string-derived models, critical points of \mathcal{W} often arise from the interplay of higher-dimensional operators suppressed by powers of M_P , and thus disappear in the limit $M_P \rightarrow \infty$.)

Not all supersymmetric vacua in supergravity have $\mathcal{W} = 0$. Vanishing F -terms imply $\partial_i \mathcal{W} = -\mathcal{W} \partial_i K$, so that equation (2.4) becomes

$$2\mathcal{W} = -\mathcal{W} \sum_i r_i \Phi_i \partial_i K . \tag{2.5}$$

This means that either $\langle \mathcal{W} \rangle = 0$ as above, or $\sum_i r_i \Phi_i \partial_i K = -2$ in the vacuum. In the following we will only consider supergravity vacua of the former type, that is with $\langle \mathcal{W} \rangle = 0$, obtained from critical points of the superpotential.

In settings with a $U(1)_R$ symmetry, the existence of supersymmetric but R -breaking vacua is by itself non-trivial (see, for instance, [9]), and will be discussed in what follows.

2.3 A note on supersymmetric vacua with broken R -symmetry

It is sometimes claimed in the literature that spontaneously broken R -symmetry in generic models implies broken supersymmetry (as part of the ‘‘Nelson-Seiberg theorem’’ [9]). More precisely, for globally supersymmetric models¹ of chiral superfields with generic superpotentials, the claim is that

¹In this section we work in global supersymmetry, but the arguments carry over to supergravity in a straightforward way.

- (i) if the model admits a supersymmetry-breaking global vacuum, then it possesses an exact $U(1)_R$ symmetry (which may be broken or unbroken in the vacuum);
- (ii) if the model possesses an exact $U(1)_R$ symmetry and a global vacuum in which it is spontaneously broken, then the vacuum also breaks supersymmetry spontaneously.

The cause for concern here is the second statement, since we are relying on models with supersymmetric vacua in which R -symmetry is spontaneously broken.

However, there is a loophole in the Nelson-Seiberg argument, which can be used to construct counterexamples [10]. Let us recall how the Nelson-Seiberg argument works: assume there are N chiral superfields Φ_i with R -charges r_i , normalized such that \mathcal{W} carries R -charge 2. Assume that Φ_N breaks R -symmetry, so $\langle \Phi_N \rangle \neq 0$ and $r_N \neq 0$. Write \mathcal{W} as

$$\mathcal{W} = \Phi_N^{2/r_N} f(\tilde{\Phi}_1, \dots, \tilde{\Phi}_{N-1}) , \tag{2.6}$$

where the $\tilde{\Phi}_j$ are chiral superfields constructed from Φ_N and from the other $\Phi_j = \Phi_1, \dots, \Phi_{N-1}$ as

$$\tilde{\Phi}_j = \Phi_j \Phi_N^{-r_j/r_N} . \tag{2.7}$$

The conditions for unbroken supersymmetry are, using that $\Phi_N \neq 0$ and $r_N \neq 0$ by assumption,

$$0 = \frac{\partial \mathcal{W}}{\partial \Phi_j} = \Phi_N^{2/r_N} \sum_{k=1}^{N-1} \frac{\partial f}{\partial \tilde{\Phi}_k} \frac{\partial \tilde{\Phi}_k}{\partial \Phi_j} = \Phi_N^{(2-r_j)/r_N} \frac{\partial f}{\partial \tilde{\Phi}_j} \quad (j = 1, \dots, N-1) , \tag{2.8a}$$

$$0 = \frac{\partial \mathcal{W}}{\partial \Phi_N} = \frac{2}{r_N} \Phi_N^{2/r_N-1} f + \Phi_N^{2/r_N} \sum_{j=1}^{N-1} \frac{\partial f}{\partial \tilde{\Phi}_j} \Phi_j \left(-\frac{r_j}{r_N} \right) \Phi_N^{-r_j/r_N-1} \tag{2.8b}$$

which is equivalent to

$$f = 0 \quad \text{and} \quad \frac{\partial f}{\partial \tilde{\Phi}_j} = 0 . \tag{2.9}$$

These are N equations in the $N-1$ variables $\tilde{\Phi}_j$ and thus do not have a solution if f is a generic function. Therefore there is either no vacuum, or supersymmetry is spontaneously broken.

The loophole is now that f is not necessarily generic even if \mathcal{W} is. For example, if r_N is not an integer fraction of 2, a constant term in f is not allowed even though it could well be formally compatible with all symmetries, because it would represent a non-polynomial piece in the superpotential. Clearly, it is sufficient that f is at least quadratic in the $\tilde{\Phi}_i$ fields; then equations (2.8) are always satisfied at $\tilde{\Phi}_i = 0$. Such f can be obtained in effective theories where all massive modes are integrated out and the superpotential \mathcal{W} starts with cubic terms; examples for such systems include the effective supergravity description of orbifold compactifications.

A simple example where it is possible to see explicitly how the argument fails has three chiral superfields X, Y, Z with the R -charges $r_X = 3, r_Y = 1$ and $r_Z = -2$. The most general superpotential compatible with these charge assignments is

$$\mathcal{W} = Y^2 + X Y Z + X^2 Z^2 + Y^4 Z + (\text{terms of order 6 and higher}) . \tag{2.10}$$

We have set all coefficients to 1, because their values are irrelevant to the argument as long as they are nonzero. Note that the R -charges are unambiguously fixed by the first four leading-order terms which we have explicitly written: the quadratic term fixes $r_Y = 1$, the quintic term then fixes $r_Z = -2$, and the other two fix $r_X = 3$.

Neglecting the higher-order terms, the F -term equations read

$$F_X : \quad Y Z + 2 X Z^2 = 0 , \tag{2.11a}$$

$$F_Y : \quad 2 Y + X Z + 4 Y^3 Z = 0 , \tag{2.11b}$$

$$F_Z : \quad X Y + 2 X^2 Z + Y^4 = 0 . \tag{2.11c}$$

They are solved by $Y = Z = 0$, with X a flat direction.² Along this flat direction, at any point $X \neq 0$, R -symmetry is spontaneously broken while SUSY is preserved. In fact any vacuum with unbroken supersymmetry but spontaneously broken R -symmetry must have a supersymmetric flat direction, because the potential must accommodate a Goldstone boson and its complex partner.

This solution remains unchanged when one takes into account higher-order terms in \mathscr{W} , since these must be at least quadratic in Z and hence give contributions to the F -term equations which are at least linear, vanishing at $Z = 0$.

Following the procedure from the previous section, we can write the superpotential as

$$\mathscr{W} = X^{2/3} \left(\tilde{Y}^2 + \tilde{Y} \tilde{Z} + \tilde{Z}^2 + \tilde{Y}^4 \tilde{Z} \right) . \tag{2.12}$$

Here $\tilde{Y} = Y X^{-1/3}$ and $\tilde{Z} = Z X^{2/3}$. The term in brackets (f in the above notation) is now not a generic polynomial — for instance it is missing a constant term, because this would correspond to a non-polynomial $X^{2/3}$ term in \mathscr{W} .

2.4 $U(1)_R$ breaking at higher order

Having seen that also in the presence of a $U(1)_R$ symmetry there can be supersymmetric solutions with nontrivial VEVs, we now turn to discuss the impact of higher order, explicit $U(1)_R$ breaking superpotential terms.

We are interested in a critical point of the exactly $U(1)_R$ -symmetric \mathscr{W} in which all field VEVs are at least slightly below the Planck scale, $\langle \Phi_i \rangle < 1$. Adding explicit R -breaking terms of some high order $\gtrsim N$ in the fields should shift the original expectation values by a small amount ϵ .

More precisely, in the exactly $U(1)_R$ symmetric case there is a flat direction in field space, which is the curve parameterizing the VEV of the R -axion. Adding explicit R -breaking terms should lead to an isolated vacuum at a distance ϵ from this curve. If the field expectation values before R -breaking were somewhat small near the new vacuum, it is self-consistent to take ϵ small as well, provided N is sufficiently large.

Assuming for simplicity that the original expectation values are all of roughly the same size, we can estimate the magnitude of ϵ . Let us write \mathscr{W} as the sum of an exactly

²They are also solved by $X = Y = 0$, with Z a flat direction, a case which can be discussed completely analogously. There are no further solutions.

R -symmetric lower-order part \mathcal{W}_0 and an R -breaking order- N piece $\Delta\mathcal{W}$,

$$\mathcal{W} = \mathcal{W}_0 + \Delta\mathcal{W} . \quad (2.13)$$

A supersymmetric vacuum for the R -symmetric truncation \mathcal{W}_0 at Φ^0 has $\partial_i\mathcal{W}_0(\Phi^0) = 0$, $\mathcal{W}_0(\Phi^0) = 0$, and $D_i\mathcal{W}_0(\Phi^0) = 0$. If it shifts by ϵ with the full superpotential, and supersymmetry is preserved, then

$$\begin{aligned} 0 &= D_i\mathcal{W}\Big|_{\Phi^0+\epsilon} = D_i\mathcal{W}\Big|_{\Phi^0} + \partial_j D_i\mathcal{W}\Big|_{\Phi^0} \epsilon_j + \partial_{\bar{j}} D_i\mathcal{W}\Big|_{\Phi^0} \bar{\epsilon}_{\bar{j}} + \mathcal{O}(|\epsilon|^2) \\ &= D_i\mathcal{W}_0\Big|_{\Phi^0} + D_i(\Delta\mathcal{W})\Big|_{\Phi^0} + \partial_j D_i\mathcal{W}_0\Big|_{\Phi^0} \epsilon_j + \partial_{\bar{j}} D_i\mathcal{W}_0\Big|_{\Phi^0} \bar{\epsilon}_{\bar{j}} + \mathcal{O}\left(|\epsilon|^2, |\Delta\mathcal{W} \epsilon|\right) \\ &= D_i(\Delta\mathcal{W})\Big|_{\Phi^0} + \partial_j \partial_i \mathcal{W}_0\Big|_{\Phi^0} \epsilon_j + \mathcal{O}\left(|\epsilon|^2, |\Delta\mathcal{W} \epsilon|\right) . \end{aligned} \quad (2.14)$$

If $\langle\Phi\rangle$ is a typical VEV, then $\langle\partial_j\mathcal{W}_0\rangle \sim \langle\Phi\rangle$ for superpotentials \mathcal{W}_0 which start at cubic order in the fields (as in models which describe the massless degrees of freedom of some string compactification), and $\langle D_i(\Delta\mathcal{W})\rangle \sim \langle\Phi\rangle^{N-1}$. We obtain the estimate

$$\epsilon \sim \langle\Phi\rangle^{N-2} . \quad (2.15)$$

This shift will cause the terms in \mathcal{W}_0 to contribute to a non-vanishing superpotential expectation value of the order $\langle\mathcal{W}_0\rangle \sim \langle\Phi\rangle^N$. The R -breaking term $\Delta\mathcal{W}$ itself will also directly induce a superpotential expectation value of the same order. The resulting $\langle\mathcal{W}\rangle \sim \langle\Phi\rangle^N$ can be hierarchically small even if $\langle\Phi\rangle$ is only moderately small but N is large.

Let us stress that the above is a very rough estimate whose details can easily change, depending on the model. However, it should be clear that with our basic assumptions (sub-Planckian field VEVs and an R -breaking term of higher order) the essential result will remain the same, namely, there is a hierarchically suppressed $\langle\mathcal{W}\rangle$. We will discuss explicit examples in section 5.

A higher-order R -breaking term turns the formerly Minkowski vacuum into an AdS vacuum; since the induced $\langle\mathcal{W}\rangle$ is hierarchically small, this translates directly into the smallness of the AdS vacuum energy. Additional (possibly non-perturbative) dynamics should then break supersymmetry to yield an “ F -term uplift” of the AdS vacuum to a local Minkowski or dS minimum of the scalar potential (see e.g. [11–13]). Since in such constructions the uplifting sector does not significantly change $\langle\mathcal{W}\rangle$, the gravitino mass in the uplifted model will be given by the pre-uplift $\langle\mathcal{W}\rangle$ and can thus naturally be of the order of a TeV.

Models which do not rely on a separate uplifting sector are also conceivable. Consider, for instance, a Kähler potential of the form [14, section 4]

$$K = -3\ln(T + \bar{T} - h(C_\alpha, \bar{C}_{\bar{\beta}})) + \tilde{K}(S_n, \bar{S}_{\bar{m}}) . \quad (2.16)$$

Here T is the usual Kähler modulus, of which the perturbative superpotential is independent, and C_α and S_n are chiral superfields. The scalar potential is

$$V = \frac{e^{\tilde{K}}}{(T + \bar{T} - h)^3} \left(\partial_\alpha \mathcal{W} \partial_{\bar{\beta}} \bar{\mathcal{W}} h^{\alpha\bar{\beta}} \left(\frac{T + \bar{T} - h}{3} \right) + D_n \mathcal{W} D_{\bar{m}} \bar{\mathcal{W}} \tilde{K}^{n\bar{m}} \right) , \quad (2.17)$$

where $h^{\alpha\bar{\beta}}$ denotes the matrix inverse of $\partial_\alpha\partial_{\bar{\beta}}h$. Assuming the T modulus to be stabilized, stationary points of V occur at

$$\partial_\alpha\mathcal{W} = D_n\mathcal{W} = 0. \tag{2.18}$$

These stationary points are, in general, not supersymmetric, and they come with vanishing vacuum energy (at least at the tree level). Evidently, if \mathcal{W} is suppressed at a point where $\partial_\alpha\mathcal{W} = \partial_n\mathcal{W} = 0$ due to an approximate R -symmetry, then it will be of the same order in the nearby vacuum (2.18).

These considerations indicate that one might ultimately even obtain an understanding of the smallness of the vacuum energy. Approximate $U(1)_R$ symmetries indeed allow us to control the vacuum energy, although not to the level needed to explain the observed cosmological constant.

Having outlined the general picture, we now turn to particular classes of models in which an approximate R -symmetry is useful to solve the μ problem.

3 Approximate R -symmetry and the μ term

3.1 The effective μ term

In the MSSM there is a single dimensionful parameter at the supersymmetric level: the Higgsino mass μ . In any model which UV-completes the MSSM at the GUT scale or at the Planck scale, one would naively expect μ to be either zero or of the order of the UV-completion scale. Vanishing μ is ruled out experimentally since massless Higgsinos would be in conflict with direct search limits. A μ parameter significantly larger than the soft SUSY-breaking Higgs masses, on the other hand, would require excessive fine-tuning to obtain the correct Z mass. This in turn would spoil one of the main motivations for low-energy supersymmetry.

To solve this so-called “ μ problem”, one should find a mechanism connecting μ with the scale of supersymmetry breaking. It should naturally give a μ parameter of the order of the gravitino mass $m_{3/2}$, which sets the scale for the soft masses. There are some well-known proposals on how to obtain a μ term of the correct size,

1. from the superpotential [4], or
2. from the Kähler potential [5].

Consider a supersymmetric extension of the MSSM by a number of chiral superfields Φ_i , which could represent e.g. string moduli or some general hidden sector fields. Let us for the moment ignore MSSM matter fields, since they will couple to the Higgs fields only by Yukawa terms, and thus be irrelevant for the Higgs potential (the coupling between the up-type Higgs and lepton doublets can be forbidden by matter parity). The Kähler potential and superpotential read, in an expansion up to quadratic order in the Higgs fields,

$$K = \mathcal{K} + \mathcal{Y}_u |H_u|^2 + \mathcal{Y}_d |H_d|^2 + (\mathcal{Z} H_u H_d + \text{h.c.}) + \dots, \tag{3.1a}$$

$$\mathcal{W} = \mathcal{W}_0 + \hat{\mu} H_u H_d + \dots, \tag{3.1b}$$

where \mathcal{K} , $\mathcal{Y}_{u,d}$, \mathcal{Z} , \mathcal{W}_0 and $\hat{\mu}$ are functions of the Φ_i (\mathcal{K} and $\mathcal{Y}_{u,d}$ are real, while \mathcal{W}_0 and $\hat{\mu}$ are holomorphic). We are assuming that none of the Φ_i carries the same quantum numbers as either H_u or H_d .

Let us take $\langle \mathcal{W}_0 \rangle$ to be real for convenience. The effective μ parameter is then given by

$$\mu = (\mathcal{Y}_u \mathcal{Y}_d)^{-1/2} \left(e^{\frac{\mathcal{K}}{2}} \mathcal{W}_0 \mathcal{Z} - \bar{F}^{\bar{i}} \frac{\partial \mathcal{Z}}{\partial \bar{\Phi}^{\bar{i}}} + e^{\frac{\mathcal{K}}{2}} \hat{\mu} \right), \quad (3.2)$$

evaluated in the vacuum, with $\bar{F}^{\bar{i}} = -e^{K/2} K^{j\bar{i}} D_j \mathcal{W}$. On the r.h.s. of equation (3.2) there are three contributions to μ with different origins. The first term is exactly the gravitino mass, up to a prefactor $\mathcal{Z}/\sqrt{\mathcal{Y}_u \mathcal{Y}_d}$ which is generically of order one. The second, ‘‘Giudice-Masiero’’-type term [5] is likewise expected to be of the correct order of magnitude since it is essentially given by F -terms. The $\hat{\mu}$ term, on the other hand, is a priori not related to SUSY breaking and can give a large contribution of the order of the fundamental scale.³

Assume now that the Higgs bilinear $H_u H_d$ is a singlet under all symmetries dictating the structure of \mathcal{W} .

The superpotential is then of the form

$$\mathcal{W} = \sum_a c_a \mathcal{M}_a(\Phi_i) + H_u H_d \sum_a c'_a \mathcal{M}_a(\Phi_i) + \dots, \quad (3.3)$$

i.e. \mathcal{W}_0 and $\hat{\mu}$ are given by

$$\mathcal{W}_0 = \sum_a c_a \mathcal{M}_a(\Phi_i), \quad (3.4a)$$

$$\hat{\mu} = \sum_a c'_a \mathcal{M}_a(\Phi_i). \quad (3.4b)$$

Here the $\mathcal{M}_a(\Phi_i)$ are normalized monomials in the Φ_i . They are singlets under all selection rules except R -symmetry. The c_a and c'_a are numerical coefficients.

There are now two possibilities to explain why $\hat{\mu}$ has the correct size:

1. The couplings c_a and c'_a may coincide up to a common factor λ of order one, so $\hat{\mu} = \lambda \mathcal{W}_0$ [15]. If, furthermore, \mathcal{W}_0 is of the order of a TeV in the vacuum (for instance, because of an approximate R -symmetry), then the same is true for μ . Such models appear naturally in heterotic orbifold constructions; they will be discussed in section 4.
2. The couplings c_a and c'_a may be completely uncorrelated, but the superpotential may be subject to an approximate R -symmetry. As we have shown, in that case $\langle \mathcal{W} \rangle$ is naturally suppressed. In what follows, we will show that, for generic c_a , this is

³ K and \mathcal{W} are defined only up to Kähler-Weyl transformations, with physical quantities depending only on the invariant G -function $G = K + \ln |\mathcal{W}|^2$. This is why it is always possible to absorb the $\hat{\mu}$ term into the Kähler potential, working with the quantities $\tilde{K} = K + f + \bar{f}$ and $\tilde{\mathcal{W}} = \mathcal{W} e^{-f}$, where $f = \hat{\mu} H_u H_d / \mathcal{W}_0$. This, of course, does not solve the μ problem but merely obscures it: in expanding the transformed Kähler potential \tilde{K} as in equation (3.1), \mathcal{Z} will now pick up a contribution $\sim \hat{\mu} / \mathcal{W}_0$. Therefore, if \mathcal{W}_0 is small, \mathcal{Z} grows large and the μ parameter resulting from equation (3.2) remains of the order of the fundamental scale.

due to each \mathcal{M}_a individually being small in the vacuum (rather than due to large cancellations between different terms in equation (3.4a)). Therefore also μ will be naturally suppressed and of the correct order of magnitude.

3.2 Models with generic superpotential coefficients

Consider a superpotential as in equation (3.3). Suppose \mathcal{W} is “generic” in the following sense: there is a set of continuous or discrete symmetries under which the fields transform, and only the terms allowed by these symmetries appear in \mathcal{W} , with no fine-tuned coefficients. We stress that if the superpotential coefficients are correlated in some manner, or if some of them accidentally vanish, then the arguments to follow must be modified. This can naturally happen if the model is subject to certain continuous or discrete non-Abelian symmetries, in which case one is to apply the subsequent analysis to invariants of such symmetries rather than the elementary fields.

If there is an exact (for now) $U(1)_R$ acting on the Φ_i , with H_u and H_d uncharged, then $\langle \mathcal{W} \rangle = 0$ as shown before. At this stage it appears to be possible that this is achieved by a cancellation of various non-zero terms in equation (3.4a). Then $\hat{\mu}$, by equation (3.4b), would give an effective contribution to μ of the order M_P (or slightly smaller if the Φ_i expectation values are slightly smaller) since the c'_a are arbitrary by assumption.

We will now show that there is in fact no such cancellation for generic superpotentials. Generic $U(1)_R$ -symmetric superpotentials vanish term by term, i.e. monomial by monomial, in their supersymmetric vacua. The proof proceeds as follows (see also [16]): write \mathcal{W} as a sum of monomials \mathcal{M}_a ,

$$\mathcal{W} = \sum_a c_a^0 \mathcal{M}_a \quad (3.5)$$

for some generic set of coefficients $c^0 = (c_a^0)$. Suppose that there is a solution to the F -term equations at $\Phi^0 = (\Phi_i^0)$:

$$\frac{\partial \mathcal{W}}{\partial \Phi_i}(\Phi_1^0, \dots, \Phi_N^0) = 0 \quad \forall i. \quad (3.6)$$

Now for each c in some open neighbourhood \mathcal{U} of c^0 in the space of coefficients, we can construct a corresponding superpotential $\mathcal{W} = \sum c_a \mathcal{M}_a$. By genericity of c^0 we can choose \mathcal{U} such that there exists a collection of solutions $(\Phi_1^0(c), \dots, \Phi_N^0(c))$ to the respective F -term equations which smoothly depends on c . Since each \mathcal{W} vanishes in its supersymmetric vacua, \mathcal{W} vanishes identically on \mathcal{U} when regarded as a function of c via

$$\mathcal{W}(c) = \mathcal{W}(\Phi_1^0(c), \dots, \Phi_N^0(c); c). \quad (3.7)$$

Hence

$$0 = \frac{d\mathcal{W}}{dc_a} = \left[\mathcal{M}_a + \sum_i \frac{\partial \mathcal{W}}{\partial \Phi_i} \frac{\partial \Phi_i}{\partial c_a} \right]_{(\Phi_1^0(c), \dots, \Phi_N^0(c))} = \mathcal{M}_a(\Phi_1^0(c), \dots, \Phi_N^0(c)), \quad (3.8)$$

which proves the assertion.

To return to the μ problem, we have shown that, as long as the superpotential is generic and $U(1)_R$ -symmetric, all \mathcal{M}_a in equation (3.3) vanish in a supersymmetric vacuum. This

implies that the potentially dangerous contribution $\hat{\mu}$ to the μ parameter in equation (3.2) also vanishes. Introducing a higher-order R -breaking term, which induces a superpotential expectation value $\langle \mathcal{W} \rangle \sim m_{3/2}$, will also induce a $\hat{\mu}$ term of the same order of magnitude, and thus give a μ parameter of the correct size.

4 The μ term in string-derived models

Recently explicit string-derived models with approximate R -symmetries have been obtained in which the combination $H_u H_d$ is a singlet w.r.t. all symmetries [17]. By the arguments of the previous section, μ would then be of the order of the gravitino mass if the superpotential couplings were generic and uncorrelated.

On the other hand, it is not really clear if arguments based on ‘genericity’ can be applied to string models, where coupling strengths are calculable and satisfy highly non-trivial consistency criteria. Yet the μ problem is solved in certain settings because the superpotential exhibits a non-trivial structure, which relates the coupling coefficients.

The question of μ terms in string-derived models has been analyzed in the past in the context of orbifold compactifications of the heterotic string, and it has been found that, indeed, scenarios incorporating a solution to the μ problem exist (although concrete models have not been presented). According to our classification in section 3.1, these scenarios fall into two classes:

1. μ from the superpotential [15];
2. μ from the Kähler potential [18, 19].

In what follows we will show that both scenarios are related, at least in certain explicit string-derived models.

Both 1 and 2 require that the pair $H_u H_d$ be vector-like (not only w.r.t. the standard model gauge group $G_{\text{SM}} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$, but also w.r.t. all other symmetries). Further, for the second scenario (2) to work, the Higgs pair has to come from the untwisted sector, specifically from an orbifold plane with \mathbb{Z}_2 symmetry. In what follows we will briefly review both scenarios and show that both emerge automatically and simultaneously from a subset of the MSSM models of the heterotic Mini-Landscape [17, 20].

Scenario 1 requires the superpotential to be of the form mentioned in point (i) of section 3.1 [15]

$$\mathcal{W} = \mathcal{W}_0 + \lambda \mathcal{W}_0 H_u H_d, \tag{4.1}$$

where \mathcal{W}_0 denotes the superpotential of the hidden sector, which is responsible for supersymmetry breakdown. It is clear that, once \mathcal{W}_0 acquires a VEV, an effective μ term $\hat{\mu} = \lambda \langle \mathcal{W}_0 \rangle$ is induced. Since in vacua with vanishing cosmological constant $\langle \mathcal{W}_0 \rangle \sim m_{3/2}$, it is automatically of the right size.

Scenario 2 relies on the special form of the Kähler potential for untwisted matter fields [18, 19, 21, 22],

$$K = -\ln [(T + \bar{T}) (Z + \bar{Z}) - (H_u + \bar{H}_d) (H_d + \bar{H}_u)] . \tag{4.2}$$

Here, T denotes the Kähler modulus of an orbifold plane with a \mathbb{Z}_2 symmetry and Z the corresponding complex structure modulus. Expanding K in the H_u and H_d fields leads to a coupling (essentially $\overline{T} H_u H_d + \text{c.c.}$) as is required for the Giudice-Masiero mechanism to work. Assuming a (dominant) VEV of the F -component of T leads to an effective μ term just like in the Giudice-Masiero mechanism. It is of the correct size provided $\widehat{\mu}$ in equation (3.2) is absent, or at most of the order of $m_{3/2}$.

Before we consider specific models, we note that any Higgs pair coming from the untwisted sector in an orbifold plane with \mathbb{Z}_2 symmetry, the superpotential has automatically the structure of equation (4.1). More specifically, we will show that then the Higgs pair H_u and H_d is neutral w.r.t. the selection rules, i.e. whenever a monomial

$$\mathcal{M} = \prod_i \Phi_i \tag{4.3}$$

denotes a superpotential term, i.e. is allowed by the selection rules [23], also the term

$$\mathcal{M} H_u H_d = H_u H_d \prod_i \Phi_i \tag{4.4}$$

is allowed. This has been noted to be the case for the Higgs field of a heterotic benchmark model of [17]. The argument turns out to be generally valid. Because of the \mathbb{Z}_2 projection, the pair is vector-like w.r.t. all gauge factors. Further, the \mathbb{Z}_2^R -charges are $(0, 0, -1)$ for both, and the corresponding discrete R -symmetry says that R charges should add to $-1 \pmod 2$ in the third component. Hence the pair $H_u H_d$ has R -charges which are equivalent to $(0, 0, 0)$, i.e. $H_u H_d$ is neutral w.r.t. the R charges. Moreover, these fields come from the untwisted sector and correspond therefore to the space group element $(\mathbb{1}, 0)$, i.e. they are neutral under the discrete symmetries representing the space group rule. Altogether we have found that the pair $H_u H_d$ is always neutral w.r.t. all selection rules, not only in the \mathbb{Z}_6 -II orbifold. For instance, the $\mathbb{Z}_2 \times \mathbb{Z}_2$ model presented in [24] exhibits this structure as well. However, this argument does not tell us that the coefficients of the terms coincide. That is, at $H_u H_d = 0$ the superpotential is as in equation (3.4a),

$$\mathcal{W} = \sum c_a \mathcal{M}_a, \tag{4.5}$$

where the \mathcal{M}_a are monomials of type (4.3), while the terms multiplying $H_u H_d$ are as in equation (3.4b),

$$\sum c'_a \mathcal{M}_a. \tag{4.6}$$

Now, if $\langle \mathcal{W} \rangle$ is small due to the cancellation between various $\langle \mathcal{M}_a \rangle$, the effective μ term $\widehat{\mu} = \sum c'_a \langle \mathcal{M}_a \rangle$ is not necessarily small, unless the c_a and c'_a are proportional to each other.

There is a simple way to see that they are indeed proportional. Our starting point is the Kähler potential (4.2). Let us expand it,

$$\begin{aligned} K &= -\ln [(T + \overline{T}) (Z + \overline{Z}) - (H_u + \overline{H_d}) (H_d + \overline{H_u})] \\ &\simeq -\ln [(T + \overline{T}) (Z + \overline{Z})] + \frac{1}{(T + \overline{T}) (Z + \overline{Z})} [|H_u|^2 + |H_d|^2 + (H_u H_d + \text{c.c.})] \\ &= -\ln [(T + \overline{T}) (Z + \overline{Z})] + [|\widehat{H}_u|^2 + |\widehat{H}_d|^2 + (\widehat{H}_u \widehat{H}_d + \text{c.c.})]. \end{aligned} \tag{4.7}$$

In the last step we switched to canonically normalized fields $\widehat{H}_{u,d}$. We see that the constants \mathcal{Y}_u , \mathcal{Y}_d and \mathcal{Z} in (3.1) all coincide. We further make the assumption that T and Z are already stabilized (see [25] for a recent discussion; alternative ideas for moduli stabilization are sketched in section 5.2).

The structure of the Kähler potential (4.2) is enforced by gauge invariance in higher dimensions. The main point is that H_u and H_d can be viewed as extra components of gauge fields in a 6D orbifold GUT limit of the setting (see [26] for a recent discussion), which by the \mathbb{Z}_2 symmetry of the orbifold plane further admits a Burdman-Nomura type [27] 5D limit. The Higgs dependence in (4.2) then follows from higher-dimensional gauge invariance [28, 29]. Gauge transformations along the generators which correspond to H_u and H_d mix H_u with \overline{H}_d ; therefore the Kähler potential can, at the gauge symmetric level, only depend on the absolute square of $H_u + \overline{H}_d$. By the same argument, the superpotential cannot depend on $H_u H_d \times (\text{SU}(2)_L \times \text{U}(1)_Y \text{ singlets})$. Higher-dimensional gauge invariance is broken by the boundary conditions of the orbifold compactification. Therefore, there are the usual logarithmic corrections to \mathcal{Y}_u , \mathcal{Y}_d and \mathcal{Z} below the compactification scale. Moreover, there are further logarithmic corrections coming from the fact that at the massless level states sitting at the fixed points do not furnish complete $\text{SU}(6)$ representations.

Higher-dimensional gauge invariance has further important implications. Because of the above arguments, the gauge-invariant superpotential is

$$\mathcal{W} = \text{independent of the monomial } \widehat{H}_u \widehat{H}_d . \tag{4.8}$$

Then this setting is, in leading order in $\widehat{H}_u \widehat{H}_d$, equivalent to a setting with

$$\widetilde{K} = -\ln [(T + \overline{T}) (Z + \overline{Z})] + [|\widehat{H}_u|^2 + |\widehat{H}_d|^2] , \tag{4.9a}$$

$$\widetilde{\mathcal{W}} = \exp(\widehat{H}_u \widehat{H}_d) \mathcal{W} , \tag{4.9b}$$

where the proportionality discussed around equations (4.5) and (4.6) is obvious. This statement can easily be verified by looking at the Kähler function G (compare the discussion in section 3.1). In leading order in $\widehat{H}_u \widehat{H}_d$ we have

$$\exp (K + \ln |\mathcal{W}|^2) = \exp \left(\widetilde{K} + \ln |\widetilde{\mathcal{W}}|^2 \right) . \tag{4.10}$$

Let us also remark that the factorized structure (4.9b) can also be obtained by a naive field-theoretic calculation of the coupling constants. As H_u and H_d come from the untwisted sector or, in other words, are extra components of the gauge fields in ten dimensions, they are bulk fields in the ten-dimensional theory. Since the product $H_u H_d$ is completely neutral under all symmetries, the profile of $H_u H_d$ is flat; this remains true if one includes effects that distort the profiles of charged fields, such as localized Fayet-Iliopoulos terms (cf. the discussion in [30]). In the naive field-theoretic approach calculating the couplings amounts to computing the overlap (for canonically normalized H_u and H_d)

$$\int d^6 y [H_u(y) H_d(y)]^n \mathcal{M}_a(\Phi_i(y)) \tag{4.11}$$

in the internal six dimensions parametrized by y_M . Due to the flatness of the profile of $H_u H_d$, the integrals (4.11) coincide for all $n \geq 0$ (up to a constant) with $\int d^6 y \mathcal{M}_a(\Phi_i(y))$, which yields the coefficient c_a of the monomial \mathcal{M}_a . Including combinatorial factors again leads to the exponential structure in equation (4.9b).

Altogether we see that the holomorphic coupling $H_u H_d \mathcal{W}$ comes, in the above supergravity formulation [18], from the Kähler potential. For small $H_u H_d$, we can write it into the superpotential.⁴ In the supergravity formulation it is easy to show that $\mu \propto \langle \mathcal{W} \rangle$, which requires additional assumptions in the “ μ from \mathcal{W} ” approach [15].

We also comment that, if T and/or Z attain non-trivial F -term VEVs, there is, as discussed below equation (4.2), an additional, non-holomorphic contribution to μ [18, 19, 22]. That is, in the string-derived MSSM models both the Kim-Nilles [4] and the Giudice-Masiero [5] mechanisms can be at work, where the former is always there and the second might or might not contribute. In particular, no F -term expectation value of the ‘radion’ T is required in order to generate the μ term.

In conclusion, although in string theory couplings are not ‘generic’ but highly constrained by consistency and, in particular, calculable, there exist simple settings in which the μ problem is solved [18] in the sense that $\mu \sim m_{3/2}$. These settings are incorporated in very promising orbifold models [6, 17, 20, 24, 31], which also exhibit the appropriate approximate R -symmetries allowing us to understand why $\langle \mathcal{W} \rangle$ is small in the first place. These approximate symmetries are a consequence of high power discrete R -symmetries, reflecting the discrete rotational symmetries of compact space. In what follows, we will illustrate the suppression of $\langle \mathcal{W} \rangle$ due to approximate R -symmetries in various examples.

5 Examples

5.1 A simple example

A very simple example to illustrate the mechanism is given by a model with only two superfields X and Y , where X carries R -charge 2 and Y has zero R -charge. At the $U(1)_R$ -symmetric level, the most general superpotential is

$$\mathcal{W} = X f(Y) , \tag{5.1}$$

where f is an arbitrary function. Suppose that \mathcal{W} can be written as

$$\mathcal{W} = X \left(\lambda Y^2 + \frac{1}{M} Y^3 + \dots \right) \tag{5.2}$$

with higher-order terms omitted. The F -term equations have a non-trivial solution at

$$\langle X \rangle = 0 , \quad \langle Y \rangle = -\lambda M . \tag{5.3}$$

⁴This is not in contradiction with the stringy calculation of allowed superpotential couplings, which are the basis of the statements around equations (4.3) and (4.4). This analysis, which is also extensively used in heterotic model building (as for instance in [17]), shows only if a holomorphic correlator between certain fields exists (or not).

We will make the assumption that $\langle Y \rangle$ is somewhat smaller than the fundamental scale. (5.3) is clearly a local minimum. Expanding around $\langle Y \rangle$, i.e. replacing Y by $\langle Y \rangle + \delta Y$, leads to

$$\mathscr{W} = m X \delta Y + \dots \quad (5.4)$$

with $m = -2\lambda^2 M$. The important point is that m is related to the fundamental scale and does not know of supersymmetry breakdown.

Now add a $U(1)_R$ violating term Y^N , i.e.

$$\mathscr{W} = X \left(\lambda Y^2 + \frac{1}{M} Y + \dots \right) + \kappa Y^N . \quad (5.5)$$

Then the above minimum undergoes a small shift, but remains a minimum if κ is sufficiently small and/or N is sufficiently large. The expectation value of the superpotential is of the order $\kappa(\lambda M)^N$. This eventually sets the scale for the gravitino mass,

$$m_{3/2} \sim \langle \mathscr{W} \rangle \sim \kappa(\lambda M)^N . \quad (5.6)$$

The masses of the fluctuations around the slightly shifted minimum are still of order m , i.e. can be much larger than $m_{3/2}$. The gravitino mass can be arbitrarily small if $\langle Y \rangle$ is slightly suppressed and N is sufficiently large.

5.2 A “string-inspired” example

Consider now a supersymmetric field theory with matter fields X , Y and Z where X and Y have R -charge 2 while Z has R -charge 0. Our superpotential is now

$$\mathscr{W} = X \left(\lambda_1(T) Z^2 + \frac{a_1}{M} Z^3 + \dots \right) + Y \left(\lambda_2(T) Z^2 + \frac{a_2}{M} Z^3 + \dots \right) . \quad (5.7)$$

Again, the higher order terms “...” will be ignored. Here we have taken into account a possible dependence of the couplings on the moduli, represented by T . That is, the true field content of our setting is $\{X, Y, Z, T\}$. Consider now the analogue of the vacuum configuration in example 5.1. We seek solutions to the F equations at $X = Y = 0$ with non-trivial Z . While the F -term equations for Z and T are trivially satisfied, the other two equations yield

$$F_X : \lambda_1(T) Z^2 + \frac{a_1}{M} Z^3 = 0 , \quad (5.8a)$$

$$F_Y : \lambda_2(T) Z^2 + \frac{a_2}{M} Z^3 = 0 . \quad (5.8b)$$

This constitutes two equations for the fields Z and T . T will be fixed by

$$\frac{\lambda_1(\langle T \rangle)}{\lambda_2(\langle T \rangle)} = \frac{a_1}{a_2} . \quad (5.9)$$

(For instance, if $\lambda_i(T) = e^{-b_i T}$, T will be fixed at $\langle T \rangle = \ln(a_1/a_2)/(b_2 - b_1)$.) Plugging this back allows us to solve for Z , $\langle Z \rangle = -\lambda_1(\langle T \rangle) M/a_1$. Again, all fields are fixed. As long as the $\lambda_i(\langle T \rangle)$ are not too small, the masses of the fields are not too far below

the fundamental scale. The important lesson here is that moduli, governing the couplings between “matter fields”, can be fixed if the F -term equations for the matter fields alone are “overconstraining”. Of course, we have to make the assumption that the λ_i and a_i are such that they admit solutions of equation (5.9) with $\lambda_i(\langle T \rangle)$ somewhat below 1. We will yet have to see if this mechanism allows us to stabilize the T -moduli in honest string-derived models. As before, adding higher-order $U(1)_R$ -breaking terms will result in a suppressed expectation value of \mathcal{W} .

In this and the previous examples, there is no symmetry principle enforcing the structure of the $U(1)_R$ -symmetric superpotential (5.2), nor the specific structure (5.5) of the R -breaking term. Arguing on the purely field-theoretic level, there is no reason why there should not be linear terms in X , Y or Z , or why the leading R -breaking term should be of some high order N . (Once such features are imposed, they are however robust under radiative corrections because of the non-renormalization theorem.) In what follows, we will present a class of generic, albeit more complicated models, in which the absence of these terms is enforced by symmetries.

5.3 A generic example in field theory

Consider three fields X, Y, Z which are charged under a $\mathbb{Z}_9 \times \mathbb{Z}_4$ symmetry with charges:

field	X	Y	Z
\mathbb{Z}_9 -charge	1	5	8
\mathbb{Z}_4 -charge	0	3	3

The corresponding superpotential reads at order 9

$$\mathcal{W} = \lambda_5 X Y^2 Z^2 + \lambda_8 X^4 Y^3 Z + \lambda'_8 X^4 Z^4 + \lambda_9 X^9. \tag{5.10}$$

If truncated at order 8, \mathcal{W} exhibits an accidental $U(1)_R$ symmetry with charges $q_R^i = (0, \frac{1}{2}, \frac{1}{2})$; the identification of such symmetries is described in appendix A. $U(1)_R$ is explicitly broken at order nine. One solution to the global F equations is

$$\langle X \rangle = -\frac{(-2)^{2/9} \lambda_5^{1/3}}{3^{1/3} \lambda_8^{2/9} \lambda_8^{1/9}}, \tag{5.11a}$$

$$\langle Y \rangle = -\frac{(-2)^{11/18} \lambda_5^{5/12}}{3^{5/12} \lambda_8^{11/18} \lambda_8^{1/18}} \lambda_9^{1/4}, \tag{5.11b}$$

$$\langle Z \rangle = \frac{(-2)^{5/18} \lambda_5^{5/12}}{3^{5/12} \lambda_8^{5/18} \lambda_8^{7/18}} \lambda_9^{1/4}, \tag{5.11c}$$

resulting in

$$\langle \mathcal{W} \rangle = -\frac{4\lambda_5^3}{27\lambda_8^2 \lambda'_8} \lambda_9. \tag{5.12}$$

If the VEVs are somewhat small, the VEV of the superpotential will be suppressed. All masses turn out to be non-vanishing.

In the $U(1)_R$ -symmetric limit $\lambda_9 \rightarrow 0$, $\langle X \rangle$ remains finite while $\langle Y \rangle$ and $\langle Z \rangle$ go to zero. Therefore $\langle \mathcal{W} \rangle$ vanishes term by term in this limit, as it should by the arguments of section 3.2. It turns out that Y and Z in fact become massless as $\lambda_9 \rightarrow 0$, with their expectation values determined by higher-power terms in the scalar potential.

5.4 Another generic example in field theory

We now show that it is possible to construct a generic model satisfying the following requirements: for the $U(1)_R$ -symmetric truncation there is a supersymmetric vacuum at sub-Planckian expectation values. This vacuum breaks $U(1)_R$ spontaneously. Therefore the Nelson-Seiberg argument must be circumvented, and the vacuum in the $U(1)_R$ symmetric truncation will possess at least one flat direction. It turns out that, in models where this flat direction includes the R -symmetric point where $U(1)_R$ is not spontaneously broken, higher-order R -breaking terms tend to stabilize the flat direction at this point. But if the VEVs of all R -charged fields vanish, higher-order terms will not induce a non-vanishing $\langle \mathcal{W} \rangle$. Therefore we focus on settings with a flat direction along which $U(1)_R$ is spontaneously broken everywhere.

The following class of examples satisfies these criteria. It comprises three chiral superfields X , Y and Z with the following R -charges:

field	X	Y	Z
R -charge	2	3	-3

The most general superpotential is

$$\mathcal{W} = X f(Y Z, X^3 Z^2) . \tag{5.13}$$

There are supersymmetric vacua in configurations with $X = 0$ and $Y Z = \alpha$, where α is determined by the condition $f(\alpha, 0) = 0$.

The superpotential up to order 10 in the fields can be written as

$$\mathcal{W} = X P(Y Z) + X^4 Z^2 Q(Y Z) + \dots , \tag{5.14}$$

where terms of order 11 and higher have been omitted, P is a quartic polynomial, and Q is a quadratic polynomial. Supersymmetric vacua appear at $X = 0$ and at the zeros of P . Assume that P has an isolated zero at $Y Z = \alpha$ with $\alpha \lesssim 1$ real. We will eventually show that it is self-consistent to neglect the higher-order terms, provided that α is somewhat small (and that their coefficients are not too large).

We now add higher-order R -breaking terms. To justify their absence at lower orders, we take the R -symmetry not to be $U(1)$ but to be given by a discrete subgroup. Let us consider \mathbb{Z}_{16} for illustration, demanding $R \equiv 2 \pmod{16}$ for each superpotential term. This allows for several more terms in \mathcal{W} ,

$$\begin{aligned} \Delta \mathcal{W} = & \lambda_1 Y^6 + \lambda_2 Y^7 Z + \lambda_3 Y^8 Z^2 + \lambda_4 Z^{10} \\ & + \beta_1 X^9 + \beta_2 X^6 Y^2 + \beta_3 X^6 Y^3 Z + \beta_4 X^3 Y^4 + \beta_5 X^3 Y^5 Z + \beta_6 X^2 Z^6 + \beta_7 X^2 Y Z^7 \\ & + \dots . \end{aligned} \tag{5.15}$$

The terms in the first line stabilize the flat direction (by contrast, the terms in the second line do not contribute to the F -terms when evaluated at $X = 0$). We find that there is a supersymmetric vacuum at

$$\langle X \rangle \approx -\frac{2(3\lambda_1)^{5/8}(5\lambda_4)^{3/8}}{P'(\alpha)}\alpha^{11/4}, \tag{5.16a}$$

$$\langle Y \rangle \approx \left(\frac{5\lambda_4}{3\lambda_1}\right)^{1/16}\alpha^{5/8}, \tag{5.16b}$$

$$\langle Z \rangle \approx \left(\frac{3\lambda_1}{5\lambda_4}\right)^{1/16}\alpha^{3/8}. \tag{5.16c}$$

The expressions for Y and Z are obtained from the equations of motion of the $U(1)_R$ -symmetric theory, up to the VEV along the flat direction. This VEV is then calculated by taking into account the higher $U(1)_R$ -breaking terms. The expression for X results from re-substituting the Y and Z VEVs into the F -term equations of Y and Z and solving them to leading order. For small α this is a self-consistent procedure, leading to exact values asymptotically as α approaches zero.

The vacuum expectation value of \mathscr{W} behaves like $\alpha^{15/4}$, with the dominant contributions coming from the λ_1 and λ_4 terms in $\Delta\mathscr{W}$. Clearly no excessive fine-tuning of coefficients is required to obtain smallish expectation values and suppressed $\langle\mathscr{W}\rangle$ (a mildly suppressed α , as we have assumed, is sufficient). At small expectation values, it is in particular safe to neglect higher-order terms in \mathscr{W} .

The superpotential VEV is not suppressed by a very large power here, but it is straightforward to extend this model to higher suppression of $\langle\mathscr{W}\rangle$, by imposing a \mathbb{Z}_N R -symmetry with N sufficiently large. However, then \mathscr{W} becomes rather cumbersome to write down explicitly. Following the procedure described above, one can determine the α -dependence of Y and Z to leading order in an asymptotic expansion, to infer the behaviour of $\langle\mathscr{W}\rangle$ at small α . The result for some select choices of N is:

N	10	13	14	16	17	19	20	22	25	28
$\langle\mathscr{W}\rangle \sim$	$\alpha^{12/5}$	$\alpha^{40/13}$	$\alpha^{20/7}$	$\alpha^{15/4}$	$\alpha^{60/17}$	$\alpha^{84/19}$	$\alpha^{21/5}$	$\alpha^{56/11}$	$\alpha^{144/25}$	$\alpha^{45/7}$

The determination of the exponents of α works as follows: the $U(1)_R$ -symmetric potential has a flat direction along the hyperbola $YZ = \alpha$. To stabilize it one needs two R -breaking terms in the superpotential, of the form $\Delta\mathscr{W} \supset Y^p + Z^q$. A Y^p term prevents a runaway towards $Z = 0$ and $Y \rightarrow \infty$; likewise, a Z^q term prevents a runaway towards $Y = 0$ and $Z \rightarrow \infty$. Therefore, the lowest powers p and q allowed by the discrete R -symmetry will essentially determine the leading α -dependence of Y and Z , and also of \mathscr{W} . In other words, given N , one just has to determine p and q in order to figure out the exponents of α in the table. We have also checked this behaviour numerically for several of the above N , using generic superpotentials with random real coefficients of order one.

Other variations of this model can be found by choosing different R -charge assignments for Y and Z . For instance, with a \mathbb{Z}_{16} R -symmetry and the R -charges

field	X	Y	Z
R -charge	2	3/4	-3/4

the operators most relevant for stabilizing the R -flat direction are Y^{24} and Z^{40} , leading to a superpotential expectation value which scales as $\langle \mathcal{W} \rangle \sim \alpha^{15}$. For a mildly suppressed α (or equivalently mildly suppressed Y and Z VEVs), a huge hierarchy is generated.

So far the discussion has been on the globally supersymmetric level, in the sense that we are identifying vacua as points where all derivatives of the superpotential vanish. Using ordinary derivatives rather than Kähler-covariant derivatives may seem not well justified, since some of the Kähler derivative terms, which we have neglected, are of lower order in the fields than some of the superpotential terms which we are relying upon. However, as already mentioned in section 2.4, the supergravity corrections to the F -terms are indeed negligible because \mathcal{W} is suppressed. We have also checked this numerically.

5.5 An example from a heterotic orbifold model

The aim of this section is to show that the ideas discussed above can also be applied to string-theoretic models. In the following we focus on the models of the ‘heterotic Mini-Landscape’ [17, 20]. These models exhibit the standard model gauge group and the chiral matter content of the MSSM. They are based on the \mathbb{Z}_6 -II orbifold with three factorizable tori (see [6, 32] for details). The discrete symmetry of the geometry leads to a large number of discrete symmetries governing the couplings of the effective field theory [23, 33] (cf. also [6, 32, 34]). Apart from various bosonic discrete symmetries, one has a

$$[\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2]_R \tag{5.17}$$

symmetry; other orbifolds have similar discrete symmetries. Further, in almost all of the Mini-Landscape models there is, at one-loop, a Fayet-Iliopoulos (FI) D -term ξ ,

$$V_D \supset g^2 \left(\sum_i q_i |\Phi_i|^2 + \xi \right)^2, \tag{5.18}$$

where the q_i denote the charges under the so-called ‘anomalous $U(1)$ ’. It turns out that, in all models with non-vanishing FI term, ξ is of order 0.1 (see [6] for an explicit example).

The first step of our analysis is to identify a set of standard model singlets Φ_i with the following properties:

- giving VEVs to the Φ_i allows to cancel the FI term;
- there is no other field that is singlet under the gauge symmetries left unbroken by the Φ_i VEVs.

These properties ensure that the Φ_i can be consistent with a vanishing D -term potential and that the F -terms of all other massless modes vanish, implying that it is sufficient to derive the superpotential terms involving only the Φ_i fields.

Given non-trivial solutions to the F -term equations,

$$\Phi_i \frac{\partial \mathcal{W}}{\partial \Phi_i} = 0, \quad \text{with } \Phi_i \neq 0, \tag{5.19}$$

one can use complexified gauge transformations to ensure vanishing D -terms as well [35]. Although D -term constraints do not fix the scale of the $\langle \Phi_i \rangle$ in general, the requirement to cancel the FI term introduces the scale $\sqrt{\xi} \sim 0.3$ into the problem. In the following we will search for solutions of $V_D = V_F = 0$ in the regime $|\Phi_i| < 1$. We will explicitly verify that for such solutions the superpotential is hierarchically small, $\langle \mathcal{W} \rangle \sim \langle \Phi \rangle^N$ where $\langle \Phi \rangle$ denotes the typical size of a VEV. A very important property of many of these configurations is that all fields acquire (supersymmetric) masses. Hereby typically only one field — the would be R -axion — has a mass of the order $\langle \mathcal{W} \rangle$ while the others are much heavier.

Turning to particular models within the Mini-Landscape we find that the corresponding superpotentials exhibit accidental $U(1)_R$ symmetries that get only broken at rather high orders N . Consequently the analysis becomes very involved, especially when many fields have to be considered. This is the case for the phenomenologically very interesting model 1 of [17], where 24 fields have to be switched on. In this model $U(1)_R$ gets broken at order 9, and the superpotential consists of 1816 terms at this order. To avoid this very complicated setup let us consider another model from the Mini-Landscape which is easier to handle but nevertheless exhibits the desired features.

The model we will consider in the following is defined by the gauge shift V and the two Wilson lines $W_{1,2}$,

$$\begin{aligned}
 V &= \begin{pmatrix} \frac{1}{3} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{6} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}, \\
 W_2 &= \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 1 & -1 & -1 & -1 & -1 & -\frac{1}{2} & -\frac{1}{2} & 2 \end{pmatrix}, \\
 W_3 &= \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & \frac{5}{3} & -2 & 2 & 2 \end{pmatrix}.
 \end{aligned}$$

The gauge group after compactification is (up to $U(1)$ factors)

$$[\text{SU}(3) \times \text{SU}(2)] \times [\text{SU}(3) \times \text{SU}(2) \times \text{SU}(2) \times \text{SU}(2) \times \text{SU}(2)],$$

where the two brackets refer to the first and second E_8 factor respectively.

We consider the following set of singlet fields with non-vanishing VEVs (in the notation of [17]):

$$\{\Phi_i\} = \{s_1, s_6, s_7, s_{10}, s_{13}, s_{14}, s_{17}, s_{22}, s_{24}, s_{33}\}.$$

The monomial to cancel the FI term is simply given by $\{s_{33}\}$. The resulting superpotential exhibits an accidental continuous R -symmetry up to order 10 which is explicitly broken at order 11. At this order the superpotential consists of 56 terms with 28 independent

coefficients,

$$\begin{aligned}
 \mathcal{W} = & \frac{1}{288} \lambda_1 s_1 s_{22}^3 s_{33}^4 (s_{10}^2 + s_{17}^2) + \frac{1}{96} \lambda_2 s_1 s_{22}^2 s_{24} s_{33}^4 (s_{10}^2 + s_{17}^2) \\
 & + \frac{1}{96} \lambda_3 s_1 s_{22} s_{24}^2 s_{33}^4 (s_{10}^2 + s_{17}^2) + \frac{1}{288} \lambda_4 s_1 s_{24}^3 s_{33}^4 (s_{10}^2 + s_{17}^2) \\
 & + \frac{1}{2} \lambda_5 s_1 s_{22} s_{33}^2 (s_{10} s_6 + s_{13} s_{17}) + \frac{1}{2} \lambda_6 s_1 s_{24} s_{33}^2 (s_{10} s_6 + s_{13} s_{17}) \\
 & + \frac{1}{144} \lambda_7 s_{22}^3 s_{33}^4 (s_{10} s_7 + s_{14} s_{17}) + \frac{1}{48} \lambda_8 s_{22}^2 s_{24} s_{33}^4 (s_{10} s_7 + s_{14} s_{17}) \\
 & + \frac{1}{48} \lambda_9 s_{22} s_{24}^2 s_{33}^4 (s_{10} s_7 + s_{14} s_{17}) + \frac{1}{144} \lambda_{10} s_{24}^3 s_{33}^4 (s_{10} s_7 + s_{14} s_{17}) \\
 & + \frac{1}{96} \lambda_{11} s_{10} s_{14} s_{17} s_{22} s_{33}^4 s_7 (s_{10} s_7 + s_{14} s_{17}) + \frac{1}{96} \lambda_{12} s_{10} s_{14} s_{17} s_{24} s_{33}^4 s_7 (s_{10} s_7 + s_{14} s_{17}) \\
 & + \frac{1}{288} \lambda_{13} s_{14} s_{22} s_{33}^4 s_7 (s_{13}^3 s_7 + s_{14} s_6^3) + \frac{1}{288} \lambda_{14} s_{14} s_{24} s_{33}^4 s_7 (s_{13}^3 s_7 + s_{14} s_6^3) \\
 & + \frac{1}{288} \lambda_{15} s_{14} s_{22} s_{33}^4 s_7 (s_{10}^3 s_{14} + s_{17}^3 s_7) + \frac{1}{288} \lambda_{16} s_{14} s_{24} s_{33}^4 s_7 (s_{10}^3 s_{14} + s_{17}^3 s_7) \\
 & + \frac{1}{2} \lambda_{17} s_{22} s_{33}^2 (s_{13} s_{14} + s_6 s_7) + \frac{1}{2} \lambda_{18} s_{24} s_{33}^2 (s_{13} s_{14} + s_6 s_7) \\
 & + \frac{1}{96} \lambda_{19} s_{13} s_{14} s_{22} s_{33}^4 s_6 s_7 (s_{13} s_{14} + s_6 s_7) + \frac{1}{96} \lambda_{20} s_{13} s_{14} s_{24} s_{33}^4 s_6 s_7 (s_{13} s_{14} + s_6 s_7) \\
 & + \frac{1}{864} \lambda_{21} s_{22} s_{33}^4 (s_{10}^3 s_7^3 + s_{14}^3 s_{17}^3) + \frac{1}{864} \lambda_{22} s_{24} s_{33}^4 (s_{10}^3 s_7^3 + s_{14}^3 s_{17}^3) \\
 & + \frac{1}{288} \lambda_{23} s_{13} s_{22} s_{33}^4 s_6 (s_{13} s_7^3 + s_{14}^3 s_6) + \frac{1}{288} \lambda_{24} s_{13} s_{24} s_{33}^4 s_6 (s_{13} s_7^3 + s_{14}^3 s_6) \\
 & + \frac{1}{288} \lambda_{25} s_{10} s_{17} s_{22} s_{33}^4 (s_{10} s_{14}^3 + s_{17} s_7^3) + \frac{1}{288} \lambda_{26} s_{10} s_{17} s_{24} s_{33}^4 (s_{10} s_{14}^3 + s_{17} s_7^3) \\
 & + \frac{1}{864} \lambda_{27} s_{22} s_{33}^4 (s_{13}^3 s_{14}^3 + s_6^3 s_7^3) + \frac{1}{864} \lambda_{28} s_{24} s_{33}^4 (s_{13}^3 s_{14}^3 + s_6^3 s_7^3) . \tag{5.20}
 \end{aligned}$$

If all these fields acquire VEVs, two U(1) factors get broken, one of them corresponding to the anomalous U(1). The charges of the ten singlets with respect to the two broken U(1) factors are

$$\begin{aligned}
 q_{\text{anom}}^i &= (6, 11, 17, 11, 11, 17, 11, 28, 28, -28) , \\
 q_{\text{U}(1)}^i &= (-2, 1, -1, 1, 1, -1, 1, 0, 0, 0) . \tag{5.21}
 \end{aligned}$$

The full quantum numbers of the s_i are (in the conventions of [17])

field	k	n_3	n_2	n_2'	q_γ	R_1	R_2	R_3	irrep	q_Y	q_2	q_3	q_4	q_5	q_6	q_7
s_1	0	*	*	*	0	-1	0	0	(1, 1; 1, 1, 1, 1, 1)	0	-1	-1	0	0	0	0
s_6	1	0	0	0	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	(1, 1; 1, 1, 1, 1, 1)	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0
s_7	1	0	0	0	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	(1, 1; 1, 1, 1, 1, 1)	0	$-\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0
s_{10}	1	0	0	0	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	(1, 1; 1, 1, 1, 1, 1)	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0
s_{13}	1	0	0	1	0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	(1, 1; 1, 1, 1, 1, 1)	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0
s_{14}	1	0	0	1	0	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	(1, 1; 1, 1, 1, 1, 1)	0	$-\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0
s_{17}	1	0	0	1	0	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	(1, 1; 1, 1, 1, 1, 1)	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0
s_{22}	2	0	*	*	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	(1, 1; 1, 1, 1, 1, 1)	0	$-\frac{1}{3}$	0	1	0	0	0
s_{24}	2	0	*	*	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	(1, 1; 1, 1, 1, 1, 1)	0	$-\frac{1}{3}$	0	1	0	0	0
s_{33}	4	0	*	*	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	(1, 1; 1, 1, 1, 1, 1)	0	$\frac{1}{3}$	0	-1	0	$-\frac{2}{3}$	0

The structure of the superpotential (5.20) is governed by a D_4 symmetry,⁵ under which 6 fields combine to 3 doublets,

$$d_1 = \begin{pmatrix} s_6 \\ s_{13} \end{pmatrix} , \quad d_2 = \begin{pmatrix} s_7 \\ s_{14} \end{pmatrix} \quad \text{and} \quad d_3 = \begin{pmatrix} s_{10} \\ s_{17} \end{pmatrix} .$$

⁵ $D_4 \subset O(2)$ is the dihedral group of order 8, the symmetry group of a square.

It further turns out that any allowed term (at order less or equal to 11) involves at least two such doublets. Another observation is that s_{33} appears only with even powers.

The solutions of the global F -term equations depend on the precise values of the λ_i coefficients. As we do not yet know how to compute these coefficients, we can only argue that point-like solutions to the F - and D -term equations exist. A consistency check for this assertion is as follows: rather than solving for the fields we solve the F -term equations for the 28 coefficients λ_i , setting the fields to ‘desirable’ values consistently with D -flatness. An example for a, thus obtained, ‘vacuum configuration’ is given by

$$\begin{aligned} \langle s_1 \rangle &= -\frac{1}{10} & \langle s_6 \rangle &= \frac{1}{10} & \langle s_7 \rangle &= \frac{1}{10} & \langle s_{10} \rangle &= \frac{1}{10} & \langle s_{13} \rangle &= \frac{1}{10} \\ \langle s_{14} \rangle &= \frac{1}{10} & \langle s_{17} \rangle &= \frac{1}{10} & \langle s_{22} \rangle &= \frac{1}{10} & \langle s_{24} \rangle &= \frac{1}{10} & \langle s_{33} \rangle &= -\frac{\sqrt{1401}}{20\sqrt{70}} \end{aligned}$$

with all $\lambda_i = 0.01$ except for

$$\begin{aligned} \lambda_1 &\sim 0.482 & \lambda_5 &\sim -0.01 & \lambda_{13} &\sim -0.29 & \lambda_{14} &\sim -0.22 \\ \lambda_{17} &\sim -0.001 & \lambda_8 &\sim 0.001 & \lambda_9 &\sim 0.001 & \lambda_{18} &\sim 0.001 . \end{aligned}$$

The coefficients $(\lambda_8, \lambda_9, \lambda_{18})$ are conveniently chosen and $(\lambda_1, \lambda_5, \lambda_{13}, \lambda_{14}, \lambda_{17})$ are fixed in terms of the remaining coefficients. The resulting vacuum expectation value for \mathscr{W} in units of M_P is given by

$$\langle \mathscr{W} \rangle \sim 1.1 \cdot 10^{-12}$$

while the mass eigenvalues resulting from the superpotential read

$$(m_i) = (1 \cdot 10^{-5}, 1 \cdot 10^{-5}, 1.4 \cdot 10^{-9}, 5.1 \cdot 10^{-10}, 2.7 \cdot 10^{-10}, 2.2 \cdot 10^{-10}, 1 \cdot 10^{-10}, 3 \cdot 10^{-11}, 0, 0) . \tag{5.22}$$

The two massless fields obtain masses from the D -term potentials corresponding to the two broken $U(1)$ factors and the absorption of the Goldstone modes, respectively. We see that the lightest mass eigenstate is of order $\langle \Phi \rangle^{N-2} \sim 10^2 \langle \mathscr{W} \rangle$ as expected.

Altogether we have argued that in the model under consideration one may obtain isolated supersymmetric field configurations with $|\Phi_i| < 1$ where the VEV of the perturbative superpotential $\langle \mathscr{W} \rangle$ is hierarchically small. It appears highly desirable to rigorously prove that such configurations exist in explicit string-derived models. This, however, requires knowledge of the coefficients λ_i , which is not yet available.

6 Conclusions

We have investigated the mechanism of generating a hierarchically small superpotential expectation value $\langle \mathscr{W} \rangle$ by an approximate R -symmetry. We have recapitulated that, in the presence of such a symmetry, $\langle \mathscr{W} \rangle$ can be highly suppressed if the typical scalar expectation values are only somewhat below the fundamental scale [3]. In the limiting case of an exact R -symmetry, we showed that there exist examples of generic models where R -symmetry is broken spontaneously in a supersymmetric vacuum. By adding higher-order polynomial

terms to the superpotential which break R -symmetry explicitly, one may then construct vacua with $\langle \mathcal{W} \rangle$ given by high powers of small field expectation values. If they can be uplifted to Minkowski minima of the scalar potential, one obtains potentially realistic vacua with naturally small gravitino mass.

The main point of this analysis is the observation that an (approximate) R -symmetry not only allows us to control $\langle \mathcal{W} \rangle$, but also the MSSM μ term, if the Higgs fields H_u and H_d are singlets with respect to all symmetries and in particular have trivial R -charges. For models with generic superpotential coefficients, we proved that $\mu \sim \langle \mathcal{W} \rangle$ in Planck units, i.e. $\mu \sim m_{3/2}$.

We have also commented on the situation in string-derived models. We analyzed scenarios in which an R -symmetry and gauge invariance in higher dimensions relate the μ term to $\langle \mathcal{W} \rangle$. We have identified explicit string-derived models with the chiral spectrum of the MSSM in which this analysis can be applied. These models indeed exhibit approximate $U(1)_R$ symmetries, deriving from high-power discrete symmetries, which can explain a highly suppressed $\langle \mathcal{W} \rangle$. We further discussed explicit examples in which such suppressed $\langle \mathcal{W} \rangle$ emerge, while all fields are stabilized. To rigorously prove that such configurations exist in string-derived models, and to study their phenomenology, however, will require a detailed understanding of the coupling strengths.

The main focus of this analysis was the role of approximate R -symmetries in generating a suppressed $\langle \mathcal{W} \rangle$ and a μ term of a similar size. However, in our examples the origin of supersymmetry breakdown remains obscure. To be able to relate μ to the MSSM soft masses properly, a better understanding of the mechanism of supersymmetry breakdown (or the so-called “uplifting”) is required.

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A Identifying accidental R -symmetries

Consider the superpotential of some fields Φ_i . Truncate it at a certain order, such that it is a sum of monomials of degree $\leq D$,

$$\mathcal{W} = \sum_{m=1}^M \Phi_1^{\nu_1^{(m)}} \dots \Phi_N^{\nu_N^{(m)}} , \tag{A.1}$$

where $\nu_i^{(m)} \in \mathbb{N}_0$. The exponent vectors $\nu^{(m)} = (\nu_1^{(m)}, \dots, \nu_N^{(m)})$ can be used for the task of identifying accidental R -symmetries. Define a matrix

$$A = \begin{pmatrix} \nu_1^{(1)} & \dots & \nu_N^{(1)} \\ \vdots & & \vdots \\ \nu_1^{(M)} & \dots & \nu_N^{(M)} \end{pmatrix}, \tag{A.2}$$

with M denoting the number of monomials appearing in (A.1). An (accidental) $U(1)$ symmetry exists only if the equation

$$A \cdot x = 0 \tag{A.3}$$

possesses a non-trivial solution. The entries of such a solution are the charges w.r.t. the (accidental) $U(1)$.

In order to identify (accidental) R -symmetries, one has to solve the equation

$$A \cdot x_R = \begin{pmatrix} 2 \\ \vdots \\ 2 \end{pmatrix}. \tag{A.4}$$

The entries of x_R denote the R -charges. It is clear that, given a solution x_R , one can always add a ‘bosonic’ solution of (A.3) to obtain ‘another’ R -symmetry. However, as the superpotential is a gauge invariant quantity, this does not change any of the conclusions presented in the main text.

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References

- [1] G. 't Hooft, *Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking*, *NATO Adv. Study Inst. Ser. B Phys.* **59** (1980) 135 [[SPIRES](#)].
- [2] E. Witten, *Dynamical Breaking of Supersymmetry*, *Nucl. Phys.* **B 188** (1981) 513 [[SPIRES](#)].
- [3] R. Kappl et al., *Large hierarchies from approximate R symmetries*, *Phys. Rev. Lett.* **102** (2009) 121602 [[arXiv:0812.2120](#)] [[SPIRES](#)].
- [4] J.E. Kim and H.P. Nilles, *The mu Problem and the Strong CP Problem*, *Phys. Lett.* **B 138** (1984) 150 [[SPIRES](#)].
- [5] G.F. Giudice and A. Masiero, *A Natural Solution to the mu Problem in Supergravity Theories*, *Phys. Lett.* **B 206** (1988) 480 [[SPIRES](#)].
- [6] W. Buchmüller, K. Hamaguchi, O. Lebedev and M. Ratz, *Supersymmetric standard model from the heterotic string. II*, *Nucl. Phys.* **B 785** (2007) 149 [[hep-th/0606187](#)] [[SPIRES](#)].
- [7] M.A. Luty and W. Taylor, *Varieties of vacua in classical supersymmetric gauge theories*, *Phys. Rev.* **D 53** (1996) 3399 [[hep-th/9506098](#)] [[SPIRES](#)].

- [8] C.D. Froggatt and H.B. Nielsen, *Hierarchy of Quark Masses, Cabibbo Angles and CP-violation*, *Nucl. Phys. B* **147** (1979) 277 [SPIRES].
- [9] A.E. Nelson and N. Seiberg, *R symmetry breaking versus supersymmetry breaking*, *Nucl. Phys. B* **416** (1994) 46 [hep-ph/9309299] [SPIRES].
- [10] S. Ray, *Supersymmetric and R symmetric vacua in Wess-Zumino models*, arXiv:0708.2200 [SPIRES].
- [11] M. Gomez-Reino and C.A. Scrucca, *Locally stable non-supersymmetric Minkowski vacua in supergravity*, *JHEP* **05** (2006) 015 [hep-th/0602246] [SPIRES].
- [12] O. Lebedev, H.P. Nilles and M. Ratz, *de Sitter vacua from matter superpotentials*, *Phys. Lett. B* **636** (2006) 126 [hep-th/0603047] [SPIRES].
- [13] F. Brümmer, A. Hebecker and M. Trapletti, *SUSY breaking mediation by throat fields*, *Nucl. Phys. B* **755** (2006) 186 [hep-th/0605232] [SPIRES].
- [14] S. Weinberg, *The cosmological constant problem*, *Rev. Mod. Phys.* **61** (1989) 1 [SPIRES].
- [15] J.A. Casas and C. Muñoz, *A Natural Solution to the MU Problem*, *Phys. Lett. B* **306** (1993) 288 [hep-ph/9302227] [SPIRES].
- [16] M. Kreuzer and H. Skarke, *On the classification of quasihomogeneous functions*, *Commun. Math. Phys.* **150** (1992) 137 [hep-th/9202039] [SPIRES].
- [17] O. Lebedev et al., *The Heterotic Road to the MSSM with R parity*, *Phys. Rev. D* **77** (2008) 046013 [arXiv:0708.2691] [SPIRES].
- [18] I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, *Effective mu term in superstring theory*, *Nucl. Phys. B* **432** (1994) 187 [hep-th/9405024] [SPIRES].
- [19] A. Brignole, L.E. Ibáñez and C. Muñoz, *Orbifold-induced mu term and electroweak symmetry breaking*, *Phys. Lett. B* **387** (1996) 769 [hep-ph/9607405] [SPIRES].
- [20] O. Lebedev et al., *A mini-landscape of exact MSSM spectra in heterotic orbifolds*, *Phys. Lett. B* **645** (2007) 88 [hep-th/0611095] [SPIRES].
- [21] M. Cvetič, J. Louis and B.A. Ovrut, *A String Calculation of the Kähler Potentials for Moduli of $Z(N)$ Orbifolds*, *Phys. Lett. B* **206** (1988) 227 [SPIRES].
- [22] A. Brignole, L.E. Ibáñez and C. Muñoz, *Soft supersymmetry-breaking terms from supergravity and superstring models*, hep-ph/9707209 [SPIRES].
- [23] L.J. Dixon, D. Friedan, E.J. Martinec and S.H. Shenker, *The Conformal Field Theory of Orbifolds*, *Nucl. Phys. B* **282** (1987) 13 [SPIRES].
- [24] M. Blaszczyk et al., *A $Z_2 \times Z_2$ standard model*, *Phys. Lett. B* **683** (2010) 340 [arXiv:0911.4905] [SPIRES].
- [25] B. Dundee, S. Raby and A. Westphal, *Moduli stabilization and SUSY breaking in heterotic orbifold string models*, arXiv:1002.1081 [SPIRES].
- [26] P. Hosteins, R. Kappl, M. Ratz and K. Schmidt-Hoberg, *Gauge-top unification*, *JHEP* **07** (2009) 029 [arXiv:0905.3323] [SPIRES].
- [27] G. Burdman and Y. Nomura, *Unification of Higgs and Gauge Fields in Five Dimensions*, *Nucl. Phys. B* **656** (2003) 3 [hep-ph/0210257] [SPIRES].
- [28] K.-w. Choi et al., *Electroweak symmetry breaking in supersymmetric gauge - Higgs unification models*, *JHEP* **02** (2004) 037 [hep-ph/0312178] [SPIRES].

- [29] F. Brümmer, S. Fichtel, A. Hebecker and S. Kraml, *Phenomenology of Supersymmetric Gauge-Higgs Unification*, *JHEP* **08** (2009) 011 [[arXiv:0906.2957](#)] [[SPIRES](#)].
- [30] H.M. Lee, H.P. Nilles and M. Zucker, *Spontaneous localization of bulk fields: The six-dimensional case*, *Nucl. Phys. B* **680** (2004) 177 [[hep-th/0309195](#)] [[SPIRES](#)].
- [31] W. Buchmüller, K. Hamaguchi, O. Lebedev and M. Ratz, *Supersymmetric standard model from the heterotic string*, *Phys. Rev. Lett.* **96** (2006) 121602 [[hep-ph/0511035](#)] [[SPIRES](#)].
- [32] T. Kobayashi, S. Raby and R.-J. Zhang, *Searching for realistic 4d string models with a Pati-Salam symmetry: Orbifold grand unified theories from heterotic string compactification on a $Z(6)$ orbifold*, *Nucl. Phys. B* **704** (2005) 3 [[hep-ph/0409098](#)] [[SPIRES](#)].
- [33] S. Hamidi and C. Vafa, *Interactions on Orbifolds*, *Nucl. Phys. B* **279** (1987) 465 [[SPIRES](#)].
- [34] T. Kobayashi, H.P. Nilles, F. Plöger, S. Raby and M. Ratz, *Stringy origin of non-Abelian discrete flavor symmetries*, *Nucl. Phys. B* **768** (2007) 135 [[hep-ph/0611020](#)] [[SPIRES](#)].
- [35] B.A. Ovrut and J. Wess, *Supersymmetric $R(\xi)$ gauge and radiative symmetry breaking*, *Phys. Rev. D* **25** (1982) 409 [[SPIRES](#)].