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Abstract—In very-large multiple-input multiple-output (MIMO) systems, the base station (BS) is equipped with very large number of antennas as compared to previously considered systems. There are various advantages of increasing the number of antennas, and some schemes require handling large matrices for joint processing (pre-coding) at the BS. The dirty paper coding (DPC) is an optimal pre-coding scheme and has a very high complexity. However, with increasing number of BS antennas, linear pre-coding performance tends to that of the optimal DPC. Although linear pre-coding is less complex than DPC, there is a need to compute pseudo inverses of large matrices. In this paper we present a low complexity approximation of down-link Zero Forcing (ZF) linear pre-coding for very-large multi-user MIMO systems. Approximation using a Neumann series expansion is opted for inversion of matrices over traditional exact computations, by making use of special properties of the matrices, thereby reducing the cost of hardware. With this approximation of linear pre-coding, we can significantly reduce the computational complexity for large enough systems, i.e., where we have enough BS antenna elements. For the investigated case of 8 users, we obtain 90% of the full ZF sum rate, with lower computational complexity, when the number of BS antennas per user is about 20 or more.

I. Introduction

Multiple-Input Multiple-Output (MIMO) techniques for wireless communication offer high data rates and reliability through the utilization of multiple transmit and receive antennas. These techniques are becoming more mature and have been incorporated in advanced standards like LTE (Long Term Evolution) Release 10 [1] to meet the International Mobile Telecommunications-Advanced (IMT-A) requirements of gigabits-per-sec data rates. Basically, the more antennas the transceivers are equipped with, the better performance can be obtained in terms of data rate, diversity (reliability) and spectral efficiency.

In [2], a Multi-User (MU) MIMO system with (an assumption of) unlimited number of base station (BS) antennas in a multi-cell environment is investigated. It is shown that all the effects of uncorrelated noise and fast fading disappear, as does the intra-cell interference. The assumption of an unlimited number of BS antennas greatly simplifies the theoretical analysis. However, it is obvious that in a practical system the number of antennas cannot be arbitrarily large due to physical, cost, and power constraints.

The theoretical analysis in [2], assumes that inner products between propagation vectors of different users grow at a slower rate than inner products of the propagation vectors with themselves when the number of antennas grow, *i.e.*, the user channels are asymptotically orthogonal. In [3], measurements in a realistic propagation environment for large array of antennas at a BS (up to 128 antennas at the BS and 26 different single antennas users) are performed. It was shown that by using reasonably large antenna arrays it is possible to decorrelate single user channels. Furthermore, in [4], residential area measurements for very-large MIMO system were performed, showing linear pre-coding sum rates of up to 98% of those achieved by dirty paper coding (DPC), for BS to Mobile Station (MS) antenna ratios as low as 10.

Although there is a clear benefit of scaling up the number of BS antennas, including an almost (near) optimal linear pre-coding, the hardware cost and signal processing complexity can be very high. When using linear pre-coding, such as Zero-Forcing (ZF), we can operate at the above mentioned BS/MS antenna ratios around 10 and the main source of complexity for ZF pre-coding becomes the inverse of a $K \times K$ matrix, where K is the number of users. The assumption of a significantly higher number of antenna elements does not affect the dimensions of this matrix, but it does offer the opportunity to carry out the matrix inverse by much simpler means than outright inversion.

There are various hardware implementations for matrix inversion using different algorithms, QR-Gram Schmidt [5], QR-Givens Rotation [6], and Gauss-Jordan [7]. While these methods are generic and work well for any type of matrix, we can exploit the special structure of matrices appearing in very-large MIMO to reduce the complexity of the linear pre-coding and make it more hardware efficient. To meet these objectives, approximations (using Neumann series) of matrix inversion is opted rather than computing the exact inverse. We describe the general setting in which our pre-coding is assumed to operate, discuss the linear class of pre-coders, and finally focus on complexity of the ZF pre-coder, when using QR-Gram Schmidt and Neumann series expansion to perform matrix inversion. We derive and compare complexity both in terms of the number of operations and the energy required to perform the computations.
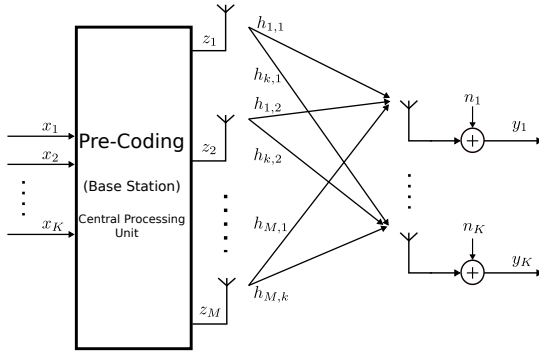
Fig. 1: System model of a MU-MIMO system with M-antenna base station and K single-antenna mobiles stations.

II. SYSTEM DESCRIPTION

The system model and the pre-coding in the following section is in line with the corresponding description in [8]. A MU-MIMO system model consists of a BS equipped with M antennas, which is simultaneously serving K single users antenna, as shown in Fig. 1. The received vector y of size $K \times 1$ is described as

$$y = \sqrt{\rho} H z + n, \tag{1}$$

where H is the $K \times M$ propagation matrix of complex-valued channel coefficients, z is an $M \times 1$ transmit vector, and n is an additive noise vector with Independent and Identically Distributed (IID) zero-mean and unit-variance complex Gaussian random variables. The scalar ρ is a measure of the Signal-to-Noise Ratio (SNR) of the link, which is proportional to the transmitted power divided by the noise-variance. Furthermore, it also absorbs various normalizing constants. The total transmit power is normalized and independent of the number of antennas M, the transmit vector z satisfies $\mathbb{E}\{||z||^2\} = 1$.

The pre-coding process at the transmit side is specified as

$$z = F x, \tag{2}$$

where F is a $M \times K$ pre-coding matrix, x a $K \times 1$ vector containing user symbols, as described in [4].

Although the very-large MU-MIMO model is similar to a standard MIMO model, the increased number of BS antennas has several consequences. Things that were random before, now start to look deterministic. For example, the distribution of the singular values of the channel matrix approaches a deterministic function [9]. Another observed property is that very wide (or tall) matrices tend to be very well conditioned.

III. LINEAR PRE-CODING SCHEMES

The optimal sum rate in the downlink of a MU-MIMO system with prefect channel state information (CSI) can be achieved by the interference pre-subtraction coding technique called DPC [10]. The optimal sum rate is given as

$$C_{\text{DPC}} = \max_{\text{Tr}(P)=1} \log_2 \det\left(I + \rho H^H P H\right), \tag{3}$$

where P is a $K \times K$ diagonal matrix for power allocation. The sum rate is maximized by optimizing the power allocation under the constraint that $\text{Tr}(P) = 1$, where $\text{Tr}(\cdot)$ is the trace operator.

Although optimal sum rate is achieved by DPC, this approach is too resource expensive to be implemented in hardware and is used as a benchmark for ZF, Minimum Mean Square Error (MMSE) and low complexity approximations of ZF. The pre-coding matrix F can be decomposed as

$$F = \frac{1}{\sqrt{\gamma}} W \sqrt{P}, \tag{4}$$

where W represents a particular linear pre-coding algorithm, and $\gamma = ||W \sqrt{(P)}||_F^2$, is a power normalization factor, where $|| \bullet ||_F$ is Frobenius Norm.

A. ZF pre-coding scheme

ZF linear pre-coding transmits user signals towards the intended user with nulls steered in the direction of other users. The ZF pre-coder is given as

$$W_{\text{ZF}} = H^\dagger, \tag{5}$$

where $H^\dagger = H^H (H H^H)^{-1}$ is the pseudo-inverse of the channel matrix H. A perfect CSI at the transmitter and nulling makes this scheme interference free, and the sum rate is given as

$$C_{\text{ZF}} = \max_{\text{Tr}(P)=1} \sum_{i=1}^{K} \log_2 \left(1 + \frac{\rho P_i}{\gamma}\right). \tag{6}$$

As the number of BS antennas M increases, H tends to have nearly orthogonal columns as the terminals are not correlated due to their physical separation. This assures that the performance of ZF pre-coding will be close to that of optimal DPC pre-coding. However, ZF pre-coding requires computation of the pseudo-inverse (in (5)), which requires the computationally expensive inversion of a $K \times K$ matrix.

B. MMSE pre-coding scheme

MMSE pre-coding can trade interference reduction for signal power inefficiency. The MMSE pre-coder is given as

$$W_{\text{MMSE}} = H^H \left(H H^H + \alpha I\right)^{-1}, \tag{7}$$

where $\alpha = K/\rho$. At low SNRs (large α) the MMSE approaches a Matched Filter (MF) pre-coder, i.e., $W_{MF} = H^H$, and at high SNRs (low α) it approaches the ZF pre-coder.

C. Low Complexity Pre-Coding

A problem with both ZF and MMSE pre-coding is the inverse operation of the $K \times K$ matrix. Since the complexity for both linear pre-coders is similar (when α is not large in (7)), in this paper we analyse impact of low complexity (approximations) only on ZF pre-coder. A standard and expensive approach would be to compute the exact inverse of the matrix $Z(\triangleq H H^H)$ in

$$W_{\text{ZF}} = H^\dagger = H^H (H H^H)^{-1} = H^H (Z)^{-1}. \tag{8}$$

However, as the number of BS and MS antennas (M and K) increases, the eigenvalues of the matrix Z converges to a fixed deterministic distribution known as the Marchenko-Pastur distribution. Now following the analysis in [8], the largest and the smallest eigenvalues of Z converge to

$$\lambda_{\max}(Z) \to \left(1 + \frac{1}{\sqrt{\beta}}\right)^2, \; \lambda_{\min}(Z) \to \left(1 - \frac{1}{\sqrt{\beta}}\right)^2,$$

where ($\beta = M/K$), as M and K grows to infinity. By scaling the Z matrix with a factor $\left(\frac{\beta}{1+\beta}\right)$, the eigenvalues are found in the region

$$\lambda_{\max}\left(\frac{\beta}{1+\beta}Z\right) \to \left(1 + 2\frac{\sqrt{\beta}}{1+\beta}\right),$$

$$\lambda_{\min}\left(\frac{\beta}{1+\beta}Z\right) \to \left(1 - 2\frac{\sqrt{\beta}}{1+\beta}\right). \quad (9)$$

Hence, the eigenvalues of $I_K - \beta/(1+\beta)Z = I_K - Z/(M+K)$ lie in the range $[-2\sqrt{\beta}/(1+\beta), 2\sqrt{\beta}/(1+\beta)]$, where I_K is an $K \times K$ identity matrix. By asymptotically increasing β, the eigenvalues of $I_K - Z/(M+K)$ lie in the range

$$\lim_{\beta \to +\infty} \left[\left(-2\frac{\sqrt{\beta}}{1+\beta}\right), \left(2\frac{\sqrt{\beta}}{1+\beta}\right)\right] \to [-0, 0]. \quad (10)$$

Therefore, as β grows, the faster is the convergence of

$$\lim_{n \to \infty} \left(I_K - \frac{1}{M+K}Z\right)^n \simeq 0_K. \quad (11)$$

It is known that if a matrix satisfies (11), its inverse can be expressed by Neumann series [11] as

$$Z^{-1} \approx \frac{\delta}{M+K} \sum_{n=0}^{L} \left(I_K - \frac{\delta}{M+K}Z\right)^n, \quad (12)$$

with equality when L grown to infinity, and $\delta < 1$ is a attenuation factor introduced, since for finite M and K the eigenvalues of channel realizations may lie outside the range specified in (9). For a feasible implementation of a matrix inversion using Neumann series the number of iterations (L) needs to be finite (or small).

The inverse of Z is approximated by a summation of powers of a matrix (or matrix multiplications) (12), which has a complexity order $\mathcal{O}((L-1) \cdot K^3)$. Although the complexity order can be equal or higher (depending on L) than computing the exact inverse (direct inversion, QR based *etc*), matrix multiplications are preferable in hardware compared to exact inversion.

The convergence of (11) is based on the fact that the eigenvalues lie in the range given by (9) as M and K grows asymptotically. However, for practical systems with finite M and K the eigenvalues may lie outside this range. In addition to what is described in [8], we introduce one modification of the Neumann series inversion. It is based on the fact that the closer the eigenvalues of our matrix are to 1, the faster the convergence of the series in (12).

The modification is described as follows. The scalar multiplication by $\delta/(M+K)$ in (12) is represented as a diagonal matrix

$$D_{\mathrm{MP}} = \frac{\delta}{M+K}I_K.$$

Using this notation, (12) is rewritten as

$$Z^{-1} \approx \sum_{n=0}^{L} \left(I_K - D_{\mathrm{MP}}Z\right)^n D_{\mathrm{MP}}, \quad (13)$$

the accuracy of the approximation, for a given number of terms (L), depend on the size of the eigenvalues of $(I - D_{\mathrm{MP}}Z)$. The smaller their magnitude, the faster the convergence. Given this, we want to pre-condition our matrix Z so that it will lead to a fast convergence for a finite M and K system.

Now, assume that we want to pre-condition it with a diagonal matrix D, with non-zero diagonal entries. In principle, we would like to calculate the eigenvalues of $(I - DZ)$ and optimize D so that the magnitudes of the eigenvalues are as small as possible. This, however, is a complex and non-trivial task. We will therefore use Gershgorins circle theorem [12] to derive an upper bound of the magnitude of the eigenvalues. By keeping this bound small, by selecting D, we can also guarantee that the magnitude of the eigenvalues are small.

In this derivation of the "optimal" D we will assume that the Hermitian matrix $Z = HH^H$ is diagonally dominant, meaning that the magnitude of the diagonal elements z_{ii} are larger than the sum of the magnitude of the off-diagonal elements in the same row, $z_{ij}, i \neq j$, namely that $|z_{ii}| > \sum_{i \neq j} |z_{ij}|$. The largest magnitude of any eigenvalue of $(I - DZ)$ is upper bounded by

$$\max_i |\lambda_i| \leq \max_i \left(|1 - d_i z_{ii}| + d_i \sum_{i \neq j} |z_{ij}|\right), \quad (14)$$

and under the condition of a diagonally dominant Z, the smallest upper bound is obtained if $d_i = 1/z_{ii}$. For this selection of D we also have that $\max_i |\lambda_i| < 1$, which guarantees convergence of the Neumann series. Hence, our final approximation of the inverse of a diagonally dominant Z is a matrix $D = \mathrm{diag}(1/z_{11}, 1/z_{22}, ..., 1/z_{kk})$, the inverse can be expressed using Neumann series as

$$Z^{-1} \approx \sum_{n=0}^{L} \left(I_K - DZ\right)^n D. \quad (15)$$

A fast (or accelerated) way to compute the series (15) and (12), up to $L = 2^P - 1$ terms, where P is an integer, is to use the identity

$$Z^{-1} \approx \sum_{n=0}^{L} \left(I_K - DZ\right)^n D = \left(\prod_{n=0}^{P-1} (I + (I - DZ)^{2^n})\right)D, \quad (16)$$

which leads to a numerical complexity proportional to the logarithm of the number or terms in the truncated series. In terms of number of matrix multiplications, the brute force computation of the inverse using (15) (or (12)) would require
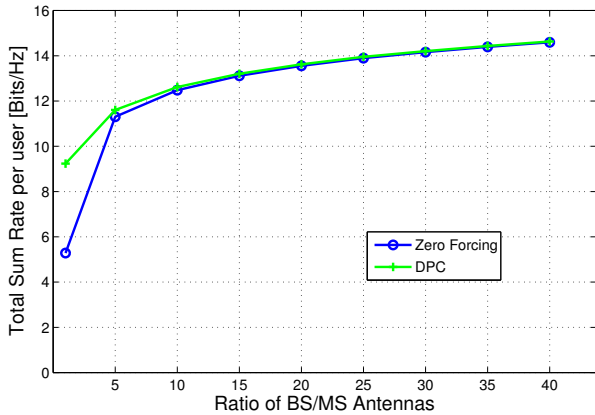
Fig. 2: Sum rate for pre-coding schemes at SNR = 30 dB, number of users ($K = 8$), in very-large MU-MIMO systems.
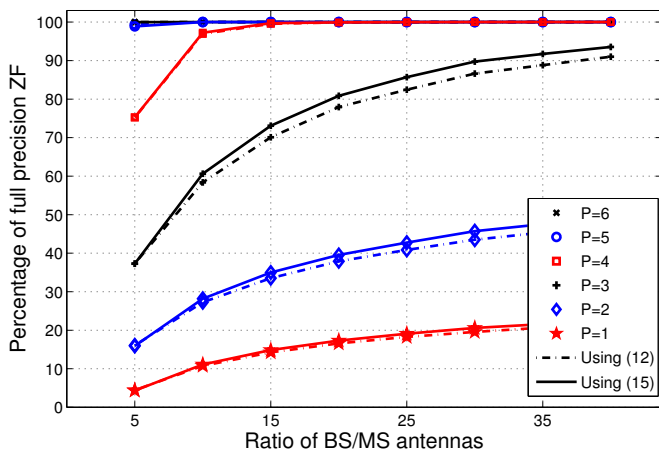

Fig. 3: Percentage of Ideal ZF sum rate achieved by approaches (12) and (15) at SNR = 30 dB.

$L - 1$, whereas computation using (16) would require only $2(P - 1)$ matrix multiplications, where $P = \log_2(L + 1)$.

In [13], a method to solve linear systems using Operator Perturbation Technique (OPT) is described. It can be shown that this method is essentially the same as our Neumann series, and the acceleration we proposed can be applied to OPT for an exponential convergence. In [14], another method to accelerate OPT is developed, and it would be interesting to merge these techniques, as part of our future work.

The accuracy of the inversion depends on the convergence of equations (16) with iterations P. Therefore, it is important to have a trade-off between complexity and the accuracy of the approximation. In the next section the impact of these low complexity approximation techniques on the overall system is analysed.

IV. PERFORMANCE SIMULATION

With an increased number of antennas at the BS, the user channels become less and less correlated, as shown in [4]. In this paper a Gaussian IID channel model is chosen to analyse the effects of low complexity pre-coders.

TABLE I: Complexity comparison for $k \times k$ matrix inversion.

		Multiplication	Division	Square Root
Neumann series	Using (12)	$2(P-1)k^3$	k [+]	0
	Modified (16)	$2(P-1)k^3+k^2$	k	0
QR	Gram-Schmidt	$3k^3 + 2k^2$ [*]	$\frac{3}{2}k^2$	k
	Householder	$\frac{3k^4+k^2}{2}$	$\frac{5}{2}k^2$	$2k$
	Givens	$\frac{8}{3}k^3$	$k^2 - k$ [#]	$k^2 - k$ [#]
Gauss-Jordan		$k^3 + k^2$	$\frac{k^2+k}{2}$	0

[+] Constant Division, implement using multiplier.
[*] Only QR complexity, does not include inversion *i.e.* ($R^{-1}Q^H$).
[#] Usually implemented as a unified cordic block.

The sum rate for different pre-coders with increasing M (and a fixed number of users $K = 8$) is shown in Fig. 2. The ideal (floating point precision) ZF pre-coder approaches optimal DPC pre-coding, as the ratio $\beta = M/K$ increases. Due to expected favourable values on β in very large MIMO systems, linear pre-coders (ZF, MMSE) are close to optimal pre-coders. The Neumann series approximations approaches ideal full precision ZF with increase in iterations (P). Computational complexity of the Neumann series approximation increases with P and beyond a certain value it becomes more complex than exact matrix inversion. However, increasing the ratio of antennas (β) the number of iterations (complexity) to reach certain level of sum rate reduces. To further analyse this trend, the ideal floating point ZF is taken as a benchmark to evaluate the Neumann series approximation (required number of iterations to reach percentage of ZF sum rate) as shown in Fig. 3. It should be noted that the computation of both the methods are performed using the identity (16), except for different approach of pre-conditioning matrices. The convergence of the modified method (using (15)) is slightly higher, *e.g.*, when $P = 3$, it reaches 90% level for β around 25 whereas the other method requires β around 35. In the next section the feasibility of this approach is compared with other matrix inversion methods, wherein various factors like order of complexity, energy efficiency, hardware cost (area) and re-usability is taken into account.

V. RESULTS AND COMPLEXITY ANALYSIS:

In Table I, complexity (# of multiplications) for matrix inversion using different algorithms is shown. Guass-Jordan elimination requires lower multiplications than other algorithms, however the mathematical properties (dynamic range, stability) would require expensive floating point implementations in hardware [7]. QR decomposition has very good mathematical properties but requires specialized circuits and operations including k^2 divisions and k square root. The approximation using Neumann series requires fewer division operations (compared to QR and Guass-Jordan) and has a variable multiplication complexity based on the iterations (P). Hence its important to analyse the impact of P on system performance.

In Fig. 4(a), a complexity comparison for reaching a certain level (90% of ZF) sum rate is shown. For $\beta > 27$, the approximations have lower complexity ($P = 3$) than a QR
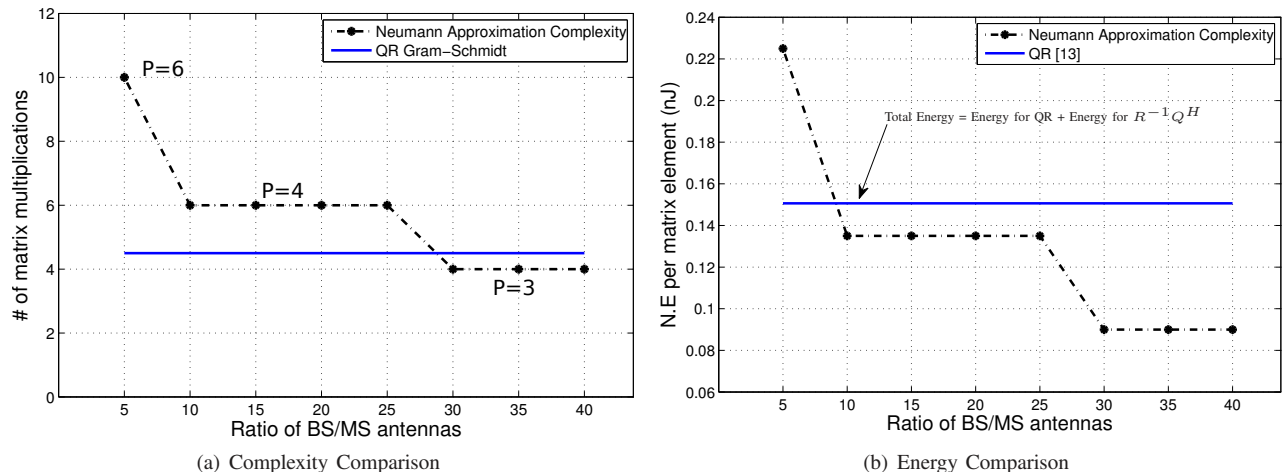
(a) Complexity Comparison

(b) Energy Comparison

Fig. 4: Performance comparison of Neumann series based approximation to achieve 90% of Ideal ZF for different β values.

based matrix inversion. Furthermore, in hardware the Neumann series approximation can be implemented by using simple matrix multiplication, whereas QR decomposition requires complicated data-flow mechanisms and specialized units (*e.g.*, square root, sin/cos functions). Therefore, in terms of area (hardware cost) and energy we expect that the crossover point would be even lower ($\beta < 27$) for Neumann series approximations over exact matrix inversion.

As a case study (for energy comparison), we choose a state of the art 4x4 QR decomposition reported in literature [15], which requires a Normalized Energy[1] (N.E) of 2.05 nJ per QR decomposition. A 4x4 matrix multiplier implemented in 65 nm CMOS technology and operating at 1.2 V, required 0.36 nJ of energy[2] per matrix multiplication (from [16]). Using these energy numbers the crossover point as expected reduced to around $\beta = 10$, see Fig 4(b). Although this case study is for a 4x4 matrix, we do expect Neumann series (implemented using matrix multiplication) to be energy efficient even for larger matrices, implying a lower crossover point.

Furthermore, the matrix multiplication can be implemented using a generic Multiply-and-Accumulate (MAC) structure, which can easily be reused for other computations in the BS. Another feature is that the lower (first) order approximations can be made available earlier where lower precision approximations can be used.

VI. Conclusion

While performance of linear pre coding schemes is known to be good for very-large MIMO systems, little is know about the computational complexity required. We have investigated

Neumann series approximation of ZF pre-coding, leading to an approximate low-complexity scheme. The performance versus complexity relation is most attractive for a high ratio between numbers of BS and terminal antennas. Counting complexity only in terms of required multiplications, the ratio has to be about 27 before the Neumann series approximation is competitive. However, when considering required computational energy, a 4x4 matrix case study showed Neumann series to be competitive for ratio around 10. There are also other complexity related issues (*e.g.*, hardware cost, throughput) not accounted in this study that may change this. The matrix multiplication used in the Neumann series approximation has a high potential of being re-used in other parts of the transceiver, it has relatively simple data flows and it does not require specialized arithmetic units, as compared to other algorithms. When these things are accounted for, we expect the Neumann series approximation to be a competitive approach for linear pre-coding in very large MIMO system. More detailed studies will be performed, where hardware architecture and implementation aspects are taken into account.

VII. Acknowledgment

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[1]Normalized Energy $= \text{Energy} \times \frac{1.2\text{V}}{\text{Voltage}} \times \frac{65\text{nm}}{\text{Technology}}$

[2]Energy simulation is performed on gate-level netlist with back annotated timing and toggle information.

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