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APPROXIMATING COMMON FIXED POINT VIA ISHIKAWA'S ITERATION

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Abstract. In this work, we approximate a common fixed point of mappings $F, G : M \cup N \to M \cup N$, satisfying the conditions

(1) $G(M) \subseteq M, G(N) \subseteq N, F(M) \subseteq M$ and $F(N) \subseteq N$;
(2) $\|Fu - Gv\| \leq \|u - v\|$ for $u \in M, v \in N$; and
(3) $\|Fu - Gv\| \leq \|u - v\|$ for $u \in N, v \in M$,

where M and N are nonempty bounded closed convex subsets of a uniformly convex Banach space. We consider Ishikawa iteration associated with F and G and von Neumann sequence associated with Ishikawa iteration to approximate the common fixed point of F and G. We prove convergent results for common fixed point of F and G. Finally, we give corollaries on common best proximity point for cyclic mappings.

Key Words and Phrases: Nonexpansive mappings, best proximity points, fixed points, Banach space, Von Neumann sequences.
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