

Seminar Paper No. 558

ARE REAL WAGES AND  
UNEMPLOYMENT RELATED?

by

Tor Jacobson, Anders Vredin and Anders Warne



INSTITUTE FOR INTERNATIONAL ECONOMIC STUDIES

University of Stockholm

ISSN 0347-8769

Seminar Paper No. 558

**ARE REAL WAGES AND UNEMPLOYMENT RELATED?**

by

**Tor Jacobson, Anders Vredin and Anders Warne**

**Seminar Papers are preliminary material circulated to stimulate discussion and critical comment.**

November 1993

**Institute for International Economic Studies  
S-106 91 Stockholm  
Sweden**

# Are Real Wages and Unemployment Related?

Tor Jacobson\* Anders Vredin† Anders Warne‡

First version: November, 1992

This version: September, 1993

## Abstract

In this paper we propose an alternative method for investigating the sources behind the behavior of real wages and unemployment. The statistical model we study is a certain structural error correction model, a so called common trends model, which has become popular in the empirical growth/business cycle literature. The system consists of real output, employment, unemployment and the product real wage and two exogenous stochastic variables, a tax wedge and a currency basket index. Based on quarterly Swedish data (1965–90) we find evidence supporting a short run but not a medium or long run relation.

KEYWORDS: Common trends, error correction, supply and demand shocks.

JEL CLASSIFICATION NUMBERS: C15, C32, C52, E24.

---

\*Department of Statistics, Uppsala University, Box 513, 751 20 Uppsala, Sweden.

†Department of Economics, Stockholm School of Economics, Box 6501, 113 83 Stockholm, Sweden.

‡Institute for International Economic Studies, Stockholm University, 106 91 Stockholm, Sweden.

We have received valuable comments from Henrik Hansen, Dennis Hoffman, Bertil Holmlund, Søren Johansen, Katarina Juselius, Grayham Mizon and participants at the Nordic workshops on cointegration in Copenhagen, June 1992, and Venice, November 1992, the conference on cointegration in Helsinki, June 1993, the ESEM in Uppsala, August 1993, and seminar participants at the Universities of Gothenburg, Lund, and Copenhagen. Financial support from *Humanistisk–Samhällsvetenskapliga Forskningsrådet* and *Tore Browalds Forskningsstiftelse* is gratefully acknowledged.

# 1 Introduction

The persistent rise in unemployment rates in the industrialised countries during the 1980's was followed by a growing interest in empirical research about labor market conditions. Part of the interest was focused on international comparisons trying to explain both similarities (e.g. rising trends in unemployment) and disparities (e.g. different levels of unemployment). In this paper we will examine the question if (and how) real wages and unemployment are related.

Unemployment is, of course, related to the level of real wages in the sense that excess supply of labor, almost by definition reflects that real wages are above their market clearing levels.<sup>1</sup> However, real wages are not sticky and in particular they rise persistently over time as labor demand increases due to technical progress. In the short run, real wages may adjust too little (or too much) to various kinds of shocks, thus giving rise to fluctuations in unemployment.

We shall analyse the relation between unemployment and real wages by considering three questions which have earlier been addressed by other researchers. First, what is the relative importance of supply and demand factors behind the development of unemployment? In a study of 18 OECD countries, which has been an important source of inspiration for many subsequent studies, Bean, Layard and Nickell (1986) report that changes in labor demand and labor supply are about equally important in explaining the excess supply of labor. Second, how quickly do real wages and employment respond to different types of shocks? Bean et al. find that labor markets seem to adjust more rapidly in countries that are more corporatist in nature. This might explain why such countries, e.g. the Nordic countries, have been able to maintain relatively low unemployment rates throughout the 1980's. Third, how strong are the long run relations between real wages and unemployment? Bean et al. find that corporatist countries generally show strong long run effects of unemployment on wages. Since the answers to these questions have strong policy implications, their robustness to alternative specifications of the econometric labor market model should be examined.

In this paper we will show how these questions about labor markets – the relative

---

<sup>1</sup>We realize that a cleared labor market may be consistent with a nonzero official rate of unemployment.

importance of supply and demand shocks, the speed of adjustment of real wages, and the long run relation between real wages and unemployment – can be approached using recent advances in multivariate time series analysis. Specifically, we analyse a vector error correction model, or, equivalently, a vector autoregressive model with cointegration constraints and a small number of exogenous regressors. The VARX model is viewed as a reduced form and the individual equations are not interpreted in economic terms. Instead, we will use the VARX model with cointegration constraints to study a so called common trends model. This model is on structural form and an economic interpretation is given to the shocks in the model. The focus is on how the endogenous variables respond to the shocks and to the observable exogenous variables. A recent paper by Bean (1992) uses a similar approach.<sup>2</sup>

In comparison with traditional single equation error correction models, the common trends modelling approach is at least complementary regarding the three questions about labor markets that were discussed above. Supply shocks may be identified with innovations to (real) stochastic trends. The short and long run response of real wages and unemployment to various disturbances may be described through impulse responses. Long run relations are captured by cointegration vectors and the coefficients on the stochastic trends.

The remainder of the paper is organised as follows. The econometric model is presented in section 2. In section 3 we compare our approach with that of earlier researchers. Our method is applied to quarterly Swedish data (1965–90), and the empirical results are reported in sections 4 and 5. The specification of the VARX model and the evidence on cointegration is discussed in section 4. Here, we conduct Monte Carlo experiments to evaluate how well the asymptotic distributions work in our small sample setting and use the results as a guide for inference on cointegration. The results from the common trends model are presented in section 5, while the conclusions are summarised in section 6. One finding is that transitory shocks seem to result in a short run relation between real wages

---

<sup>2</sup>The equivalence of the vector error correction, VAR and common trends representations has been established by Engle and Granger (1987) and Stock and Watson (1988). Macroeconomic applications have been presented by e.g. Blanchard and Quah (1989), Englund et al. (1992), King et al. (1991), and Mellander et al. (1992).

and unemployment. In the medium and in the long run, however, the two variables do not appear to be related.

## 2 Common Trends and Error Correction

In this section we will first provide a general description of the relation between vector error correction and common trends models. We will then specify a common trends model of the labor market.

### 2.1 The Statistical Model

Let  $x_t$  denote an  $n$  dimensional vector time series with endogenous variables and  $z_t$  an  $m$  dimensional vector with exogenous variables. The vector  $z_t$  can be decomposed into  $[d_t' s_t']'$ , where  $d_t$  is an  $m_d$  vector of deterministic regressors (e.g. a constant and centered seasonal dummies) and  $s_t$  is an  $m_s$  vector of stochastic regressors. We shall assume that  $s_t$  is jointly wide sense stationary and ergodic.

Suppose that  $x_t$  is generated by the VARX model:

$$\Pi(L)x_t = P(L)z_t + \varepsilon_t, \quad t = 1, \dots, T. \quad (1)$$

$\Pi(\lambda)$  is an  $n \times n$  matrix polynomial of order  $p$ , i.e.  $\Pi(\lambda) = I_n - \sum_{j=1}^p \Pi_j \lambda^j$ ,  $\lambda$  is a complex number, and  $L$  is the lag operator. It is assumed that  $\det[\Pi(\lambda)] = 0$  if and only if  $|\lambda| > 1$  or  $\lambda = 1$ . Hence, we limit our attention to  $x_t$  processes that are integrated. The  $n \times m$  matrix polynomial  $P(\lambda)$  is of order  $p$ , with  $P(\lambda) = \sum_{j=0}^p P_j \lambda^j$ . Also, we decompose  $P_j$  analogously with  $z$ , i.e.  $P_j = [P_{d,j} P_{s,j}]$  where  $P_{d,j} = 0$  for  $j = 1, \dots, p$ . In the empirical analyses we also restrict  $P_{s,0}$  to be zero. Hence, the deterministic regressors only appear contemporaneously with  $x$  while the stochastic exogenous regressors are lagged (from 1 to  $p$ ). Finally, the initial conditions  $\{x_0, \dots, x_{1-p}\}$  are fixed, while the innovations  $\varepsilon_t$  are i.i.d. Gaussian with zero mean and positive definite covariance matrix  $\Sigma$ .<sup>3</sup>

---

<sup>3</sup>Note that it is straightforward to interpret the system in equation (1) as a subsystem of a VAR model for  $X_t := [x_t' s_t']'$ . For that model  $x$  does not Granger cause  $s$  and the innovations to  $s$  are independent of  $\varepsilon$ . In our empirical application below we will not study this “full system” since one of the stochastic exogenous regressors is difficult to model. Also, if  $P_s(\lambda) = 0$  it follows that (1) is just a standard VAR model for  $x_t$ .

Let  $\Gamma(\lambda) = I_n - \sum_{i=1}^{p-1} \Gamma_i \lambda^i$  and  $\Pi$  be defined from  $\Gamma_i = -\sum_{j=i+1}^p \Pi_j$  and  $\Pi = \Pi(1)$ , respectively. The VARX model (1) may now be rewritten as

$$\Gamma(L)\Delta x_t = P(L)z_t - \Pi x_{t-1} + \varepsilon_t, \quad (2)$$

where  $\Delta$  is the first difference operator. With the additional assumption that  $\Pi = \alpha\beta'$  has rank  $r < n$  (where  $\alpha$  and  $\beta$  are  $n \times r$  matrices) we arrive at the error correction model:

$$\Gamma(L)\Delta x_t = P(L)z_t - \alpha\beta'x_{t-1} + \varepsilon_t. \quad (3)$$

The version of Granger's representation theorem in Johansen (1991) give conditions which imply that  $\beta'x_t$  and  $\Delta x_t$  are integrated of order zero (denoted by I(0)), whereas  $x_t$  is I(1). Whenever  $r > 0$  we say that  $x_t$  is cointegrated of order (1,1) (denoted by CI(1,1)) and the deviation of  $\beta'x_t$  from its mean is interpreted as a long run "equilibrium error". The  $r$  columns of  $\beta$  are called cointegration vectors, and the elements of  $\alpha$  are interpreted as "adjustment coefficients". At this stage it should be emphasized that the model in (3) is written in reduced form in the sense that a specific economic interpretation is given neither to the individual equations nor to the innovations.

In this paper we will use the reduced form error correction model (3) to estimate a common trends model, which, in the present setting, is given by

$$x_t = x_0 + A\tau_t + \Phi(L)\nu_t + B\sum_{i=1}^t z_i + D(L)z_t. \quad (4)$$

The  $k$  dimensional vector  $\tau_t$  (with  $k := n - r$ ) is a random walk, i.e.

$$\tau_t = \tau_{t-1} + \varphi_t. \quad (5)$$

The  $n \times m$  matrix  $B$  and matrix polynomial  $D(\lambda)$  are determined from  $C(\lambda)P(\lambda) = B + D(\lambda)(1 - \lambda)$ , where  $C(\lambda)$  is defined below. We assume that  $\Phi(\lambda)$  and  $D(\lambda)$  are absolutely summable.

From equation (4) we find that the  $n$  variables in  $x_t$  have an I(1) (permanent) component,  $x_{P,t} := (A\tau_t + B\sum_{i=1}^t z_i)$ , and an I(0) (transitory) component,  $x_{T,t} := (\Phi(L)\nu_t + D(L)z_t)$ . Since  $\beta'x_t$  is I(0) it follows that  $\beta'x_{P,t} = 0$ . That is,  $\beta'A = 0$  and  $\beta'B = 0$  so both  $A$  and  $B$  depend on  $\beta_\perp$ . Furthermore, for each component the two terms are independent since  $\nu_t$  and  $s_t$  are assumed to be independent. In the empirical section we

shall mainly concentrate on the permanent component conditional on  $z$ , i.e. on  $A\tau_t$ , and on the transitory component conditional on  $z$ , i.e. on  $\Phi(L)\nu_t$ . Clearly, the independence assumption makes such conditioning meaningful from an economic point of view.

Conditional on  $z$  the long run properties of  $x_t$  are determined by the  $k$  independent stochastic trends and the loading matrix  $A$ . The trends are common in the sense that each stochastic trend, at least in principle, is allowed to influence any variable. (Restrictions on  $A$  will be discussed below). Note that if all elements of  $s_t$  are  $I(0)$ , e.g. white noise, then the stochastic exogenous regressors yield  $m_s$  additional stochastic trends which are also allowed to be common for all variables in  $x_t$ .<sup>4</sup>

Some restrictions are needed to identify the common trends model (4) from the reduced form error correction model (3). Following standard practise in analyses of VAR models, we first assume that the covariance matrix of  $\nu_t$  is diagonal, so that these innovations can be interpreted as structural. For convenience, we normalize all variances to unity. The  $n$  residuals  $\varepsilon_t$  are linear combinations of the structural innovations. That is,

$$\varepsilon_t = F\nu_t = F \begin{bmatrix} \varphi_t \\ \psi_t \end{bmatrix}, \quad (6)$$

where  $\varphi$  are the  $k$  trend innovations and  $\psi$  are  $r$  transitory innovations. By transitory we mean that the innovations do not affect the permanent component of  $x_t$  in (4).

Just like standard VAR models, common trends models may be characterized in terms of “impulse response functions”, which are charts of the coefficients on  $\nu$  in the moving average representation

$$\Delta x_t = C(L)P(L)z_t + R(L)\nu_t. \quad (7)$$

Here,  $C(\lambda) = R(\lambda)F^{-1}$  is obtained by inverting the error correction model in (3). Naturally, the common trends model may also be characterized in terms of variance decompositions, i.e. by decomposing the variance of the forecast error for each variable (conditional on  $z$ ), at various horizons, into the contributions from different shocks.

As shown by Warne (1991)<sup>5</sup> the  $n \times n$  matrix  $F$ , needed to exactly identify the struc-

---

<sup>4</sup>If  $s_t$  is  $I(-1)$ , then clearly  $\sum_1^t s_i$  is  $I(0)$  and the term  $B \sum_1^t z_i$  does not incorporate any additional stochastic trends.

<sup>5</sup>See also the appendix in King et al. (1987).



tural model (4) from the reduced form model (3) is given by:

$$F = \begin{bmatrix} \Sigma C' A (A' A)^{-1} & \zeta (Q')^{-1} \end{bmatrix}. \quad (8)$$

The matrix  $\zeta$  is  $n \times r$  with  $\zeta \in \text{sp}(\alpha)$ , e.g.  $\zeta = \alpha(u'\alpha)^{-1}$  so that  $\zeta$  is a suitable normalisation of  $\alpha$ , while the  $r \times r$  (lower triangular) matrix  $Q$  is obtained from  $Q Q' = \zeta' \Sigma^{-1} \zeta$ . Furthermore,  $C = C(1) = \beta_{\perp} (\alpha'_{\perp} \Psi \beta_{\perp})^{-1} \alpha'_{\perp}$ , with  $\Psi := -(d\Pi(\lambda)/d\lambda)|_{\lambda=1}$ , as shown in Johansen (1991). From these properties it is straightforward to show that  $F^{-1} \Sigma (F')^{-1} = I_n$  and that  $R(1) = [A \ 0]$  (see Warne (1991, Theorem 2)). Finally,  $\Phi(\lambda)$  is obtained from  $R(\lambda) = R(1) + (1 - \lambda)\Phi(\lambda)$ .

Since identification of the common trends model is obtained via  $F$ , it is easy to see that a “structural” error correction model may be derived by premultiplying the reduced form systems by  $F^{-1}$ . We then find that the structural error correction model is

$$\Gamma^*(L)\Delta x_t = P^*(L)z_t - \alpha^* \beta' x_{t-1} + \nu_t. \quad (9)$$

The innovations in (9) are, of course, the structural innovations in the common trends model. Moreover,  $\Gamma^*(0) = F^{-1}$  and  $\alpha^* = F^{-1}\alpha$ , while the cointegration vectors are still given by  $\beta$ . In other words, the contemporaneous relations between the endogenous variables are given by the identification matrix  $F^{-1}$ , as they should, while the structural form “adjustment coefficients”,  $\alpha^*$ , depend both on  $F^{-1}$  and on the reduced form adjustment coefficients. The cointegration vectors, however, are just as structural in the reduced form model (3) as in the structural form model (9).<sup>6</sup>

It is interesting to note that the matrix with structural adjustment coefficients is here given by

$$\alpha^* = \begin{bmatrix} 0 \\ Q' u' \alpha \end{bmatrix}. \quad (10)$$

Hence, the first  $k$  equations in (9) describe the common trends and the remaining  $r$  equation represent the error correcting or I(0)-ness properties of the system.<sup>7</sup>

---

<sup>6</sup>Most economists are probably comfortable once we think about  $\beta$  as being normalized, e.g.  $\beta' = [\tilde{\beta}' \ I_r]$ . For this normalisation the coefficients in  $\tilde{\beta}$  are uniquely determined. Moreover, we can often construct examples such that these vectors have an economic interpretation.

<sup>7</sup>Structural error correction models of this type have previously been studied by Boswijk (1992) to

From the moving average representation in (7) it follows that changes in the endogenous variables are always the result of changes in the exogenous, i.e. the  $z$ 's and the  $\nu$ 's. Moreover, in terms of parameters, such changes are measured (over time) by the parameters of this system. Since these parameters are highly nonlinear functions of the parameters of the error correction model (9) it is difficult to provide an economic interpretation of the coefficients in the matrices  $\Gamma_i^*$ ,  $P_j^*$  and  $\alpha^*$ . Furthermore, the coefficients in  $\alpha^*$  (or  $\alpha$ ) are sometimes considered to contain information about long run responses in the endogenous variables to the equilibrium errors. Clearly, this is not the case. The only long run information to be found in  $\alpha^*$  is that the common trends in (9) are given by  $\alpha_{\perp}^{*'} = [I_k \ 0]$ . (See Warne (1993) for a more detailed discussion, and Gonzalo and Granger (1992) for an alternative interpretation of common trends.)

## 2.2 A Labor Market Model

Now, the question is: How can we make identifying assumptions such that the common trends model has an economic interpretation? To address this let us consider the labor market model we shall investigate in sections 4 and 5. Let  $x_t = [(Y_t - E_t) \ E_t \ (L_t - E_t) \ W_t]'$ , where  $Y_t$  is real output,  $E_t$  employment,  $L_t$  the labor force, and  $W_t$  the product real wage. All variables are expressed in natural logarithms and, without loss of generality, we delete the exogenous variables from the discussion here.

We conjecture that the data generating process is driven by two common trends (i.e.  $k = 2$ ): A technology trend ( $\tau_{y,t}$ ), which lies behind the long run development of productivity ( $Y_t - E_t$ ) and the real wage, and a labor supply trend ( $\tau_{l,t}$ ). If there are two common trends, there must be two cointegration vectors. Our primary candidate for  $\beta$  will be

$$\beta' = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

This means that unemployment ( $L_t - E_t$ ) and labor's share of value added ( $W_t + E_t - Y_t$ ) are  $I(0)$ . Most economists would probably agree that these are meaningful hypotheses about

---

test for, e.g., cointegration. There the final  $r$  rows of  $\alpha^*$  form a diagonal matrix. For example, by letting  $\beta^{*'} = Q' u' \alpha \beta'$  and  $\alpha^{**} = [0 \ I_r]'$  we find that  $\alpha^* \beta' = \alpha^{**} \beta^{*'}$ . Thus, the common trends model is fully consistent with the structural error correction model in Boswijk.

the cointegration vectors, although there is no strong theoretical support for either.

While cointegration implies that  $\beta'A = 0$ , i.e.  $rk = 4$  restrictions on  $A$ , to identify all 8 elements of  $A$  in (4) we must at least impose  $k(k-1)/2 = 1$  additional restriction.<sup>8</sup> Let  $a_{ij}$  denote the element in the  $i$ :th row and  $j$ :th column of  $A$ , and let  $\tau_t = [\tau_{l,t} \ \tau_{y,t}]'$  so that the labor supply trend is ordered first and the technology trend last. We may, e.g., assume that the size of the labor force in the long run is independent of the technology trend. Since  $(L_t - E_t)$  is  $I(0)$  it follows that  $L_t$  and  $E_t$  must be driven by the same trend, i.e.  $a_{22} = 0$ . Using the condition  $\beta'A = 0$  and the identifying assumption we get the following common trends model (ignoring, for the moment, the components related to the exogenous variables):

$$x_t = \begin{bmatrix} Y_t - E_t \\ E_t \\ L_t - E_t \\ W_t \end{bmatrix} = x_0 + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & 0 \\ 0 & 0 \\ a_{41} & a_{42} \end{bmatrix} \begin{bmatrix} \tau_{l,t} \\ \tau_{y,t} \end{bmatrix} + \Phi(L)\nu_t. \quad (12)$$

The identifying assumptions made so far are sufficient to derive the permanent structural shocks  $\varphi_t$  from the residuals  $\varepsilon_t$ . To identify the transitory structural shocks  $\psi_t$  we make assumptions about their contemporaneous effects on  $\varepsilon_t$  (and, hence, on  $x_t$ ), i.e. put restrictions on  $Q$  (see equation (8)).

Our identifying assumptions are motivated in terms of Figure 1, where the D line traces out the labor demand schedule, S represents labor supply and  $\bar{W}_t$  may be interpreted as an institutionally determined real wage level. We have already assumed that there are two shocks which have permanent effects on  $L_t$ ,  $E_t$  and  $W_t$ : a labor supply shock (which permanently shifts S) and a technology shock (which permanently shifts D).  $\bar{W}_t$  is assumed to adjust to the market clearing level in the long run. It is natural to interpret one of the two transitory shocks as a wage shock  $\psi_{w,t}$ , which involves a shift in  $\bar{W}_t$  and contemporaneously alters  $Y_t$ ,  $L_t$ , and  $E_t$ . The other transitory shock may be interpreted as an aggregate demand shock ( $\psi_{d,t}$ ), a transitory shift in D, which affects  $Y_t$ ,  $L_t$  and  $E_t$  but not  $W_t$  (since  $W_t = \bar{W}_t$  in the short run). These assumptions are sufficient to obtain an exactly identified common trends system. It should be emphasized that identification

---

<sup>8</sup>The remaining  $k(k+1)/2 = 3$  free coefficients can be obtained from the fact that  $AA' = C\Sigma C'$ . For details see Englund et al. (1992), Mellander et al. (1992), Quah (1992), or Warne (1991).

of the trend shocks and of the transitory shocks is independent. That is, independent of which identifying assumptions we make about the transitory shocks, the reaction of the system to the trend shocks is the same and vice versa.

We may now return to the three questions about the labor market discussed in section 1 and explain how they may be answered in our common trends framework. Regarding the first question, about the relative importance of supply and demand factors, we will follow Blanchard and Quah (1989) and others and identify “supply shocks” as shocks to the stochastic trends, i.e. the innovations to the labor supply and technology trend. The transitory innovations are viewed as “demand shocks”.<sup>9</sup> The importance of these shocks may be analysed by investigating the impulse response functions (sequences of coefficients in the structural moving average representation).<sup>10</sup> These functions will also be exploited for answering the second question, about speeds of adjustment.

How to answer the third question, about the long run relation between the real wage and unemployment, is less obvious. If the former variable is  $I(1)$  while the latter is  $I(0)$ , there is no cointegration relation between the real wage and unemployment. In this sense, there is no long run relation between the two variables. Another way to express the same thing is to note that the long run effects on the real wage and unemployment from the permanent shocks are given by the coefficients in the  $A$  matrix of equation (12) (transitory shocks have, by definition, zero long run effects on all variables). The effects on the real wage are given by the coefficients in the fourth row, while the effects on unemployment are given by the third row, which is zero. “Long run relations” between the real wage and unemployment can however be found by comparing the impulse responses for these variables (for various shocks) at a certain distant, but finite, horizon.

---

<sup>9</sup>There are many legitimate objections to such an interpretation; see Blanchard and Quah (1989) for a careful discussion.

<sup>10</sup>To study the relative importance of the different shocks, forecast error variance decompositions are usually computed in the empirical VAR literature. However, since we do not have a model for the stochastic exogenous variables, we cannot assess the relative importance of these versus the shocks. Accordingly, we will not present any variance decompositions below.

### 3 Comparisons with Earlier Models

Earlier papers which have addressed the questions we are dealing with have most often been conducted within a single (or two) equation(s) error correction framework. To facilitate the interpretation of our results, this section is devoted to a comparison between our approach and previous studies. We let the latter be represented by the influential paper by Bean et al. (1986).

The econometric system which is studied in Bean et al. can be represented by the following two equations:

$$\begin{aligned}\Delta E_t &= \delta_1(E_{t-1} - K_t) + \delta_2 W_t + \delta_3 t + \delta_4 t^2 + \delta_5 \Delta E_{t-1} + \eta_{E,t}, \\ \Delta W_t &= \gamma_1(L_t - K_t) + \gamma_2 W_{t-1} + \gamma_3 t + \gamma_4 t^2 + \gamma_5(L_t - E_t) + \eta_{W,t}.\end{aligned}\tag{13}$$

This system is subject to the cross equation restrictions  $\gamma_i = c\delta_i$  for  $i = 1, \dots, 4$  for some nonzero constant  $c$  (e.g.  $\gamma_i = (\gamma_2/\delta_2)\delta_i$  for  $i = 1, 3, 4$  so that  $c = \gamma_2/\delta_2$ ). These restrictions imply that changes in employment and the real wage are affected by the same equilibrium relations between the real wage, employment, the capital stock ( $K_t$ ) and the deterministic trend. Bean et al. explicitly view  $\beta'_1 = [0 \ -\delta_1 \ \delta_2 \ \delta_3 \ \delta_4 \ \delta_5]$  as a cointegration vector, where we have ordered the variables as  $X_t = [E_t \ K_t \ W_t \ t \ t^2 \ L_t]'$ . Furthermore, Bean et al. argue that their model possesses a natural rate of unemployment, which may be taken to mean that unemployment is also  $I(0)$ , i.e. that another cointegration vector is given by  $\beta'_2 = [-1 \ 0 \ 0 \ 0 \ 0 \ 1]$ . Using these cointegration relations we may rewrite the model in (13) on error correction form as

$$\begin{aligned}\Delta E_t &= \delta_3 + \delta_4(2t - 1) + \beta'_1 X_{t-1} - \delta_1 \beta'_2 X_{t-1} - \delta_1 \Delta K_t + \delta_2 \Delta W_t \\ &\quad + \delta_5 \Delta E_{t-1} + \eta_{E,t}, \\ \Delta W_t &= \gamma_3 + \gamma_4(2t - 1) + c\beta'_1 X_{t-1} + \gamma_5 \beta'_2 X_{t-1} + (\gamma_1 + \gamma_5) \Delta L_t - \gamma_1 \Delta K_t \\ &\quad - \gamma_5 \Delta E_t + \eta_{W,t}.\end{aligned}\tag{14}$$

As discussed in the previous section, the common trends model also has an implied error correction representation on structural form, (9). In what important respects do the two approaches differ? Disregarding minor differences in included variables one could view the Bean et al. model as partial with respect to our model.<sup>11</sup> They arrive at their

---

<sup>11</sup>Bean et al. include some additional exogenous variables, e.g. the capital stock, while productivity enters endogenously in our model.

specification by imposing restrictions on the dynamics and, more importantly, through exogeneity assumptions. While zero restrictions on the dynamics may be regarded as an empirical issue, the question whether certain variables can be treated as exogenous or not cannot be settled on empirical grounds. The latter claim (at least in practise) also applies to the question of whether trends should be modelled as deterministic or stochastic. Bean et al. handle the trends driving labor supply and technology (captured by a time trend,  $t$ ) as deterministic, while our model is based on the assumption that these are stochastic (although they are allowed to have a drift via  $d_t$ ). Our analysis centers on ideas which are common in modern business cycle analysis, while the approach taken by Bean et al. is characteristic of most empirical labor market models.

Turning to the three questions which are the focus of this paper, Bean et al. first measure the importance of supply (and demand) factors by the influence from the different exogenous variables which are assumed to be associated with supply (or demand), such as the labor force and the time trend. In the common trends model, we primarily measure the importance of supply and demand *shocks*.<sup>12</sup> Second, Bean et al. measure the speed of adjustment of the real wage by one coefficient,  $(1 + \gamma_2)/\gamma_2$ , while we investigate the adjustment of the real wage to each type of shock, i.e. look at the impulse responses. The third question, about the long run relation between unemployment and the real wage, again requires a more detailed discussion.

Bean et al. calculate a “long run effect of unemployment of the real wage” as  $-\gamma_5/\gamma_2$ . More generally, if we look at an equation of the form

$$\gamma_1(L)W_t = \gamma_2(L)(L_t - E_t) + \gamma_3(L)Y_t + \eta_{W,t},$$

a “long run effect” of  $(L_t - E_t)$  on  $W_t$  may be calculated as  $\gamma_2(1)/\gamma_1(1)$  ( $Y_t$  is used

---

<sup>12</sup>The importance of our observable exogenous variables, a currency basket index and a tax wedge, will not be examined in any detail. The main reason for this is that a thorough analysis should include a model of these variables as well. To understand why this is a relevant concern, suppose that the exogenous variables have mean zero and are generated from independent AR(1) models. Hence, a shock to one of the exogenous variables will never take the form of one unit increase at some point in time and then a zero change afterwards. Rather, they will only gradually revert to zero. Hence, it is not meaningful to conduct an experiment in the common trends model where the exogenous stochastic regressors are shocked by one unit at a point in time and then set to zero afterwards.

to denote any other included variable). The question is what this measure *means*. If unemployment is viewed as an exogenous variable we can indeed speak of a long run “effect” from  $(L_t - E_t)$  on  $W_t$ . It deserves to be emphasized that in that case it does not matter if unemployment is regarded as an  $I(0)$  time series while the real wage is treated as  $I(1)$  as long as  $\gamma_1(1) \neq 0$ . A transitory change in unemployment can have a permanent effect on the real wage, just like the innovations to the stochastic trends have permanent effects although the innovations themselves are i.i.d. If unemployment is (autocorrelated or) *not* exogenous, it makes little sense to speak of a “*ceteris paribus*” change in unemployment or of an “effect” from  $(L_t - E_t)$  on  $W_t$ . The long run response of the real wage will be different, depending on what shock that creates the change in unemployment.<sup>13</sup>

One seemingly important difference between our approach and that of Bean et al. is that model (13) is viewed as describing a labor demand and a wage setting equation. In contrast, we focus on the common trends model (4) and on interpreting the innovations. It may then seem as if our approach is “less structural” than that of Bean et al. In a recent paper Manning (1992) discusses the problems of identification in wage equations such as the one in (13). Manning argues that formal identification *per se* is a question of imposing sufficient restrictions, but that the heart of the matter is whether economic theory can justify the chosen set of restrictions, and that set only. This point is illustrated by considering an often desired quantity: the estimated wage elasticity with respect to unemployment. Manning shows that two perfectly reasonable routes to identification will come up with estimated elasticities of different signs. The conclusion drawn is that the estimated coefficients are functions of ‘deep’ economic theory parameters and these cannot, in general, be unambiguously retrieved. We like to think that we are as “structural” as the empirical problem requires.

Most econometricians probably agree that time series models such as VAR and error correction models often provide a useful framework for a good description of macroeconomic data. But these models are far from optimal when our interest in the data goes beyond that of pure data description. First, the number of coefficients tends to rapidly become very large. With the limited amount of data at our disposal, it follows that the

---

<sup>13</sup>Blanchard and Quah (1989) make the same objection against “Okun’s law”.

standard errors of the estimated coefficients tend to be “too large”. Second, the coefficients generally have no clear economic interpretation. Economic theory can clearly play a major role when we address these problems in practice, and theoretical relationships may be tested as restrictions on the coefficients of a VAR or an error correction model. But we interpret Manning (1992) as saying that there is no economic theory which motivates the restrictions imposed in (14).

The approach taken in this paper focuses on a good description of the data. The restrictions considered concern long run relations (cointegration vectors and identifying assumptions on long run impulse responses). A time series model with these properties should be able to provide empirically relevant answers to the three questions discussed above.

A recent paper by Bean (1992) is in the same spirit as our. However, there are some minor differences regarding the specification of the econometric model. His data set (for the British labor market) includes the nominal wage and the price level, which makes it possible to identify and analyse the effects of purely nominal shocks. Such a model may be preferable to ours on *a priori* grounds, but our experience from earlier studies (Vredin and Warne (1991), Englund et al. (1992), Mellander et al. (1992)) is that a model with only real endogenous variables has more satisfactory statistical properties.<sup>14</sup> Another difference between our model and that of Bean (1992) is that we make the identifying assumption that technology shocks have no long run effect on the labor force, while Bean assumes that labor supply shocks have no long run effect on productivity. Both assumptions seem plausible, and, hence, one cannot discriminate between them on theoretical grounds. It should be noted that the distinction between the two supply (permanent) shocks has no implication for the relative importance of supply and demand (transitory) shocks.

---

<sup>14</sup>See also Blakemore and Hoffman (1993) for a common trends study on U.S. labor market data using e.g. nominal wages and prices.



## 4 Empirical Evidence on Cointegration: A Monte Carlo Study

As frequently noted above, there are several equivalent representations of a cointegrated system. In this section we shall focus on the estimation of the error correction system in (3). Our primary concern will be to estimate the cointegration rank  $r$  for our empirical labor market model and to test if the cointegration vectors in (11) are consistent with data. The output from this analysis will then be used in the common trends model, discussed in section 5.

The data set we shall analyse consists of quarterly observations (1965–90) from the Swedish labor market on (the natural logarithm of) real GDP for the private sector ( $Y_t$ ), and total employment in hours for the private sector ( $E_t$ ). The unemployment series,  $(L_t - E_t)$ , is equal to  $\ln[1/(1 - u_t/100)]$ , where  $u_t$  is the unemployment rate, in percent, for the economy. Note that this, implicitly, gives us data on the labor force ( $L_t$ ) in hours for the private sector. The product real wage series ( $W_t$ ) for the private sector is computed from the average nominal wage per hour (in the private sector), the pay roll tax rate, and the producer price index for manufacturing industries.

The deterministic variables include a constant (to allow for linear deterministic trends in  $x$ ) and centered seasonal dummies. Moreover, two exogenous stochastic variables, a tax wedge and a currency basket index, are included in our model. The inverse tax wedge ( $V_t$ ) is calculated as  $\ln[(1 - \theta_t^i)/(1 + \theta_t^p)(1 + \theta_t^{\text{vat}})]$ , where  $\theta_t^i$  is the average income tax rate,  $\theta_t^p$  is the pay roll tax rate, and  $\theta_t^{\text{vat}}$  is the value added tax rate. Hence,  $V_t$  is decreasing in all tax rates. Finally, the currency basket index ( $S_t$ ) is based on the 15 exchange rates in the old Swedish currency basket index (1979–91). The weights in the basket are taken from March 1979, and the observations of the exchange rates are the notations of the Central Bank of Sweden for the last month of each quarter. The index is set to 100 for 1965:1, and the series we examine is the natural logarithm of this index. Since both  $V_t$  and  $S_t$  are mean reverting in first differences, but not in levels, the  $s_t$  vector is given by  $[\Delta V_t \ \Delta S_t]'$ .<sup>15</sup>

---

<sup>15</sup>We are grateful to Bertil Holmlund, Uppsala University, for letting us use the data on GDP, unemployment, the average nominal wage per hour, the producer price index and the average income tax. We

The data is depicted in Figure 2. Here we have graphed the quarterly series in yearly terms, i.e. for each variable we graph one series with only first quarter observations, one with second quarter observations, etc. As pointed out by Franses (1992) this graph can be useful for evaluating whether the seasonality in the data is well modelled by dummies. For example, if two quarters lie on different levels and “change places” after some period, this may be an indication that seasonality is better represented by a stochastic model rather than a deterministic. Based on the graphs in Figure 2 it seems reasonable to consider the simple dummy model of seasonality.

Prior to the cointegration analysis we need to establish an appropriate lag order for the underlying VARX. We have considered lag lengths ranging from 1 to 8 and the results are reported in Table 1. The preferred order is  $p = 4$  since the residuals in this case seem less (if at all) disturbed. Using a test size of five percent the multivariate Portmanteau test (see Hosking (1980)) rejects the null of no serial correlation in the residuals for all lag orders but 4. Given the two normality tests (based on estimated skewness and kurtosis and due to Mardia (1970)) we reject normality for  $p = 1$  only. As for the order determination criteria, we find that the Akaike (1969) information criterion (AIC) does not converge to a minimum within the investigated range of orders, whereas the law-of-the-iterated-logarithm criterion (LIL; Hannan and Quinn (1979)) prefers 5 lags and the more conservative Bayesian information criterion (BIC; Schwarz (1978)) chooses 2 lags. All in all, these results, together with the univariate residual analysis (not reported) with respect to serial correlation, ARCH-effects and normality, suggest  $p = 4$  as a reasonable choice.

The above misspecification analysis is standard for empirical VAR's; see e.g. Lütkepohl (1991). As a further means to check the specification we have calculated the roots of the estimated polynomial matrix  $\hat{\Pi}(\lambda)$ . The roots must be outside the unit circle or equal to one in order for the estimated error correction model to describe a stable system. We find that all roots to  $\hat{\Pi}(\lambda)$  have modulus greater than 1. However, three roots are close to unity (1.0163 and  $2 \times 1.0526$ ), informally suggesting that the unit root approach is appropriate for the data at hand.

---

are also grateful to Lena Svensson, Trade Union Institute for Economic Research (FIEF), for providing us with the exchange rate data. These and the remaining series are taken from the SNEP data base.

Statistical analysis of the error correction model (3) involves for a given  $\beta$  linear estimation of coefficients on  $I(0)$  processes only. In contrast, estimation of (2) is nonlinear in the coefficient matrices  $\alpha$  and  $\beta$ . The method of estimation is maximum likelihood (reduced rank regression) and the procedures involved have been developed by Søren Johansen; see Johansen (1991,1992) and Johansen and Juselius (1990,1992) for details. Inference about the cointegration rank  $r$  in (2) is carried out using a likelihood ratio test, known as the *trace test* for cointegration rank. The asymptotic distributions for this test are tabulated e.g. in Osterwald-Lenum (1992).

According to the evidence in Table 2 the estimated cointegration rank is 2. The estimation strategy is to first test the null  $r = 0$  against the alternative  $1 \leq r \leq 4$ . For a test size of five percent this null is not supported by the data. Next, the null  $r \leq 1$  is rejected against the alternative  $2 \leq r \leq 4$ , while the null hypothesis  $r \leq 2$  is not rejected against  $3 \leq r \leq 4$ . However, choosing  $r = 3$  is by no means unreasonable since the p-value for the previous test is around seven percent. In order to make a closer investigation of the behavior of the trace test we have undertaken a simulation study. The purpose is not primarily to resolve the rank determination problem but rather to gain insights about how well the asymptotic distributions approximate in the small sample case at hand.

The design of the experiment is as follows. Five data generating processes are constructed by taking the coefficients of the estimated model and setting the cointegration rank  $r$  to 0, 1, 2, 3, and 4 respectively.<sup>16</sup> For each DGP we generate 10,000 samples by adding pseudo-normal errors that have been transformed according to the estimated covariance matrix.<sup>17</sup> Trace test statistics are calculated under the four possible null hypotheses  $r = 0$ ,  $r \leq 1$ ,  $r \leq 2$ , and  $r \leq 3$ . These statistics are ordered into empirical distributions that can be compared with critical values published by Osterwald-Lenum

---

<sup>16</sup>Note that all the exogenous processes and the initial conditions are fixed.

<sup>17</sup>Simulation experimentation as outlined here is also known as ‘parametric’ bootstrapping. The procedure is closely related to the more common ‘nonparametric’ bootstrap. Whereas the latter resamples by drawings with replacement from e.g. the estimated residuals, the former method resamples by drawing from e.g. zero mean pseudo-normal variates having a covariance matrix equal to the one originally estimated. Hence, the parametric bootstrap relies on specific distributional assumptions while the nonparametric does not. The simulations are carried out in FORTRAN 77 and, specifically, the standard normal variates are generated from a ratio-of-uniforms routine.

(1992). The results from these exercises are given in Tables 3 and 4.

In panels A–D of Table 3 the 50, 80, 90, and 95 percent quantiles are compared with the corresponding critical values of Osterwald–Lenum. The focus is here on the cases when the null hypothesis is true. We find that the empirical distributions are skewed to the right of the asymptotic, i.e. the test is oversized so that we may reject a true null at some higher level than the desired, say, five percent level. It is striking how similar the patterns of right–skewness are manifested in all four empirical distributions of the true null hypotheses. Note that the critical values of the 80 percent quantiles of the empirical distributions are almost equal to those from the 95 percent quantiles of the Osterwald–Lenum distributions. In other words, when we use the five percent critical values of the asymptotic distributions the trace tests display empirical sizes of roughly 20 percent for the four nulls. Figure 3 illustrates how the density functions of the empirical and asymptotic distributions relate.<sup>18</sup>

A consequence of these results is that one may reconsider the preferred rank  $r$  in the formal testing in Table 2 and reject only the hypothesis  $r = 0$ , i.e. choose the estimated cointegration rank  $r = 1$ . The test statistic of 78.66 is in the extreme upper tail of the empirical distribution and, hence, the null  $r = 0$  can safely be rejected. However, the simulations also provide empirical distributions for the test statistics when the null hypothesis is not true. Using this information modifies the interpretation as we shall demonstrate below.

Panel A of Table 4 describes how successful the trace test procedure is in selecting the correct cointegration rank. We see that the proportion of correctly determined ranks decreases as the true rank increases. The difference is only marginal for true ranks of zero and one (79.95 versus 78.09 percent), whereas the proportion for a true  $r = 4$  is substantially smaller. An interesting break in the pattern can be noted for  $r = 3$ ; the trace test procedure favors two cointegration vectors to the true three in the DGP. We interpret this as a sign of a lesser significance for the third estimated vector.

---

<sup>18</sup>The density functions are calculated from a smoothed Rosenblatt histogram estimator. The asymptotic distributions are simulated using a procedure which is equivalent to the one used by Osterwald–Lenum. The difference is that our densities are based on 10,000 replications, whereas the Osterwald–Lenum values are based on 6,000 replications.

Panel B of Table 4 gives size (in the bold faced “diagonal”) and power (cells/grids to the right of the “diagonal”) estimates, whereas panel C provides the size adjusted power estimates. The message from panel D is how plausible the outcome of the testing is in light of the simulation experiment. Focusing on panel D we find that if the cointegration rank is zero, there is only a chance of 0.07 percent of recording a statistic larger than 78.66 when testing the null  $r = 0$ . Likewise, the probability of surpassing 30.29 when testing the null  $r \leq 1$  is 2.12 percent. In other words, if the true rank is zero, then the outcome from the tests (see Table 2) is very improbable (except for the null  $r \leq 3$ ). Moving to the next column in panel D, where the true cointegration rank is one, we see that the outcome is much less extreme were it not the null  $r \leq 2$ . For this hypothesis the probability of obtaining a value larger than 14.66 is only 4.11 percent. Skipping one column we find for a true cointegration rank of 3 that the situation is reversed. The observed statistics for the nulls  $r = 0$  and  $r \leq 1$  are down in the lower tail of the simulated distributions. Based on these exercises we conclude that the experiments do *not* lend convincing support for rejection of the formal results in Table 2. Hence, in what follows we condition our analysis on  $r = 2$ .

The two estimated and suitably normalized cointegration vectors have the following appearance:

$$\hat{\beta}'_N = \begin{bmatrix} -.0195 & .0862 & 1.0000 & .0000 \\ -1.0561 & -1.5358 & .0000 & 1.0000 \end{bmatrix}.$$

By imposing linear restrictions on the vectors we may investigate if there is empirical support for the two candidates in equation (11); labor’s share of value added ( $W_t + E_t - Y_t$ ) and unemployment ( $L_t - E_t$ ). In other words, are the vectors  $\beta'_1 = [-1 \ 0 \ 0 \ 1]$  and  $\beta'_2 = [0 \ 0 \ 1 \ 0]$  contained in the estimated cointegration space? The test procedure is likelihood ratio and the asymptotic distribution of the test is  $\chi^2$  with degrees of freedom depending on the number of restrictions imposed. For the individual tests the degrees of freedom is two and for the joint test it is four. As in the case of the trace test above we augment the formal testing by simulations in order to evaluate how valid the asymptotic approximation is in this specific small sample application.

The formal test results as well as those from the simulations are presented in Table 5. For the individual hypotheses, i.e.  $\beta_1 \in \hat{\beta}$  and  $\beta_2 \in \hat{\beta}$ , the values of the test statistics

are 2.65 and 1.57, respectively, while the corresponding asymptotic p-values are .27 and .46. Further, for the joint hypothesis the observed value of the  $LR$  test is 21.22 and its corresponding p-value is almost zero. The explanation for these results is that each hypothesized vector is consistent with the eigenvector of the second largest eigenvalue ( $\hat{e}_2 = .146$ ; see Table 2), but not with the eigenvector to the largest eigenvalue ( $\hat{e}_1 = .386$ ). Given the large difference between the two eigenvalues it is therefore not surprising that the individual test statistics have small values while the joint test statistic is large.

From the simulation results in Table 5 we also find that the  $LR$  tests are oversized. Based on the critical values from the 95 percent quantiles of the  $\chi^2$  distributions, the probability of obtaining larger values is approximately 30 percent when the empirical distributions are considered. The final column in Table 5 contains the p-values of the tests. Since the empirical distributions are skewed to the right of the asymptotic, the empirical p-values are larger than the asymptotic. For example, the test of the joint hypothesis that unemployment and labor's share of value added are  $I(0)$  now has a p-value of 3.16 percent. Below we shall therefore condition the analysis on the two cointegration vectors in equation (11). The two cointegration relations are portrayed in Figure 4.

Before we proceed with the common trends analysis, it is interesting to examine the evidence in Table 6. Here we study the rate at which the empirical distribution, for the null that  $\beta$  in equation (11) is consistent with the data, converges to the asymptotic  $\chi^2_4$ . The evidence in this Table only concerns the 95 percent critical value of 9.488 from the asymptotic distribution. Since we do not have a model for the exogenous stochastic variables we have chosen to exclude these from the DGP. Relative to the DGP we used in Table 5 for the current null, we have not changed the values of the coefficients on the remaining regressors in the DGP. For a sample size of 100 observations, the probability of exceeding 9.488 is approximately 41 percent. This is roughly consistent with the 35 percent we obtained for the same hypothesis in Table 5.<sup>19</sup> As we increase the sample size, the nominal size of the test decreases, first rapidly, then more slowly. At 400 to 500 observations, the nominal size is still high, about 10 percent, and not until we consider a sample size of 2,000 does the nominal size approximate the test size (5 percent) well. Hence, the evidence presented here suggests that we should be very careful when we test

---

<sup>19</sup>The sample 1966:2–90:4 consists of 99 observations.

linear restrictions on  $\beta$  and use the asymptotic distribution to conduct inference, even for large samples.

## 5 Unemployment and Real Wages in a Common Trends Framework

In this section we shall discuss the common trends evidence on Swedish labor market data. In section 5.1 we study the empirical evidence from a common trends model based on the identification procedure outlined in section 2. In section 5.2 we present results based on the Bean (1992) identification scheme and compare those results with our benchmark model. Also, we try to determine which of the two models is preferred for the data at hand. In section 5.3 we examine the importance of the exogenous variables for describing the behavior of real wages and unemployment.

### 5.1 The Benchmark Model

The common trends model we will study in this section is based on the identifying assumptions we discussed in section 2. That is, employment is determined solely by the labor supply trend in the long run, while the trending behavior in real wages and productivity is given by the technology *and*, possibly, by the labor supply trend. To identify the transitory shocks it is assumed that the first, “aggregate demand” shock has zero contemporaneous influence on the real wage. Below we shall compare the identifying assumption about the transitory innovations to the case when productivity has a zero contemporaneous response to the aggregate demand shock.

In Table 7 the estimated matrix of common trends coefficients,  $\hat{A}$ , and the estimated  $\alpha$  matrix are presented. Standard errors of the estimated coefficients are reported within parenthesis. Focusing first on the common trends coefficients we see that a one standard deviation increase in the technology trend raises real wages (and productivity) by .6 percent in the long run. Similarly, a unit increase in the labor supply trend results in a .9 percent long run response in employment (and in the labor force). Furthermore, real wages (and productivity) are lowered by .8 percent in the long run from such a shock. In

fact, it is meaningful to test the hypothesis that  $a_{11} = 0$  (see equation (12)). The value of the Wald statistic is 8.11, clearly in excess of 6.63 from the 99 percent quantile of the  $\chi_1^2$  distribution. Hence, conditional on our identifying assumptions, we conclude that the labor supply trend matters, at least in the long run, for explaining the behavior of real wages (and productivity).

It is interesting to compare the above results with the type of conclusions typically drawn from estimates of  $\alpha$ . The first column of  $\alpha$  contains the coefficients on unemployment while the coefficients on labor's share of value added are given in the second column. In the real wage equation (row four) the coefficient on unemployment is approximately equal to  $-2$ . The minus sign is, of course, due to the minus sign in front of  $\alpha$  in equation (3). Since the coefficients on lagged changes of the real wage are small<sup>20</sup> this result is broadly consistent with what other researchers have found. However, it does *not* mean that a one percent increase in unemployment leads to a two percent long run drop in real wages. Rather, the correct interpretation of the coefficient is that the information in unemployment is useful for making predictions about real wages. That is, unemployment Granger causes (in the standard sense) real wages.

Also, from Table 7 we find that the coefficient on labor's share of value added in the unemployment equation (row three) is not significantly different from zero at the ten percent level ( $t$ -ratio is approximately 1.17). Still, one of the coefficients on lagged changes of real wages is significantly different from zero, informally suggesting that real wages Granger causes unemployment. In fact, the interesting  $\alpha$  coefficient in the unemployment equation is the first. Conditional on the other variables in the  $x$  vector being  $I(1)$  and the coefficient on labor's share of value added being zero, the  $\alpha$  coefficient on unemployment measures the degree of mean reversion in unemployment (the coefficients in the third row of  $\hat{\Gamma}(\lambda)$  are small and may be neglected). The larger the  $\alpha$  coefficient is (as long as it is below 1), the faster unemployment reverts back to its mean. A value of .16 suggests that the mean reversion is slow, which is consistent with the general observation in the empirical macro labor literature of persistence in unemployment.

In Figures 5 to 8 the responses in  $x$  (over ten years) from the four shocks are portrayed. For the two permanent shocks we find no significant effect on the unemployment rate. The

---

<sup>20</sup>All estimated coefficients of the error correction model are available from the authors on request.



uncertainty is, however, quite large for the first 8 quarters. Similarly, we find no significant response in employment beyond the first ten quarters. This is particularly disturbing for the labor supply shock. After approximately four years, the point estimates have reached the long run level. The long run 95 percent confidence interval is  $[\text{.004}; \text{.014}]$ , which may be compared with the 10 year 95 percent confidence interval of  $[-\text{.007}; \text{.025}]$ . The latter interval is approximately three times as wide as the former.

The responses in real wages and productivity from the permanent shocks are more in line with what we expect. After 3 to 4 years the responses in productivity has reached a neighborhood of the long run responses and the confidence intervals do not include the origin. The real wage seems to respond more slowly, reaching the long run level after about 5 to 7 years. At this point the confidence intervals are narrow enough not to include the origin. In addition, the 10 year confidence intervals of real wages and productivity are relatively close to the long run confidence intervals.

Turning to the transitory shocks, it can be seen in Figure 7 that the contemporaneous response in employment from the first transitory shock (the “aggregate demand” shock) is about minus 1 percent (i.e. a negative demand shock). At the same time unemployment increases by about .1 percent. Accordingly, the labor force decreases by roughly .9 percent. Real wages are, by assumption, not affected contemporaneously. After about 2 years the effects on employment and unemployment die out, while short run negative effects are recorded in real wages. The effect after three quarters is a .15 percent increase in unemployment, while after nine quarters real wages have dropped by .7 percent. For these quarters real wages thus decrease by approximately 5 units per unit increase in unemployment. Note, however, that for any given quarter at least one of the variables is not (significantly) influenced by the aggregate demand shock.

The fact that unemployment responds more quickly to an aggregate demand shock than real wages explains (at least in part) why unemployment Granger causes real wages. Theoretical work which may explain this picture of real wage stickiness and quantity adjustment in the short run include models with trade unions, efficiency wages, or (firm-specific) human capital.

Note that although some of the short run responses in productivity from a negative shock are positive (in terms of the confidence bands), total output decreases but not as

much as employment. This result is clearly reasonable as it is consistent with the elasticity of output with respect to labor (in a production function) being less than unity.

The second transitory shock is interpreted as a wage shock. From the response functions in Figure 8 we find that such a shock leaves employment and unemployment relatively unchanged, while real wages increase and productivity falls within the first year.

To investigate the robustness of the results for the transitory shocks we have also estimated the responses in  $x$  when the contemporaneous effect on productivity from the first transitory shock is set to zero. Since the results from this identifying assumption are virtually identical to those presented above we do not report them here. It is, however, worth pointing out that a large number of alternative identifying assumptions are possible. For example, one may wish to consider an aggregate demand shock which does not have a contemporaneous effect on the labor force. In that case, the contemporaneous effect on employment is equal to minus the effect on unemployment. We will not pursue the matter further here.

## 5.2 The Bean Model

In his study on U.K. labor market data, Bean (1992) identifies labor supply and technology trends by assuming that the labor supply shock has a zero long run impact on productivity. The technology trend, in contrast, is allowed to influence the long run behavior of employment. Since our labor supply variable has been constructed from data on aggregate unemployment and employment in the *private* sector, the otherwise reasonable assumption that technology shocks do not affect labor supply in the long run may seem too strong.<sup>21</sup> There are thus good reasons to examine the differences in results from applying Bean's identifying assumptions.

Given our cointegration vectors (see equation (11)), these mean that the trend coeffi-

---

<sup>21</sup>Bertil Holmlund has pointed this out to us.

cient matrix takes the form:

$$A = \begin{bmatrix} 0 & a_{12} \\ a_{21} & a_{22} \\ 0 & 0 \\ 0 & a_{12} \end{bmatrix}.$$

Like in section 5.1 above the labor supply trend is ordered first, while the technology trend is ordered last. For the transitory shocks we keep the identifying assumption that the real wage is not contemporaneously affected by the first transitory shock. Since the identifying assumptions about the transitory shocks are independent of which identifying restrictions are imposed on the permanent shocks (and vice versa), it follows that the impulse response functions for the transitory shocks are those depicted in Figures 7 and 8.<sup>22</sup>

In Table 8 we report the estimated coefficients of the  $A$  matrix, along with standard errors. Here we find that a unit technology increase raises the long run real wage (and productivity) by roughly 1 percent. Similarly, a unit increase in the labor supply trend pulls up employment by .5 percent in the long run. Additionally, employment (and the labor force) drops by approximately .8 percent from a technology shock.

The impulse response functions (for the first 10 years) along with 95 percent confidence bands from the trend shocks are depicted in Figures 9 and 10. In terms of unemployment, the two common trends models generate almost identical results. Furthermore, the estimated responses in employment are very uncertain for the Bean model (like we observed for our benchmark model). In fact, if we compare the confidence intervals from the two models it seems like those for the real wage (and productivity) have “jumped up” relative to the benchmark, whereas those for employment have “fallen down”.

To investigate the model selection issue further, we follow the suggestion made by e.g. Blakemore and Hoffman (1993) and compare the estimated trends with “reasonable” measures of what the trends are supposed to represent. The labor supply trend is therefore depicted with the labor force measure (i.e.  $L_t$ ), while the technology trend is compared with productivity. Alternatively we may consider representing technology with Solow residuals (see e.g. King, Plosser, Stock and Watson (1991)). Since such residuals depend

---

<sup>22</sup>It should be pointed out that exact identification is sufficient but not necessary for this result to hold.

on a particular choice of production function, we have decided not to construct such a measure of technology.

In Figure 11 the labor supply trend/labor force and technology trend/productivity series from the benchmark model are portrayed. Similarly, the corresponding series from the Bean model are given in Figure 12.<sup>23</sup> For the benchmark model it can be seen that both trends (the dashed lines) describe the trending behavior of the observed series quite well. Given the identifying assumption we expect the labor supply trend to replicate the labor force series well, but there is no guarantee that the technology trend will resemble the productivity series. For the Bean model the reverse can be expected, i.e. that the technology trend follows the trending behavior of productivity well, while the labor supply trend may or may not represent the trend in the labor force with accuracy. From Figure 12 we find this latter concern to be noteworthy. While the labor force decreases over a large section of the sample, the labor supply trend is increasing. Hence, despite the lack of formal statistical analysis we think that these graphs lend support to our benchmark model.<sup>24</sup>

### 5.3 The Role of the Exogenous Variables

Thus far we have only been able to establish a weak relationship between real wages and unemployment. For the first transitory shock, the so called aggregate demand shock, we noted a negative “dynamic correlation”. Short run increases in output and employment

---

<sup>23</sup>In order to make this comparison easier, we have adjusted the variances and the levels of the estimated trends. Specifically, for the labor supply trend we let the standard deviation of the innovation equal the coefficient on the trend in the employment equation of the common trends model (these are, of course, different for the two models). Similarly, for the technology trend we let the standard deviation of the innovation be equal to the coefficient on the technology trend in the productivity equation. In terms of both models, this means that the standard deviation of the labor supply trend innovation is  $a_{21}$ , while that of the technology trend innovation is  $a_{12}$ . From Tables 7 and 8 we know that the point estimates of these coefficients are different in the two models. Furthermore, we have adjusted the levels of the trends by adding  $(Y - E)_{1966:1}$  to the technology trend, and  $L_{1966:1}$  to the labor supply trend.

<sup>24</sup>Bertil Holmlund suggests that a more appropriate measure for the labor force series would be one based on the whole economy. Such a series is fairly well described by the labor supply trend in the Bean model.

and decreases in unemployment were followed by short run increases in the real wage.

In Figure 13 we depict the estimated transitory components,  $\hat{x}_{T,t}$ , for the endogenous variables. It is striking how alike the transitory components of productivity and unemployment appear. Similarly, the behavior of the components in employment and the real wage also display a common pattern. Furthermore, in comparison to the cointegration relations in Figure 4, the transitory component of the real wage resembles the series on labor's share of value added. In addition, it seems like the real wage component is decreasing when the unemployment series is increasing and vice versa. While the aggregate demand shock can account for some of this negative comovement, it is plausible that some further correlation is unaccounted for. In this section we shall investigate the coefficients on the stochastic exogenous variables. Hopefully, this will shed some more light on possible relations between real wages and unemployment.

In Table 9 estimated coefficients from  $B$  are given (cf. equation (4)). The columns of the submatrix  $B_s$ , where  $B = [B_d \ B_s]$ , contain the coefficients on the tax wedge and on the currency basket index.<sup>25</sup> Since unemployment is modelled as  $I(0)$ , it follows that the third row of  $B$  is zero. Furthermore, with labor's share of value added also being  $I(0)$ , the coefficients in the first and fourth rows are equal across the two equations.

For the currency basket index (second column of  $B_s$ ) the point estimates are not significantly different from zero at the 5 percent level. Likewise, for the tax wedge no single coefficient is significantly different from zero at the 5 percent level. However, for the real wage (and, thus, for productivity) the Wald test of the null  $b_{s,41} = 0$  takes on the value 3.083, corresponding to a p-value of .079. In order to test the null hypothesis that  $B_s = 0$  we first observe that this null is true if and only if  $P_s = \alpha \tilde{P}_s$ , where  $\tilde{P}_s$  is an  $r \times m_s$  matrix. Hence, the null involves exactly  $(n - r)m_s = 4$  restrictions on  $P_s$ . To test the hypothesis we use the  $LR$  statistic within the reduced form error correction model (3).

To understand how the test is performed, let  $P(\lambda) = [P_d(\lambda) \ P_s(\lambda)]$ , where  $P_d(\lambda) = P_{d,0}$ . Next, define  $G_s(\lambda)$  from  $P_s(\lambda) = P_s\lambda + G_s(\lambda)(1 - \lambda)\lambda$ . The error correction model

---

<sup>25</sup>Since  $B = CP$  it can easily be related to  $A$ , the common trends coefficients, and to  $P^* = P^*(1)$  from the structural error correction model. With  $R = R(1) = [A \ 0] = CF$  and  $P^* = F^{-1}P$ , we have that  $B = AP_k^*$ , where  $P_k^*$  is a  $k \times m$  matrix and  $P^* = [P_k^{*'} \ P_r^{*'}]'$ .

can now be written:

$$\Gamma(L)\Delta x_t = P_{d,0}d_t + P_s s_{t-1} + G_s(L)\Delta s_{t-1} - \alpha\beta' x_{t-1} + \varepsilon_t.$$

Under the null  $P_s = \alpha\tilde{P}_s$  so that

$$\alpha\tilde{P}_s s_{t-1} - \alpha\beta' x_{t-1} = \alpha \begin{bmatrix} -\beta' & \tilde{P}_s \end{bmatrix} \begin{bmatrix} x_{t-1} \\ s_{t-1} \end{bmatrix}.$$

In other words, to estimate  $P_s$  under the null, we can use the same idea as when we estimate the coefficients on the constant term, in the error correction model, under the condition that this term does not result in a linear deterministic trend in  $x_t$ .<sup>26</sup> For the labor market model we get an  $LR$  statistic of 6.831, with p-value equal to .145. Hence, we may conclude that the exogenous stochastic variables do not influence the permanent component of  $x_t$ . This result is not unreasonable. The currency basket index is a nominal variable, and our results support the view that real variables are neutral, in the long run, with respect to nominal influences. The result that the product real wage is unaffected by the tax wedge in the long run suggests that tax incidence falls entirely on labor.

In Figures 14 and 15 we have plotted estimated coefficients of the matrix polynomial  $C(\lambda)P(\lambda)$  from equation (7) along with 95 percent confidence intervals. Note that these graphs should *not* be interpreted as impulse response functions. The reason for this is that  $s_t$  need not be i.i.d. As long as  $s_t$  shows some form of serial dependence it is not meaningful to conduct the experiment: shock  $s$  by one standard deviation at time zero and let  $s$  equal zero thereafter.

Instead, an interpretation of the graphs is that they indicate which point estimates are significantly different from zero (in terms of 5 percent  $t$ -tests). Hence, for the  $\Delta(Y_t - E_t)$  equation, no single coefficient on  $\Delta V_t$ , from lags 1 to 20, is significant. In fact, for the tax wedge only the third lag in the  $\Delta W_t$  equation is significant. Since the confidence interval is below the origin it suggests that increases in the (inverse) tax wedge, i.e. decreases in any of the three tax rates, tend to be followed by decreases in the product real wage. It is also noteworthy that the point estimates in the  $\Delta(L_t - E_t)$  and  $\Delta W_t$  equations tend

---

<sup>26</sup>Note that we also estimate the error correction model under the restriction that  $\beta$  is as given in equation (11).

to follow one another for the first 4 or 5 lags. Still, the individual coefficients in the unemployment equation are not significantly different from zero.<sup>27</sup>

Furthermore, for the currency basket index,  $\Delta S_t$ , few of the individual coefficients are significant. In the employment equation the first three lags have confidence intervals that do not include the origin. A similar behavior is found in the productivity equation. It is, however, difficult to interpret the coefficients. For the real wage and unemployment equations the point estimates seem to move in the opposite direction. However, only one coefficient is significant at the 5 percent level.

## 6 Concluding Comments

The sources of labor market fluctuations and, in particular, the nature of the relation between real wages and unemployment have been examined in several empirical studies in recent years. Questions have been raised about the relative importance of supply and demand shocks, about speeds of adjustment in labor markets, and about the long run relation between real wages and unemployment. These questions have often been addressed within the context of single equation structural error correction models.

In this paper we take a new look at these questions through a common trends model (a vector error correction model with cointegration constraints). Our main motives for following this alternative approach is, first, that we believe that an empirical model of real wages, employment, output and labor supply should take these as being endogenous. The question about the response of real wages to a unit increase in unemployment *ceteris paribus* is not very interesting, since real wages and unemployment *both* react to some underlying disturbances, and the joint behavior of these two variables differs between different kinds of disturbances. Second, we believe that real wages, employment, output and labor supply are governed by common stochastic trends, while unemployment may very well be stationary. If so, one cannot really speak of a long run relation between real

---

<sup>27</sup>One may, of course, argue that for a 5 percent level, we should expect 1 out of 20 point estimates to be significant. However, the point estimates are not independent and, more importantly, we are primarily interested in the first 4 to 8 lags. In addition, the exogenous regressors are very important when it comes to verifying the statistical assumptions which underlie the analysis. Among other things, they account for heteroskedasticity and lag order determination problems that will otherwise occur in this data set.

wages and unemployment. Third, the potential advantage with structural error correction models, that they are more easily interpreted in economic terms, seldom seems to be fulfilled in practice. Manning (1992) argues that although formal identification prevails for the “wage equations” that have been estimated, theoretical support for the restrictions involved is often weak, making identification rather ambiguous.

We find evidence which suggests that real wages and unemployment Granger cause each other. Real wages, employment, output and the labor force seem to be driven by two common stochastic trends. We interpret one of these trends as a technology trend and the other as a labor supply trend. Unemployment appears to be stationary, i.e. employment and the labor force are cointegrated. The only evidence of “hysteresis” in unemployment is that the series shows signs of persistence. Our conclusions are drawn after careful testing, where Monte Carlo experiments have been performed in order to examine the properties of the standard cointegration tests in a small sample like ours.

We also find that much of the medium run (and all of the long run) fluctuations in real wages can be attributed to permanent shocks, i.e. shocks to the stochastic labor supply and technology trends. Short run fluctuations, however, are primarily due to transitory shocks. Such shocks also have effects on unemployment that last for 2 to 3 years. The high (implicit) “elasticity” of real wages with respect to changes in unemployment that has been recorded in earlier studies is a reflection of the fact that unemployment responds much less to various kinds of shocks than real wages. For temporary aggregate demand shocks there is a negative correlation between the responses of real wages and unemployment. For different kinds of shocks, real wages reach a long run equilibrium level within 5 to 7 years.

A transitory demand shock that increases output leads to an even larger response of employment, i.e. productivity goes down. Real wages are “sticky” in the sense that they are significantly affected by most shocks after some time. On average, the transitory component of productivity is positively correlated with unemployment, while the transitory component of the real wage is negatively correlated with unemployment. Hence, productivity appears to be counter-cyclical while real wages are pro-cyclical.



Table 1: Multivariate determination of lag order for the labor market model, 1965:1–90:4.

Statistic	Lags							
	1	2	3	4	5	6	7	8
AIC	-34.428	-34.987	-35.385	-35.702	-36.040	-35.965	-36.111	-36.414*
BIC	-34.016	-34.158*	-34.134	-34.024	-33.930	-33.417	-33.119	-32.973
LIL	-34.261	-34.651	-34.879	-35.023	-35.187*	-34.935	-34.902	-35.024
Port.	289.85	221.76	184.91	151.92	143.49	137.13	153.09	146.32
p-value	.000	.001	.012	.073	.024	.004	.000	.000
Skewness	3.32	1.68	1.46	1.22	1.42	1.44	1.30	1.29
$Q_{skew}$	56.46	28.26	24.27	20.04	23.11	23.29	20.83	20.38
p-value	.000	.103	.231	.455	.283	.275	.407	.435
Kurtosis	29.88	24.96	25.40	24.46	23.82	23.56	24.22	24.06
$Q_{kurt}$	18.35	.49	1.03	.11	.02	.10	.02	.00
p-value	.000	.484	.311	.742	.899	.753	.877	.964

Notes: The Portmanteau test has an asymptotic  $\chi^2$  distribution with  $n^2(12 - p)$  degrees of freedom. The skewness and kurtosis tests,  $Q_{skew}$  and  $Q_{kurt}$ , are asymptotically distributed as  $\chi^2$  with 20 and 1 degrees of freedom, respectively.

Table 2: Cointegration analysis of VAR(4) with two exogenous stochastic regressors (4 lags), 1966:2–90:4.

$H_0$	$\hat{e}$	$LR_{tr}$	.90	.95	.99
$r = 0$	.386	78.66	43.95	47.21	54.46
$r \leq 1$	.146	30.29	26.79	29.68	35.65
$r \leq 2$	.134	14.66	13.33	15.41	20.04
$r \leq 3$	.005	.46	2.64	3.76	6.65

$$\hat{\beta}' = \begin{bmatrix} 16.23 & -.95 & -214.37 & -11.41 \\ -13.83 & -18.84 & 11.12 & 12.89 \end{bmatrix}$$

Notes: Critical values from the .90, .95, and .99 quantiles of the simulated asymptotic distribution are taken from Osterwald-Lenum (1992, Table 1).

Table 3: Simulated percentiles for the trace test for cointegration rank in comparison with critical values from Osterwald-Lenum.

Panel A					Panel B				
Testing $H_0: r = 0$ against $H_A: r \geq 1$ .					Testing $H_0: r \leq 1$ against $H_A: r \geq 2$ .				
percentiles:	.50	.80	.90	.95	percentiles:	.50	.80	.90	.95
crit. values:	33.60	40.15	43.95	47.21	crit. values:	18.70	23.64	26.79	29.68
DGP rank					DGP rank				
4	100.98	115.71	124.21	131.08	4	45.46	54.75	60.09	64.74
3	98.06	112.73	121.13	128.08	3	43.25	52.07	57.15	61.55
2	86.25	101.55	110.29	117.98	2	33.75	42.11	46.97	51.22
1	76.80	91.02	99.28	105.76	1	<b>23.03</b>	<b>29.65</b>	<b>33.63</b>	<b>37.27</b>
0	<b>38.63</b>	<b>47.23</b>	<b>52.21</b>	<b>56.35</b>	0	16.83	21.57	24.39	27.13

Panel C					Panel D				
Testing $H_0: r \leq 2$ against $H_A: r \geq 3$ .					Testing $H_0: r \leq 3$ against $H_A: r = 4$ .				
percentiles:	.50	.80	.90	.95	percentiles:	.50	.80	.90	.95
crit. values:	7.55	11.07	13.33	15.41	crit. values:	.44	1.66	2.69	3.76
DGP rank					DGP rank				
4	18.13	23.86	27.25	30.07	4	3.36	6.68	8.58	10.22
3	16.23	21.38	24.37	26.96	3	<b>1.54</b>	<b>4.40</b>	<b>6.22</b>	<b>7.92</b>
2	<b>11.05</b>	<b>15.48</b>	<b>18.08</b>	<b>20.51</b>	2	1.08	3.21	4.66	5.84
1	7.00	10.35	12.31	14.14	1	.48	1.67	2.57	3.48
0	5.25	7.82	9.48	10.79	0	.33	1.15	1.86	2.56

Notes: Bold faced numbers signifies percentile values of the empirical distribution when the null hypothesis is true. Critical values from the simulated asymptotic distribution are taken from Osterwald-Lenum (1992, Table 1).

Table 4: Monte Carlo evidence on the small sample behavior of the trace test for cointegration rank in the labor market model, 1966:2–90:4.

<b>Panel A</b>						
Frequencies of preferred cointegration rank $r$						
Number of cointegration vectors in the DGP						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	
Preferred rank						
$r = 0$	79.95 %	2.05 %	.31 %	.03 %	.01 %	
$r = 1$	17.51 %	78.09 %	31.94 %	6.23 %	4.41 %	
$r = 2$	2.19 %	17.01 %	47.46 %	38.12 %	28.84 %	
$r = 3$	.12 %	1.68 %	11.18 %	34.68 %	26.24 %	
$r = 4$	.23 %	1.17 %	9.11 %	20.94 %	40.50 %	

<b>Panel B</b>						
Rejection rates for different $H_0$ in a 5 % trace test						
Number of cointegration vectors in the DGP						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$C_{AS,0.05}$
Hypothesis						
$H_0: r = 0$	20.05 %	97.95 %	99.69 %	99.97 %	99.99 %	47.21
$H_0: r \leq 1$	2.55 %	19.86 %	67.75 %	93.74 %	95.57 %	29.68
$H_0: r \leq 2$	.40 %	2.94 %	20.41 %	55.66 %	66.80 %	15.41
$H_0: r \leq 3$	1.41 %	3.94 %	15.37 %	24.51 %	45.46 %	3.76

<b>Panel C</b>						
Size adjusted rejection rates for different $H_0$ in a 5 % trace test						
Number of cointegration vectors in the DGP						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$C_{EM,0.05}$
Hypothesis						
$H_0: r = 0$	5.00 %	91.28 %	97.57 %	99.81 %	99.90 %	56.35
$H_0: r \leq 1$	.11 %	5.00 %	34.97 %	73.11 %	79.43 %	37.27
$H_0: r \leq 2$	.01 %	.24 %	5.00 %	23.83 %	36.07 %	20.51
$H_0: r \leq 3$	.04 %	.10 %	1.18 %	5.00 %	13.07 %	7.92

<b>Panel D</b>						
Rates of simulated test statistics surpassing the estimated						
Number of cointegration vectors in the DGP						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$LR_{tr}$
Hypothesis						
$H_0: r = 0$	.07 %	45.29 %	67.90 %	88.80 %	91.87 %	78.66
$H_0: r \leq 1$	2.12 %	18.09 %	65.05 %	92.63 %	94.86 %	30.29
$H_0: r \leq 2$	.68 %	4.11 %	24.35 %	60.80 %	71.25 %	14.66
$H_0: r \leq 3$	42.76 %	50.98 %	67.67 %	72.52 %	86.53 %	.46

Notes: For Panel A; frequencies of preferred cointegration rank are based on the outcome of the trace test procedure for DGP's with different cointegration ranks and are evaluated against the 95 percent quantile values in Osterwald-Lenum (1992, Table 1).

Table 5: Simulations of  $LR$  tests of linear restrictions on  $\beta$ .

Hypothesis	$LR$	Distribution	Percentiles				Size	p-value
			.50	.80	.90	.95		
$\beta_1 \in \hat{\beta}$	2.646	$\chi_2^2$	1.386	3.219	4.605	5.991	5.00 %	.266
		Emp.	3.748	8.142	10.993	13.566	31.70 %	.620
$\beta_2 \in \hat{\beta}$	1.571	$\chi_2^2$	1.386	3.219	4.605	5.991	5.00 %	.456
		Emp.	3.581	7.506	10.301	13.029	28.95 %	.742
$\beta_1, \beta_2 \in \hat{\beta}$	21.224	$\chi_4^2$	3.357	5.989	7.779	9.488	5.00 %	.000
		Emp.	7.231	12.523	16.013	19.204	34.81 %	.031

Notes: To obtain empirical distributions for the tests,  $LR$  statistics have been calculated, under the null, for 10,000 samples and have been ordered from the smallest to the largest.  $\beta_1' = [-1 \ 0 \ 0 \ 1]$  and  $\beta_2' = [0 \ 0 \ 1 \ 0]$ .

Table 6: Simulations of  $LR$  test for  $H_0: \beta_1, \beta_2 \in \hat{\beta}$ . The 5 percent critical value from the  $\chi_4^2$  distribution is related to empirical distributions for different sample sizes.

	Sample size, $T$								
	50	75	100	200	300	400	500	1000	2000
empirical size	82.55 %	55.67 %	40.76 %	18.68 %	12.39 %	10.34 %	9.08 %	6.84 %	5.64 %

Notes: To construct data for different sample sizes we have excluded to exogenous stochastic regressors from the DGP, i.e. their observed values are set to zero. The coefficients in the DGP are taken from the model with  $r = 2$  and whose cointegration vectors are given in equation (11).

Table 7: Estimated common trends coefficients in the benchmark model and  $\alpha$  coefficients in the error correction model, 1966:2–90:4.

$$\hat{A} = \begin{bmatrix} -.0082 & .0059 \\ (.0029) & (.0011) \\ .0093 & .0000 \\ (.0025) & (--) \\ .0000 & .0000 \\ (--) & (--) \\ -.0082 & .0059 \\ (.0029) & (.0011) \end{bmatrix}, \quad \hat{\alpha} = \begin{bmatrix} -1.353 & -.125 \\ (.457) & (.035) \\ -.067 & .087 \\ (.447) & (.034) \\ .156 & .004 \\ (.052) & (.004) \\ 2.054 & .194 \\ (.718) & .054 \end{bmatrix}.$$

Notes: The estimated standard errors are reported within parenthesis. The asymptotic distribution of the estimated coefficients is normal and for the common trends coefficients the standard errors are based on Warne (1991, Theorem 3).

Table 8: Estimated common trends coefficients in the Bean model, 1966:2–90:4.

$$\hat{A} = \begin{bmatrix} .0000 & .0101 \\ (--) & (.0024) \\ .0054 & -.0076 \\ (.0012) & (.0028) \\ .0000 & .0000 \\ (--) & (--) \\ .0000 & .0101 \\ (--) & (.0024) \end{bmatrix}.$$

Table 9: Estimated  $B_s$  matrix with coefficients on the stochastic exogenous regressors in the common trends model.

$$\hat{B}_s = \begin{bmatrix} -.223 & -.080 \\ (.127) & (.071) \\ .165 & .046 \\ (.137) & (.078) \\ .000 & .000 \\ (--) & (--) \\ -.223 & -.080 \\ (.127) & (.071) \end{bmatrix} .$$

$$H_0 : B_s = 0 \quad LR = 6.831 \quad \text{p-value} = .145$$

**Notes:** The estimated standard errors are reported within parenthesis. The asymptotic distribution of the estimated coefficients is normal and the standard errors are based on Warne (1991, Theorem 3). The coefficients in the first column are those on the tax wedge while those on the currency basket index are given in the second column.

Figure 1: A simple description of the identifying assumptions used in the common trends models of the labor market.

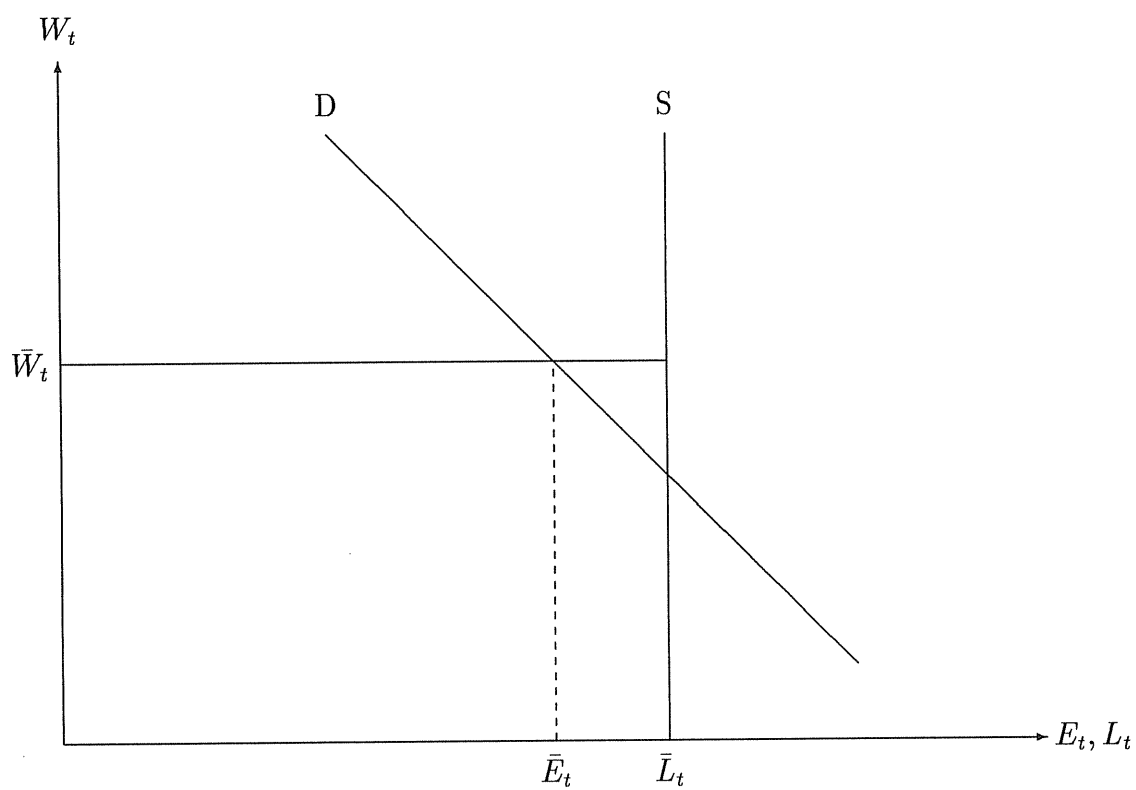


Figure 2: Yearly observations of the four quarters for the endogenous,  $x_t$ , variables and the exogenous,  $s_t$ , variables; 1965–90.

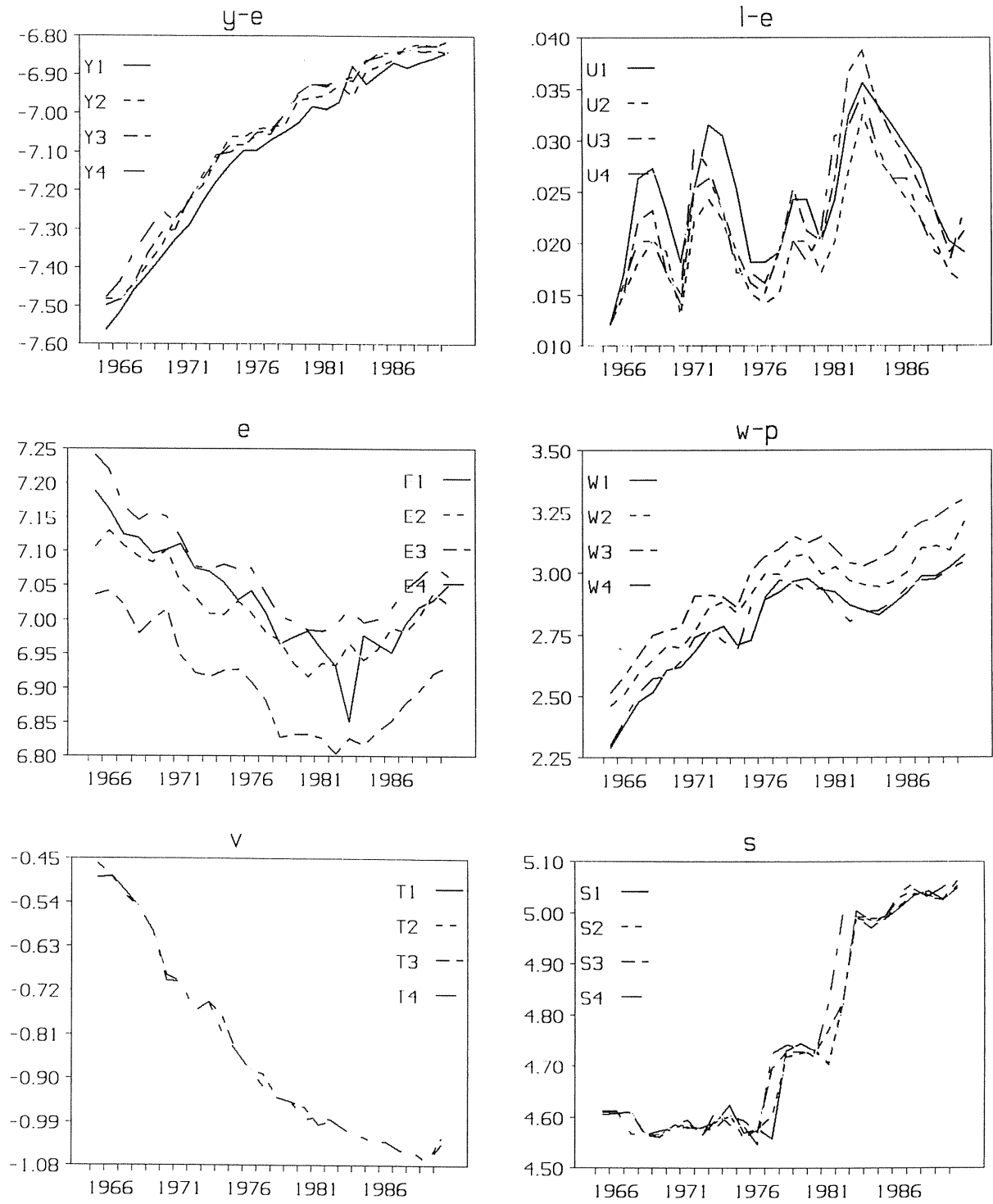




Figure 3: Density functions of the asymptotic and empirical distributions of the trace test for cointegration rank.

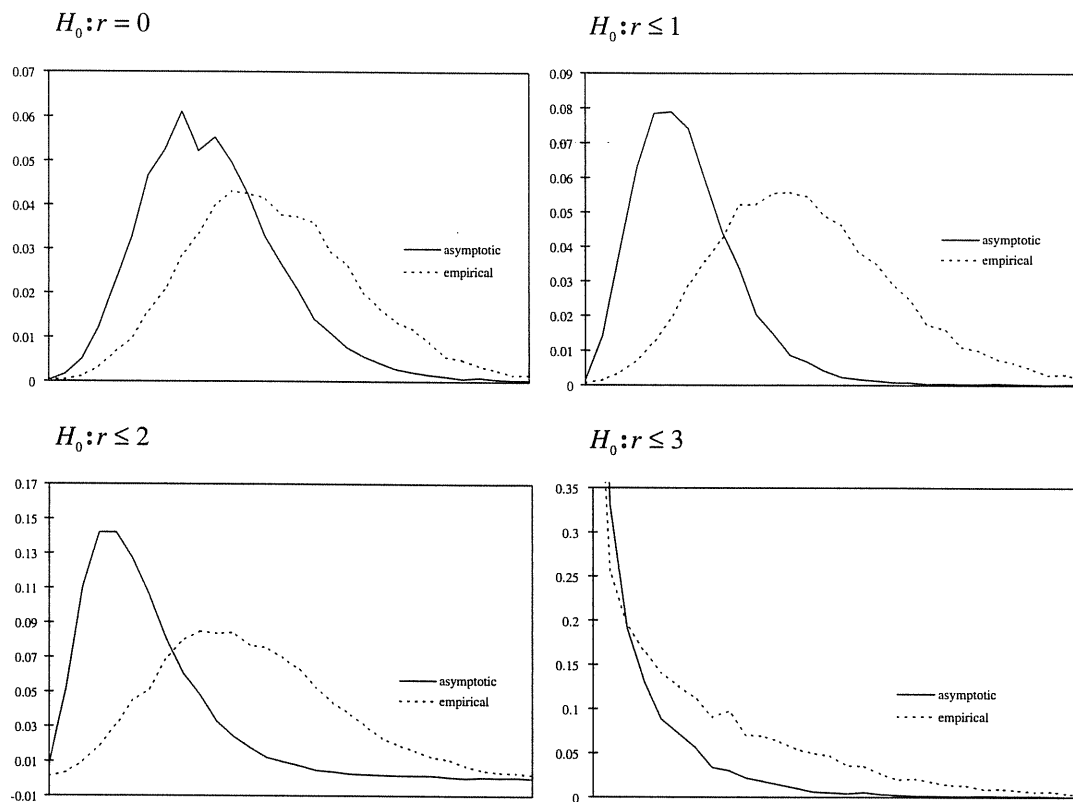


Figure 4: Quarterly observations of the cointegration relations; 1965:1–90:4.

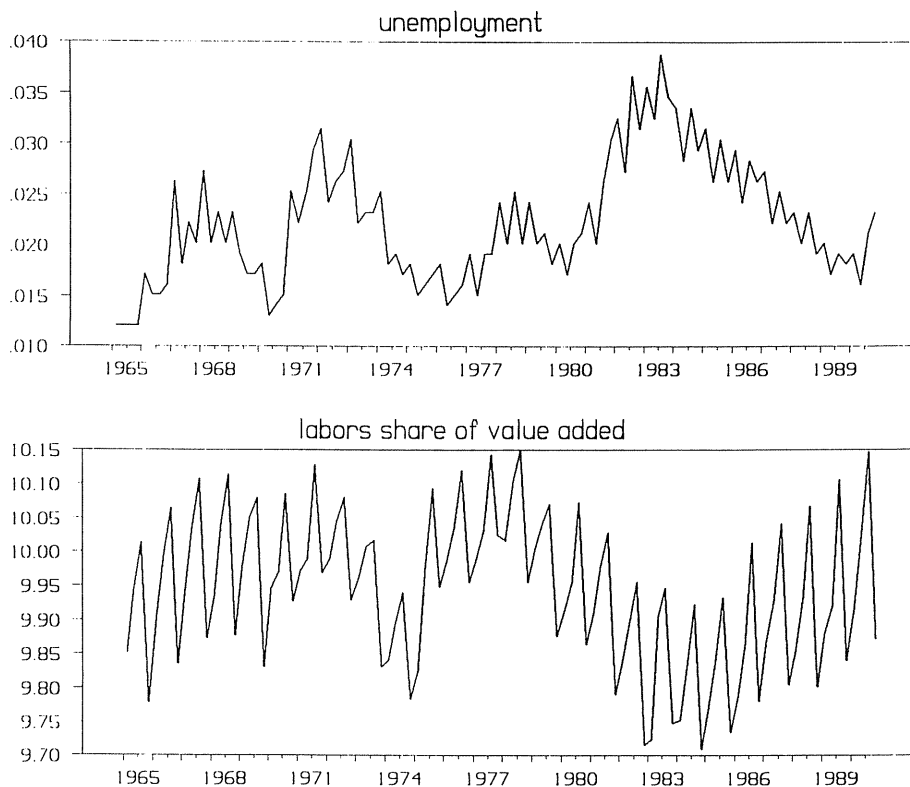


Figure 5: Impulse response functions with 95 percent confidence intervals from a one standard deviation shock to the labor supply innovation in the benchmark model.

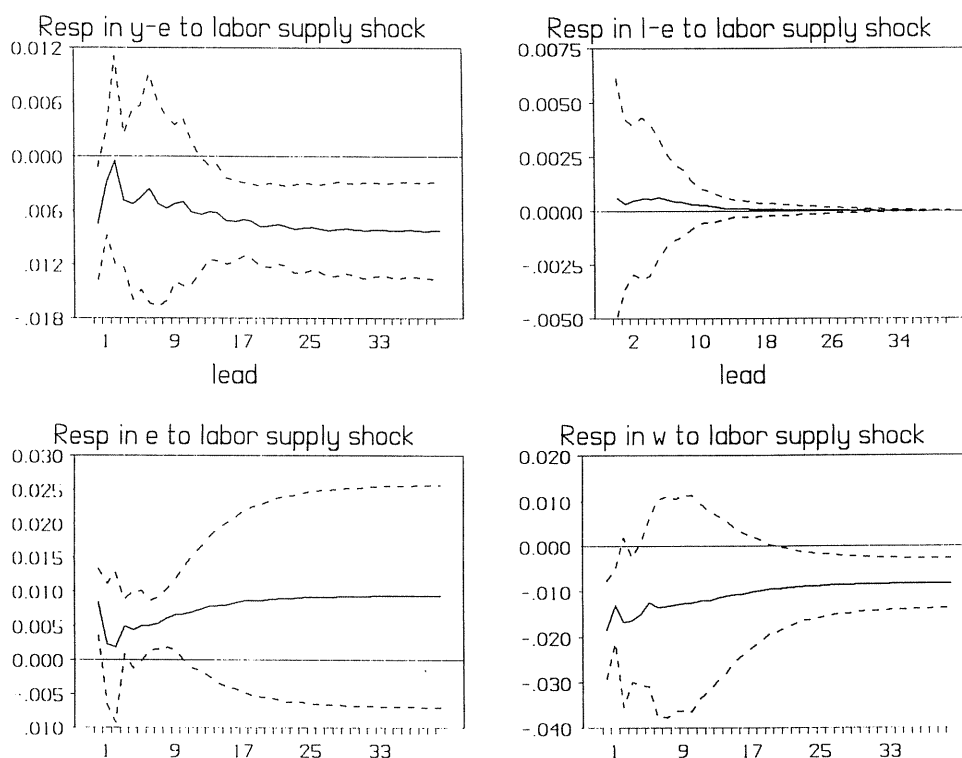


Figure 6: Impulse response functions with 95 percent confidence intervals from a one standard deviation shock to the technology innovation in the benchmark model.

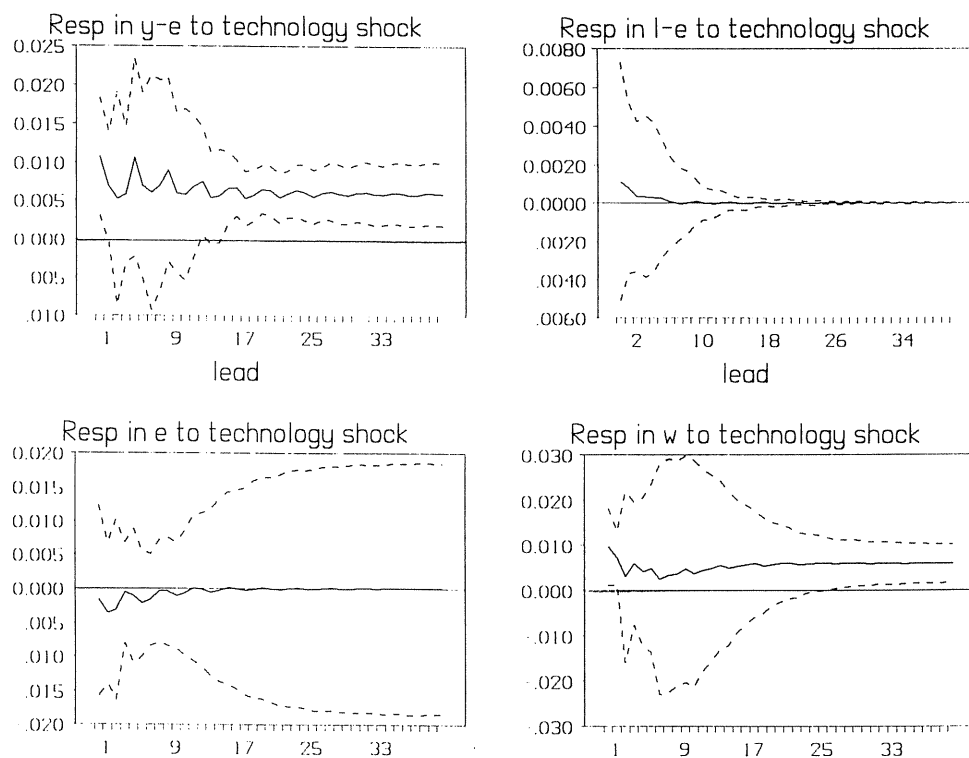


Figure 7: Impulse response functions with 95 percent confidence intervals from a one standard deviation shock to the aggregate demand innovation.

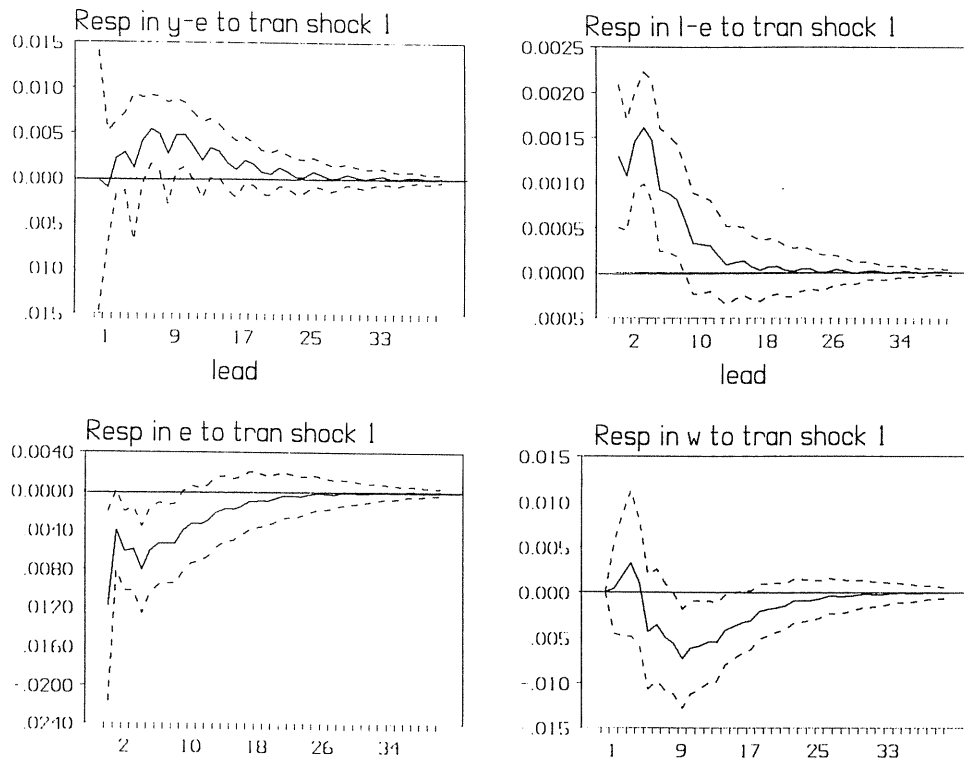


Figure 8: Impulse response functions with 95 percent confidence intervals from a one standard deviation shock to the wage innovation.

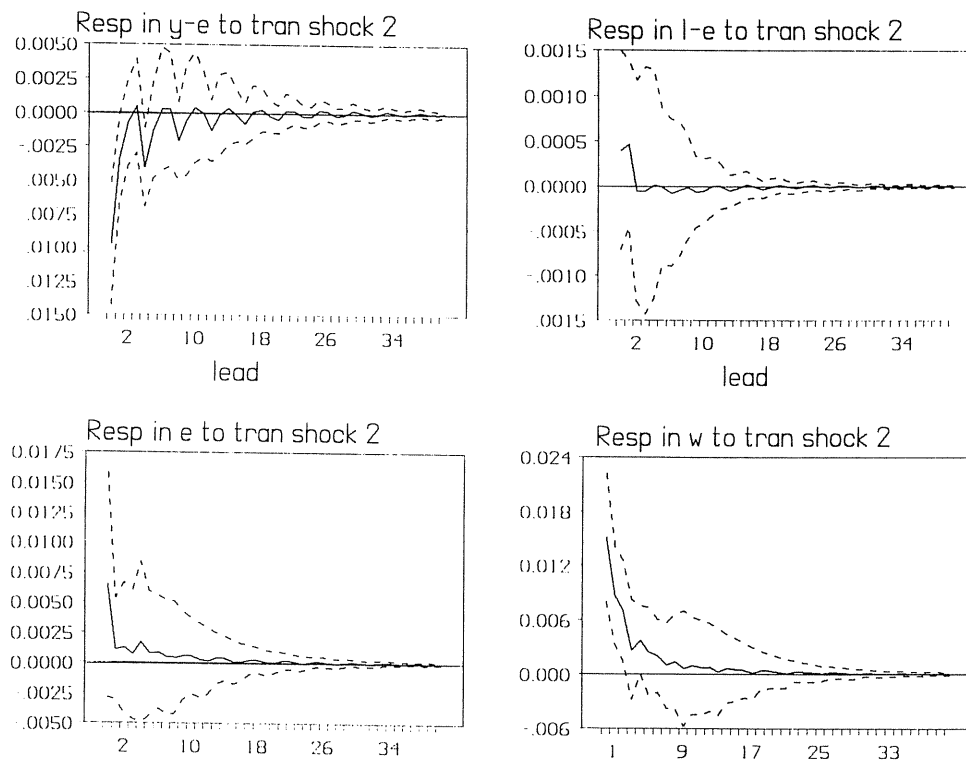


Figure 9: Impulse response functions with 95 percent confidence intervals from a one standard deviation shock to the labor supply innovation in the Bean model.

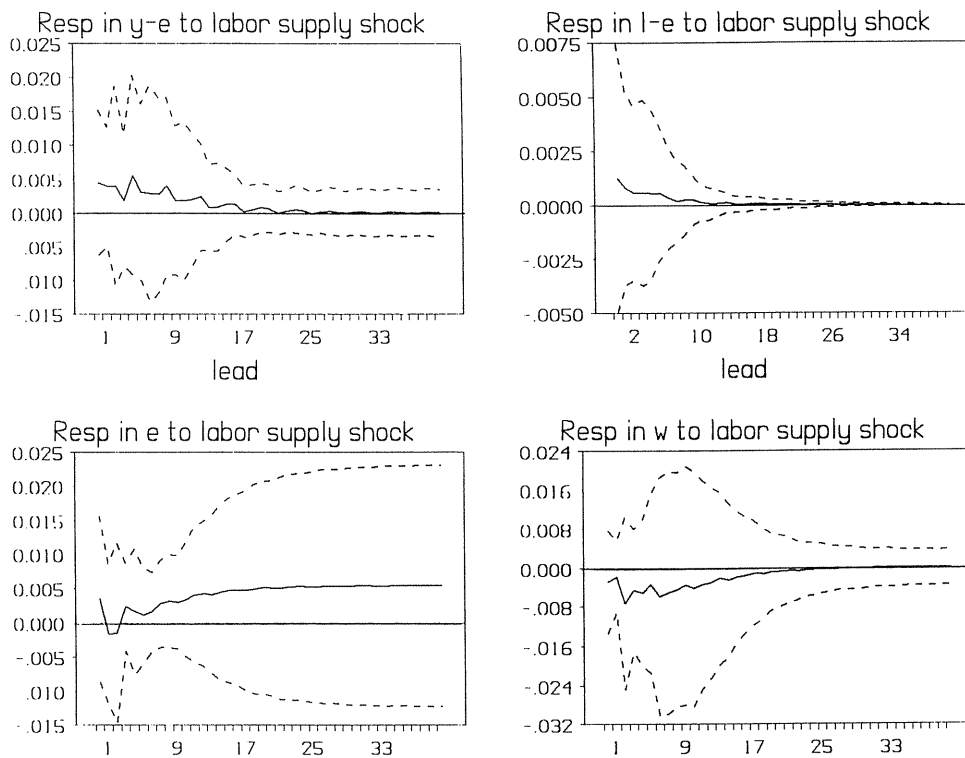


Figure 10: Impulse response functions with 95 percent confidence intervals from a one standard deviation shock to the technology innovation in the Bean model.

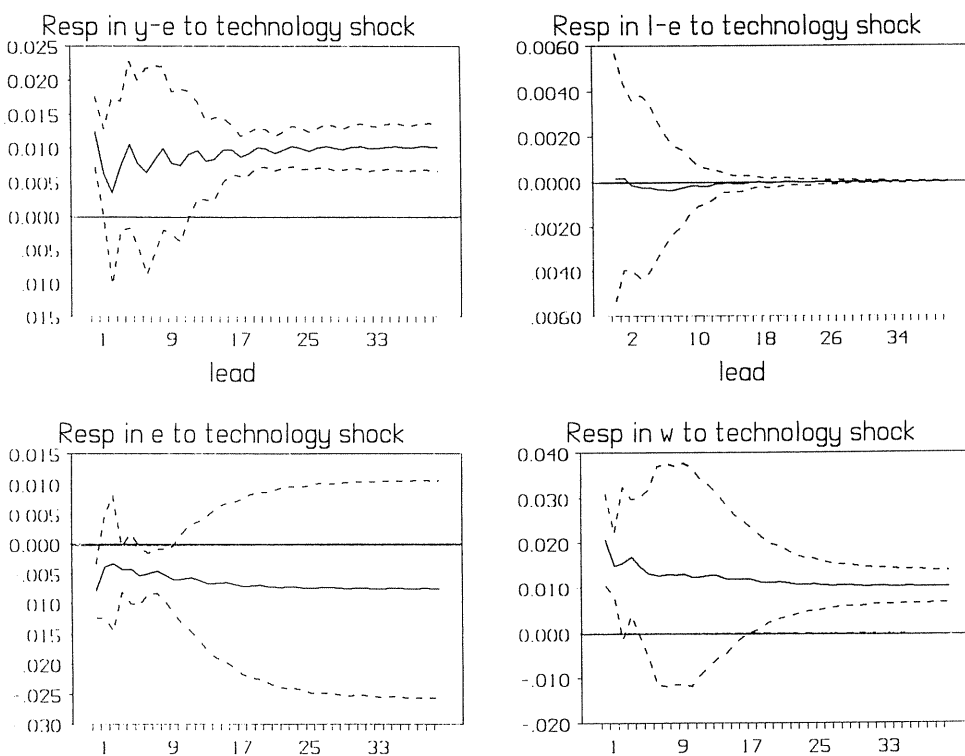


Figure 11: Estimated stochastic trends with drift (dashed lines) and compared series (solid lines) in the benchmark model, 1966:2–90:4.

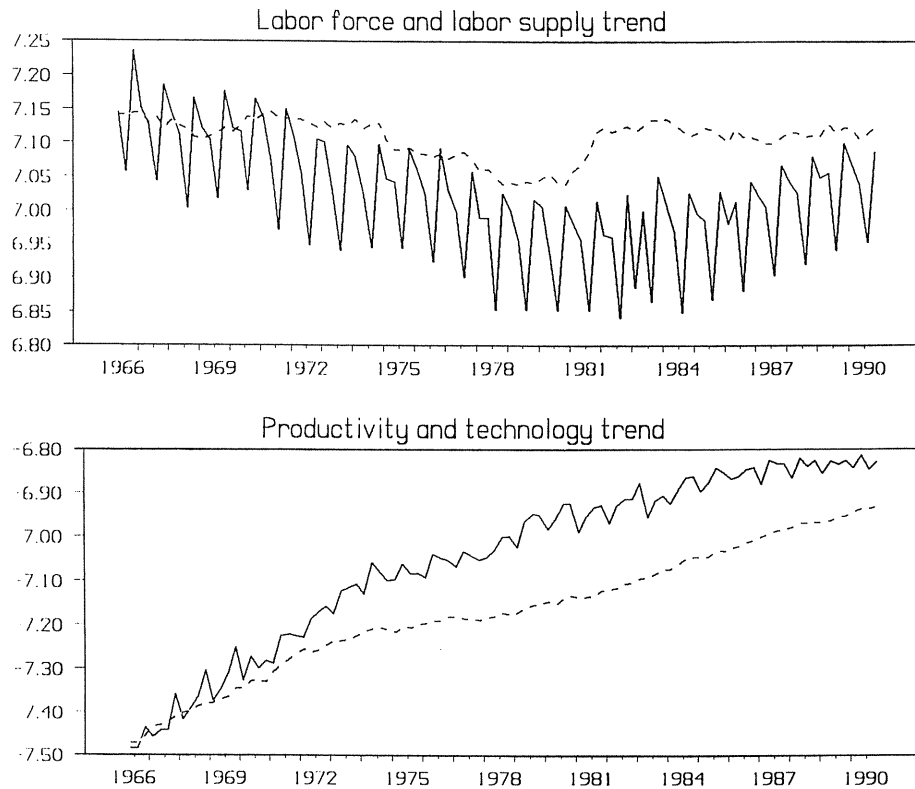


Figure 12: Estimated stochastic trends with drift (dashed lines) and compared series (solid lines) in the Bean model, 1966:2–90:4.

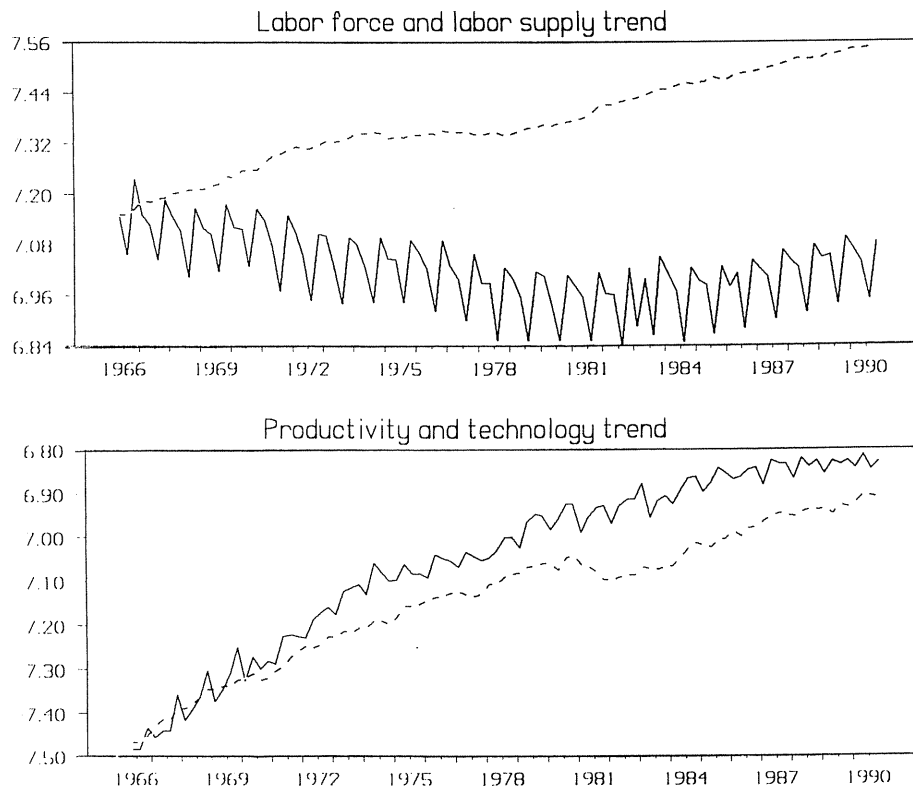


Figure 13: Estimated transitory components, 1966:2-90:4.

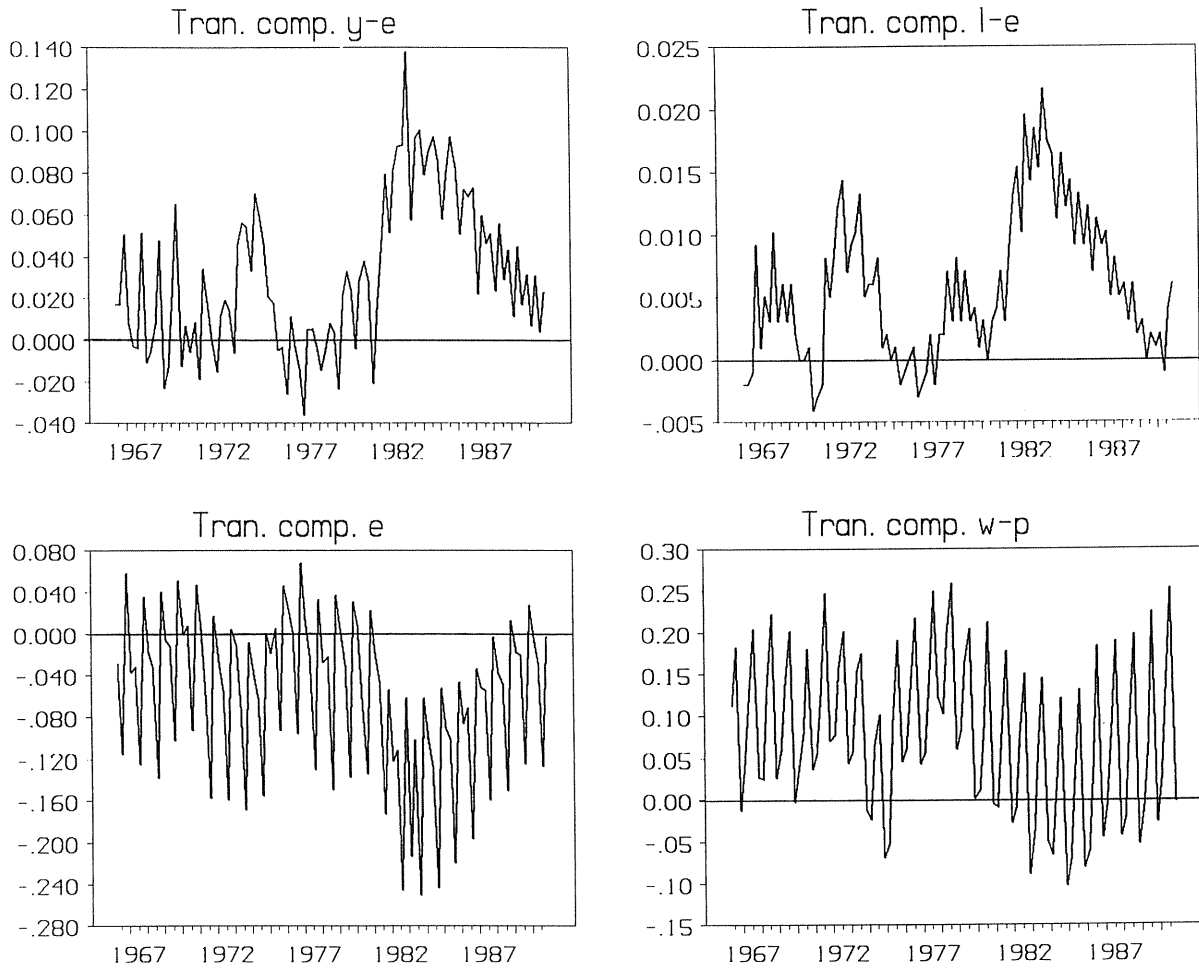


Figure 14: Estimated coefficients on lags of  $\Delta V_t$  in the  $\Delta x_t$  equations of the structural moving average representation.

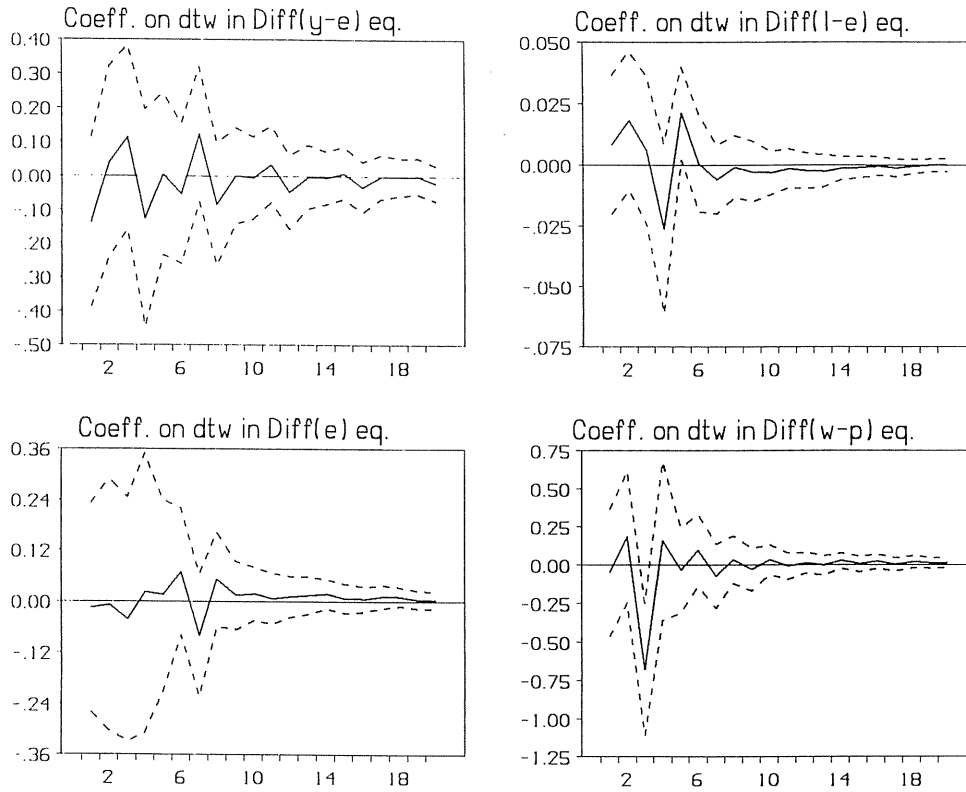
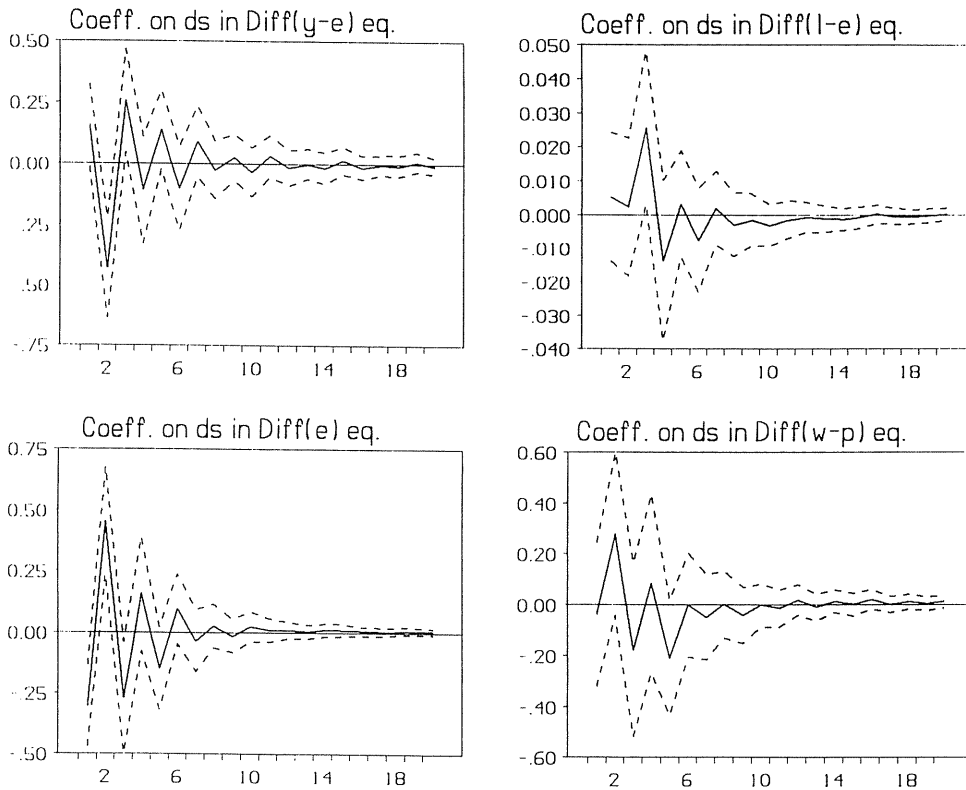


Figure 15: Estimated coefficients on lags of  $\Delta S_t$  in the  $\Delta x_t$  equations of the structural moving average representation.



## References

- [1] Akaike, H. (1969), "Fitting Autoregressive Models for Prediction", *Annals of the Institute for Statistical Mathematics*, 21:243–247.
- [2] Bean, C. R. (1992), "Identifying the Causes of British Unemployment", Working Paper No. 276, London School of Economics.
- [3] Bean, C. R., Layard, P. R. G., and S. J. Nickell (1986), "The Rise in Unemployment: A Multi-country study", *Economica*, Supplement, 53:1–22.
- [4] Blakemore, A. E. and D. L. Hoffman (1993), "Nonstationary Behavior in the Labor Market and the Intertemporal Substitution Effect", Unpublished Manuscript, Arizona State University.
- [5] Blanchard, O. J. and D. Quah (1989), "The Dynamic Effects of Supply and Demand Disturbances", *American Economic Review*, 79:655–673.
- [6] Boswijk, H. P. (1992), *Cointegration, Identification and Exogeneity*, PhD Thesis, Tinbergen Institute Research Series, No. 37.
- [7] Engle, R. F. and C. W. J. Granger, (1987), "Co-Integration and Error Correction: Representation, Estimation and Testing", *Econometrica*, 55:251–276.
- [8] Englund, P., Vredin, A., and A. Warne (1992), "Macroeconomic Shocks in an Open Economy: A Common Trends Representation of Swedish Data 1871–1990", *FIEF Studies in Labor Markets and Economic Policy*, vol. 5, Clarendon Press, Oxford (forthcoming).
- [9] Franses, P. H. (1992), "A Multivariate Approach to Modelling Univariate Seasonal Time Series", *Journal of Econometrics*, forthcoming.
- [10] Gonzalo, J. and C. W. J. Granger (1992) "Estimation of Common Long-Memory Components in Cointegrated Systems", Unpublished Manuscript, University of California, San Diego.



- [11] Hannan, E. J. and B. G. Quinn (1979), “The Determination of the Order of an Autoregression”, *Journal of the Royal Statistical Society, Series B*, 41:190–195.
- [12] Hosking, J. R. M. (1980), “The Multivariate Portmanteau Statistic”, *Journal of the American Statistical Association*, 75:602–608.
- [13] Johansen, S. (1991), “Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models”, *Econometrica*, 59:1551–1580.
- [14] Johansen, S. (1992), “Determination of Cointegration Rank in the Presence of a Linear Trend”, *Oxford Bulletin of Economics and Statistics*, 54:383–397.
- [15] Johansen, S. and K. Juselius (1990), “Maximum Likelihood Estimation and Inference on Cointegration — With Applications to the Demand for Money”, *Oxford Bulletin of Economics and Statistics*, 52:169–210.
- [16] Johansen, S. and K. Juselius (1992), “Testing Structural Hypotheses in a Multivariate Cointegration Analysis of the PPP and the UIP for U.K.”, *Journal of Econometrics*, 53:211–244.
- [17] King, R. G., Plosser, C. I., Stock, J. H., and M. W. Watson (1987), “Stochastic Trends and Economic Fluctuations”, NBER Working Paper No. 2229.
- [18] King, R. G., Plosser, C. I., Stock, J. H., and M. W. Watson (1991), “Stochastic Trends and Economic Fluctuations”, *American Economic Review*, 81:819–840.
- [19] Lütkepohl, H. (1991), *Introduction to Multiple Time Series Analysis*, 1st Edition, Springer-Verlag, Berlin.
- [20] Manning, A. (1992), “Wage Bargaining and the Phillips Curve: The Identification and Specification of Aggregate Wage Equations”, Discussion Paper No. 62, Center for Economic Performance, London School of Economics.
- [21] Mardia, K. V. (1970), “Measures of Multivariate Skewness and Kurtosis with Applications”, *Biometrika*, 57:519–530.

- [22] Mellander, E., Vredin, A., and A. Warne (1992), “Stochastic Trends and Economic Fluctuations in a Small Open Economy”, *Journal of Applied Econometrics*, 7:369–394.
- [23] Osterwald–Lenum, M. (1992), “A Note with Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics: Four Cases”, *Oxford Bulletin of Economics and Statistics*, 54:461–472.
- [24] Quah, D. (1992), “Identifying Vector Autoregressions”, *FIEF Studies in Labor Markets and Economic Policy*, vol. 5, Clarendon Press, Oxford (forthcoming).
- [25] Schwarz, G. (1978), “Estimating the Dimension of a Model”, *Annals of Statistics*, 6:461–464.
- [26] Stock, J. H. and M. W. Watson (1988), “Testing for Common Trends”, *Journal of the American Statistical Association*, 83:1097–1107.
- [27] Vredin, A. and A. Warne (1991), “Current Account and Macroeconomic Fluctuations”, *Scandinavian Journal of Economics*, 93:511–530.
- [28] Warne, A. (1991), “A Common Trends Model: Identification, Estimation and Asymptotics”, Unpublished Manuscript, Stockholm School of Economics.
- [29] Warne, A. (1993), “Common Trends and Structural Error Correction”, Unpublished Manuscript, Institute for International Economic Studies, Stockholm University.

SEMINAR PAPER SERIES

The Series was initiated in 1971. For a complete list of Seminar Papers, please contact the Institute.

1990

459. *Thorvaldur Gylfason:* Exchange Rate Policy, Inflation, and Unemployment: The Experience of the Nordic EFTA Countries. 44 pp.
460. *Nils Gottfries and Barry McCormick:* Discrimination and Open Unemployment in a Segmented Labour Market. 30 pp.
461. *Lars Calmfors and Ragnar Nymoen:* Real Wage Adjustment and Employment Policies in the Nordic Countries. 93 pp.
462. *Carl B. Hamilton:* The Nordic EFTA Countries and the European Community: Options for the 1990's. 31 pp.
463. *Anders Forslund:* Wage Setting in Sweden: An Empirical Test of a Barebones Union Model. 28 pp.
464. *Jaime de Melo, Carl B. Hamilton and L. Alan Winters:* Voluntary Export Restraints: A Case Study Focussing on Effects in Exporting Countries. 28 pp.
465. *Paul Söderlind:* The Swedish Tax Reform from an Intertemporal Perspective. 64 pp.
466. *Lars E.O. Svensson:* The Term Structure of Interest Rate Differentials in a Target Zone: Theory and Swedish Data. 53 pp.
467. *Michael Dooley and Lars E.O. Svensson:* Policy Inconsistency and External Debt Service. 15 pp.
468. *Carl B. Hamilton:* The New Silk Road to Europe. 41 pp.
469. *Lars E.O. Svensson:* The Simplest Test of Target Zone Credibility. 23 pp.
470. *Jörgen W. Weibull:* On Self-Enforcement in Extensive-Form Games. 39 pp.
471. *Kala Krishna:* Auctions with Endogenous Valuations, the Snowball Effect, and Other Applications. 43 pp.
472. *Kala Krishna:* Auctions with Endogenous Valuations: The Persistence of Monopoly Revisited. 48 pp.
473. *Peter Englund, Torsten Persson and Lars E.O. Svensson:* Swedish Business Cycles: 1861-1988. 34 pp.
474. *Assar Lindbeck and Dennis J. Snower:* Interactions Between the Efficiency Wage and Insider-Outsider Theories. 6 pp.
475. *Lars E.O. Svensson:* The Foreign Exchange Risk Premium in a Target Zone with Devaluation Risk. 29 pp.