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Research Article

Approximate Solutions of Klein-Gordon Equation with Kratzer Potential

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Approximate solutions of the D-dimensional Klein-Gordon equation are obtained for the scalar and vector general Kratzer potential for any l by using the ansatz method. The energy behavior is numerically discussed.

1. Introduction

The Kratzer potential is amongst the most attractive physical potentials as it contains a degeneracy-removing inverse square term besides the common Coulomb term. It appears in a wide class of physical and chemical sciences including the atomic and molecular physics providing quite motivating results [1–7]. When we deal with this potential within the framework of Schrödinger equation, the problem is simply solved via the analogy with familiar example of 3-dimesnional Coulomb Hamiltonian or many other techniques including series expansions, supersymmetry quantum mechanics (SUSY) [8–10], the Nikiforov-Uvarov (NU) [11], point canonical transformation (PCT) [12–14], and so forth. Such investigations have been done by many authors in the annals of wave equations [15–24]. The problem just arises when we intend to study the problem via the Klein-Gordon (KG) equation. This is because we have to deal with an equivalent potential which includes Coulomb, inverse square, inverse cubic and inverse quadric terms. Until now, no exact analytical solution has been reported for the problem. Within the present study, we study the problem via an Ansatz approach proposed by Dong [25] and numerically report the results.

2. *D*-Dimensions Klein-Gordon Equation

The radial Klein-Gordon equation for a spherically symmetric potential in D-dimensions is

$$-\frac{d^2R_{n,l}(r)}{dr^2}-\frac{d-1}{r}\frac{R_{n,l}(r)}{dr}+\left[\frac{l(l+D-2)}{r^2}+(m(r)+S(r))^2-(E_{n,l}-V(r))^2\right]R_{n,l}(r)=0. \tag{2.1}$$

For the scalar and vector potentials we choose

$$V(r)=\frac{V_0}{r}+\frac{V_1}{r^2},\qquad S(r)=\frac{S_0}{r}+\frac{S_1}{r^2}, \tag{2.2}$$

where r denotes the hyperradius and V_0, S_0, V_1, and S_1 are constant coefficients. For the mass, instead of constant one, we consider a position-dependent mass of the form

$$m(r)=m_0+\frac{m_1}{r}. \tag{2.3}$$

The transformation $R_{n,l}(r)=r^{-(D-1)/2}U_{n,l}(r)$, after inserting (2.2) brings (2.1) into the form

$$\begin{aligned}&\left\{\frac{d^2}{dr^2}+\frac{-2E_{n,l}V_0+2m_0m_1-2m_0S_0}{r}\right.\\&\quad+\frac{V_0^2-2E_{n,l}V_1-\ m_1^2-S_0^2-2m_0S_1-2m_1S_0-(D+2l-1)(D+2l-3)/4}{r^2}\\&\quad\left.+\frac{2V_0V_1-2S_0S_1-2m_1S_1}{r^3}+\frac{V_1^2-S_1^2}{r^4}+\left(E_{n,l}{}^2-m_0^2\right)\right\}U_{n,l}(r)=0.\end{aligned} \tag{2.4}$$

Choosing

$$\begin{aligned}F&=2E_{n,l}V_0-2m_0m_1+2m_0S_0,\\C&=-V_0^2+2E_{n,l}V_1+m_1^2+S_0^2+2m_0S_1+2m_1S_0+\frac{1}{4}(D+2l-1)(D+2l-3),\\B&=-2V_0V_1+2S_0S_1+2m_1S_1,\\A&=-V_1^2+S_1^2,\\\varepsilon_{n,l}&=\left(E_{n,l}{}^2-m_0^2\right).\end{aligned} \tag{2.5}$$

Equation (2.4) is more neatly written as

$$\left\{\frac{d^2}{dr^2}-\frac{F}{r}-\frac{C}{r^2}-\frac{B}{r^3}-\frac{A}{r^4}+\varepsilon_{n,l}\right\}U_{n,l}(r)=0. \tag{2.6}$$

The Schrödinger analogue of this problem has been analyzed by Dong [25]. We choose [25]

$$U_{n,l}(r) = h_n(r)\exp[k_l(r)], \tag{2.7}$$

where

$$h_n(r) = \begin{cases} 1 & n = 0 \\ \prod_{i=1}^{n}(r - \delta_i^n) & n \geq 1, \end{cases} \tag{2.8}$$

$$k_l(r) = \frac{a}{r} + br + c\ln r, \quad a < 0,\ b < 0. \tag{2.9}$$

Substitution of (2.9), (2.8), and (2.7) in (2.4), after equating the corresponding coefficients on both sides, gives

$$\begin{aligned} a &= -\sqrt{A}, \\ b &= -\sqrt{-\varepsilon_{0,l}}, \\ 2a(1-c) &= B, \\ -c + c^2 - 2ab &= C, \\ 2bc &= F. \end{aligned} \tag{2.10}$$

From (2.5) and (2.10), the energy of the nodeless state is obtained as

$$\left({E_{0,l}}^2 - m_0^2\right) = -\frac{1}{16A}\left[C \pm \sqrt{C^2 - 2BF}\right], \tag{2.11}$$

with its corresponding eigenfunction being obtained by substitution of (2.8), (2.9), and (2.10) in (2.7) as

$$U_{0,l}(r) = N_{0,l}r^{-F/2\sqrt{-\varepsilon_{0,l}}}\exp\left[\frac{-\sqrt{A}}{r} - \sqrt{-\varepsilon_{0,l}}r\right]. \tag{2.12}$$

In Table 1, we have reported the eigenvalues for Ds and ls. Repeating the same procedure for the first node, the eigenvalues are found as

$$\left({E_{1,l}}^2 - m_0^2\right)_{\pm} = -\left(\frac{C \pm \left[C^2 - 4F\left(\delta_1^{(1)} + \sqrt{A}\right)\left(1 + \left(B/\sqrt{4A}\right)\right)\right]^{1/2}}{4\left(\delta_1^{(1)} + \sqrt{A}\right)}\right)^2, \tag{2.13}$$

Table 1: Energy for various D and m_0s.

D	$m_1 = 0$	$m_1 = 0.3$	$m_1 = 0.6$	$m_1 = 0.9$
		$E_{0,0}$		
0	−1.97449	−1.91048	−1.82626	−1.73275
1	−1.98553	−1.94026	−1.87175	−1.78922
2	−1.98766	−1.947	−1.88311	−1.80423
3	−1.98553	−1.94026	−1.87175	−1.78922
4	−1.97449	−1.91048	−1.82626	−1.73275
5	−1.88838	−1.78173	−1.67846	−1.57745
6	−1.90592	−1.3237	−0.94114	−1.04893
7	−1.98238	−1.87584	−1.45224	−0.63401
8	−1.98238	−1.95096	−1.84827	−1.50584
9	−1.98238	−1.95096	−1.91782	−1.8237
10	−1.98238	−1.95096	−1.91782	−1.88619

Table 2: Energy for $l = 1$ and various Ds.

D	1 s		1 p		1 d		1 f	
				$E_{1,l}$				
1	−1.70	1.98	−1.77	1.98	−1.70	1.98	−1.05	1.98
2	−1.75	1.96	−1.75	1.96	−1.54	1.96	−0.30	1.95
3	−1.77	1.93	−1.70	1.93	−1.05	1.93	−1.31	1.93
4	−1.75	1.91	−1.54	1.91	−0.30	1.91	−1.75	1.90
5	−1.70	1.89	−1.05	1.89	−1.31	1.88	−1.87	1.87
6	−1.54	1.86	−0.30	1.86	−1.75	1.85	−1.87	1.84
7	−1.05	1.83	−1.31	1.83	−1.87	1.82	−1.87	1.80
8	−0.30	1.80	−1.75	1.79	−1.87	1.78	−1.87	1.76
9	−1.31	1.77	−1.87	1.76	−1.87	1.74	−1.87	1.71
10	−1.75	1.73	−1.87	1.71	−1.87	1.69	−1.87	1.65

where

$$\delta_1^1 = \frac{\left(\left(B/\sqrt{4A}\right)+2\right)}{D}\left\{C - \left(1+\frac{B}{\sqrt{4A}}\right)\left(2+\frac{B}{\sqrt{4A}}\right)\right\} - \sqrt{A}. \tag{2.14}$$

And the corresponding eigenfunction is

$$U_{1,l}(r) = N_{1,l}\left(r - \delta_1^{(1)}\right) r^c \exp\left[\frac{a}{r} + br\right]. \tag{2.15}$$

Also in Table 2, as well as Figures 1 and 2, we have reported the energy behavior for various conditions. The figures well illustrate the symmetries of energy relation.

3. Conclusion

Approximate analytical solutions of Klein-Gordon equation are reported for the Kratzer potential using the Ansatz method. The behavior of energy eigenvalues on dimension and

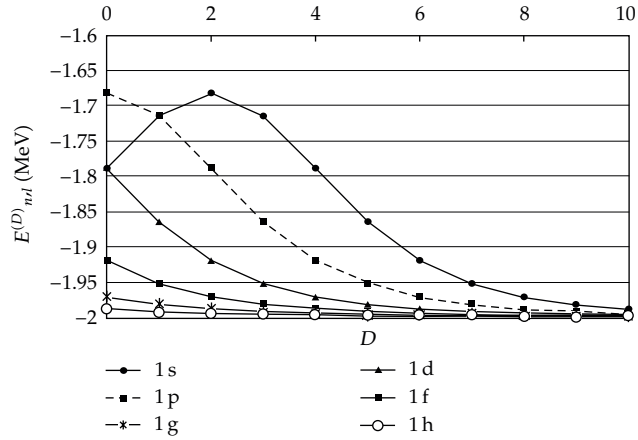

Figure 1: Energy versus dimension for $l = 1$.

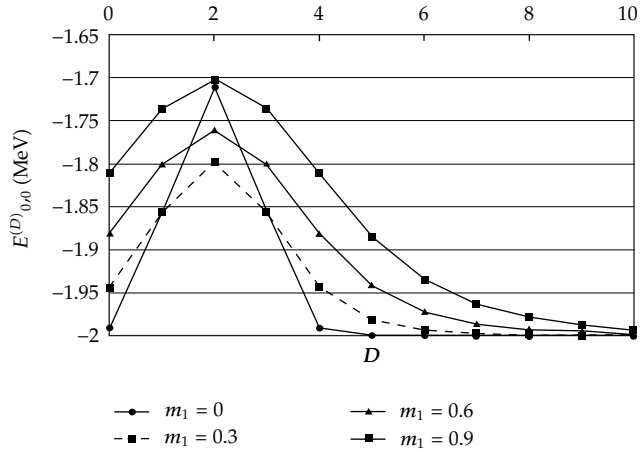

Figure 2: Energy versus dimension for various m_1s.

quantum numbers is numerically calculated. The results are applicable to some branches of physics, particularly atomic, molecular, and chemical physics, where a spin-0 system is being investigated.

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