Aristotle and the Mathematical Tradition on *diastēma* and *logos*: An Analysis of *Physics* 3.3, 202a18–21

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ARISTOTLE'S *PHYSICS* 3.3 contains interesting evidence of an open debate in mathematics, concerning the interchangeability of the notions of *diastēma* and *logos* in the theory of harmonics. Because of the standard interpretation of the passage, however, this reference to harmonics has gone unnoticed: a slightly different understanding is proposed in this paper, which restores the relevance of the passage and its place in the contemporary debate.

In §1 Aristotle's text is analysed. The context is that of motion: in order to justify the apparently counterfactual assertion that a single motion corresponds to two different objects, for it is the actuality of the movable (by the mover) but also of the mover (in the movable), Aristotle resorts to a simile: likewise a single *diastēma*, namely the interval between two notes, corresponds to two different *logoi*; both the ratio (*logos*) of the higher note to the lower and the ratio of the lower note to the higher can be used to describe the same interval.

The historical relevance of the *diastēma/logos* example is discussed in §2: although both concepts are employed in harmonics, the *diastēma* is a purely harmonic-theoretical concept, while the strictly connected notion of *logos* primarily pertains to proportion theory, so that the way of relating them is far from straightforward. In the so-called Pythagorean tradition, the notion of *diastēma* is not defined, but the interval between two notes is identified with their ratio (*logos*). The further unspoken convention of taking the notes in a precise order—the ratio is of the higher note to the lower and not *vice versa*—leads to a one-

to-one correspondence between *diastēma* and *logos*, leaving no room for ambiguities.

But this correspondence is a result of Pythagorean conventions, and different traditions rely on different assumptions, so much so that in the Aristoxenian theory, for instance, the two notions of *diastēma* and *logos* become completely separate. It is reasonable to assume that when a text from the Pythagorean tradition, where the two terms are used interchangeably, is read according to a more Aristoxenian perspective, or, more generally, according to any philosophical rather than strictly mathematical perspective, ambiguities like the one mentioned by Aristotle will arise.

The emergence of a discussion on this subject is particularly evident in the well-known passage about the harmony of the world-soul in the *Timaeus*: where Plato comfortably employs *diastēma* as a synonym for *logos*, interpreters like Theon or Eratosthenes feel uncomfortable, and try to assess the question within a harmonic-theoretical context, asking for example why the ratio of the higher note to the lower will be preferable to the ratio of the lower note to the higher in order to single out an interval.

In fact, there is no reason other than convention for preferring one *logos* to the other, and once the situation is clear, the ambiguity is perfectly acceptable, even if the acceptance needs some sort of discussion. My claim is that Aristotle's passage must be read in this perspective, as evidence of a wider debate, some parallel testimonies for which I list and discuss in §3.

1. *Aristotle's text*

Chapters 1–3 of *Physics* Book 3 deal with motion (*kinēsis*). At the very beginning motion is defined as "the actuality of what potentially is, *qua* such."[1] The definition is then restated as "the actuality of the movable, *qua* movable," and the long discussion that follows is concerned with the problems that this raises. In particular, the whole of 3.3 is devoted to illustrating the seem-

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[1] ἡ τοῦ δυνάμει ὄντος ἐντελέχεια, ᾗ τοιοῦτον (201a10–11).

ingly paradoxical consequence that motion is a shared actu-
ality: since 'movable' is a relative term (πρός τι) it cannot be
defined, *qua* movable, without its correlative term, namely the
mover. But if the movable is always a movable-by-a-mover,
motion is the actuality of the movable, but also of its mover.
But what does it mean that motion, while being a single ac-
tuality, is the actuality of two objects?

To make the statement acceptable to his audience, Aristotle
sets forth an analogy between motion and *diastēma* (202a18–21):[2]

> T1: ὥστε ὁμοίως μία ἡ ἀμφοῖν ἐνέργεια ὥσπερ τὸ αὐτὸ διά-
> στημα ἓν πρὸς δύο καὶ δύο πρὸς ἕν, καὶ τὸ ἄναντες καὶ τὸ
> κάταντες· ταῦτα γὰρ ἓν μέν ἐστιν, ὁ μέντοι λόγος οὐχ εἷς·
> ὁμοίως δὲ καὶ ἐπὶ τοῦ κινοῦντος καὶ κινουμένου.

> So that one is the actuality of both, similarly to the way in which
> it is the same interval "one to two" and "two to one," and the
> ascending and the descending: for they are one but the *logos* is
> not one: and similarly about the moving thing and the moved.[3]

In short, we have the following proportion (P):

> P)   *diastēma* : two *logoi* :: *kinēsis* : two *logoi*

The current interpretation of passage T1, ultimately trace-
able to Themistius,[4] reads *logos* as an abbreviation of "*logos* of

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[2] Unless otherwise stated, translations are Hussey's, slightly modified.

[3] Insofar as motion is concerned, Aristotle can speak indifferently of the
movable/mover relation or of the moved thing/moving one relation. For
when the movable (κινητόν, what potentially is moved) is in actuality, it is a
moved thing (κινούμενον: present participle, meaning it is actually moved),
but then its correlative term is no longer a mover (κινητικόν, what poten-
tially moves), but a moving thing (κινοῦν, that actually moves). As Aristotle
says at the beginning of 3.3, "the mover is such for its power, but is moving
for its actuality, and it is actualizing the movable" (202a16–18; cf. *Metaph*. Δ
15, 1021a14–21).

[4] Themistius' rendering of T1 is as follows: "The actuality of the moving
thing and the moved is one whenever the former moves and the latter is
moved, and they are one in their underlying subject, since in their definition
and in essence they are two. Compare uphill and downhill: in their case the
interval is the same but if you start from here it is uphill, from there down-
hill, and because of this it is two in definition. Likewise with motion, the

the essence" (ὁ λόγος ὁ τοῦ τί ἦν εἶναι), namely the definition, and opposes it to *diastēma*, namely the generic interval, or distance, between the two terms, understood as the substratum of the definition.[5] To sum up, the proportion may be read as follows:

P1)   *diastēma* (generic interval) : two *logoi* (definitions) ::
        kinēsis (actuality) : two *logoi* (definitions)

Undoubtedly this is an admissible reading, perfectly consistent with the conclusion Aristotle will reach at the end of his analysis: motion as a substratum is one, but it can be defined in two ways: as the actuality of the movable (*by* the mover) but also as the actuality of the mover (*in* the movable) (202b19–22):

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actuality of the moving thing and the moved one is one with respect to the underlying subject": μία μὲν ἀμφοῖν ἡ ἐνέργεια τοῦ κινοῦντος καὶ τοῦ κινουμένου, ὅταν τὸ μὲν κινῇ τὸ δὲ κινῆται, μία δὲ τῷ ὑποκειμένῳ. τῷ γὰρ λόγῳ δύο καὶ τὸ τί ἦν εἶναι, ὥσπερ τὸ ἄναντες καὶ τὸ κάταντες. ὡς γὰρ ἐπὶ τούτων τὸ μὲν αὐτὸ διάστημα, ἀλλ᾽ ἔνθεν μὲν ἀρχομένῳ ἄναντες, ἔνθεν δὲ κάταντες, καὶ διὰ τοῦτο τῷ λόγῳ δύο· οὕτως καὶ ἐπὶ τῆς κινήσεως κατὰ μὲν τὸ ὑποκείμενόν ἐστι μία ἡ ἐνέργεια τοῦ κινοῦντος καὶ τοῦ κινουμένου (Themist. *In Phys.* p.76.15–21 Schenkl; transl. Todd, modified). In turn, Themistius' reading seems to follow the Aristotelian compiler of *Metaphysics* K, who in his simplified version of the passage directly replaces λόγος with εἶναι (*Metaph.* K 9, 1066a31–34; on the question of the inauthenticity of Book K see for example P. Aubenque, "Sur l'inauthenticité du livre K de la Métaphysique," in P. Moraux and J. Wiesner [eds.], *Zweifelhaftes im Corpus Aristotelicum: Studien zu einigen Dubia* [Berlin/New York 1983] 318–344.

[5] In the most recent editions, we find the following translations: "As the same interval is the interval between one and two and between two and one. It is a case of there being one thing definable in two ways" (W. D. Ross, *Aristotle, Physics* [Oxford 1936] 361); "As it is the same interval from 1 to 2 and from 2 to 1, and as the uphill and the downhill. These are one, yet the definition is not one" (E. Hussey, *Aristotle Physics. Books III and IV* [Oxford 1983] 4); "Just as there is the same interval from 1 to 2 as there is from 2 to 1, just as uphill and downhill are identical (for these things are the same, although their definitions are different)" (R. Waterfield, *Aristotle, Physics* [Oxford 1996] 60); "à la manière dont c'est le même intervalle qu'il y a entre un et deux et entre deux et un, et à la manière de la montée et de la descente. Car toutes ces choses sont une, bien que leurs définitions ne soient pas une" (P. Pellegrin, *Aristote, Physique* [Paris 2000] 167–168).

T2:   ὅλως δ’ εἰπεῖν οὐδ’ ἡ δίδαξις τῇ μαθήσει οὐδ’ ἡ ποίησις
τῇ παθήσει τὸ αὐτὸ κυρίως, ἀλλ’ ᾧ ὑπάρχει ταῦτα, ἡ κίνησις· τὸ
γὰρ τοῦδε ἐν τῷδε καὶ τὸ τοῦδε ὑπὸ τοῦδε ἐνέργειαν εἶναι ἕτε-
ρον τῷ λόγῳ.

Generally speaking, teaching is not the same as learning, in the
primary sense, nor acting as suffering, but that to which they be-
long, namely motion. To be the actuality of this one in this one,
and to be the actuality of this one by this one is different in *logos*.

However, this is not the only admissible reading of T1. First of
all, it is not only consistent but actually coincident with the
conclusion T2, where it is reasonable to expect that T1 con-
tains more information: indeed, within the framework of chap-
ter 3.3, T1 does not play the role of a simple anticipation of the
conclusion, but is intended as a first step in the argument that
will lead to the conclusion T2 itself.

As Aristotle makes clear, motion can be the act of two differ-
ent "things" only because these "things" are relations:[6] motion
is the act of the "movable *by* the mover" but also of the "mover
in the movable." To understand not only that we have two
different definitions, but also that these definitions are con-
nected with two different relations, is indispensable for un-
derstanding Aristotle's argument. Indeed, Aristotle not only
explains why the movable and the mover can share the same
actuality, but also why this does not mean that the mover, and
more generally the acting object, also suffers the same passion
that it acts, when it acts. Both conclusions are reached by re-
sorting to the properties of relatives.

[6] Aristotle never speaks in term of relations, but only of properties like
being relative-to-something (πρός τι). In this way he makes clear that no ad-
ditional entities must be introduced beyond the relative and its correlative.
For the sake of simplicity and in accordance with current practice, I will
notwithstanding use the noun "relation" and the corresponding adjective
"relational." By saying that the movable and the mover are in relation, or
that there is a relation between the movable and the mover, I simply com-
bine the two assertions that the movable is a relative to a mover and a
mover is a relative to a movable.

For this reason, I suggest first of all recovering the interpretation of those commentators who, like Simplicius or Philoponus, maintain the polysemy of *logos*, which in T1 does mean both definition and ratio.[7] In addition, I propose keeping in mind that the definitions at issue are definitions expressed in terms of relatives, and finally I suggest identifying the particular *diastēma*

[7] See in particular Simplicius: "As the interval from one 'to two and from two to one is one and upwards and downwards' are one in substrate, but two in definition, the ratio from one to two being a half and from two to one double, so is the relation of change in the case of the changer and the changed, since they are one in substrate, but is viewed one way in the changer, another way in the changed," ὡς οὖν τὸ διάστημα τοῦ ἑνὸς "πρὸς δύο καὶ δύο πρὸς ἓν καὶ τὸ ἄναντες καὶ τὸ κάταντες" ἓν μέν ἐστι τῷ ὑποκειμένῳ, δύο δὲ κατὰ τὸν λόγον, ἀπὸ μὲν τοῦ ἑνὸς πρὸς τὰ δύο ἥμισυ, ἀπὸ δὲ τῶν δύο πρὸς τὸ ἓν διπλάσιον, οὕτως ἔχει καὶ ἐπὶ τοῦ κινοῦντος καὶ κινουμένου ἡ κίνησις, μία μὲν οὖσα τῷ ὑποκειμένῳ, ἄλλως δὲ ὡς ἀπὸ τοῦ κινοῦντος καὶ ἄλλως ὡς ἐν τῷ κινουμένῳ θεωρουμένη (*In Phys.* pp.439.33–440.1 Diels; transl. Urmson; cf. p.439.5–8); and Philoponus: "That it is not surprising for there to be one and the same actuation of two things which differ only in relation he confirms by the examples. 'Just as', he says, 'there is the same interval between one and two as between two and one'. For this is one and the same thing in respect of the substratum. For what is the difference between a change from two to one and a change from one to two? That is, to say what ratio two has to one does not differ from saying what ratio one has to two, excepting only in the relation, while as to the substratum they are one and the same. But if there is one interval, it is the relations then that differ. For two has to one the ratio of being double, while one has to two the ratio of being half. Likewise, the upward and downward are different relations, while the distance is single," καὶ ὅτι οὐ θαυμαστὸν μίαν καὶ τὴν αὐτὴν ἐνέργειαν δύο εἶναι πραγμάτων τῇ σχέσει μόνον διαφέρουσαν, τοῖς παραδείγμασι πιστοῦται. "ὥσπερ," φησί, "τὸ αὐτὸ διάστημα ἓν πρὸς δύο καὶ δύο πρὸς ἕν"· ἓν γὰρ τοῦτο καὶ ταὐτὸν κατὰ τὸ ὑποκείμενον. τί γὰρ διαφέρει ἐκ τῶν δύο πρὸς τὸ ἓν κινηθῆναι, καὶ ἐκ τοῦ ἑνὸς πρὸς τὰ δύο; οἷον τὸ λέγειν ὅτι ὃν λόγον ἔχει τὰ δύο πρὸς τὸ ἕν, οὐδὲν διαφέρει τοῦ εἰπεῖν ὃν λόγον ἔχει τὸ ἓν πρὸς τὰ δύο, εἰ μὴ μόνον τῇ σχέσει, τῷ ὑποκειμένῳ δὲ ταὐτὸν καὶ ἕν. ἀλλ' εἰ καὶ μία ἡ διάστασις, ἀλλ' οὖν αἱ σχέσεις διάφοροι· τὰ μὲν γὰρ δύο πρὸς τὸ ἓν διπλασίονα λόγον ἔχει, τὸ δὲ ἓν πρὸς τὰ δύο τὸν ἥμισυν. ὁμοίως δὲ καὶ τὸ ἄναντες καὶ τὸ κάταντες σχέσεις μὲν διάφοροι, διάστασις δὲ μία (*In Phys.* pp.375.25–376.5 Vitelli; transl. Edwards).

that Aristotle is referring to with the harmonic interval, avoiding to equate it with a generic substratum.

To sum up, I suggest summarizing P in the following way:

P2) *diastēma* (musical interval) : two *logoi* (numerical ratios) ::
     kinēsis (actuality) : two *logoi* (definitions in term of relatives)

On the left horn *diastēma* is the musical interval and the two *logoi* are the two possible ratios between the values of the two terms, which define the *diastēma*: namely the pitches of a higher and a lower note, one *pros* two and two *pros* one. It is worth noting that it is not by chance that this particular example was chosen, for the interval two-one is the interval of an octave, or diapason, namely the most perfect accord.[8] On the right horn *kinēsis* is the actuality and the two *logoi* are the two ways of relating the two terms, which define the *kinēsis*: namely the mover *pros* the movable, and the movable *pros* the mover (or—which amounts to the same thing—the moving thing *pros* the moved one and the moved thing *pros* the moving one).

Of course, this reading too is compatible with Aristotle's conclusion T2, although it does not coincide with it: in this case the conclusion is not immediate, but is reached in two steps. The first step is to set the philosophical opposition between *kinēsis* and *logos* in analogy with the more handleable mathematical opposition between *diastēma* and *logos*. The second step is to observe that both the mathematical *logos* of the last example and the philosophical *logos* of the first one are in fact *logoi* also as far as the definition is concerned.[9]

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[8] Even those commentators who, like Simplicius and Philoponus, recognise the ambiguity of *logos*, fail to recognise the harmonic content of the example (see n.7): to them one and two have nothing to do with any pitch, but are simply the end points of a generic *diastēma*, in the sense of spatial distance. On the Aristotelian tradition of reading of T1 see M. Ugaglia, "Alcune osservazioni sull'uso di λόγος in Aristotele," in F. Franco Repellini and G. Micheli (eds.), *La scienza antica e la sua tradizione* (Milan 2011) 81–117.

[9] *Phys*. 2.3, 194b26–29: "According to another 'way of speaking' the form or model is a cause, and this is the definition (the *logos* of the essence, ὁ λόγος ὁ τοῦ τί ἦν εἶναι) and its genera, thus the cause of an octave is the two

Just as two ratios give rise to the same musical interval—for example the quoted interval of an octave can be defined by either the ratio "two *pros* one" or the ratio "one *pros* two"—so two ways of putting the movable and the mover in relation give rise to the same motion. Indeed, motion can be defined either in terms of the moved *pros* the moving thing or of the moving thing *pros* the moved one, depending on whether one says that it is the actuality of the "movable *by the* mover" or that it is the actuality of the "mover *in the* movable." As requested for proving that the mover does not suffer the same passion that it acts, when it acts, this happens without the two relations necessarily coinciding: for just as it is not the same to state that "one is *half of* two" and that "two is the *double of* one," it is not the same to state that "the moved *is being moved by the* moving" and that "the moving *is moving the* moved." However—and this is the crucial point—the two statements are not independent. On the contrary, they entail each other: if the one holds, the other necessarily holds as well, and it is impossible to think of them separately.

In this case the terms ἄναντες and κάταντες too acquire a quasi-technical meaning in T1: aside from their more evident affinity with Heraclitus' roads,[10] they find a natural—albeit less philosophically evocative—counterpart in the notions of ascending and descending intervals in harmonic theory.[11]

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to one (τὰ δύο πρὸς ἕν), and more generally number."

[10] Heraclitus 22 в 60 D.-K. Cf. Aristotle's reference to the road from Thebes to Athens, set in contrast to the road from Athens to Thebes, in *Phys*. 3.3, 202b10–16.

[11] See in particular Aristoxenus *Harm.*, 10.11–2 Meibom.: "Stretching is the continuous motion of the voice from a deeper position to a higher one, tightening from a higher position to a deeper one," ἡ μὲν οὖν ἐπίτασίς ἐστι κίνησις τῆς φωνῆς συνεχὴς ἐκ βαρυτέρου τόπου εἰς ὀξύτερον, ἡ δ' ἄνεσις ἐξ ὀξυτέρου τόπου εἰς βαρύτερον. The terminology was traditional, the ascending motion being called ἐπίτασις, which literally denotes stretching (usually that of a string), the descending motion being called ἄνεσις, literally loosening. Aristoxenus redefined these terms, freeing them from any mechanical connotation.

A reason for preferring the second reading is that it does justice to Aristotle's philosophical argument, restoring the original complex articulation of the analogy, impoverished by the standard interpretation. In the standard interpretation, in order to clarify the way in which a single subject, namely motion, has two possible definitions, Aristotle would have simply produced an additional example, the one of *diastēma*, standing on the same footing as the one of movement. In other words, we have nothing but two perfectly equivalent instances of the same relationship, and one may wonder why Aristotle thinks that it would be simpler to understand that the *logoi* (= definitions) of a single interval can be two different ones than to posit that the *logoi* (= definitions) of a single state of motion can be two different ones.

On the contrary, in the interpretation I suggest the parallel case of the *diastēma* has a stronger cognitive function: it moves onto a new level, introducing a technical mathematical simile, which is actually different, but also—and most importantly—simpler, than the philosophical counterpart that it is intended to clarify.

Indeed, Aristotle frequently resorts to mathematics when he needs to illustrate a particularly difficult philosophical point, as though mathematics were an unproblematic realm that can help clarify more complex states of affair.[12] In this case, in addition to Aristotle's general penchant for mathematical sim-

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[12] See for instance the use of the construction of the right angle in a semicircle in order to explain the procedure of knowledge in *Metaph.* Θ 9 (1051a27–29) and the notion of *epagoge* in *An.Post.* 1.1 (71a17–b5), or the mention of the line cutting a plane figure parallel to its side, used for clarifying the role of the middle term as a "reason" in a syllogism in *Top.* 8.3 (158b29–34). More generally, some twenty-odd mathematical examples can be found in *Metaphysics* (excluding the "mathematical" books **M** and **N**), roughly the same number in *Physics*, a dozen or so in *De anima*, and a few in the biological works. For an analysis of Aristotle's philosophical use of mathematical examples see M. Ugaglia, "Knowing by Doing: the Role of Geometrical Practice in Aristotle's Theory of Knowledge," *Elenchos* 36 (2015) 45–88.

iles, the choice of the specific example has a particular *raison d'être*, for the opposition *diastēma/logos* explicitly refers to a well-known lexical controversy in ancient Greek mathematics, which presumably would have been familiar to an educated reader in Aristotle's time.

As we show in the next section, the extant Greek sources on harmonic theory bear witness to the emergence of a terminological discussion concerning the notion of *diastēma*, a purely harmonic-theoretical notion, and the strictly connected notion of *logos*, which primarily pertains to proportion theory. In short, the question concerns the equivalence of the two concepts, which is warranted in the so-called Pythagorean tradition but denied in the Aristoxenian one.

Aristotle's statements, chronologically intermediate between these two poles, testify to the difficulties connected with the confusion which inevitably goes with the existence of such opposite points of view, and of the discussions accompanying it.

2. *Harmonic background*

It is customary to gather under the improper label of 'Pythagoreans' a wide family of Greek musical theorists who, though belonging to different philosophical schools and periods, share a mathematical approach which can in some way be traced back to the work of Pythagoreans like Archytas and Philolaus.[13] According to the Pythagorean tradition, harmonics is not a self-standing discipline, and its objects of study—namely, the elements out of which a melody is composed—are not independent entities. More specifically, sounds are viewed as the result of particular motions of physical instruments, so that notes, which are sounds with a particular pitch, are as-

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[13] On this basis, the musical theories that are implicitly or explicitly in the background of the expositions of Plato, Theon of Smyrna, Nicomachus, Ptolemy, Aristides Quintilianus, and also Euclid's *Sectio Canonis* are conveniently labelled as 'Pythagorean'. On this question see, for example, the introduction and accompanying bibliography in A. Barker, *Greek Musical Writings* II *Harmonic and Acoustic Theory* (Cambridge 1989).

sociated with the physical properties of the instrument (the diameter, length or tension of the strings). In their turn, physical properties can be quantified and associated with arithmetical objects, so that any harmonic notion can be read in mathematical terms. In particular, the intervals between notes, understood as the relation between the corresponding pitches, are represented by the ratios between their numerical values: an interval is a consonance when it is represented by a ratio of simple numbers.[14]

This ultimate reduction to arithmetical terms makes it unnecessary to define harmonic notions in an independent way. In particular, we can note the absence of a Pythagorean definition of interval, the term *diastēma* being employed merely as the harmonic equivalent of the arithmetical *logos* (= ratio).[15] By convention, the ratio is always the ratio of the higher note to the lower one, and not *vice versa*.

In contrast, the Aristoxenian tradition considers harmonics an independent discipline, grounded on sensory data and accountable in terms of purely harmonic entities.[16] Although the elements forming a melody are the objects of study of harmonics, they are not the basic entities of the theory, the main

[14] A systematic account of such an approach is presented, for example, in the opening sections of the first book of Ptolemy's *Harmonics*.

[15] In the Euclidean *Sectio Canonis*, for example, which chiefly deals with harmonic ratios, we find only six occurrences of the term *logos*, the harmonic ratios always being designated by the term *diastēma*. Leaving aside the three occurrences of *logos* in the Introduction, whose authenticity is doubtful, and the one at the end of Prop. 12, where it is part of a sentence whose spurious character is evident, the remaining two occurrences are in Prop. 3, and they are induced by a change in the paradigm of reference. The second occurrence is contained in a verbatim quotation of *Elements* 8.8 —i.e., a proposition from the arithmetic books (a fact that should in any case raise some doubts as to the authenticity of the quotation): as a consequence, one must resort to the terminology of arithmetic. The first occurrence paves the way to the second.

[16] See in particular the beginning of Book 2 of Aristox. *Elem.Harm.*, pp. 32.31–34.32 Meibom.

object of interest being the so-called musical motion. This is a kind of Aristotelian local *kinēsis*, where the contrary places between which motion arises are not up and down, but high and low (pitch). In the case of voice, for instance, the subject of motion makes a continuous motion in speaking, but a discontinuous one in singing. During this discontinuous motion, which Aristoxenus calls διαστηματική, "intervallic,"[17] the voice again and again breaks this process of ascending or descending motion by settling on a note, which is then read as a point of discontinuity of motion. In such a picture, an interval is that which separates two notes. As a consequence, it is a part of motion, in the same way as, for Aristotle, what separates two 'nows' in time is still a part of time.[18] It is clear that within this perspective there is no need to relate intervals to any kind of ratio;[19] therefore, an explicit definition of interval must be provided. Aristoxenus defines it as "what is determined by two notes not having the same tension."[20] Similar definitions are offered in later musical writings.[21]

In sum, there is a crucial difference between the Pythagorean point of view and the Aristoxenian: the tacit convention of reading a musical *diastēma* as an ordered relation builds a one-to-one correspondence between the notions of *diastēma* and *logos*, so that although the two terms apply to two different theoretical fields, in a purely Pythagorean context there is no room for ambiguities.

[17] *Elem.Harm.* pp.8.14–10.10 Meibom.

[18] See for example *Phys.* 4.10 (218a16–21), 11 (219a22–30).

[19] In fact, the notes themselves, which as dimensionless points are devoid of any quality, cannot be considered in any reciprocal relation, the interval itself aside. On this point see for instance Ptolemy's critique in *Harm.* 1.9 (20.3–9 Düring).

[20] τὸ ὑπὸ φθόγγων ὡρισμένον μὴ τὴν αὐτὴν τάσιν ἐχόντων (*Elem.Harm.* p. 15.22–23 Meibom).

[21] See Cleonides *Harm.* p.179.11–12, Nicomachus *Harm.* p.261.8, Bacchius *Harm.* p.292.20–21, Gaudentius *Harm.* p.329.23–24 (page numbers of C. Jan, *Musici scriptores graeci* [Leipzig 1899]).

On the contrary, no mention of any privileged direction is to be found in the Aristoxenian definition of *diastēma* as "what is determined by two notes." In this perspective, any attempt to relate a *diastēma* to a *logos* necessarily gives rise to ambiguities: even assuming that a *logos* can be associated with the two notes defining an interval, why would the *logos* associated with the relation of the higher note to the lower be preferable to the *logos* associated with the relation of the lower note to the higher?

As a consequence of all of this, ambiguities may arise in texts such as those of the Pythagorean tradition, where *diastēma* and *logos* are used interchangeably, if the two terms are read in an Aristoxenian perspective, or, more generally, in any philosophical rather than strictly mathematical perspective, thereby breaking the one-to-one correspondence between *diastēma* and *logos*. The ambiguity arising from an Aristoxenian reading of Pythagorean texts is exactly what Aristotle alludes to in T1.

3. *Parallel texts*

The remark in T1 can be found, expressed in the very same terms, in other passages in the Greek musical corpus.

In the first passage, extracted from Nicomachus' *Enchiridion*, we find the example of the one-two interval, as in Aristotle's T1, together with an attempt to clarify the *diastēma*/*logos* ambiguity by referring it to the διαφορά/σχέσις pair, where a *diaphora* is defined as the excess or defect of two notes, and a *schesis*, or *logos metrētikos*, as its (double) measure (*Harm.Ench.* 12 [p.261.8–17 Jan]):

> T3: διάστημα δ' ἐστὶ δυοῖν φθόγγων μεταξύτης. σχέσις δὲ λόγος ἐν ἑκάστῳ διαστήματι μετρητικὸς τῆς ἀποστάσεως· διαφορὰ δὲ ὑπερβολὴ ἢ ἔλλειψις φθόγγων πρὸς ἀλλήλους. κακῶς γὰρ οἴονται οἱ νομίζοντες διαφορὰν καὶ σχέσιν τὸ αὐτὸ εἶναι. ἰδοὺ γὰρ τὰ δύο πρὸς τὸ ἓν διαφορὰν μὲν ἔχει τὴν αὐτήν, ἣν ἓν πρὸς δύο, σχέσιν δὲ οὐ τὴν αὐτήν. τὰ μὲν γὰρ δύο διπλάσια, τὸ δὲ ἓν ἥμισυ. καὶ πάλιν ἐν πᾶσιν ἀριθμητικῆς μεσότητος ὅροις τρισὶν ἢ καὶ πλείοσι διαφορὰ μὲν ἡ αὐτὴ ἐν πᾶσι, σχέσις δὲ ἄλλη καὶ ἄλλη.

An interval is what lies between two notes. A relation is the ratio which measures the distance in any interval: and a difference is the excess or deficiency of one note with respect to another. Those who think that relation and difference are the same are wrong: for as you will understand, the difference between 2 and 1 is the same as that between 1 and 2, whereas their relation is not the same. For 2 is double 1, while 1 is half of 2. Or again, the difference between three or more terms in arithmetic proportion is the same in each case, but the relation is different in each. (transl. Barker)

The same example of the two-one interval can be found in the *Expositio rerum mathematicarum ad legendum Platonem utilium* by Theon of Smyrna,[22] where the notions of *diastēma* and *logos* are contrasted (*Exp.* p.81.6–16 Hiller):

T4:   διαφέρει δὲ διάστημα καὶ λόγος, ἐπειδὴ διάστημα μέν ἐστι τὸ μεταξὺ τῶν ὁμογενῶν τε καὶ ἀνίσων ὅρων, λόγος δὲ ἁπλῶς ἡ τῶν ὁμογενῶν ὅρων πρὸς ἀλλήλους σχέσις. διὸ καὶ τῶν ἴσων ὅρων διάστημα μὲν οὐδέν ἐστι μεταξύ, λόγος δὲ πρὸς ἀλλήλους εἷς καὶ ὁ αὐτὸς ὁ τῆς ἰσότητος· τῶν δὲ ἀνίσων διάστημα μὲν ἓν καὶ τὸ αὐτὸ ἀφ᾽ ἑκατέρου πρὸς ἑκάτερον, λόγος δὲ ἕτερος καὶ ἐναντίος ἑκατέρου πρὸς ἑκάτερον· οἷον ἀπὸ τῶν β′ πρὸς τὸ ἓν καὶ ἀπὸ τοῦ ἑνὸς πρὸς τὰ β′ διάστημα ἓν καὶ τὸ αὐτό, λόγος δὲ ἕτερος, τῶν μὲν δύο πρὸς τὸ ἓν διπλάσιος, τοῦ δὲ ἑνὸς πρὸς τὰ β′ ἥμισυς.

An interval and a ratio are different, for an interval is what lies between homogeneous and unequal terms, while a ratio is quite generally the reciprocal relation of homogeneous terms. For this reason, while there is no interval between equal terms, their reciprocal ratio is one and the same, namely, that of equality; on the other hand, the interval between unequal terms, from any of them to the other, is one and the same, but the ratio of any of them to the other is different and the opposite: for example, from 2 to 1 and from 1 to 2 there is one and the same interval, while the ratio is different: double that of 2 to 1, one half that of

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[22] On the sources of the passage see the Introduction in F. M. Petrucci, *Teone di Smirne: Expositio rerum mathematicarum ad legendum Platonem utilium* (Sankt Augustin 2012), with a detailed analysis of the passage at 388–389.

the 1 to 2.

Here the context is more general, but the fact that the two-one example comes from harmonics is suggested by the possible origin of Theon's analysis. Some very similar remarks are repeated a few lines later, but now they are explicitly ascribed to Eratosthenes (*Exp.* pp.81.17–82.5): [23]

T5: Ἐρατοσθένης δὲ ἐν τῷ Πλατωνικῷ φησι, μὴ ταὐτὸν εἶναι διάστημα καὶ λόγον, ἐπειδὴ λόγος μέν ἐστι δύο μεγεθῶν ἡ πρὸς ἄλληλα ποιὰ σχέσις· γίνεται δ' αὕτη καὶ ἐν διαφόροις <καὶ ἐν ἀδιαφόροις>.[24] οἷον ἐν ᾧ λόγῳ ἐστὶ τὸ αἰσθητὸν πρὸς τὸ νοητόν, ἐν τούτῳ δόξα πρὸς ἐπιστήμην, καὶ διαφέρει καὶ τὸ νοητὸν τοῦ ἐπιστητοῦ ᾧ καὶ ἡ δόξα τοῦ αἰσθητοῦ. διάστημα δὲ ἐν διαφέρουσι μόνον, ἢ κατὰ τὸ μέγεθος ἢ κατὰ ποιότητα ἢ κατὰ θέσιν ἢ ἄλλως ὁπωσοῦν. δῆλον δὲ καὶ ἐντεῦθεν, ὅτι λόγος διαστήματος ἕτερον· τὸ γὰρ ἥμισυ πρὸς τὸ διπλάσιον <καὶ τὸ διπλάσιον πρὸς τὸ ἥμισυ>[25] λόγον μὲν οὐ τὸν αὐτὸν ἔχει, διάστημα δὲ τὸ αὐτό.

In his *Platonicus*, Eratosthenes says that an interval and a ratio are not the same, for a ratio is a qualified reciprocal relation between two magnitudes. It comes to be between different <and similar terms>; for instance, the sensible is to the intelligible in the same ratio as opinion is to science, and the intelligible differs from the knowable as opinion differs from the sensible. An interval, instead, is given only between terms that are different either in magnitude, in quality, in position, or in any other way. It is clear that a ratio is also different from an interval from this point of view: the half to its double <and the double to its half> do not have the same ratio, but do have the same interval.

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[23] This is one of the only two surviving passages containing an explicit reference to the Πλατωνικός, and it has proven crucial for the attempt to reconstruct the contents of that lost work of Eratosthenes. The other passage, again in Theon of Smyrna (*Exp.* p.2.3–12), concerns the problem of the duplication of the cube. On these passages and on the content of Eratosthenes' work see for example B. Vitrac, "Eratosthène et la théorie des médiétés," in Ch. Cusset and H. Frangoulis (eds.), *Eratosthène: un athlète du savoir* (Saint-Etienne 2008) 77–103.

[24] *Add.* Hiller; ἀδιαφόροις *perperam corr.* Düring.

[25] *Add.* Hiller.

Without going into the general problem of the reliability of Theon as a witness to Eratosthenes' mathematical achievements, one must observe that the compilatory nature of the *Expositio*, together with its philosophical rather than mathematical aims, generally does not allow us to precisely distinguish Eratosthenes' statements from material coming from other sources. In some cases, however, it is possible to clarify the situation by comparing Theon's report and analogous accounts by other authors. Concerning the passage at issue, a happy coincidence provides a parallel text dealing with the same topic and referring to the same examples. The argument, now framed within a harmonic context, is explicitly ascribed to Eratosthenes by Porphyry in his *Commentary on Ptolemy's Harmonics* (*In Harm.* pp.90.24–91.9 and 94.2–4 Düring):

T6: "λόγος δὲ λέγεται δύο μεγεθῶν ὁμογενῶν ἡ κατὰ πηλικό-τητα ποιὰ σχέσις," κατὰ δὲ τοὺς Ἀριστοξενείους "τὸ περιεχό-μενον ὑπὸ δύο φθόγγων ἀνομοίων τῇ τάσει"· καὶ ἄλλοι ἄλλως ἐδόξασαν περὶ τοῦ διαστήματος. Ἐρατοσθένης μὲν οὖν φησιν ἕτερον εἶναι διάστημα λόγου· ἐν γὰρ ἐνὶ διαστήματι δύο λόγοι γίνονται. ὁ δὲ λόγος δὶς φέρεται, ὅ τε τοῦ μείζονος πρὸς τὸ ἔλαττον καὶ τοῦ ἐλάττονος πρὸς τὸ μεῖζον καὶ κοινὴ διαφορὰ ὑπεροχῆς καὶ ἐλλείψεως ὡς τῆς διαφορᾶς δηλονότι τὸ διάστημα ποιούσης. διπλασίου τε γάρ φησι πρὸς ἥμισυ καὶ ἡμίσεος πρὸς διπλάσιον ὁ μὲν λόγος ἕτερος, τὸ αὐτὸ δὲ διάστημα […] ὅτι μὲν τοίνυν ὁ λόγος ἐν διαφόροις γίνεται ὅροις, ὁμογενέσι δὲ πάντως, καὶ ἐν ἀδιαφόροις, ὡς Εὐκλείδῃ δοκεῖ, δειχθήσεται· διάστημα δ' ἐν τοῖς διαφέρουσι μόνον, φανερόν.

"Ratio is said to be a qualified relation in respect of size between two homogeneous magnitudes," and following the Aristoxenians "what is contained by two notes dissimilar in pitch"; others have held different opinions on intervals. Eratosthenes says that an interval is different from a ratio, for in a single interval two ratios come to be. Ratio presents itself in two ways: that of the greater to the smaller and that of the smaller to the greater, and the interval of excess and deficiency is common, for, obviously, it is the difference that makes the interval. Indeed, he says that the ratio of the double to its half and of the half to its double is different, but the interval is the same. […] It will be shown that a ratio comes to be between different yet still totally homo-

geneous terms, or between similar terms, as Euclid thinks. It is clear, instead, that an interval comes to be only between different terms.

What clearly emerges here is Eratosthenes' intention to stress the non-coincidence between the notions of *diastēma* and *logos*. As to the meaning of *logos*, both Theon and Porphyry have no doubts: insomuch as they make Eratosthenes quote the definition in terms of the *schesis* found in Euclid *Elem.* 5.def.3, a *logos* is a sort of relation. As for the *diastēma*, whether it is understood as an excess or a difference, as that which separates two terms or as that which puts them in relation,[26] the problem is that a single *diastēma* corresponds to two *logoi*, or to two ways of reading it, and this is what Eratosthenes, like Aristotle before him, points out.

The main difference is that while Aristotle accepts the ambiguity and simply turns the one-to-one correspondence into a one-to-two correspondence—the same *diastēma* corresponds to two *logoi*, which both can be used to define it—Theon, Nicomachus, and Eratosthenes are more interested in the negative side of the ambiguity, that is, in the fact that the two terms refer to two distinct objects. Aristoxenus will carry the situation to the extreme consequence of regarding only one of them, the *diastēma*, as a true harmonic notion.

4. *Conclusions: Plato's original sin*

According to Porphyry, who claims to be following here the criticism of "Demetrius and Panaetius the mathematicians,"[27] many authors too hastily conflated the meanings of the terms *diastēma* and *logos*. Besides the Pythagoreans in general, he lists

[26] According to Porphyry (*In Harm.* pp.91.10–92.13), Eratosthenes did not give an explicit definition of *diastēma*, even though he set it in contrast to *logos*. It is perhaps for this reason that interpreters, both before and after him, defined the interval as an excess (ὑπεροχή), or a difference between notes, as did Philolaus and Thrasyllus.

[27] *In Harm.* p.92.19–21.

Plato,[28] and in particular Plato's discussion about the division of the world-soul by means of harmonic intervals (*Ti.* 35C–36B):

T7:   μετὰ δὲ ταῦτα συνεπληροῦτο τά τε διπλάσια καὶ τριπλά-
σια διαστήματα, μοίρας ἔτι ἐκεῖθεν ἀποτέμνων καὶ τιθεὶς εἰς τὸ
μεταξὺ τούτων, ὥστε ἐν ἑκάστῳ διαστήματι δύο εἶναι μεσότη-
τας, τὴν μὲν ταὐτῷ μέρει τῶν ἄκρων αὐτῶν ὑπερέχουσαν καὶ
ὑπερεχομένην, τὴν δὲ ἴσῳ μὲν κατ' ἀριθμὸν ὑπερέχουσαν, ἴσῳ
δὲ ὑπερεχομένην. ἡμιολίων δὲ διαστάσεων καὶ ἐπιτρίτων καὶ
ἐπογδόων γενομένων ἐκ τούτων τῶν δεσμῶν ἐν ταῖς πρόσθεν
διαστάσεσιν, τῷ τοῦ ἐπογδόου διαστήματι τὰ ἐπίτριτα πάντα
συνεπληροῦτο, λείπων αὐτῶν ἑκάστου μόριον, τῆς τοῦ μορίου
ταύτης διαστάσεως λειφθείσης ἀριθμοῦ πρὸς ἀριθμὸν ἐχούσης
τοὺς ὅρους ἓξ καὶ πεντήκοντα καὶ διακοσίων πρὸς τρία καὶ τετ-
τάρακοντα καὶ διακόσια.

After this he went on to fill the double and triple intervals by cutting off still more portions from the mixture and placing these between them, in such a way that in each interval there were two middle terms, one exceeding the first extreme by the same fraction of the extremes by which it was exceeded by the second, and the other exceeding the first extreme by a number equal to that by which it was exceeded by the second. These connections produced intervals of 3/2, 4/3, and 9/8 within the previous intervals. He then proceeded to fill all the 4/3 intervals with the 9/8 interval, leaving a small portion over every time. The terms of this interval of the portion left over are as the numbers 256/243. (transl. Zeyl, modified)

If one reads Plato's words in a purely Pythagorean perspective, the text does not leave any room for misinterpretation: the interval between two terms is the ratio of the greater to the lesser: the hemiolic interval is the ratio of 3 to 2, the epitritic of 4 to 3 and so on, until the minimal interval, or lemma, which is

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[28] *In Harm.* p.92.13–18. The other quoted authors are Dionysius of Halicarnassus (pp.92.28 and 94.25), Archytas (92.29, 93.4–19, 94.24), Euclid (92.29–93.4, 94.25), and Eratosthenes himself (92.24–25). On Porphyry's passage and his sources see M. Raffa, "The Debate on *logos* and *diastēma* in Porphyry's *Commentary* on Ptolemy's *Harmonics*," *Greek and Roman Musical Studies* 1 (2013) 243–252.

the ratio of 256 to 243. On the other hand, texts like T4–6 show a certain unease on the part of the interpreters, who try to eliminate an ambiguity which they evidently perceive.

Something has changed, between Plato's writing and Eratosthenes' reading: something that has to do with the emergence of a different way to approaching harmonic notions. Eratosthenes' testimony, just like Aristotle's before it,[29] bears witness to the strengthening of this approach, which will eventually take the concrete shape of the Aristoxenian theory of music.

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[29] The hypothesis that Eratosthenes' exposition of the problem depends on Aristotle, as the extreme similarity between the texts suggests, is corroborated by another fact. Theon's quotation is part of a longer text extracted from Eratosthenes' Πλατωνικός. In addition to the quotation, the text contains another terminological discussion (p.82.6–21 Hiller), aimed at avoiding the confusion between the notions of ἀναλογία (proportion) and μεσότης (mean): as in the case of *diastēma* and *logos* in *Phys.* 3.3, a textual comparison shows a strong similarity with an Aristotelian passage in *Eth.Nic.* 5.3: here too, Aristotle's intention is to use a mathematical example to clarify a philosophical concept, in this case the notion of justice and its forms. For a comparison between Eratosthenes' work and the passage in Aristotle's *Nicomachean Ethics* see B. Vitrac, "La Définition V. 8 des *Eléments* d'Euclide," *Centaurus* 38 (1996) 97–121.