# ARIEL III8-6i: A VERY CLOSE BINARY SYSTEM? 

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## SUMMARY

The hypothesis is considered that the $6 \cdot 75-\mathrm{min}$ periodicity observed in the transient X-ray source Ariel iri8-61 is due to orbital motion. Such a binary system would consist of a white dwarf transferring matter on to a compact object. The average mass transfer rate is $\sim 10^{-7} M_{\odot} \mathrm{yr}^{-1}$ and is driven almost entirely by gravitational radiation. This rate of mass transfer obscures the X-ray source for most of the time and a transient X-ray source is seen when the mass transfer temporarily drops below this level.

## I. INTRODUCTION

The transient X-ray source recently observed by Ariel 5 (Ives, Sanford \& Bell-Burnell 1975) displays a regular variation present throughout the 10 days or so for which the X-ray source was observed, with a period of $6 \cdot 75 \mathrm{~min}$. This period is unique among those observed in X-ray sources so far, lying between the observed orbital periods (typically days but, for Cygnus $\mathrm{X}-3,4.8 \mathrm{hr}$ ) and the short pulse periods ( 4.84 s for Cen $\mathrm{X}-3$ ) which are associated with the rotation of a neutron star. We regard the most plausible explanation of the $6.75-\mathrm{min}$ period to be that it is an orbital period (we discuss other possibilities in Section 4) and that the binary system conforms to the canonical model for binary X-ray sources; that is, it consists of a star filling its Roche lobe transferring matter to its compact companion (either a neutron star or a black hole). In Section 2 we discuss the consequences of this assumption and in Section 3 consider the transient source.

## 2. THE BINARY SYSTEM

### 2.1 Orbital parameters and mass transfer rate

If we interpret the observed X-ray periodicity as an orbital period $P=6.75 \mathrm{~min}$, then by Kepler's third law the semi-major axis, $a$, of the system is given by

$$
\begin{equation*}
a=8.2 \times 1^{9} 9\left(\frac{m_{1}+m_{2}}{m_{\odot}}\right)^{1 / 3} \mathrm{~cm} \tag{2.1}
\end{equation*}
$$

where $m_{2}$ is the mass of the (compact) X-ray star (secondary) and $m_{1}$ the mass of its companion (primary).

We assume that the primary fills its Roche lobe. The Roche lobe equivalent radius is given approximately by

$$
\begin{equation*}
R_{\mathrm{L}}=0.46\left(\frac{m_{1}}{m_{1}+m_{2}}\right)^{1 / 3} a \tag{2.2}
\end{equation*}
$$

provided that $0 \leqslant m_{1} / m_{2} \leqslant 0.8$ (Paczynski 1971). Combining equations (2.1) and (2.2), we find that $R_{\mathrm{L}}$ is just a function of $m_{1}$, independent of $m_{2}$. In fact, it can be shown that for any mass ratio,

$$
R_{\mathrm{L}}<6.7 \times \mathrm{Io}^{9}\left(\frac{m_{1}}{m_{\odot}}\right)^{1 / 3} \mathrm{~cm}
$$

It is therefore clear that the primary is at least as compact as a white dwarf: it cannot, for example, be a helium star.

The mass-radius relation for cold, non-rotating, non-relativistic white dwarfs consisting entirely of helium is given by:

$$
\begin{equation*}
R_{1}=7.7 \times 10^{8} x^{0.3767-0.00605 \log x} \mathrm{~cm} \tag{2.4}
\end{equation*}
$$

where

$$
x=\frac{1 \cdot 44 m_{\odot}}{m_{1}}-\mathrm{I} \quad(\text { Webbink 1975a). }
$$

Thus setting $R_{\mathrm{L}}$ equal to $R_{1}$ yields a unique value for the mass of the white dwarf:

$$
\begin{equation*}
m_{1}=0.12 \mathrm{I} m_{\odot} . \tag{2.5}
\end{equation*}
$$

We must regard this mass as an upper bound to the white dwarf mass, as there are a number of effects which may conspire to increase $R_{1}$ beyond the value in equation (2.4): tidal and rotational distortion, non-degeneracy in the envelope, and X-ray heating of the surface. Of these effects, we estimate that the accretion luminosity of the secondary cannot enlarge the primary by more than 0.5 per cent by heating, and the remaining effects are unlikely to amount to more than a few per cent unless the star is rotating much faster than synchronously. Any such decrease in the primary mass has the effect of reducing the mass transfer rate.

For such a close binary system as this it is clear that gravitational radiation can be the chief driving force for mass transfer (Kraft, Matthews \& Greenstein 1962). The rate at which the orbital angular momentum

$$
\begin{equation*}
H=\left(\frac{G a m_{1}{ }^{2} m_{2}^{2}}{m_{1}+m_{2}}\right)^{1 / 2} \tag{2.6}
\end{equation*}
$$

is radiated away is given in the weak field limit of the theory of General Relativity (Zeldovich \& Novikov 197I) by

$$
\begin{equation*}
\frac{d H}{d t}=-\frac{64}{5}\left(\frac{G^{4}}{c^{5}}\right) m_{1}^{3} m_{2}^{2} / a^{3} \tag{2.7}
\end{equation*}
$$

Since mass transfer is taking place, we have assumed that the orbit is circular. Suppose that the secondary accretes a fraction $\alpha(\leqslant \mathrm{I})$ of the transferred material, and expels the remainder with angular momentum per unit mass

$$
\begin{equation*}
h_{2}=\left[\frac{G a m_{1}^{4}}{\left(m_{1}+m_{2}\right)^{3}}\right]^{1 / 2} \tag{2.8}
\end{equation*}
$$

corresponding to its own orbital motion. A straightforward calculation, assuming that the white dwarf always fills its Roche lobe and that its radius is given by equation ( 2.4 ) then yields the result

$$
\frac{d m_{1}}{d t}=-\frac{64}{5}\left(\frac{G^{3}}{c^{5}}\right) m_{1}^{2} m_{2}\left(m_{1}+m_{2}\right) a^{-4} \beta^{-1}
$$



Fig. I. The mass loss rate of the white dwarf ( $\dot{m}_{1}$ ), as a consequence of gravitational radiation, is drawn as a function of secondary mass $m_{2}$ for two cases. The continuous line assumes that the secondary accretes all the transferred material and the dotted line assumes that the secondary expels all the transferred material with specific angular momentum corresponding to its own angular momentum
where

$$
\begin{equation*}
\beta=\frac{5 m_{2}^{2}+(3-4 \alpha) m_{1} m_{2}-6 m_{1}^{2}}{3^{m_{2}}\left(m_{1}+m_{2}\right)}-\frac{0.533 m_{\odot}}{\mathrm{I} \cdot 44 m_{\odot}-m_{1}} . \tag{2.9}
\end{equation*}
$$

This mass transfer rate is shown in Fig. I as a function of $m_{2}$ (using $m_{1}=0 \cdot 121 m_{\odot}$ ) for the cases $\alpha=\mathrm{o}$ (dotted line) and $\alpha=\mathrm{I}$. We note that the conditions for stability of the white dwarf against dynamical mass loss are:

$$
m_{2}>0.192 m_{\odot} \quad \text { for } \quad \alpha=1
$$

and

$$
m_{2}>0.126 m_{\odot} \text { for } \alpha=0
$$

Note also that unless $m_{2} \gg 100 m_{\odot}$ this mass transfer rate is in excess of that which can be swallowed by the compact object, radiating at the Eddington limit.

### 2.2 Supercritical accretion

Since the mass transfer takes place through the Roche lobe, the accretion on to the compact object must take place via an accretion disc. We assume that the compact object is a neutron star-or, more precisely, that any material accreted by it must liberate a constant fraction, $f$, of its rest mass energy. It is not yet clear whether or not this condition is satisfied if the accreting object is a black hole
(Meszaros \& Rees 1975). The Eddington limiting luminosity at which radiation pressure on free electrons balances gravity is (Zeldovich \& Novikov 1971)

$$
\begin{equation*}
L_{\mathrm{E}}=\mathrm{I} \cdot 38 \times \mathrm{IO}^{38}\left(\frac{m_{2}}{m_{\odot}}\right) \mathrm{erg} \mathrm{~s}^{-1} . \tag{2.10}
\end{equation*}
$$

Thus the maximum rate of accretion by the compact object is

$$
\dot{M}_{\mathrm{E}}=L_{\mathrm{E}} / f c^{2} .
$$

We define (cf. Shakura \& Sunyaev 1973 (SS))

$$
\dot{m}=\dot{M} / \dot{M}_{E}
$$

where $\dot{M}$ is the accretion rate in the disc. Then (see SS) the accretion disc is thickened by radiation pressure at a critical radius

$$
\begin{equation*}
R_{\mathrm{C}}=\mathrm{Io}^{6} \dot{m}\left(\frac{m_{2}}{m_{\odot}}\right) \mathrm{cm} . \tag{2.11}
\end{equation*}
$$

For $\dot{m}>\mathrm{I}$, it is reasonable to assume that the excess accretion flux $\dot{M}-\dot{M}_{\mathrm{E}}$ is blown outwards in a spherically symmetric manner, while a flux $\dot{M}_{\mathrm{E}}$ is still accreted by the compact object, which therefore continues to radiate at the Eddington limit. The excess mass flux is blown out from radius $R_{\mathrm{c}}$, reaching a terminal velocity roughly equal to the escape velocity, $V$, from that point. Thus,

$$
\begin{equation*}
V=\left(2 G m_{2} / R_{\mathrm{c}}\right)^{1 / 2}=5 \cdot 16 \times 10^{9} \dot{m}^{-1 / 2} \mathrm{~cm} \mathrm{~s}^{-1} . \tag{2.12}
\end{equation*}
$$

Assuming spherically symmetric outflow we may evaluate the density as a function of radius:

$$
\begin{align*}
\rho(R) & =\left(\dot{M}-\dot{M}_{\mathrm{E}}\right) / 4 \pi R^{2} V \\
& =1 \cdot 54 \times 10^{7}(\dot{m}-1) \dot{m}^{1 / 2} R^{-2} F_{18} \mathrm{~g} \mathrm{~cm}^{-3} \tag{2.13}
\end{align*}
$$

where we have written $\dot{M}_{\mathrm{E}}=\left(\mathrm{Io}^{18} \mathrm{~g} \mathrm{~s}^{-1}\right) F_{18}$. Thus the electron scattering optical depth from infinity to a radius $R$ is

$$
\begin{align*}
\tau_{\mathrm{es}}(R) & =\int_{R}^{\infty} \kappa_{\mathrm{es}} d R \\
& =\mathrm{I} \cdot 22 \times 10^{7}(\dot{m}-\mathrm{I}) \dot{m}^{1 / 2} F_{18} R^{-1} \tag{2.14}
\end{align*}
$$

We define the radius $R_{\mathbb{T}}$, by $\tau_{\mathrm{es}}\left(R_{\mathrm{T}}\right)=\mathrm{I}$.
These expressions are really only valid for $R \gg R_{\mathrm{c}}$. Near $R \approx R_{\mathrm{c}}$, the excess material has just risen up out of the disc and is in the process of being accelerated outwards by the radiation from the central object. Here the density is approximately equal to that in the accretion disc (at the same radius) and therefore exceeds $\rho\left(R_{\mathrm{c}}\right)$ by a factor $\sim V / V_{\mathrm{r}}$ where $V_{\mathrm{r}}$ is the inward radial velocity in the accretion disc. Since the effective optical depth through the inner region of the disc is, in general, greater than unity (SS) we may assume that the X-ray photons emitted by the compact object are thermalized by the gas at $R \approx R_{\mathrm{c}}$. We assume initially that free-free absorption may be neglected for $R>R_{\mathrm{c}}$. At each radius we define an effective photon diffusion velocity $V_{\mathrm{d}}(R) \sim c / \tau_{\mathrm{es}}(R)$. We define the radius $R_{\mathrm{d}}$ by $V_{\mathrm{d}}\left(R_{\mathrm{d}}\right)=V$. Then

$$
\frac{R_{\mathrm{d}}}{R_{\mathrm{c}}} \sim \frac{2 \cdot \mathrm{I}(\dot{m}-\mathrm{I}) F_{18}}{\dot{m}\left(m_{2} / m_{\odot}\right)} .
$$

For $\dot{m} \gg \mathrm{I}, R_{\mathrm{d}} \sim R_{\mathrm{c}}$ and in the region $R \widetilde{<} R_{\mathrm{c}}$ the photons diffuse outwards through the outflowing gas at a speed greater than $V$. Thus the number of collisions, $N_{\mathrm{s}}$, a photon undergoes on its way out is roughly

$$
\begin{aligned}
N_{\mathrm{S}} & \sim\left[\tau_{\mathrm{es}}\left(R_{\mathrm{c}}\right)\right]^{2} \\
& =\mathrm{I} \cdot 5 \times \mathrm{IO}^{2}(\dot{m}-\mathrm{I})^{2} \dot{m}^{-1}\left(m_{2} / m_{\odot}\right)^{-2} F_{18^{2}} .
\end{aligned}
$$

Since $N_{\mathrm{s}} \gg c / V$ and $N_{\mathrm{s}} \gg m_{\mathrm{e}} c^{2} / k T$ (for $T \sim 10^{8} \mathrm{~K}$ ) X-ray photons lose all their energy through adiabatic losses and Compton collisions before they can reach $R_{\mathrm{T}}$, even if free-free absorption can be neglected. A proper calculation should take account of bound-free and free-free processes as well as the ionization-and velocity-structure of the outflowing material. In any event we may conclude that for most of the time the secondary does not emit hard (keV) X-rays.

### 2.3 Reflection effect

The optical luminosity of the material surrounding the compact object can be small. However, that part of the flux from this material that strikes the white dwarf can be absorbed and then reradiated mostly in the optical or ultraviolet. The luminosity of the white dwarf, due to the reflection effect, is then

$$
L_{1} \leqslant L_{\mathrm{E}}\left(\frac{\Omega}{4 \pi}\right)
$$

where $\Omega$ is the solid angle subtended by the white dwarf from the compact object. For large $m_{2} / m_{1}$ we may write approximately

$$
\frac{\Omega}{4 \pi}=\frac{2 \pi R_{1}^{2}}{4 \pi a^{2}}=0.026\left(\frac{m_{1}+m_{2}}{m_{\odot}}\right)^{-2 / 3}
$$

and hence,

$$
L_{1} \leqslant 3.58 \times \mathrm{Io}^{36}\left(\frac{m_{2}}{m_{\odot}}\right)\left(\frac{m_{1}+m_{2}}{m_{\odot}}\right)^{-2 / 3} \mathrm{erg} \mathrm{~s}^{-1}
$$

Thus the effective temperature of the side of the white dwarf facing its companion is

$$
T_{1}=\left(\frac{L_{1}}{2 \pi R_{1}^{2} \sigma}\right)^{1 / 4} \leqslant 2.32 \times 1^{5}\left(\frac{m_{2}}{m_{\odot}}\right)^{1 / 4}\left(\frac{m_{1}+m_{2}}{m_{\odot}}\right)^{-1 / 6} \mathrm{~K}
$$

Here $\sigma$ is Stefan's constant.

### 2.4 Magnetic neutron star

If the compact object is a neutron star with a strong ( $\gtrsim 10^{9} G$ ) magnetic field, the foregoing analysis of supercritical accretion must be modified. Let us define the Alfvén radius $R_{\mathrm{M}}$, as the radius where the magnetic field of the central star disrupts the accretion disc. If $R_{\mathrm{M}} \ll R_{\mathrm{c}}$, the above analysis holds. If $R_{\mathrm{M}} \gtrsim R_{\mathrm{c}}$, it seems likely that the spherically symmetric outflow of material takes place from this radius and at a velocity corresponding to the escape velocity from $R_{M}$.

Because, in this case, accretion takes place only on to the magnetic polar caps of the neutron star, the luminosity can be modulated with a period equal to the neutron star's rotation period. If the modulation occurs with a period of a few seconds, it is, in general, not smeared out by effects of time delay or electron
scattering. Thus the radiation output from the whole system could pulse on a time scale of a few seconds.

### 2.5 Period change

The time scale for the rate of change of the orbital period is of the order of $\tau_{\mathrm{P}} \sim m_{1} /\left|\dot{m}_{1}\right| \sim 5 \times 10^{5} \mathrm{yr}$. If our model is correct the period change should be an increase.

We note that if a theory of gravitation in which dipole gravitational radiation is permitted is correct, the time scale $\tau_{\mathrm{P}}$ could be much shorter than this and the change of period could be observed at the next outburst of this source (cf. Eardley 1975). However, the average mass transfer rate would then be correspondingly higher, making the probability of seeing the object as a transient X-ray source correspondingly less.

## 3. THE TRANSIENT X-RAY SOURCE

The hypothesis that the $6.75-\mathrm{min}$ period is indeed an orbital period leads us inevitably to conclude that on average the mass transfer rate is so high in the binary system that the X-rays emitted from the compact star are extinguished by the excess incoming material. To produce the observed transient X-ray source we must further hypothesize that the mass transfer rate must decrease sufficiently (by a factor $\dot{m}$ ) or cease entirely for the io days or so for which the transient source was observed. There are strong physical reasons (Webbink 1975b) for supposing that mass-loss rate from any star with a non-negligible superadiabatic surface convection zone will be pulsationally unstable in a Bath-type mode (Bath 1969 , 1972). Such a zone undoubtedly exists in the hypothetical white dwarf, but the period of the instability will depend sensitively on its intrinsic luminosity. It remains to be shown that the amplitude is as large as the mean mass transfer rate.

The peak in the observed X-ray intensity should correspond to radiation from the compact object at the Eddington limit. This gives us an estimate for the distance of the source and we find

$$
D \sim \mathrm{I} 2\left(\frac{m_{2}}{m_{\odot}}\right)^{1 / 2} \mathrm{kpc} .
$$

The variation of X-ray flux before and after maximum must be due to the combined variation of the mass transfer flux and of the amount of matter in the line of sight. For example, if the rate of mass transfer is decreased to zero (cf. Pringle 1973) the increase in X-ray flux could be caused by the clearing of the line of sight to the compact object, whereas the slow decline could be due to the evolution of the remaining accretion disc around the compact object (cf. Lynden-Bell \& Pringle 1974).

During the outburst the X-rays come from the compact object itself. If the compact object is a black hole, the flux comes solely from the accretion disc. If the compact object is a neutron star with a weak ( $\lesssim \mathrm{I}^{9} G$ ) magnetic field, comparable amounts of flux come from the disc and from the boundary layer at the disc/star interface. The contribution from the boundary layer is emitted at a higher temperature than that from the disc (Lynden-Bell \& Pringle 1974). If the compact object is a neutron star with a strong magnetic field, it is possible that the X-ray flux is significantly pulsed with a period corresponding to the rotation period of the
neutron star. If this pulsing were observed in a subsequent outburst of this source, it would be strong support for the model proposed here. In general, whatever the compact object, we expect the temperature to be high at X -ray maximum and to decrease with the X-ray flux.

If the X-ray eclipses were simply due to the occultation of the compact object by the white dwarf, the maximum duration of such eclipses would be

$$
t_{\mathrm{eclipse}}=\frac{R_{1}}{\pi a} \times P=59\left(\frac{m_{1}}{m_{1}+m_{2}}\right)^{1 / 3} \mathrm{~s} .
$$

We note that $t_{\text {eclipse }}$ is less than the integration time of the X-ray observations and that we would therefore not expect Ives et al. to have observed 'total' eclipses of the source.

The quasi-sinusoidal $X$-ray light curve of the source is reminiscent of the system Cygnus X-3. Theoretical models for Cygnus X-3 (Davidsen \& Ostriker 1974; Pringle 1974) indicate that the X-ray variation is caused by the variation with orbital phase of the line of sight optical depth to the X-ray component of the binary system. Such a variation can be caused by the presence of material in the system-for example a strong stellar wind (possibly excited by the presence of the X-ray source) emanating from the ' normal' star which is transferring matter. It seems probable that we are observing a similar effect in this system.

## 4. OTHER POSSIBILITIES

The two obvious other mechanisms for producing the observed $6.75-\mathrm{min}$ period are pulsation and rotation.
(i) Pulsation. To produce the required pulse period we require the average density of the star to be roughly $\sim\left(4 G P^{2}\right)^{-1} \sim 23 \mathrm{~g} \mathrm{~cm}^{-3}$. Thus the star will be too large to be able to produce X-rays of its own accord and must have a compact binary companion accreting a modulated amount of material from it. In order that this modulation not be smeared out in the mass transfer process we then also require the binary period to be comparable to $P$, and our original hypothesis follows.
(ii) Rotation. Accretion on to a magnetized star, rotating with a period of 6.75 min can produce the observed effect, $c f$. DQ Her (Bath, Evans \& Pringle 1974) and HZ Her (Blumenthal \& Tucker 1974). Since the accreting star must be in a binary system, the detection of an orbital period $\gg P$ in the X-ray flux would strengthen this hypothesis. Such a long ( 6.75 min ) rotation period may be somewhat improbable for a neutron star (see, however, Fabian 1975; Illarionov \& Sunyaev 1975). Confirmation of the suggested identification of Ariel in 18-16 with the Mira-type variable RS Cen (Fabian, Pringle \& Webbink 1975) would considerably strengthen the rotation hypothesis.

## 5. SUMMARY

We have considered the hypothesis that the $6 \cdot 75-\mathrm{min}$ period observed recently in a transient X-ray source is an orbital period. We deduce that the binary system must consist of a white dwarf of mass $0.12 m_{\odot}$, transferring matter because of gravitational radiation at an average rate of $\sim 10^{-7} m_{\odot} \mathrm{yr}^{-1}$ on to a compact companion. The appearance of a transient X-ray source was caused by a temporary reduction in this mass transfer rate and the assumption that this X-ray source then
radiates at a rate close to the Eddington limiting luminosity yields a distance of $\sim$ Io kpc for this source. For most of the time, however, the hard X-rays from the compact object are smothered by the transferred material, and although the source can be luminous ( $\sim 10^{38} \mathrm{erg} \mathrm{s}^{-1}$ ) it radiates mostly at soft X-ray or ultraviolet wavelengths. The high column density to the hard X-ray source (Ives et al. 1975) may make it difficult to observe the source at these wavelengths. If it is observable, we expect a $6.75-\mathrm{min}$ periodicity to be apparent. If the compact companion is a strongly magnetized neutron star there may be a periodicity with a time scale of seconds as well. Because such a strong ( $\sim \mathrm{IO}^{-7} m_{\odot} \mathrm{yr}^{-1}$ ) wind is emitted by the compact object (and, possibly, by the white dwarf as well) there is the possibility that the source could display a Cyg X-3 type variation in the infrared (Becklin et al. 1973; Davidsen \& Ostriker 1974; Pringle 1974). This may be the only means of detecting this source at non-X-ray wavelengths.

Finally we note that if the transient X-ray source discussed here is a short period binary system, it is the first binary system discovered in which the evolution of the system is undeniably wholly due to gravitational radiation.

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