Approximation of the `True Unfalsified Set'

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Jeroen van Helvoort, Bram de Jager and Maarten Steinbuch
Technische Universiteit Eindhoven
Mechanical Engineering, Control Systems Technology
PO Box 513, WH -1.125, 5600 MB Eindhoven, The Netherlands
Email: j.j.m.v.helvoort@tue.nl

1 Introduction

Typically, control design focusses on finding one value for the controller parameters, for a given problem. Unfalsified Control (Safonov and Tsao, 1997), however, tries to find the entire set of controller parameters, which are able to satisfy a given performance requirement.

Definition 1 (True Unfalsified Set) *The set of controller parameters, which satisfies the performance requirements (for a given plant)*

In Ellipsoidal Unfalsified Control (EUC) (Van Helvoort, De Jager and Steinbuch, 2005), an outer-bounding approximation is made of the True Unfalsified Set, using an easily computable ellipsoidal approximation. In a simulation, a comparison is shown between the True Unfalsified Set and the approximation thereof.

2 Simulation

Consider the plant $y(t) = P(s) * u(t)$, with

$$P(s) = \frac{0.01s^2 + 0.4s + 100}{s^2 + 4s + 100} \tag{1}$$

Also consider the controller

$$u(t) = \frac{1}{\theta_1}\left(r(t) - \theta_2 \frac{s+20}{s+2} * y(t)\right) \tag{2}$$

Here, θ_1 and θ_2 are the controller parameters and $r(t)$ is the reference $r(t) = \sin(\pi t)$. Then the closed loop steady state response $Y(s)/R(s)$ is given by

$$\frac{Y(s)}{R(s)} = \frac{\frac{1}{\theta_1}P(s)}{1 + \frac{\theta_2}{\theta_1}\frac{s+20}{s+2}P(s)} \tag{3}$$

The performance requirement is given by $1 - \Delta \leq Y(s)/R(s) \leq 1 + \Delta$, with $\Delta \geq 0$ a threshold on the allowable tracking error.

2.1 True Unfalsified Set

The performance requirement can be rewritten as

$$1 - \Delta \leq Y(s)/R(s) \leq 1 + \Delta \Leftrightarrow \tag{4}$$

$$-\Delta \leq Y(s)/R(s) - 1 \leq \Delta \Leftrightarrow \tag{5}$$

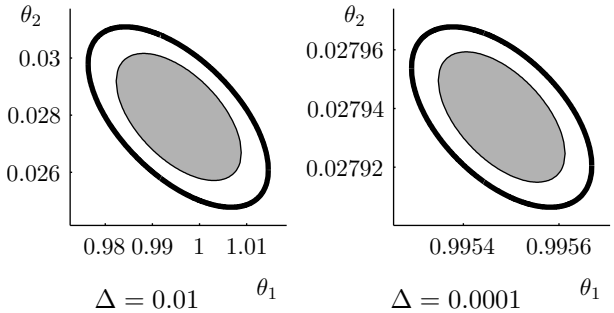
$$(Y(s)/R(s) - 1)'(Y(s)/R(s) - 1) \leq \Delta^2 \tag{6}$$

The performance requirement is evaluated at $s = j\pi$ (since $r(t) = \sin(\pi t)$). From (3) and (6), it can be seen that the True Unfalsified Set is an ellipsoid in the controller parameter space, with size dependent on Δ.

2.2 Ellipsoidal Unfalsified Control

EUC is a data-driven, model-free controller design method, which recursively falsifies controller parameter sets that fail to meet the specified performance requirement. An outer-bounding ellipsoidal approximation is used to describe the set of unfalsified controllers.

2.3 Results



True Unfalsified Set (grey, solid ellipsoid) and the outer bounding approximation, computed by EUC (black, open ellipsoid), for two values of Δ.
The distinction between the two sets originates from the outer-bounding approach of the EUC algorithm.

3 Remark

In EUC, the approximation of the True Unfalsified Set is used to select suitable controller parameters for implementation.

References

Van Helvoort, J., De Jager, B. and Steinbuch, M. (2005), Unfalsified control using an ellipsoidal unfalsified region applied to a motion system, *in* 'Proc. IFAC World Congress', Prague, Czech Republic, pp. CD–ROM.

Safonov, M. and Tsao, T.-C. (1997), 'The unfalsified control concept and learning', *IEEE Trans. Automatic Control* **42**(6), 843–847.