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Are preschoolers who spontaneously create patterns better in mathematics

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Nore Wijns, Bert De Smedt, Lieven Verschaffel, Joke Torbeyns

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Title: ARE PRESCHOOLERS WHO SPONTANEOUSLY CREATE PATTERNS BETTER IN MATHEMATICS?

Short title: *SPONTANEOUS CREATION OF PATTERNS*

Nore Wijns*[1], Bert De Smedt[2], Lieven Verschaffel[1], and Joke Torbeyns[1],

[1]Centre for Instructional Psychology and Technology, KU Leuven, Belgium

[2]Parenting and Special Education Research Unit, KU Leuven, Belgium

*Corresponding author information: Nore Wijns, KU Leuven, Centre for Instructional Psychology and Technology, Dekenstraat 2, Postbox 3773, 3000 Leuven, Belgium. (e-mail: nore.wijns@kuleuven.be).

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Abstract:

Background: Early patterning competence has recently been identified as an important precursor of mathematical development. Whereas the focus of this research has been on children’s ability regarding repeating patterns, children might also differ in their spontaneous attention to patterns.

Aims: The present study aimed to explore four- to five-year olds’ Spontaneous Focusing On Patterns (SFOP) and its association with their patterning and mathematical ability.

Sample: Participants were 378 children (M_{age} = 4y10m; 191 boys) from 17 preschools.

Methods: SFOP was measured with a construction task in which children had to build a tower with 15 blocks of three different colours. The constructions of the children were grouped into three categories (i.e., pattern, random, and sorting). We additionally administered tasks assessing their patterning ability, mathematical ability, spatial ability, and visuospatial working memory.

Results: When building a tower, 37% of the preschoolers spontaneously created a pattern, 49% made a random construction, and 14% sorted the blocks per colour. Preschoolers who spontaneously created a pattern had better patterning and mathematical ability than children in the random group. Group differences in patterning ability and spatial skills accounted for the difference in mathematical ability.

Conclusions: The current data suggest that children's spontaneous attention to patterns is an important component of their mathematical ability. Children's spontaneous pattern constructions may provide opportunities to discuss and practice patterns in preschool settings or at home, but more research is required to further analyse the role of SFOP in early mathematical development.

Keywords:

Preschool; Mathematics; Spontaneous focusing tendencies; Patterning

1 Introduction

A growing body of research addresses young children's tendency to spontaneously attend to mathematical elements in their environment (Hannula-Sormunen, 2015; Verschaffel, Rathé, Wijns, De Smedt, & Torbeyns, 2018). This research distinguishes between children's *spontaneous tendency* to think and behave mathematically, and their mathematical *ability*, i.e., the mathematical knowledge and skills they demonstrate when explicitly guided towards mathematical elements (Mulligan et al., 2018; Verschaffel et al., 2018; Verschaffel, Torbeyns, & De Smedt, 2017). Hannula and Lehtinen (2005) described one such tendency as Spontaneous Focusing On Numerosities (SFON), which is "a separate mental process, one which refers to the child's tendency to spontaneously focus on the aspect of numerosity and utilize his or her enumeration skills in various activity situations" (p. 239). This SFON concept has been extensively researched and its importance for children's mathematical development has been emphasized (Hannula-Sormunen, 2015; Rathé, Torbeyns, Hannula-Sormunen, De Smedt, & Verschaffel, 2016).

Researchers have started to explore children's spontaneous tendency to focus on mathematical aspects other than numerosity, including number symbols (SFONS; Rathé, Torbeyns, De Smedt, & Verschaffel, 2019) and quantitative relations (SFOR; Degrande, Verschaffel, & Van Dooren, 2017; McMullen, Hannula-Sormunen, & Lehtinen, 2014). Some researchers suggested the existence of a Spontaneous Focus On Patterns (SFOP) tendency and its role in early mathematical development (Mulligan et al., 2018; Sarama & Clements, 2009; Sharir, Mashal, & Mevarech, 2015; Verschaffel et al., 2018, 2017). The current study focuses on this SFOP concept, which can be described as the tendency to spontaneously look for, notice, or create patterns. The importance of early patterning for later mathematical development has been found repeatedly (Lüken, 2012; Rittle-Johnson, Fyfe, Hofer, & Farran, 2017; Rittle-Johnson, Zippert, & Boice, 2018), yet these studies focused on children's pattern ability. Few attempts have been made to systematically investigate SFOP. We addressed this gap by exploring young children's SFOP and its association with their patterning and mathematical abilities.

1.1 Early patterning

Patterns have been suggested to form the core of mathematics (Steen, 1988; Wittmann & Müller, 2007) and recent research supports this idea. Early patterning ability has been identified as a precursor for later mathematical development, even after controlling for domain-specific (e.g., numerical ability, geometry; Nguyen et al., 2016; Rittle-Johnson et al., 2017) and domain-general abilities (e.g., spatial skills, verbal ability, and verbal working memory; Rittle-Johnson et al., 2018). There are different types of patterns, including repeating (e.g., ABABAB), growing (e.g., 1 3 5), and spatial structure patterns (e.g., :::). The main focus, both in research and in practice with preschoolers, has been on repeating patterns, which are the most easy and accessible for young children (Author, 2019, for a review). Repeating patterns consist of a unit that repeats (e.g., AB). Understanding this unit of repeat is mathematically valuable, because it is a stepping-stone towards multiplicative reasoning and algebraic thinking (Warren & Cooper, 2007).

Children's patterning ability has been measured by activities in which they are explicitly oriented towards a given pattern and asked to do something with it (e.g., copying or extending the pattern). In line with other spontaneous focusing tendencies, young children may also differ in their SFOP, as a few observational studies have documented. Fox (2005) and Garrick, Threlfall and Orton (2005) described exemplary instances of preschoolers who spontaneously create patterns in a free play context. Seo and Ginsburg (2004) found that pattern and shape activities (e.g., name or create shapes, create patterns) were the most frequently occurring mathematical activities in four- to five-year-olds' free play. Older children more frequently engaged in mathematical activities in general, and pattern and shape activities specifically, whereas gender or income level did not have an impact on the frequency or type of activity (Seo & Ginsburg, 2004).

Researchers have suggested that children's SFOP might be associated with their patterning ability (McKillip, 1970) and, by extension, their mathematical ability (Mulligan & Mitchelmore, 2009). Mulligan and Mitchelmore (2009) introduced the construct "Awareness of Mathematical Patterns and Structures" (AMPS), which consists of "a cognitive and a meta-cognitive component". The cognitive component, described as "knowledge of structure", refers to patterning ability, while the meta-cognitive component, described as the "tendency to seek and analyse patterns" (p. 38), is closely related to the above-mentioned notion of SFOP. The authors claim that children with such a tendency will have better mathematical thinking skills. In another study, Mulligan, Mitchelmore, and Stephanou (2015) found an association between AMPS and mathematical ability, but their instrument to assess young children's AMPS (i.e., the Pattern and Structure Assessment) did not involve tasks measuring this metacognitive component. The instructions of these tasks clearly *guided* children towards patterning and, consequently, this measure cannot be considered as a measure to address children's tendency to *spontaneously* focus on patterns.

1.2 Spontaneous focusing tendencies

Five task criteria have been identified to measure SFON (Hannula-Sormunen, 2015; Rathé et al., 2016). With a few modifications related to the specific domain, these criteria are applicable for tasks that measure other spontaneous focusing tendencies. First, the task has to be novel and not explicitly mathematical. Second, there cannot be any indication of the mathematical nature of the task based on the instruction, previous tasks, or the feedback given by the experimenter. These two criteria suggest that the task should allow children to focus on non-mathematical aspects of the task, besides the mathematical ones. Third, the experimenter needs to have the child’s full attention to prevent that task performance is determined by attention skills. Fourth, the task should not exceed children’s working memory capacity, visuo-motor skills, or verbal skills, to avoid that these general skills affect task performance. Finally, only small numbers of items should be used to avoid the impact of children’s number recognition skills. Tasks that aim to measure other spontaneous mathematical tendencies should similarly require only limited impact of the related mathematical ability.

Using such carefully-designed tasks, researchers have identified individual differences in children’s spontaneous mathematical focusing tendencies, which correlate with their mathematical abilities: Associations are found between SFON(S) and early numerical skills (Hannula-Sormunen, 2015; Hannula-Sormunen, Lehtinen, & Räsänen, 2015; Rathé et al., 2019), as well as between SFOR and fraction knowledge (McMullen et al., 2014). One mechanism underlying these associations is the possibility of *self-initiated practice:* Children with a spontaneous tendency to focus on mathematical elements in their environment will have more opportunities to practice their mathematical abilities and therefore improve them (Hannula-Sormunen, 2015).

1.3 Measuring SFOP

To our knowledge, only one study has tried to explicitly measure SFOP. Sharir, Mashal, and Mevarech (2015) investigated four- to six-year olds' Recognition Of Mathematical Structures (ROMS). They differentiated between recognition of quantities (e.g., □□□), mathematical patterns (e.g., □□ □□ □□), and arithmetic series (e.g., □ □□ □□□). Each of these aspects of ROMS was measured with three tasks. Two tasks asked children to describe a picture of abstract geometric shapes or a real-world situation. In the third task, the experimenter put coloured discs into a box and the children had to do exactly what the experimenter did. Preschoolers were able to spontaneously recognize exact quantities in the ROMS for quantities tasks, as well as – to a lesser extent – mathematical patterns and arithmetic series in their respective ROMS tasks (Sharir et al., 2015). ROMS explained 34% of the variance in mathematical reasoning (measured by a short curriculum-based 15-item test), above age and educational level of the mother.

These ROMS tasks do not meet the criteria for tasks addressing spontaneous mathematical focusing tendencies (Hannula-Sormunen, 2015). The spontaneous nature of the two picture tasks is questionable, as there were hardly any non-mathematical elements in the situation to focus on. The behavioural task highly depended on children's working memory, particularly the mathematical patterns and arithmetic series tasks vs. the quantities task. In view of the absence of an appropriate SFOP measure, we developed a new task. Driven by the evidence from abovementioned exploratory observational studies (Fox, 2005; Garrick et al., 2005; Seo & Ginsburg, 2004), we simulated a free play session and observed the extent to which the child spontaneously created a repeating pattern. We imposed some restrictions to ensure that the setting was the same for each child and to allow for adequate scoring. Specifically, we provided a limited set of carefully chosen materials and the instruction gave no indication of the mathematical nature of the task.

1.4 The current study

Young children might not only differ in their patterning ability, but also in the extent to which they spontaneously focus on patterns. These differences in SFOP might be associated with children's patterning ability (McKillip, 1970), because children who spontaneously look for, notice, or create patterns may have more opportunities to practice their patterning ability, which might improve their patterning ability (cfr. self-initiated practice). These individual differences in SFOP might also be related to general measures of mathematical ability (Mulligan & Mitchelmore, 2009), given the associations between patterning ability and mathematical development (e.g., Nguyen et al., 2016; Rittle-Johnson et al., 2017, 2018). SFOP in itself can have a positive influence on children's mathematical development, given that the search for patterns is at the heart of mathematics (Steen, 1990; Wittmann & Müller, 2007).

We investigated individual differences in young children's SFOP and their association with patterning and mathematical ability. To grasp early traces of SFOP before the start of formal mathematics instruction, we focused on four- to five-year olds. Our research questions were:

(1) Are there individual differences in four- to five-year olds' SFOP tendency?
(2) Is there an association between four- to five-year olds' SFOP tendency and their patterning ability?
(3) Is there an association between four- to five-year olds' SFOP tendency and their mathematical ability?

Several studies on early mathematical development have controlled for background characteristics such as age, gender, or SES, because of their potential influence on mathematical ability and its related correlates (Hawes, Moss, Caswell, Seo, & Ansari, 2019; Jordan, Kaplan, Nabors Olah, & Locuniak, 2006; Starkey, Klein, & Wakeley, 2004). Seo and Ginsburg (2004) found that age, but not gender or income level, was also related to the frequency children spontaneously engage in mathematical activities, including patterning. If any of the above-mentioned background characteristics correlated with performance on our SFOP task, these characteristics might explain associations between SFOP and patterning or mathematical ability. We therefore included age, gender, and SES as control variables. Research thus far has revealed an association between preschoolers' patterning ability and their spatial skills, including visuospatial working memory, spatial ability, and form perception (Collins & Laski, 2015; Rittle-Johnson et al., 2018). Associations have been found between spatial and mathematical skills (Gilligan, Hodgkiss, Thomas, & Farran, 2019; Hawes et al., 2019). We therefore verified whether differences in SFOP were associated with differences in spatial skills, and whether spatial skills affected the associations between SFOP, patterning, and mathematical ability.

2 Method

2.1 Participants

This study is part of a project on young children's development of mathematical competencies [blinded for review] in about 400 preschoolers, starting at the age of four in a 5-year longitudinal design. This study uses the same sample as in (Author, 2019). Most children (98%) in [blinded for review] go to preschool between the age of two-and-a-half and six. Preschool is federally funded and consists of three years. We selected 17 schools on the basis of the relative number of children in each school that received a study allowance and/or whose mother did not obtain a certificate of secondary school, to represent the whole range of socio-economic backgrounds. We received informed consent for 410 children. Missing data included 14 children for the patterning and mathematical tasks and 18 children for the spatial tasks, leading to a final sample of 378 participants with complete data (187 girls; M_{age} = 4 years, 10 months, range 4 years, 3 months to 5 years, 7 months). SES data were not available for another 29 children, therefore analyses with SES were performed with n = 349. Children with missing SES data are likely from lower SES backgrounds (LeFevre et al., 2009). Exploratory analyses indicated that these children had significantly lower patterning ability, mathematical ability, and spatial ability, which may affect the generalizability our findings.

A post-hoc power analysis indicated that assuming α = .05, our study had a power of 0.99 to find a medium effect (f = 0.25). The study was approved by the university's Social and Societal Ethics Committee [blinded for review].

2.2 Procedure

We administered the patterning and mathematical tasks individually in a quiet room at the children's schools when children were in the last semester of their second preschool year (Spring 2017). There were three test sessions of approximately 30 minutes each. Only tasks administered in session 1 and 2 are included in this study. Session 1 involved the assessment of children's SFOP and their ability to handle patterns. In session 2 we measured mathematical ability. Both sessions were presented to all children in the same order on two different days (*Mdn* = 2 days between session 1 and session 2). To guarantee children's spontaneous tendency to attend to patterns, the SFOP task was always administered first. After the summer break (Fall 2017), spatial ability and visuospatial working memory were measured, as well as other tasks not further discussed. We also administered a parent questionnaire to collect information on the educational degree of the mother to index SES. There were seven categories of educational degree: “primary education”, “lower secondary education”, “upper secondary level education”, “lowest level tertiary education”, “lower degree level tertiary education”, “higher degree level tertiary education”, and “other”.

2.3 Materials

2.3.1 SFOP measure.

Children were presented with a set of 15 building blocks in three colours (five per colour). We used building blocks because they provide the opportunity to make patterns and are available in most preschool settings, but are not frequently used in patterning activities in preschool in [blinded for review - location (Author, 2019)]. The instruction was simple: “Please build a tower that goes straight up, using all of the building blocks provided.” Before and during this instruction, the experimenter was not allowed to use the word “pattern” or any other associated term. Afterwards the experimenter took a picture of the child’s tower. Based on that picture, the child’s tower was scored into two categories: pattern or random.

Based on a rational analysis, a tower was categorized as *pattern* when there was a sequence of at least two full units of a pattern and the start of a third unit (e.g., ABABA or ABCABCA). This sequence of two units and the start of a third one could be anywhere in the tower. Towers that did not have a pattern as defined above, were considered to be *random*. During the categorization of children’s constructions, we noticed that several children sorted all the blocks per colour (e.g., 5 blue blocks, 5 red blocks, 5 yellow blocks). This sorting has been found as a common mistake in patterning activities (e.g., copying a pattern; Collins & Laski, 2015; Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013). Observational studies also found indications of both spontaneous patterning and sorting behaviour (Seo & Ginsburg, 2004). We therefore created a third category, i.e., *sorting,* for constructions in which all available blocks were grouped per colour and re-categorized those trials that were initially coded as random, but fell within this new category. Two independent raters scored 50 constructions and agreed on all of them, indicating a perfect inter-rater reliability ($\kappa = 1.00$). No analyses were performed until all trials were coded based on this final categorization.

2.3.2 Patterning ability.

Patterning was investigated with the same measure as in (Author, 2019). It consisted of three types of patterning activities: extending, translating, and identifying. These activities are commonly used in patterning research and cover various patterning abilities (Lüken, 2016; Rittle-Johnson et al., 2013; Sarama & Clements, 2009). Each activity had 6 patterns, leading to 18 items. Instructions for each task and examples are provided in Figure 1. The internal consistency of this measure in our sample was good ($\alpha = .83$; see Field, 2009).

INSERT FIGURE 1

2.3.3 Mathematical ability.

We used a measure that consisted of 88 items covering eight aspects of numerical ability (see Author, 2018 for more information). The selection of the items for this measure was based on recent research with children of a similar age range (see Andrews & Sayers, 2015; Jordan et al., 2006; Purpura & Lonigan, 2013). We included verbal counting (i.e., “Count as high as you can”, $n = 1$), dot enumeration (e.g., “Count the dots”, $n = 9$), object counting (e.g., “Give me N stones”, $n = 8$), symbolic and non-symbolic comparison (e.g., “Which number is the largest” , $n = 12*2$), number order (e.g., “Which number comes before/after N?” , $n = 8$), number recognition (e.g., “Which number is this?” , *n* = 30 with stopping rule), and verbal arithmetic (e.g., “I put N stones in a box and add/subtract M, how many stones are in my box?”, *n* = 8). We created one overall score for mathematical ability, because we did not have any hypotheses regarding more specific associations between SFOP and different mathematical tasks. This overall score also reflects the range of skills preschoolers are confronted with in early educational settings. To have each of the eight aspects equally represented in our mathematical measure, we standardized the performance on each aspect and used the sum of these standardized scores. The internal consistency of this measure in our sample was high (α = .93).

2.3.4 Spatial ability.

Spatial ability was measured by the Block Design subtest of the WPPSI-III-NL (Hendriksen & Hurks, 2002). Children had to make a certain construction of red and white blocks that was presented by the researcher within a given time limit. The assessment terminated after three consecutive wrong trials. This task had an acceptable internal consistency (α = .76; Hendriksen & Hurks, 2002).

2.3.5 Visuospatial working memory.

We used the Corsi span task from De Smedt et al. (2009), in which the children had to reproduce a sequence of blocks that the experimenter tapped in a random order. There were three trials of each sequence length (2 – 9 blocks), and the assessment was terminated if all three trials of one sequence length were wrong. The internal consistency of this task is acceptable (α = .77; De Smedt et al., 2009).

3 Results

3.1 Individual differences in SFOP

Children differed in their SFOP tendency: About 38% of the children spontaneously created a pattern, 48% made a random arrangement, and 14% sorted the blocks per colour. Table 1 presents the descriptive statistics of the children per SFOP group. We also observed individual differences in the length of the (units of the) patterns that were created by the children from the pattern group (Appendix, Table A.1).

INSERT TABLE 1

We explored whether the SFOP groups differed in age, gender, and SES. There were no significant group differences in age ($F(2, 375) = 2.12$, $p = .122$, $\eta^2_{partial} = .011$). There were gender differences ($\chi^2(2) = 13.70$, $p = .001$): girls made more patterns (46% vs. 29%) and fewer random arrangements (39% vs. 58%) than boys. No differences in sorting behaviour were found (16% vs. 13%). There were no significant group differences in SES ($\chi^2(10) = 8.57$, $p = .573$). As the SFOP groups only differed in gender, only this variable was controlled in subsequent analyses.

3.2 Association between SFOP and patterning ability

Table 1 presents the patterning ability of all SFOP groups. There were significant group differences in patterning ability ($F(2, 375) = 13.90$, $p < .001$, $\eta^2_{partial} = .069$). Bonferroni-corrected post-hoc *t*-tests indicated that children in the random group had significantly lower patterning ability than children in the pattern group ($p < .001$, $g = 0.56$) or the sorting group ($p = .002$, $g = 0.55$), while there were no significant differences between the latter two ($p > .99$, $g = 0.01$). The differences in patterning between SFOP groups remained when controlling for gender ($F(2,374) = 12.04$, $p < .001$, $\eta^2_{partial} = .061$).

3.3 Association between SFOP and mathematical ability

Table 1 presents the mathematical ability of the children per SFOP group. There were significant group differences ($F(2, 375) = 5.27$, $p = .006$, $\eta^2_{partial} = .027$). Bonferroni-corrected post-hoc *t*-tests indicated that the pattern group had significantly better mathematical ability than the random group ($p = .006$, $g = 0.35$), while there were no differences between the sorting group and the pattern ($p = .120$, $g = 0.31$) or the random group ($p > .99$, $g = 0.02$). These differences remained when controlling for gender differences ($F(2,374) = 5.61$, $p = .004$, $\eta^2_{partial} = .029$).

We explored whether the association between SFOP and mathematical ability could be explained by patterning ability by including patterning as a covariate in our analysis. The effect of SFOP remained significant ($F(2,374) = 4.34$, $p = .014$, $\eta^2_{partial} = .023$), but patterning ability was a significant covariate ($F(1,374) = 287.30$, $p < .001$, $\eta^2_{partial} = .434$). Bonferroni-corrected post-hoc *t*-tests indicated that the sorting group now had significantly lower mathematical ability than the pattern (Adjusted Mean difference = 1.79, $p = .024$, $g = 4.17$) or random group (Adjusted Mean difference = 1.85, $p = .016$, $g = 4.74$), while there was no significant difference anymore between the pattern group and the random group (Adjusted Mean difference = 0.06, $p > .99$, $g = 0.17$, see Figure A1b in Appendix).

3.4 Spatial ability and visuospatial working memory

We observed significant group differences in spatial ability ($F(2,375) = 7.25$, $p = .001$, $\eta^2_{partial} = .037$). Bonferroni-corrected post-hoc *t*-tests indicated that the sorting group had significantly better spatial ability than the pattern ($p = .032$, $g = 0.40$) or random group ($p = .001$, $g = 0.60$), while there was no significant difference between the pattern group and the random group ($p = .357$, $g = 0.18$). There were also significant group differences in visuospatial working memory ($F(2,375) = 4.67$, $p = .010$, $\eta^2_{partial} = .024$). Children in the pattern group performed significantly better than children in the random group (p = .031, g = 0.29), while there were no significant differences between the sorting group and the pattern group (p > .99, g = 0.08) or the random group (p = .054, g = 0.37).

Due to these differences in spatial skills and because both spatial skills were associated with patterning and mathematical ability (see Appendix, Table A.2), we further verified if the group differences in patterning and mathematical ability remained when these differences in spatial skills were controlled for by including spatial ability and visuospatial working memory as covariates. For patterning ability the effect of SFOP remained significant ($F(2,373) = 8.93$, $p < .001$, $\eta^2_{partial} = .046$), and both spatial ability ($F(1,373) = 64.43$, $p < .001$, $\eta^2_{partial} = .147$) and visuospatial working memory ($F(1,373) = 28.80$, $p < .001$, $\eta^2_{partial} = .072$) were significant covariates. The difference between the pattern group and the random group remained significant (Adjusted Mean difference = 1.54, $p < .001$, g = 6.01), whereas the difference between the sorting and the random group was no longer significant (Adjusted Mean difference = 0.83, p = .32, g = 2.75, see Figure A1a in Appendix).

For mathematical ability, the effect of SFOP also remained significant ($F(2, 373) = 6.62$, $p = .001$, $\eta^2_{partial} = .034$), and both spatial ability ($F(1\ 373) = 46.41$, $p < .001$, $\eta^2_{partial} = .111$) and visuospatial working memory ($F(1, 373) = 45.25$, $p < .001$, $\eta^2_{partial} = .108$) were significant covariates. The difference between the pattern and the random group was no longer significant (Adjusted Mean difference = 1.12, p = .110, g = 3.62), whereas the difference between the pattern and the sorting group became significant (Adjusted Mean difference = 2.71, p = .001, g = 5.60, see Figure A1c in Appendix).

4 Discussion

We developed a new task to explore whether young children differ in the extent to which they display a spontaneous focus on patterns (SFOP) tendency. We found differences in children's SFOP: About one third of the children spontaneously created a pattern, whereas the other children did not. The patterns that were constructed differed in terms of length and unit. This suggests that the tower task allows us to capture differences in children's spontaneous tendency to attend to patterns in their environment.

4.1 The pattern vs. the random group

Individual differences in SFOP were associated with children's patterning ability. Children who spontaneously created a pattern demonstrated better patterning ability than children who made a random arrangement. This difference was not explained by gender, SES, age, or spatial skills. These results echo research on other mathematical focusing tendencies (Hannula-Sormunen, 2015; McMullen et al., 2014; Rathé et al., 2019) and support previous suggestions about the potential role of SFOP in the development of patterning ability (McKillip, 1970; Mulligan & Mitchelmore, 2009). One mechanism that might explain this association between SFOP and patterning ability is self-initiated practice.

We also found an association between SFOP and mathematical ability. Children who SFOP demonstrated better mathematical ability than those who made a random arrangement. This was not explained by gender, SES or age. Differences in patterning ability and spatial skills accounted for this group difference. This suggests that children in the pattern group had better mathematical ability than children in the random group due to their better patterning and spatial skills. The impact of SFOP on mathematical ability seems therefore indirect, through its impact on patterning ability and spatial skills. This might be due to the operationalization of our early math measure, which only captures basic numerical skills. Other aspects of mathematical ability, such as algebraic or statistical reasoning, might be more related to SFOP, but are difficult to measure in young children.

Spatial skills played a role in children's mathematical development, and contributed to the association between SFOP and mathematical ability. This aligns with previous research that revealed associations between spatial skills, patterning and mathematical ability (Collins & Laski, 2015; Rittle-Johnson et al., 2018). Future studies are required to fully grasp the complex role of both patterning and spatial skills in children's mathematical development (Rittle-Johnson et al., 2018).

4.2 The sorting group

During the categorization of children's behaviour on the SFOP task, we observed that some children systematically sorted the blocks per colour. Because their behaviour neither fitted our definition of patterning, nor could be conceived as random, we grouped them into the sorting group. This decision was supported by the finding that the sorting group had higher spatial ability than the random group. Children from the sorting group had strong spatial and patterning skills, but relatively low mathematical ability, especially when taking into account their spatial and patterning skills. Two conclusions can be drawn from these results. First, although the sorting and pattern group had similar patterning ability, their behaviour on the SFOP task was different. Variation in behaviour on the SFOP task could therefore not entirely be explained by variation in patterning skills, suggesting that SFOP is distinct from patterning ability. Second, children in the pattern and the sorting group both had strong spatial and patterning skills, but the pattern group had better mathematical skills. This suggests that the impact of SFOP on mathematical ability cannot be merely explained by spatial or patterning skills.

One explanation for the strong patterning skills of the sorting group might be that children who sort apply some kind of very basic structure (i.e., all similar elements together) which might be a first step towards patterning. Sorting requires seeing similarities and differences between elements, which according to Mulligan and Mitchelmore (2012, p. 530) “plays a critical role in the development of pattern and structure”. More research is needed to understand the cognitive characteristics of children who spontaneously sort, compared to those who focus on pattern and those who show no systematic behaviour at all.

4.3 Educational implications

Young children encounter free play situations in which they can spontaneously create patterns on a daily basis. Early childhood educators should be aware of these spontaneous pattern creations and the learning opportunities they provide. By talking about these spontaneous constructions (e.g., “Look, you created a repeating pattern. I can see a unit that repeats over and over.”) and using them as stepping-stones for new patterning activities (e.g., “Could you make the same pattern, but using blocks in different colours?”), early childhood educators can enrich children’s patterning experiences (Fox, 2005). Most preschool settings have, for example, building corners in which children can make different kinds of constructions, including patterns. Early childhood educators could provide building blocks in different colours, shapes or lengths, and a few illustrative patterns to inspire children.

4.4 Future research

The current study indicates that children differ in their spontaneous attention towards patterns and that these differences correlate with their mathematical development. Future research should evaluate the psychometric properties of our SFOP task. This could include validating children's behaviour on our SFOP task with classroom observations or teacher ratings of children's spontaneous patterning behaviour.

At least four other issues require further investigation. First, studies need to unravel the origins and early development of SFOP. This work should focus on the contribution of child characteristics (e.g., spatial skills, attentional skills) and children's home and classroom environment (e.g., frequency and types of patterning and mathematical activities; Rathé et al., 2016). Second, the developmental association between SFOP and patterning ability requires further study. It remains unclear whether SFOP fosters the development of patterning ability, or vice versa, or whether there is a bidirectional association between them, as has been found for other spontaneous mathematical focusing tendencies (Hannula-Sormunen, 2015; Verschaffel et al., 2018). Panel longitudinal studies that focus on this bidirectional association between SFOP and patterning ability could shed a light on this issue. Third, it remains unclear whether SFOP is distinct from other spontaneous mathematical focusing tendencies, or whether they all represent a general spontaneous tendency toward mathematical elements, patterns, and relations (Verschaffel et al., 2018). Fourth, intervention studies should explore whether and how this spontaneous interest in patterns can be stimulated, and whether this stimulation leads to improved patterning and/or mathematical ability, which would provide evidence for causal effects.

References

Author (2019)

Author (2019)

Author (2018)

Andrews, P., & Sayers, J. (2015). Identifying opportunities for grade one children to acquire foundational number sense: Developing a framework for cross cultural classroom analyses. *Early Childhood Education Journal*, *43*(4), 257–267. https://doi.org/10.1007/s10643-014-0653-6

Collins, M. A., & Laski, E. V. (2015). Preschoolers' strategies for solving visual pattern tasks. *Early Childhood Research Quarterly*, *32*, 204–214. https://doi.org/10.1016/j.ecresq.2015.04.004

De Smedt, B., Janssen, R., Bouwens, K., Verschaffel, L., Boets, B., & Ghesquière, P. (2009). Working memory and individual differences in mathematics achievement: A longitudinal study from first grade to second grade. *Journal of Experimental Child Psychology*, *103*(2), 186–201. https://doi.org/10.1016/j.jecp.2009.01.004

Degrande, T., Verschaffel, L., & Van Dooren, W. (2017). Spontaneous Focusing on Quantitative Relations: Towards a characterization. *Mathematical Thinking and Learning*, *19*(4), 260–275. https://doi.org/10.1080/10986065.2017.1365223

Field, A. (2009). *Discovering statistics using SPSS* (Third). London, UK: SAGE Publications.

Fox, J. (2005). Child-initiated mathematical patterning in the pre-compulsory years. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp.

313–320). Melbourne: PME.

Garrick, R., Threlfall, J., & Orton, A. (2005). Pattern in the nursery. In A. Orton (Ed.), *Pattern in the teaching and learning of mathematics* (pp. 1–17). London: Continuum.

Gilligan, K. A., Hodgkiss, A., Thomas, M. S. C., & Farran, E. K. (2019). The developmental relations between spatial cognition and mathematics in primary school children. *Developmental Science*, (March). https://doi.org/10.1111/desc.12786

Hannula-Sormunen, M. M. (2015). Spontaneous focusing on numerosity and its relation to counting and arithmetic. In R. C. Kadosh & A. Dowker (Eds.), *The Oxford handbook of numerical cognition* (pp. 275–290). Oxford: Oxford University Press.

Hannula-Sormunen, M. M., Lehtinen, E., & Räsänen, P. (2015). Preschool children's spontaneous focusing on numerosity, subitizing, and counting skills as predictors of their mathematical performance seven years later at school. *Mathematical Thinking and Learning*, *17*(2–3), 155–177. https://doi.org/10.1080/10986065.2015.1016814

Hannula, M. M., & Lehtinen, E. (2005). Spontaneous focusing on numerosity and mathematical skills of young children. *Learning and Instruction*, *15*(3), 237–256. https://doi.org/10.1016/j.learninstruc.2005.04.005

Hawes, Z., Moss, J., Caswell, B., Seo, J., & Ansari, D. (2019). Relations between numerical, spatial, and executive function skills and mathematics achievement: A latent-variable approach. *Cognitive Psychology*, *109*, 68–90. https://doi.org/10.1016/j.cogpsych.2018.12.002

Hendriksen, J., & Hurks, P. (2002). Wechsler Preschool and Primary Scale of Intelligence (WPPSI-III-NL).

Jordan, N. C., Kaplan, D., Nabors Olah, L., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development*, *77*(1), 153–175. https://doi.org/10.1111/j.1467-8624.2006.00862.x

LeFevre, J.-A., Skwarchuk, S.-L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioural Science/Revue Canadienne Des Sciences Du Comportement*, *41*(2), 55–66. https://doi.org/10.1037/a0014532

Lüken, M. M. (2012). Young children's structure sense. *Journal Für Mathematik-Didaktik*, *33*(2), 263–285. https://doi.org/10.1007/s13138-012-0036-8

Lüken, M. M. (2016). Repeating patterning competencies in 3- and 4-year old kindergartners. In *13th International Congress on Mathematical Education* (pp. 1–4). Hamburg.

McKillip, W. D. (1970). "Patterns" - A mathematics unit for three- and four-year-olds. *The Arithmetic Teacher*, *17*(1), 15–18.

McMullen, J., Hannula-Sormunen, M. M., & Lehtinen, E. (2014). Spontaneous focusing on quantitative relations in the development of children's fraction knowledge. *Cognition and Instruction*, *32*(2), 198–218. https://doi.org/10.1080/07370008.2014.887085

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal*, *21*(2), 33–49. https://doi.org/10.1007/BF03217544

Mulligan, J., & Mitchelmore, M. (2012). Developing Pedagogical Strategies to Promote Structural Thinking in Early Mathematics. In J. Dindyal, L. P. Cheng, & S. F. Ng (Eds.), *Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia, eBook)* (pp. 529–536). Singapore: MERGA, Inc.

Mulligan, J., Mitchelmore, M., & Stephanou, A. (2015). PASA Teacher guide. ACER Press.

Mulligan, J., Verschaffel, L., Baccaglini-Frank, A., Coles, A., Gould, P., He, S., … Yang, D.-C. (2018). Whole Number Thinking, Learning and Development: Neuro-cognitive, Cognitive and Developmental Approaches. In M. Bussi & X. Sun (Eds.), *Primary mathematics study on whole numbers (ICMI Study 23)* (pp. 137–167). New York: Springer. https://doi.org/10.1007/978-3-319-63555-2_7

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D., Sarama, J., Wolfe, C., & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly*, *36*, 550–560. https://doi.org/10.1016/j.ecresq.2016.02.003

Purpura, D. J., & Lonigan, C. J. (2013). Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *American Educational Research Journal*, *50*(1), 178–209. https://doi.org/10.3102/0002831212465332

Rathé, S., Torbeyns, J., De Smedt, B., & Verschaffel, L. (2019). Spontaneous focusing on Arabic number symbols and its association with early mathematical competencies. *Early Childhood Research Quarterly*, *48*, 111–121. https://doi.org/10.1016/j.ecresq.2019.01.011

Rathé, S., Torbeyns, J., Hannula-Sormunen, M. M., De Smedt, B., & Verschaffel, L. (2016). Spontaneous focusing on numerosity: a review of recent research. *Mediterranean Journal for Research in Mathematics Education*, *15*, 1–25.

Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early math trajectories: Low-income children's mathematics knowledge from age 4 to 11. *Child Development*, *88*(5), 1727–1742. https://doi.org/10.1111/cdev.12662

Rittle-Johnson, B., Fyfe, E. R., McLean, L. E., & McEldoon, K. L. (2013). Emerging understanding of patterning in 4-year-olds. *Journal of Cognition and Development*, *14*(3), 376–396. https://doi.org/10.1080/15248372.2012.689897

Rittle-Johnson, B., Zippert, E. L., & Boice, K. L. (2018). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly*, *46*, 166–178. https://doi.org/10.1016/j.ecresq.2018.03.006

Sarama, J., & Clements, D. (2009). Other content domains. In *Early childhood mathematics education research* (pp. 319–334). New York: Routledge. https://doi.org/10.4324/9780203883785

Seo, K.-H., & Ginsburg, H. (2004). What is developmentally appropriate in early childhood mathematics education? Lessons from new research. In D. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics* (pp. 91–104). Mahwah, NJ: Lawrence Erlbaum.

Sharir, T., Mashal, N., & Mevarech, Z. R. (2015). To what extent can young children spontaneously recognize mathematical structures? Paper presented at the

Bienial Conference of the European Assocation for Research on Learning and Instruction. Limasol, Cyprus.

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly*, *19*(1), 99–120. https://doi.org/10.1016/j.ecresq.2004.01.002

Steen, L. A. (1988). The Science of Patterns. *Science*, *240*(4852), 611–616.

Steen, L. A. (1990). Pattern. In L. A. Steen (Ed.), *On the Shoulders of Giants: New Approaches to Numeracy* (pp. 1–10). Washington, DC: National Academy Press. https://doi.org/10.1002/tie.21668

Verschaffel, L., Rathé, S., Wijns, N., De Smedt, B., & Torbeyns, J. (2018). Young children's early mathematical competencies: The role of mathematical focusing tendencies. Invited plenary lecture. Kristiansand, Norway.

Verschaffel, L., Torbeyns, J., & De Smedt, B. (2017). Young children's early mathematical competencies: Analysis and stimulation. Plenure lecture presented at the congress of European research in mathematics education (CERME), Dublin, Ireland.

Warren, E., & Cooper, T. (2007). Repeating patterns and multiplicative thinking: Analysis of classroom interactions with 9-year-old students that support the transition from the known to the novel. *The Journal of Classroom Interaction*, *41.2*(7), 7–17.

Wittmann, E. C., & Müller, G. N. (2007). Muster und strukturen als fachliches grundkonzept. In G. Walther, M. van den Heuvel-Panhuizen, D. Granzer, & O.

Köller (Eds.), *Bildungsstandards für die Grundschule: Mathematik konkret* (pp. 42–65). Berlin: Cornelsen. https://doi.org/10.18452/3121

Table 1

Descriptive statistics for each SFOP group

SFOP	Pattern	Random	Sorting
Number of Children	141	183	54
Number of Girls (% within SFOP group)	85 (60.3%)	73 (39.9%)	29 (53.7%)
Number of Boys (% within SFOP group)	56 (39.7%)	110 (60.1%)	25 (46.3%)
Age (SD) [range]	*M* = 4y10m (3m) [4y4m; 5y7m]	*M* = 4y10m (4m) [4y3m; 5j4m]	*M* = 4y9m (3m) [4y4m; 5y3m]
SES[a]	(1) 6 (2) 12 (3) 32 (4) 30 (5) 16 (6) 38 (7) 0 Missing: 7	(1) 4 (2) 15 (3) 53 (4) 36 (5) 16 (6) 40 (7) 0 Missing: 19	(1) 1 (2) 6 (3) 8 (4) 13 (5) 5 (6) 18 (7) 0 Missing: 3
Spatial ability (SD) [range]	*M* = 22.11 (5.69) [6; 38]	*M* = 21.15 (5.32) [4; 32]	*M* = 24.37 (5.49) [5; 35]
Visuospatial working memory (SD) [range]	*M* = 6.31 (2.15) [0; 11]	*M* = 5.70 (2.11) [0; 11]	*M* = 6.48 (2.10) [3; 11]
Patterning ability (SD) [range]	*M* = 9.58 (3.97) [1; 18]	*M* = 7.51 (3.53) [1; 18]	*M* = 9.54 (4.15) [1; 17]

Mathematical ability	*M* = 1.37 (5.83)	*M* = -0.58 (5.25)	*M* = -0.47 (5.88)
(SD) [range]	[-10.97; 14.99]	[-11.64; 12.18]	[-11.86; 12.73]

Note. The measure of mathematical ability represents the sum of the z-scores of performance on each of the eight aspects that were measured. a: (1) “primary education”, (2) “lower secondary education”, (3) “upper secondary level education”, (4) “lowest level tertiary education”, (5) “lower degree level tertiary education”, (6) “higher degree level tertiary education”, (7) “other”.

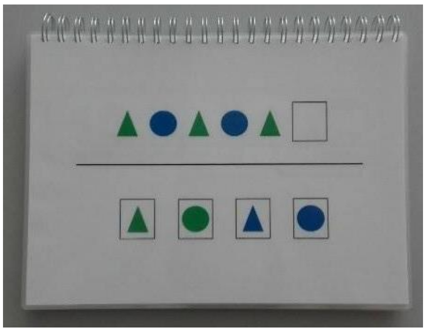
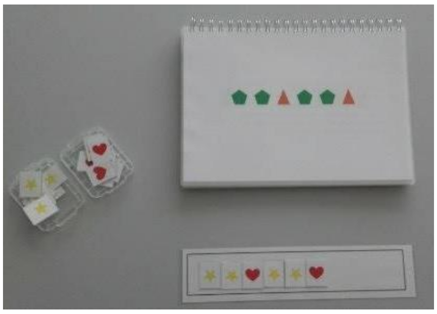
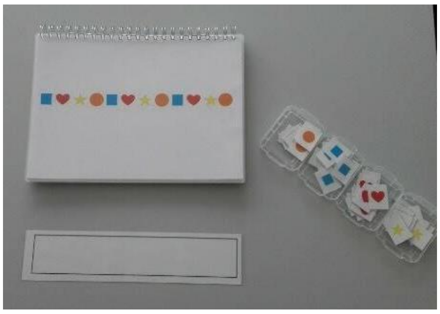
Activity Instruction	**Example item**
Extending "Look carefully at this row. There is a pattern in it. At the end there is something missing. One of these figures has to be placed in the empty spot. Do you know which of these has to be on the empty spot?"	
Translating "Look carefully at this row. There is a pattern in it. Please make the same pattern with your figures on the paper strip."	
Identifying "You will soon see a row with a pattern in it. You will have to look very closely at it and try to remember the pattern. After a short period I will hide the pattern and you will have to reconstruct it."	

Figure 1. Instruction and example for each patterning task.

Appendix

Table A.1

Length, frequency, relative frequency, and examples of the patterns created by the pattern group

Length	Frequency	Relative frequency	Examples
5	29	20.6%	**AB**ABA
6	9	6.4%	**AB**ABAB
7	25	17.7%	**AB**ABABA, **ABC**ABCA, **AAB**AABA
8	10	7.1%	**AB**ABABAB, **ABC**ABCAB, **AAB**AABAA
9	9	6.4%	**ABC**ABCABC, AABBAABBA, ABBCABBCA
10	14	9.9%	**AB**ABABABAB, **ABC**ABCABCA, **ABAC**ABACAB
11	1	0.7%	**ABC**ABCABCAB
12	7	5.0%	**ABC**ABCABCABC
13	4	2.8%	**ABCA**BCABCABCA, **ABCBC**ABCBCABC, **AABBCC**AABBCCA
15	33	23.4%	**ABC**ABCABCABCABC

Table A.2

Correlation matrix for patterning ability, mathematical ability, age, spatial ability, and visuospatial working memory

	Patterning ability	Mathematical ability	Spatial ability	Visuospatial working memory
Patterning ability	-	.661 **	.484**	.415**
Mathematical ability		-	.429**	.442**
Spatial ability			-	.381**
Visuospatial working memory				-

Note. * $p < .05$; ** $p < .001$.

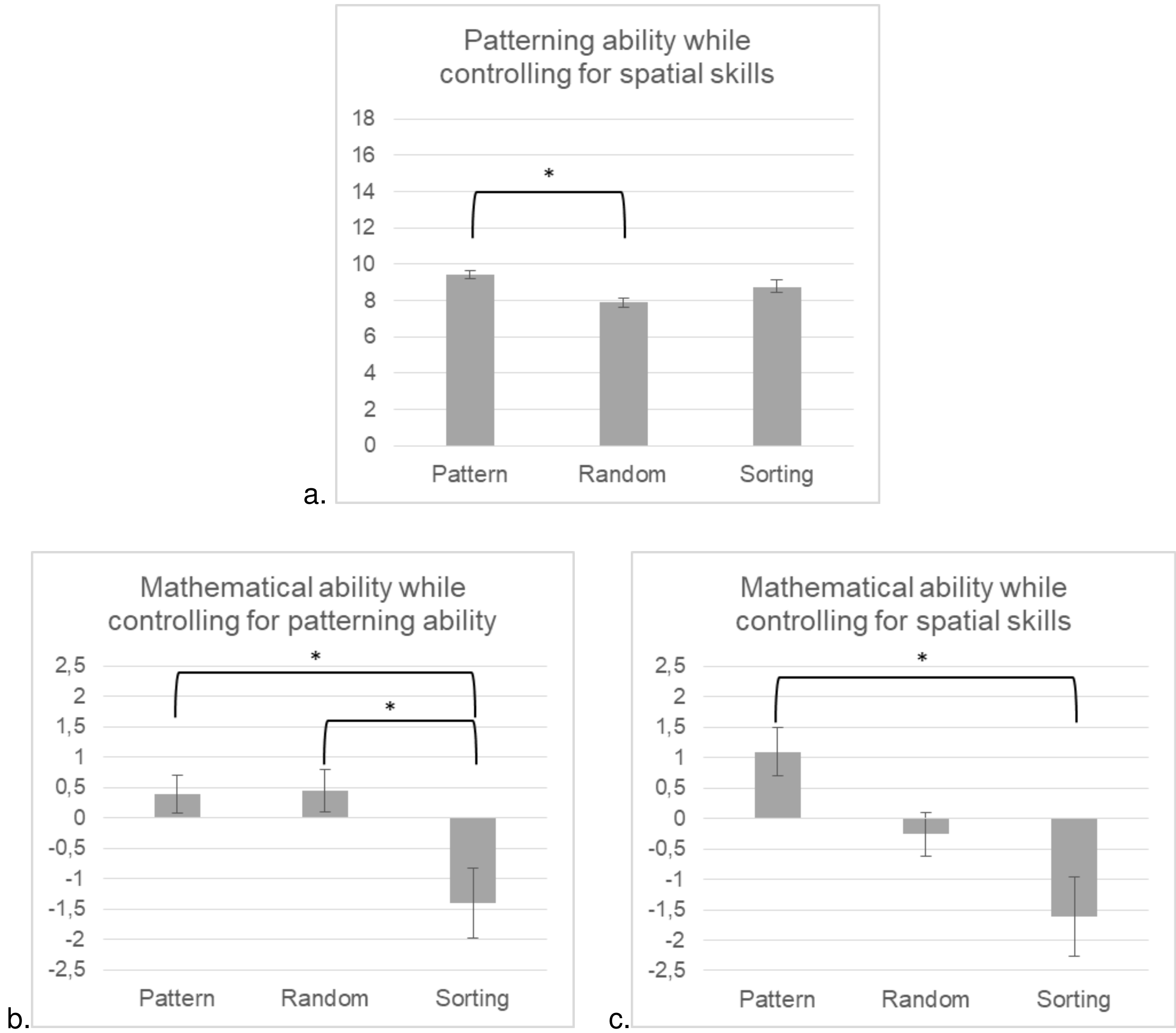


Figure A1. Bar chart showing patterning ability while controlling for spatial skills (a) and mathematical ability while controlling for patterning ability (b) or spatial skills (c) for the three SFOP groups. Error bars represent standard errors.