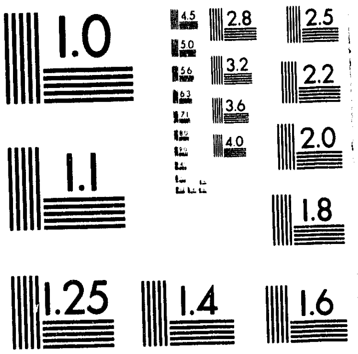
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ARE THE INVARIANCE PRINCIPLES REALLY TRULY LORENTZ COVARIANT?

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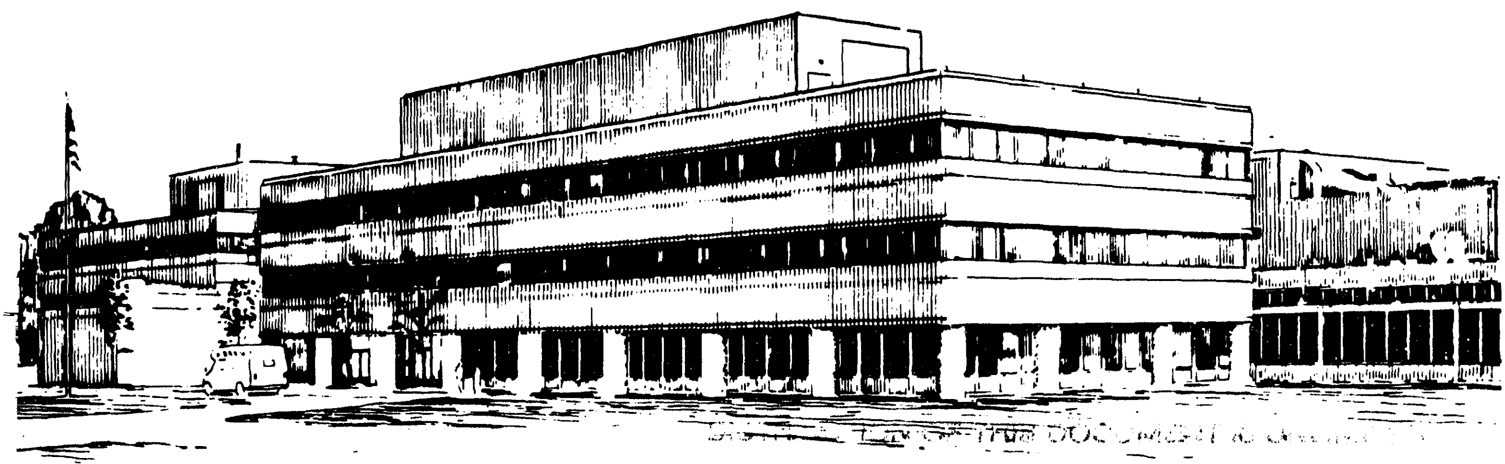

ARE THE INVARIANCE PRINCIPLES REALLY TRULY LORENTZ COVARIANT?

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ABSTRACT

It is shown that some sections of the invariance (or symmetry) principles such as the space reversal symmetry (or parity P) and time reversal symmetry T (of elementary particle and condensed matter physics, etc.) are not really truly Lorentz covariant. Indeed, I find that the Dirac-Wigner sense of Lorentz invariance is not in full compliance with the Einstein-Minkowski requirements of the Lorentz covariance of all physical laws (i.e., the world space Mach principle).

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The invariance (or symmetry) principles[1-3] play a significant role in many branches of physics. In particular, they are of considerable interest in group theory, elementary particle and condensed matter physics, quantum field theory, string theory, etc. The basic reason for such a significance is that the invariance under a symmetry operation is always associated with a conservation law for some physical quantity. Indeed, a large number of Nobel prizes have been awarded to both the theoretical and experimental works relating to these invariance principles. However, we have failed to ask the question: Are these invariance principles really truly Lorentz covariant? It is my aim in this letter to show that some sections of the invariance principles such as the space reversal symmetry (or parity **P**) and time reversal symmetry **T** are not really truly Lorentz covariant while the *world parity or the proper parity* **W** (i.e., the space-time reversal symmetry **PT**) is a truly Lorentz covariant concept. Because of the fundamental significance and its huge implications to the foundations of physics and to the future developments of Lorentz covariant unified field theories, I felt that it is important to draw attention to this problem by writing this letter.

It is well known that in classical mechanics[4] there are two types of relativistic formalisms: One that is Lorentz invariant where we choose the Hamiltonian H_1 as the total energy E of the particle which is the generator of the particle motion with the ordinary time t and is given by $H_1 = E = (c^2\mathbf{p}^2 + m^2c^4)^{1/2}$, and the other that is Lorentz covariant where we take the Hamiltonian H_2 as the negative of 1/2 the rest energy ($-E_0/2$) of the particle which is the generator of the particle motion with the proper time τ (i.e., the generator of the particle's world line) and is given by $H_2 = (2m)^{-1}\,\mathbf{p}^2 = (2m)^{-1}\,\Sigma_\lambda\, p_\lambda p_\lambda = -\,mc^2/2 = -\,E_0/2$. Here $p_1 = p_x$, $p_2 = p_y$, $p_3 = p_z$, and $p_4 = iE/c$. However, in the Lorentz invariant relativistic quantum mechanics there is no Hamiltonian formalism for spin zero particles,[5,6] but there are these analogous two types of relativistic Hamiltonian formalism for spin 1/2 particles: One that is Lorentz invariant where the Hamiltonian $H_3 = E = -c\alpha.\mathbf{p} - \beta mc^2$ is the Dirac Hamiltonian[5,6,3] and this Hamiltonian is the generator of the particle motion again with the ordinary time t , and the other that is Lorentz covariant where the Hamiltonian is H_2 for spin zero particles and is[3] $H_4 = (2m)^{-1}\,(\gamma.\mathbf{p})^2 = (2m)^{-1}\,(\Sigma_\lambda\,\gamma_\lambda p_\lambda)\,(\Sigma_\mu\,\gamma_\mu p_\mu) = -\,mc^2/2 = -\,E_0/2$ for the spin 1/2 particles and these Hamiltonians are the generators of the particle motion with the proper or world time τ (i.e., the generators of the particle's world lines) *and here the ordinary time t is an operator and not a parameter* . The α, β, and γ are the familiar spin matrices of the Dirac's theory of the electron.[5,3,6] In the Lorentz invariant quantum mechanics one usually studies the eigenvalue equation (i.e., the Schrodinger or the Klein-Gordon relativistic wave equation) of the form $E^2\,\psi(x,y,z,t) = (c^2\mathbf{p}^2 + m^2c^4)\,\psi(x,y,z,t)$ for spin zero particles, where $E = i\hbar\,\partial/\partial t$ and $p_j = -\,i\hbar\,\partial/\partial x_j$ for j = 1, 2 and 3, and $H_3\psi(x,y,z,t) = E\psi(x,y,z,t)$ for spin 1/2 particles with the same replacement for E and p_j for the Schrodinger wave equation type formalism, and the fundamental commutation relations $[x_i, p_j] = i\hbar\,\delta_{ij}$, $[x_i, x_j] = [p_i, p_j] = 0$ for i, j = 1, 2 and 3 in conjunction with the Heisenberg equation of motion for any matrix operator

M as given by $dM/dt = \partial M/\partial t + (i\hbar)^{-1}[\mathbf{M}, \mathbf{H}_3]$ for the Heisenberg matrix operator type formalism. It may be noted that if **M** does not depend explicitly on the ordinary time t and commutes with $\mathbf{H}_3$ then it is a constant of motion in that particular Lorentz frame. This operator **M** and the associated constant of motion is, in general, not Lorentz covariant since for the Dirac Hamiltonian $[\mathbf{H}_3, \mathbf{L}] \neq 0$ and when $[\mathbf{M}, \mathbf{H}_3] = 0$ by Jacobi's identity[4,6] we get $[\mathbf{H}_3, [\mathbf{L}, \mathbf{M}]] = -[\mathbf{M}, [\mathbf{H}_3, \mathbf{L}]]$. Thus $[\mathbf{M}, \mathbf{L}]$ can be zero only in the special situation (i.e., in the special Lorentz inertial frame) where $[\mathbf{M}, [\mathbf{H}_3, \mathbf{L}]] = 0$, which may not be true in the general cases. Here **L** is the Einstein's Lorentz transformation matrix or the boost[1,3,5]. For spin zero particles, however, there is no Lorentz invariant Heisenberg matrix operator type formalism since there exist no corresponding Hamiltonian to begin with. In the Lorentz covariant quantum mechanics one starts studying the world-space eigenvalue equation of the form[3] $H\,\psi(x,y,z,ict,\tau) = E_0\,\psi(x,y,z,ict,\tau)$, where $E_0 = -\,mc^2/2$, regardless of its spin. Here H is either H_2 or H_4. For the world space Schrodinger wave equation type formalism we adopt the substitution $E_0 = i\hbar\,\partial/\partial\tau$, and $p_\mu = -\,i\hbar\,\partial/\partial x_\mu$ for $\mu = 1, 2, 3$ and 4; and we use the world space fundamental commutation relations $[x_\nu, p_\mu] = i\hbar\,\delta_{\nu\mu}$, and $[x_\nu, x_\mu] = [p_\nu, p_\mu] = 0$ for $\nu, \mu = 1, 2, 3$, and 4 in conjunction with the world space Heisenberg equation of motion for any *four-matrix operator* **M** as given by

$$dM/d\tau = \partial M/\partial\tau + (i\hbar)^{-1}[\mathbf{M}, \mathbf{H}] \tag{1}$$

for the world space Heisenberg matrix operator mechanics type formalism. It may be noted from Eq. (1) that if the four-operator **M** does not depend explicitly on the world or proper time τ and commutes with both the Lorentz covariant Hamiltonians **H** (i.e., $[\mathbf{M}, \mathbf{H}] = 0$ with $[\mathbf{H}, \mathbf{L}] = 0$) and the boost transformation matrix **L** (i.e., $[\mathbf{M}, \mathbf{L}] = 0$) then it is a truly manifestly Lorentz covariant operator that is a constant of motion in the world or the Minkowski space, and is form-invariant under the boost transformation **L**, i. e., it is a truly Lorentz covariant symmetry operation.

In the Lorentz invariant quantum mechanics we usually talk of two pictures,[1,3] namely, the Heisenberg picture where the state vectors are time independent and the operators are time dependent, and the Schrodinger picture where the state vectors are time dependent while the operators are time independent. In a similar way in the Lorentz covariant quantum mechanics[3] one has the Heisenberg world picture in which all operators are world time dependent while the state vectors are world time independent, and the Schrodinger world picture in which all operators are world time independent while the state vectors are world time dependent. In the Wignerian analysis of the relativistic invariance under the symmetry group one usually works with the Dirac Hamiltonian where the time t enters as a parameter marking the dynamical evolution of the particle rather than an operator, and the quantum states are described in the Heisenberg picture.[1] The symmetry operations or the invariance principles are those that preserve the transition probabilities $|(\varphi,\psi)|^2$ between the two quantum states φ and ψ, where (φ,ψ) is the scalar product of the two state vectors φ and ψ. This requirement of the relativistic invariance in the conventional Lorentz invariant quantum mechanics leads to the existence of ray representations up to a phase factor of modulus one of the inhomogeneous Lorentz or Poincare group.[1]

We have thus far discussed the quantum dynamics of a single particle. However, the quantization of this wave field ψ (i.e., the *second quantization*) converts this one-particle theory into the corresponding many-particle theory[6] (i.e. the field theory). When the particle is in an electromagnetic (e.m.) field described by a four-vector potential **A** with $A_1 = A_x$, $A_2 = A_y$, $A_3 = A_z$, and $A_4 = i\phi/c$, then all these Hamiltonians undergo the replacement of **p** by the expression $(\mathbf{p} - q\mathbf{A}/c)$. Thus, combining the second quantization of the single-particle wave field ψ with the first quantization of the e.m. field **A**, one obtains the conventional many-body quantum field theory. Hence, any symmetry operation that is Lorentz covariant with respect to the singe-particle quantum dynamics descibed in terms of the proper time τ (keeping the ordinary time t as an operator and not as a parameter) will also be Lorentz covariant in the corresponding quantum field theory and *vice versa.*

We now wish to examine the mapping associated with the symmetry operations in the four-dimensional space-time continuum known as the world space or the Minkowski space. The familiar relativistic Lorentz transformation is an orthogonal transformation in this world space whose coordinates are $x_1 = x$, $x_2 = y$, $x_3 = z$, and $x_4 = ict$. Let us consider the two inertial frames F and F' whose axes of coordinates are parallel to each other, and the primed system is moving with a uniform velocity $\mathbf{v} = \beta c \mathbf{i}_x$ along the x direction with respect to the unprimed system. Then the pure proper orthochronous (or Einstein) Lorentz transformation (i. e., the boost transformation) may be written[1,3] $\mathbf{x}' = \mathbf{L}\,\mathbf{x}$ and $\mathbf{x} = \mathbf{L}^{-1}\mathbf{x}'$, where $L_{11} = L_{44} = \gamma$, $L_{22} = L_{33} = 1$, $L_{14} = - L_{41} = i\beta\gamma$, all other $L_{\mu\nu} = 0$ for $\mu,\nu = 1, 2, 3$, and 4, $\gamma = (1 - \beta^2)^{-1/2}$, and $\beta = v/c$. An event A in the F frame is given by the tip of the position four-vector $\mathbf{x}(a)$ whose Minkowskian coordinates are $x_1(a)$, $x_2(a)$, $x_3(a)$, and $x_4(a) = ict(a)$. Thus in the F frame the mapping of an event A [i.e., the space-time point $\mathbf{x}(a)$] onto an event B [i.e., the space-time point $\mathbf{x}(b)$] may be written $\mathbf{x}(b) = \mathbf{M}\mathbf{x}(a)$, where $\mathbf{M}$ is the mapping matrix with elements $M_{\mu\nu}$. In the F' frame the corresponding mapping of the event A' [i.e., the space-time point $\mathbf{x}(a')$] onto the event B' [i.e., the space-time point $\mathbf{x}(b')$] is $\mathbf{x}(b') = \mathbf{M}'\mathbf{x}(a')$, where $\mathbf{M}'$ is the corresponding mapping matrix in the F' frame with elements $M'_{\mu\nu}$. But by the boost Lorentz transformation, $\mathbf{x}(a') = \mathbf{L}\mathbf{x}(a)$, and $\mathbf{x}(b') = \mathbf{L}\mathbf{x}(b)$. Hence $\mathbf{x}(b') = \mathbf{L}\mathbf{x}(b) = \mathbf{L}\mathbf{M}\mathbf{x}(a) = (\mathbf{L}\mathbf{M}\mathbf{L}^{-1})\mathbf{L}\mathbf{x}(a) = (\mathbf{L}\mathbf{M}\mathbf{L}^{-1})\,\mathbf{x}(a')$. Thus, $\mathbf{M}' = \mathbf{L}\mathbf{M}\mathbf{L}^{-1}$. That is, the mapping matrix $\mathbf{M}'$ of the F' frame is related to the corresponding mapping matrix $\mathbf{M}$ of the F frame by the usual similarity or the equivalence transformation. The *principle of relativity* states that all inertial frames are equivalent. Hence according to the principle of relativity, we find that any symmetry operation or mapping matrix $\mathbf{M}$ is *manifestly Lorentz covariant* (i.e., will appear form invariant) if and only if $\mathbf{M} = \mathbf{M}' = \mathbf{L}\mathbf{M}\mathbf{L}^{-1}$. That is,

$$[\mathbf{M}, \mathbf{L}] = \mathbf{M}\mathbf{L} - \mathbf{L}\mathbf{M} = 0 \qquad (2)$$

Hence any symmetry operation or mapping matrix $\mathbf{M}$ *is manifestly Lorentz covariant (i.e., will appear form invariant) if and only if it commutes with the Einstein Lorentz transformation matrix* *(i.e., the boost)* **L**. This is the necessary and sufficient condition for the manifestly Lorentz covariance of any physically meaningful symmetry operation and the associated world space mapping. Hence according to the Einsteinian principle of relativity, any dynamical laws of symmetry or *covariance principle* must, of course, satisfy the kinematic symmetry requirements of Eq. (2).

Let us first examine the Lorentz covariance of the Newtonian concept of simultaneity. We may define the simultaneity matrix **S** in the F frame by taking the matrix elements of **S** as given by $S_{44} = 1$, $S_{4i} = S_{i4} = 0$, and all other elements S_{ij} are finite, where i, j = 1, 2, and 3. Since $\mathbf{x}(b) = \mathbf{S}\mathbf{x}(a)$, it is clear that $x_4(b) = ict(b) = x_4(a) = ict(a)$. That is, t(b) = t(a) in the F frame. Note that $[\mathbf{S}, \mathbf{L}] \neq 0$ and since for a Lorentz covariant Hamiltonian $[\mathbf{H}, \mathbf{L}] = 0$ we find that by the Jacobi's identity $[\mathbf{H}, \mathbf{S}]$ can be zero only in the special situation where $[\mathbf{H}, [\mathbf{L}, \mathbf{S}]] = - [\mathbf{L}, [\mathbf{S}, \mathbf{H}]] = 0$ which may not be true in the general cases. This implies that in the general cases **S** is not a constant of motion in the Minkowski space [according to Eq. (1)]. In the F' frame $\mathbf{x}(b') = \mathbf{S}'\mathbf{x}(a')$, where $\mathbf{S}' = \mathbf{L}\mathbf{S}\mathbf{L}^{-1}$. It is relatively easy to show that $x_4(b') \neq x_4(a')$, i.e., $t(b') \neq t(a')$. *Hence, the Galilean-invariant concept of simultaneity is not a truly Lorentz covariant concept for inertial observers,* a well-known fact from the early days.

We now wish to examine the Lorentz covariance of the Newtonian concept of spatial inversion symmetry or parity. We may define the spatial inversion matrix or the parity operator **P** in the F frame by taking the matrix elements of **P** as given by $P_{ii} = -1$ for i = 1, 2, and 3, $P_{44} = 1$, and all other elements $P_{\mu\nu} = 0$. Note that $[\mathbf{P}, \mathbf{L}] \neq 0$ and since $[\mathbf{H}, \mathbf{L}] = 0$ for a Lorentz covariant Hamiltonian we again find that by the Jacobi's identity $[\mathbf{H}, \mathbf{P}]$ can be zero only in the special situation where $[\mathbf{H}, [\mathbf{L}, \mathbf{P}]] = - [\mathbf{L}, [\mathbf{P}, \mathbf{H}]] = 0$ which again may not be true in the general cases. This implies that in the general cases **P** is not a constant of motion in the world space [according to Eq. (1)]. *It may be noted that the parity operator* **P** *is a special case of the simultaneity operator* **S**. In the F' frame $\mathbf{P}' = \mathbf{L}\mathbf{P}\mathbf{L}^{-1}$. The matrix elements of **P'** are given by $P'_{44} = - P'_{11} = \gamma^2(1 +$

β^2), $P'_{22} = P'_{33} = -1$, $P'_{14} = P'_{41} = i2\beta\gamma^2$, and all other elements $P_{\mu\nu} = 0$. This is clearly not a spatial inversion or parity operator in the F' frame. *That is the Galilean-invariant concept of spatial inversion symmetry or parity is not a truly Lorentz covariant concept for inertial observers.* This result is in disagreement with the familiar Dirac-Wigner invariance principle analysis. The physical reason for this disagreement is due to the fact that the Dirac Hamiltonian H_3 is not Lorentz covariant since it is not an invariant world scalar and is the generator of the particle motion with the ordinary time t (in space and time and not in the space-time continuum); and since it is linear in $\alpha.\mathbf{p}$ where α is an operator in the four dimensional spin space, while $\mathbf{p}$ is an operator in the ordinary space and time (and also in the space-time continuum), there exists an intrinsic coupling between the spin space and the real space (implying also an intrinsic coupling between the spin space and the Minkowski space) *even for a free particle.* This artificially introduced coupling yields some peculiar nonphysical behavior for a Dirac free particle. For example, its velocity operator $\mathbf{v} = d\mathbf{x}/dt = c\,\alpha$ is independent of $\mathbf{p}$, acts only on the spin space and takes the eigenvalues $\pm$ c, whereas for a particle of finite mass m classical velocity cannot be equal to $\pm$ c. Note the explicit coupling between the $\mathbf{x}$-space and the α-space by the relation $d\mathbf{x}/dt = c\alpha$. Also this velocity oscillates rapidly (the *Zitterbewegung*), whereas its momentum is a strict constant of motion. Further, neither its orbital angular momentum nor its spin angular momentum is a constant of motion but only its total (orbital plus spin) angular momentum is a constant of motion, clearly and explicitly exhibiting this nonphysical coupling *even for a free particle.* All these behavior of the Dirac free particle are contrary to those expected from both the Lorentz invariant and the Lorentz covariant formulations of classical mechanics.[4] A truly covariant Hamiltonian must satisfy the requirements of Eq. (2), i. e., $[\mathbf{H}, \mathbf{L}] = 0$, since it must be the generator of the particle (motion with the proper time τ) world line and *should not couple these two spaces for a free particle and according to the correspondence principle should yield results that reduce to the classical results in the limit $\hbar \to 0$.* Indeed, it is shown elsewhere[3] that the truly Lorentz covariant Hamiltonians H_2

and H_4 do not show this nonphysical coupling between the four dimensional spin space and the four dimensional Minkowski space for a free quantum particle and these quantum results are completely consistent with the corresponding classical results.[4,3]

A theory based on the total energy as the Hamiltonian in which time time is treated differently from the space coordinates is not manifestly Lorentz covariant, since $[\mathbf{H}, \mathbf{L}] \neq 0$. This does not imply that the theory is incorrect. Here, since $[\mathbf{H}, \mathbf{L}] \neq 0$, we should not (and it is inappropriate to) require the symmetry operators **M** such that $\mathbf{M}\psi = \pm\psi$ to satisfy the requirements of Eq. (2). We only require the results of the theory (e.g, the transition probabilities $|(\phi,\psi)|^2$) to be Lorentz invariant. This is clearly true for the Dirac-Wigner invariance principle analysis. However, the theory with mathematical beauty in which space and time are considered as entirely similar coordinates in the Minkowski or world space should be based on a Hamiltonian H that is a Lorentz-invariant world scalar [e.g H_2 and H_4], and this theory is manifestly Lorentz covariant since $[\mathbf{H}, \mathbf{L}] = 0$ where $\mathbf{H} = H\mathbf{I}$ and **I** is the identity mapping matrix in the world space. Here, we not only require the results of this theory [e.g., $|(\phi,\psi)|^2$] to be Lorentz invariant , but also require that all physically meaningful symmetry operators **M** such that $\mathbf{M}\psi = \pm\psi$ satisfy the Lorentz-covariant requirements of Eq. (2), since **H** itself satisfies Eq.(2) [i.e., *covariant symmetry principles in contrast to the present day invariant symmetry principles*]. The basic difference is that in the former theory one would have to prove that its results are Lorentz invariant before they could be accepted as correct, while in the latter covariant theory the Lorentz invariance of its results should be explicitly apparent from its mathematical structure.

Let us now examine the Lorentz covariance of the newtonian concept of time reversal symmetry. We may define the time reversal matrix **T** in the F frame by taking the matrix elements of **T** as $T_{44} = -1$, $T_{ii} = 1$ for i = 1, 2, and 3, and all other elements $T_{\mu\nu} = 0$. Note that $[\mathbf{T}, \mathbf{L}] \neq 0$ and since for a Lorentz covariant Hamiltonian $[\mathbf{H}, \mathbf{L}] = 0$ we again find that by the Jacobi's identity $[\mathbf{H}, \mathbf{T}]$ can be zero only in the special situation where $[\mathbf{H}, [\mathbf{L}, \mathbf{T}]] = -[\mathbf{L}, [\mathbf{T}, \mathbf{H}]] = 0$ which again may not be true in the general cases. This implies that in the general cases **T** is not a constant of motion in the world space [according to Eq. (1)]. In the F' frame $\mathbf{T}' = \mathbf{LTL}^{-1}$. The matrix elements of $\mathbf{T}'$ are given by $T'_{44} = T'_{11} = -\gamma^2(1 + \beta^2)$, $T'_{22} = T'_{33} = 1$, $T'_{14} = T'_{41} = -i2\beta\gamma^2$, and all other elements $T'_{\mu\nu} = 0$. This is clearly not a time reversal operation in the F' frame. *Thus we see that the Galilean invariant concept of time reversal symmetry is not a truly Lorentz covariant concept for inertial observers.* Here, again, this result is in disagreement with the usual Dirac-Wigner invariance principle analysis.

It is known in mechanics that the momentum canonical to the time coordinate t is the total energy E. Thus in quantum mechanics the energy operator $E = i\hbar\, \partial/\partial t$. Hence $\mathbf{T}\, E = -E = i\hbar\, \partial/\partial(-t)$. That is a pure time reversal **T** corresponds to negative energy states. But in nonrelativistic quantum mechanics the energy cannot be negative. This led Wigner to suggest that the proper quantum mechanical time reversal operator $\mathbf{T_W} = \mathbf{TK}$, where **K** is the complex conjugation operator (that changes i to -i). Then $\mathbf{T_W}\, E = \mathbf{TK}E = E = -i\hbar\, \partial/\partial(-t)$. However in relativistic quantum mechanics Feynman[5] proposed that the negative energy states should be interpreted as states representing particles moving backward in time. That is, the time-reversed negative energy states of a particle correspond to the positive energy states of the corresponding antiparticle. Further it is well known that the Maxwell equations and their consequences lend themselves very simply to a Lorentz covariant discription and are invariant with respect to change in sign of the charge density. This leads one naturally to consider charge conjugation **C** on an equal footing with time reversal in quantum electrodynamics. However, this charge conjugation operation does not fall directly within the framework of world space mapping. But this does lead us to the famous **CPT** theorem,[1,2] which states that if a local Lagrangian theory (which may contain derivative couplings to any high but finite orders) is invariant under proper orthochronous Lorentz transformation, it is invariant under the product **CPT** (and its permutations **PCT**, etc.), although the theory may not be separately invariant under each of these operators **C**, **P**, and **T**. It should be noted that for the **CPT** theorem, we require the invariance of the Lagrangian theory under only the restricted Lorentz group and not the other three components of the full Lorentz group.[1] Strictly speaking, for an ideally correct theory in the Einstein-Minkowski sense, we should require the

Lagrangian and the Hamiltonian operators of the theory by themselves to be Lorentz covariant with respect to this restricted Lorentz group of transformations. Then **P** and **T** separately have no place in this manifestly Lorentz-covariant theory.

It should be noted that in nature there are both charged (i.e., for example, electrons, protons, $\mu^{\pm}$ –mesons, $\pi^{\pm}$ –mesons, $\tau^{\pm}$ –mesons, charged hyperons, etc., and their antiparticles) and uncharged (i.e., for example, neutrons, π^{0} -mesons, θ^{0} -mesons, uncharged hyperons, etc., and their antiparticles) particles of finite mass. The **CPT** theorem applies to the field theory of charged particles. However, in the field theory of uncharged particles the **CPT** theorem must reduce to a **PT** theorem as it should since the particles carry no charge. We will soon see that **PT** is a Lorentz covariant kinematical symmetry property of the Minkowski or world space. Therefore **PT** seems as fundamental to the field theory of uncharged particles as **CPT** is to the field theory of charged particles. Further, the Lorentz covariant Hamiltonians $H_5 = (2m)^{-1}(\mathbf{p} - q\mathbf{A}/c)^2 = (2m)^{-1} \Sigma_\lambda (p_\lambda - qA_\lambda/c)(p_\lambda - qA_\lambda/c) = (2m)^{-1} \Sigma_\lambda u_\lambda u_\lambda = - mc^2/2 = - E_0/2$ for the spin zero particles and $H_6 = (2m)^{-1} \{\gamma . (\mathbf{p} - q\mathbf{A})/c\}^2 = (2m)^{-1} \Sigma_\lambda\{\gamma_\lambda (p_\lambda - qA_\lambda/c)\, \gamma_\lambda (p_\lambda - qA_\lambda/c)\} = (2m)^{-1} \Sigma_\lambda\{(\gamma_\lambda u_\lambda)(\gamma_\lambda u_\lambda)\} = - mc^2/2 = - E_0/2$ for the spin 1/2 particles are both Lorentz invariant world scalars. Hence, in the total Hamiltonian $H = (H_{particle} + H_{interaction}) + H_{field}$ for a charged particle in the e.m. field, H_{field} is Lorentz covariant and is also invariant under the **C** operation while $(H_{particle} + H_{interaction})$ is Lorentz covariant under the **PT** operation, since $(H_{particle} + H_{interaction})$ is H_5 for the spin zero particles and is H_6 for the spin 1/2 particles.

We now examine the Lorentz covariance of the Minkowskian concept of the *world parity* or *the proper parity* operator **W** (i.e., the space-time reversal operator **PT**). It is relatively easy to show that **W** = **PT** = - **I**. That is, $W_{\mu\mu} = - 1$, and all the other elements $W_{\mu\nu} = 0$, where μ, ν = 1, 2, 3, and 4. Note that [**W, L**] = 0 and since for a Lorentz covariant Hamiltonian [**H, L**] = 0 we find that [**H, W**] = 0 implying that **W** is a strict constant of motion in the Minkowski space [according to Eq. (1)]. The corresponding mapping matrix **W'** in the F' frame is **W'** = $\mathbf{LWL}^{-1}$ = - **I**. This is clearly a world parity or proper parity (i.e., a space-time reversal mapping) in the F' frame. Note that **W** = **W'** = - **I**, the same operator for all inertial observers, i.e., a truly manifestly Lorentz covariant operator. *Thus the concept of world parity or proper parity (i.e., the space-time reversal symmetry) is indeed a truly Lorentz covariant concept for all inertial observers.* This result is in accordance with the spirit of the world space Mach principle:[2,3] the laws of physics should not depend on the particular Minkowskian or world geometrical coordinate system we happen to choose. In summary, since the covariant Hamiltonians H_2, H_4, H_5 and H_6 will satisfy the commutation relation [**H, L**] = 0 and since [**W, L**] = 0 while [**P, L**] ≠ 0 and [**T, L**] ≠ 0, it is clear [**H, W**] = 0 while, in general, [**H, P**] and [**H, T**] may not be zero. That is, in any truly covariant formalism the world or proper parity **W** is always conserved while the ordinary parity **P** and the ordinary time reversal symmetry **T** may not, in general, be conserved. However, in the special cases (i.e., in the special Lorentz inertial frames) where [**L**, [**P, H**]] = 0 and [**L**, [**T, H**]] = 0, the ordinary parity **P** and the ordinary time reversal symmetry **T** are also conserved.

Although we have only illustrated the application of our mathematical results of the theory of Lorentz covariant world space mapping to the relativistic quantum theory, it is apparent that these results apply equally well to *any world eigenvalue problem* in the Minkowski space-time continuum. In essence, the bottom line is this: according to both the Einstein's special theory of relativity and the laws of causality[7] (which states that the effect should not precede the cause for particles of positive real mass moving with speeds less than c, i. e., tardyons or bradyons with *timelike four-vectors), in any truly Lorentz-covariant theory of any physical phenomena,* one cannot change the space leaving the time invariant (as is usually done in simultaneity, parity, rigid-body constraints, action-at-a-distance forces, etc.) and vice versa (as is usually done in time reversal). In the literature[8] the possible existence of tachyons, i.e., particles with imaginary mass moving faster than the speed of light c with *spacelike four-vectors*, has been postulated. Here, we will only consider the mechanics of tardyons. As Minkowski said,[9] "Space of itself, and time of itself will sink into mere shadows, and only a kind of union between them shall survive." Hence in the spirit of Minkowski's remark, we conclude that space-reversal symmetry (**P**) by itself and time reversal symmetry (**T**) by itself will sink into mere shadows, and only a kind of union between them, the world parity or the proper parity (**W** = **PT**) shall survive in any properly formulated manifestly Lorentz-covariant theory of any physical phenomena, regardless of the nature of the interaction. It is, of course, the duty of a correct theory of any physical phenomenon to provide Lorentz-covariant expressions for the forces involved from which one can construct the covariant Lagrangian and the covariant Hamiltonian that will satisfy the requirements of Eq. (2). Unfortunately, at present however, we do not have covariant theories of all the possible forces in nature such as electromagnetic, nuclear (both the strong and weak), gravitational, etc. Only the electromagnetic theory provides a covariant force equation. However, as pointed out by Goldstein,[4] the transformation properties must be the same for all forces no matter what their origin. For example, the statement "a particle is in equilibrium under the influence of two forces" must hold true in all Lorentz systems which can only be the case if all forces transform in the same manner. Hence, we must be able to formulate a fully satisfactory manifestly Lorentz covariant unified field theory of all the forces in nature no matter what their origin. Although according to Dirac and Wigner it is necessary and sufficient for any correct relativistic theory to predict only Lorentz invariant results, but according to Mach, Einstein, Minkowski and Maxwell all correct relativistic theories must be Lorentz covariant. I strongly believe that we should follow the latter view. Then and only then we have a good chance of producing the fully Lorentz covariant grand unified field theory of all the forces in nature no matter what their origin with all its mathematical beauty and glory. Clearly, one cannot mix simply Lorentz invariant but not Lorentz covariant theories of strong, weak and gravitational interactions with the manifestly Lorentz covariant Maxwell's e.m. theory and obtain any meaningful and fully satisfactory (i.e., a truly Lorentz covariant) unified field theory.

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