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## ALstract

The results of a rigorously controlled sampling cf mathematics computaticnal skills, as possessed by grades 1 - 12 students attending private and purlic schools, in a rural semi-urban, contiguous area in Connecticut, are contained in this refcrt. Status study data is presented ir a graphic form. The data reforted is derived from an item analysis of the mathematical computaticnal sut-secticn of the wide Range Achievement Test (WRAT), 1965 revisicn. Nc reasons or causes of success or failure are given, kut scme suggestions frcm recent research are cffered as fcssible contributing factors. A related document is RC 00342 . (SW)

## ARITHMETICS:-

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George M. Murphy
Box 176
Pleasant Valley,
Connecticut 06063

The major portion of the raw data analysed in this report was originally obtained under grants from the U.S.Office of Education, (P.A.C.E.), \& the Connecticut State Department of Education, and was reported in summary form in another publication. All material and interpretation contained herein is the sole responsibility of the author and does not necessarily reflect the opinions or positions of any other persons, or of any agency.

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## INTRODUCTION

The proof of any pudding is in the eating. So it should be with any educational program, its success should be measured by the relative degree of success demonstrated by those who have experienced same. It is for the above reason that this author strongly feels that basic computational facility should be the measure of success in any basic mathematics program(Murphy,1963).

Of what utility is broad based understandings, if concepts cannot be used in generating procedures to solve specific problems? Mathematics is a specific tool area. Granted, it does have some applicability in higher education, but it has a more functional and wider relevance for all citizens. In today's complex world, a citizen who cannot compute, cannot compete, and in reality is educationally handicapped in at least one dimension.

The international study on mathematics education, (Husén, 1967), which disclosed the United States as placing twelfth of twelve countries, caused many reactions on the local scene with much breast beatings and solemn protestations..." not here....not our schools....must be someplace else.... not in our system."

Yet, an impartial analysis of local mathematics achievement, on the basis of United States national norms, appears to be consistently lower than expected, (Murphy,1968). Figure One, on the next page, pictorially represents relative placements of area students and their growth in computational skills over a year's time. The "Grade Score" scale on the left of the figure indicates the expected score for that grade at that point in time, (United States norms). The growth gradients within the body of the figure denote the actual mean computational grade score placement of students. The numeral placed at the beginning and end of each growth gradient refers to general grade level of students tested. As one might easily infer, after a careful study of Figure One, it appears that effective mathematics instruction, in terms of United States standards, has ceased at about the end of the eighth grade, as reported previously, (Murphy, 1967).

If national norms are unsatisfactory, in terms of the international study, what does significantly lower local mathematics achievement, in terms of national norms, say to the local educator? There are several alternatives one might profitably explore:-

## 1.THE INTERNATIONAL STUDY IS A LOT OF USELESS INFORMATION!

An exhaustive study of the material in the international report would indicate that it appears to have been handled in a rather careful and complete manner, and such a statement relative to its questionable value seems to be indefensible.

FIGURE ONE
mean achievement levels, arithmetic computation, BY GRADE, BY TIME PERIOD, (NATIONAL NORMS).

2. NATIONAL NORMS ARE UNREALISTIC, AND EVEN THOUGH THEY ARE LOWER THAN THE INTERNATIONAL STUDY, THEY DO NOT represent a true picture begause they understate the CASE.

Again, a careful study of the norming procedures and data behind any good standardized test will reveal exacting experimental design and controls with broadbased samples, leading to the almost undisputable
conclusion, by any unbiased observer, that national norms are relevant for the United States in the main.
3. LOWERED LOCAL SUCCESS, IN TERMS OF A NATIONAL SAMPLE IS AN UNFAIR COMPARISON, BECAUSE WE ARE NOW USING MODERN MATH !

Not all schools use modern math, and there are enough samples of both modern and traditional approaches to mathematics in recently standardized instruments to adequately measure a satisfactory amount of both. The international study included the "new" math as well.
4. LOCAL EDUCATIONAL SCHOOL SYSTEMS ARE OPERAIING ON THE BASIS OF AN EDUCATIONAL PHILOSOPHY WHICH IS COMPLETELY DIFFERENT FROM THAT OF ALL THE SCHOOL SYSTEMS THAT CONTRIBUTED TO THE NATIONAL NORMS SAMPLES.

While this may be true in part, it is difficult to accept the fact that all of the school systems participating in a national norm sample have educational philosophies which are dissimilar to those of the local schools. However, it is also true that many of the local systems do not have a well defined explicitly stated philosophy of educational approach, covering the entire operation of their educational effort.
5. ON THE LOCAL LEVEL, OVERALL, WE ARE NOT SUCCEEDING AS WELL AS WE MIGHT IN THE AREA OF MATHEMATICS INSTRUCTION !

It would seem that this is the only alternative left open, after an examination of all the evidence, taking into consideration the random approach to curriculum improvement which seems to be the rule rather than the exception, as mentioned in Alternative Four, above.

When the original data in this report was published in a preliminary form, (Murphy, 1967), the area teachers were up in arms, and requested a further refinement of the analysis, that they might know where gaps, if any, existed. This paper is in answer to their request.

## ORIGIN OF THE DATA

The data reported herein is derived from an item analysis of the mathematical computational submsection of the Wide Range Achievement Test, (WRAT), 1965 revision, (Jastak \& Jastak, 1965), used in a rigorously controlled sampling procedure.

Every child in each grade in both public and private schools, grade one through grade twelve, in a six town region in Northwestern

Connecticut was assigned a number in serial sequence, from 001-up for each grade. Thirty-five (35) numbers were chosen randomly, from a table of random numbers, for each grade, for each sample, (beginning-middle-end year), producing thirty-six (36) separate sets of randomly chosen numbers. Children who had begun to receive service through the Cooperative Educational Services Center, (CESC), were not included in the sample, nor were any children chosen more than once for testing, to prevent contamination of the data due to a child's possible experience with the instrument.

The children corresponding to the randomly chosen sets of numbers were tested on the WRAT, (at the appropriate point for their particular sample), by the psychological \& guidance services and learning resources sub-sections of the CESC. Said staff scored and cross checked each other's results, and distribution charts were made. Along with the translation of national normative data into local New England norms, (Murphy,1968), an item analysis for each example on the mathematical sub-test was carried out for each of the three time samples, beginning-middle-end year.

Since there appeared to be no significant differences across each grade for individual item percentages passed, by time sample, the total number passing or failing each jitem at each grade was computed, a percent passing each item for each grade was derived, and pass-fiail curves were drawn based upon the above, (Figures Two through Thirteen).

## SOME SPECIAL CONDITIONS OF DESIGN

Since the 1965 WRAT is presented in two levels, as a part of the research design Level I was administered up through grade five, and Level II from grade six and beyond, for it was determined that the appropriate age-level break should happen somewhere between the end of the fifth grade and the beginning of the sixth grade for the majority of students. Admittedly this is not consistent with the 1965 restandardization of the WRAT, but it was felt to be more useful for local purposes, since the items of interest concerned computational achievement, by grade, rather than achievement of individual children as individuals.

## CONGRUENCE WITH OTHER GEOGRAPHICAL AREAS

While it is true that the data reported in this study has direct application only to the six Connecticut towns involved, (Barkhamsted, Cole'orpok, Hartland, New Hartford, Norfolk, and Winchester/Wincted), the local population characteristics are such that the results of the study may very well apply generally to most of rural-semi urban New England, thus lending wider relativity to the implications of the analysis. (Please see Appendix for population and area profiles).

## SUGGESTIONS FOR TEACHERS

This study does not attempt to say that certain techniques should be taught at certain grade levels. It only attempts to
generalize, from a very carefully controlled sampling procedure, as to the situation that exists within the universe of students in a particular locale. Teachers of mathematics will have to make their own decisions as to whether or not this status report has significant meaning for them. If the particular techniques represented in the figures are not important to their instructional area of involvement, then they should not concern themselves.

However, if it is felt that a technique has been taught and learned adequately by the majority of pupils, and the status study shows otherwise, then such teachers might very well be concerned as to their own approaches to instruction, methodology, and other and perhaps more efficient ways of assuring that material that has been taught will be retained.

There has been no attempt to "bake a pie" in this document, only to cut a slice of the pre-existing pie, and to show all who are interested of what the ingredients actually consist. "Verbum sat sapienti."

## BASIC OPERATIONS

Simple Addition and Subtraction:- An interesting facet appears almost immediately when one examines Figure Two, below, and compares it with Figure Three, in that some subtraction skills appear

to be learned as rapidly, if not more $s o$, than the addition skills, with the exception of "one + one". Except for a minor variation,it would appear that most children are able to satisfactorily operate with the basic skills of subtraction, by the end of grade three, but not until the end of grade four with the skills of addition.

FIGURE THREE

(Guidelines for figure interpretation)-- In the figures above and in those that follow, one may notice that at the $20 \%$ and $80 \%$ level a line goes completely across the figure. It is felt that, below the $20 \%$ pass, if there has been saccess, then either the children have not experienced adequate formal instruction, or have learned the skill by accident. Above the $80 \%$ pass, it is assumed that rather thorough exposure has taken place in the procedure being tested, and the majority have the procedure well in hand. Some failure at beyond the $80 \%$ pass (mastery level) is to be expected in some cases in any normal classroom.

The variable dotted line at the $50 \%$ pass level might well be the optimum place for intensive instruction in the skill under consideration, provided that such a level is reached. The area between the $20 \%$ and the $80 \%$ levels might appropriately be considered as the area of instructional concern for mathematics teachers.

A number of ex post facto explanations, for the differences in acquisition speed between addition and subtraction, have been offered by those who have reviewed this data prior to publication. One administrator pointed out that the child's early experiences with money at the neighborhood store, wherein he might spend only part of his nickel or dime, quickly leads to greater interest and facility with the subtractive process,ie: how much does he have left to spend at another time? A local first grade teacher comented that while the above might be true, the figures simply indicated to her that "emphasis" was the important factor. She related that teachers have long felt subtraction was a more difficult concept than addition, and therefore took greater pains to insure that it was mastered.

This writer feels that probably both factors contribute greatl.y to the difference-- a more direct relevancy to the child'3 experience with money manipulation and a measureable difference in the motivational approach by inscructors.

Multiplication \& Division:- Figure Four, below, ind!cates that
FIGURE FOUR
MULTIPLICATION OF WHOLE NUMBERS

simple multiplication reaches mastery level at grade three with simple numbers, at grade four with a complex multiplicand, and with a complex multiplier and muitiplicand-does not reach the $80 \%$ pass mark at all through grade twelve. In simple division, as shown on Figure Five, below, the $80 \%$ mastery level is reached by grade four. Short Division, with a single numeral divisor, does not

FIGURE FIVE
DIVISION OF WHOLE NUMBERS

pass into the instructional zone until grade five, or over the $80 \%$ pass until grade eight.

Long division competency is difficult to determine. Item( $C$ ) in Figure Five begins to show positive movement, but because of the decign of the study, evaluation of response is cut off at the end of grade five. Item(E), in Figure Six (next page), is very closely related to ( $C$ ) above, but since decimals are an additional factor, interpretation is not clear. A re-examination of the raw data indicated that if one ignores decimal placement, Figure Six(E) still does not reach the mastery level at all, but places its zenith at $72 \%$ pass at grade twelve, (not graphed because of possible interpretive
problems mentioned above).

## DECIMALS AND FRACTIONS

Decimal Operations:- The addition of the decimal factor does not appear to have too important an effect on student success. For example, compare the items (B)-(D) curve in Figure Six, below, with the items (C)-(D) curve in Figure Four. The only notable

FIGURE SIX

variation due to decimals seen here is a flattening of the curve through the junior high grades, the end result being the same in both cases at grade twelve.

In general, with the exception of decimals in long division as commented upon previously, decimals appear to retard the percent passing curve by about one grade level.
Addition and Subtraction of Fractions:- As is the case with basic addition and subtraction, addition and subtraction of fractional
elements appear to indicate differential response on the part of students, but in a reverse manner than was seen previously.

FIGURE SEVEN


In this case also, teachers seem to expend more effort in the exploration of the subtractive process, irut the per-cent pass curves show lowered success of subtraction of fractions as compared to addition of same. Please note that mixed numbers and unlike fractions are represented in both addition and subtrac: tion of fractions.

Only the simpler problems show early success, and the $80 \%$ pass mastery level is reached rather late in the majority of items, if indeed it is reached at all.

FIGURE EIGHT

SUBTRACTION OF FRACTIONS


One interesting feature of most of the figures is portrayed very clearly in Figure Eight, above. Note the movement of item (C) and item (D) from grade eight onward. A study of the items throughout will generally reveal a dip, or regression, somewhere in the grade seven to grade ten area. Could it be related to the shrinking of population size at the upper grades due to dropouts, or the "push-outs" as this author prefers to call them?

If push-outs were less able than the general population, would not the figures indicate steeper gradients from that seen prior to grade six or seven? Could it be that many of the pushouts are from the more able but less organized segments of society, and therefore less acceptable to the middle-class orientation and philosophy of the typical school ?

Multiplication of Fractions:- From an examination of Figure Nine, below, it appears tiat the major portion of techniques in the

FIGURE NINE
multiplication of fractions

multiplication of fractions have difficulty moving up through the $20 \%$ pass lower boundary of the instructional zone. Only one technique has demonstrated sufficient success past the $50 \%$ level through to mastery at the $80 \%$ pass mark.

Notice here too, there appears to be the plateau or regression in the grade seven to grade ten area, as mentioned on the previous page. With very few exceptions, the same is true of all of the figures, from Figure Two to Figure Thirteen.

Conversions:- In the broad sense, conversion of a mathematical statement from one type of unit to another, without change in the value of the statement, can be considered as usually involving

FIGURE TEN
CONVERSIONS: DECIMALS,FRACTIONS

fractional or decimal operations, and therefore should follow naturally in sequence.

By examination of Figure Ten above, it becomes immediately apparent that at the higher grade levels, only half of the decimalfraction conversions make it to the $50 \%$ pass point, and only one of those climbs briefly to the $80 \%$ pass mastery level. Many of the basic conversions in the early grades show definite signs of positive success, but again- due to the nature of the Wrat and the design of the study, evaluation of said items is cut off after grade five.

Along with comparisons in the decimal-fraction area, there are other conversions usually taught in basic mathematics programs which consider changes in units, many times difficult to classify under any one category. Figure Eleven, below, might appropriately be listed as miscellaneous conversions, and encompasses all items which do not fit easily into any other class.

## FIGURE ELEVEN

CONVERSIONS : MEASURES


The items (B)-(F) curve appears to be the only type of problem that meets the criteria for mastery, but not until grade eleven. One might also question the relevancy of instruction in the change of Roman numerals to Arabic in a basic mathematics program, of if such is being taught, question the value of such teaching from an analysis of the items (E)-(J) curve. Further, a serious look should be taken at the curve for item (H).

## COMBINED OPERATIONS

The items represented in Figure Twelve,below, indicate problems that require more than one technique or process, used in a particular sequence, in order that a solution to the problem might be derived.

FIGURE TWELVE
COMBINED OPERATIONS


A rather disturbing note is engendered by the items (A)-(C) curve. It begins to be difficult to accept that a rather straight forward averaging problem, with five values, does not reach the mastery level at all, (through grade twelve), after reaching the $50 \%$ pass three times previously, at grades eight, nine, and eleven. Please note that plateaus and regressions at the upper levels are still very much in evidence.

## ADVANCED TECHNIQUES

In Figure Thirteen, below, please find those items tested on the WRAT that scored high enough to be portrayed on a graph, and fit the general theme of higher mathematics, (college preparatory programs), although items (A), (B) \& (C) are approached many times in basic mathematics courses, especially in "modern math" schemas.

## FIGURE THIRTEEN

- ALGEBRAIC OPERATIONS - FACTORING - EXPONENTS -


Consideration, to the fact that about half of the students surviving to grade twelve are in programs with heavy academic orientations, should temper the reader's reaction to Figure Thirteen. With such a restricted population from which a sample may be drawn, there should be differentiation in the instructional zone criteria. In the figure above, please note that, for the academic segment of the population, the instructional zone might Well range from $10 \%$ pass to $40 \%$ pass, with the middle value of 25\% pass shown as an additional variable line, (percentages referred to relate to the total population of the grade, academic and general
students combined in toto). Even on this restricted basis, only one item, ( $D$ ), temporarily reaches mastery level. Several other items within the scope of the more advanced high school mathematics programs are also tested by the WRAT, but sample student performance indicated lower than two per-cent success overall, at all grade levels, and thus were not included in the figures.

## SOME POSSIBLE CONTRIBUTING FACTORS

Recent research into the nature of factors essential for success in mathematics has isolated some interesting areas which appear to account for over $80 \%$ of the variance in mathematical achievement. Jastak $\varepsilon$ Jastak, (i965), performed a clinical factor analysis by successive regressions a.d score transformations in such a manner as to obtain individual variances for each factor, as extracted from test comparisons on both the WRAT, the Wechsler Intelligence Scales, and other standard achievement tests. The basic transformations yielded a group of factors which the WRAT arithmetic sub-test shared in common with the other subtests, and several that either were specific or mostly related to arithmetic alone, or had little or no bearing on mathematical success. Table One, below, indicates that portion of the factor separation which bears on the results of this study.

IABLE ONE
Factors and their Variances for three WRAT subtests

II. Verbal Factor $=$ a personality variable, unrelated to intelligence, but usually confused with intellect and cognition.
III. Motivation Factor $=$ a personality factor, in reality independent of I \& II above, and more specifically related to attention, recall, recognition, persistence, aspiration level, distractability and memory.
IV. Somatic Factor = muscular efficiency of the body, and degree of inertia displayed by a person getting ready to act.
V. Cognitive Factor $=$ often misinterpreted as central to intelligence, is more closely related to judgment \& choice, generalizations, and the categories of reasoning.
VI. Specific Factor ${ }^{*}=$ related to that amount of variance that is specific to the subject area under consideration. For example, the specific variance for Arithmetic is completely unrelated to the specific variances of the other two areas, and is of value only when used to assist in an understanding of mathematics scores.
VII. Error Factor = residual variance which remains after all of the ciearly identifiable factors have been extracted, and related to the reliability of test scores. (note, $* *$ there appears to be numerical differences in in the error factor, although empirically there are no significant differences in this factor across all three areas)

As one may determine by an analysis of Table One, mathematical skills depend the heaviest (of the three areas) on motivation, the least on verbal skills, and is the sole area involved with the factor of cognition. Figure Fourtoen, below, is a more graphic display of the contributing factors relating to success in mathematics, based on the above.

FIGURE FOURTEEN
Factors Involved in Computational Success

Roman Numerals relate to the Factors and their descriptions as previously reported.


A tentative conclusion based on the foregoing analysis would seem to indicate that, of those factors that are at least partially subject to control by instruction, the combination of the motivational and specific factors plays the largest role in mathematics success or failure.

In another recent landmark effort, Worthen (1968) demonstrated aiso that motivation and/or instructional stimulation to materials appeared to be one of the most significant variables in mathematics success, and was perhaps the keystone of the puzzle.

The question might well be asked, what is the relative amount of attention paid to the motivational factor in area mathematics instruction ? It would appear from ali of the above that mere lip service or the stating of a rationale for a particular procedure might not be quite enough. The research seems to indicate that the motivational theme must pervade the entire operation if the students are to meet satisfactory levels of achievement. Again, the reader will have to judge for himself.

## SUMMARY

This report presented the results of a rigorously controlled sampling of mathematics computational skills, as possessed by grade l-12 students att anding private and public schools, in a rural-semi urban contiguous area in New England, (Connecticut). The status study data was presented as it appeared, but cast in a graphic form for easier interpretation.

No attempt was made to search for reasons or causes of success or lack of same, but some suggestions from recent research were offered as possible contributing factors.

## REFERENCES

HUSEN, Torsten, Ed., International Study of Achievement in MathematicsA Comparison of Twelve Countries, New York, John Wiley \& Sons,1967.

JASTAK,J.F \& JASTAK,S.R., The Wide Range Achievement Test-Manual of Instructions, (Rev.Ed.) Wilmington, Delaware, Guidance Associates, 1965.

MURPHY, George M., One Down and Two to Go--End of Budget Period ReportProject 2098, Winsted, Connecticut, Cooperative Educational Services Center, 1967.

MURPHY, George M., Northeastern Educational Placement Guide in Reading, Arithmetic \& Speliing., Winsted, Connecticut, Cooperative Educational Services Center, 1968

MURPHY, George M., An Investigation of the Utility of the Mathematics Subtest from the Wide Range Achievement Test as Applied to Intermediate Level Mathematics Groups., Litchfield, Connecticut, Iitchfield Public Schools, 1963.

WORTHEN, Blaine R., Discovery and Expository Task Presentation in Elementary Mathematics., New York, Monograph Supplement-Journal of Educational Psychology, Vol.59,I,2.,American Psychological Association, 1968.

APPENDIX
GEOGRAPHIC CHARACTERISTICS

| TOWN | SQUARE MILES OF AREA | MILES OF PAVED STATE ROADS | $\begin{aligned} & \text { MILES OF } \\ & \text { PAVED } \\ & \text { TOWN ROADS } \\ & \hline \end{aligned}$ | MILES OF UNIMPROVED ROADS | PRINCIPAL. <br> INDUS"RIES |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Barkhamsted | 38.9 | 27.50 | 36.49 | 1.14 | Agriculture |
| Colebrook | 33.5 | 16.42 | 29.44 | 10.94 | Agriculture |
| Hértland | 33.7 | 23.80 | 22.50 | .26 | Agriculture |
| New Hartford | 37.4 | 19.49 | 52.35 | 15.38 | Agriculture, Small Manufacturing |
| Norfolk | 46.0 | 18.37 | 40.14 | 15.57 | Agriculture, Summer Resort |
| Winchester | 36.0 | 23.16 | 69.89 | 12.56 | Small <br> Manisfacture |

The above data obtained from The Connecticut State Highway Department, and/or The
Town Clerk, lst Selectman, or Street Department Superintendent of the involved Towns.

FUNDING DATA FOR PUBLIC EDUCATION IN THE SIX TDWN REGION

| TOWN | NET GRAND LIST OCT. 1,1964 | INDEBTEDNESS JAN. 1,1965 | POPULATION ESTIMATE | PER PUPIL COST <br> (INC。 TRANS.) | $\begin{array}{r} \text { PER PUPIL } \\ \text { TRANS.CCST } \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Barkhamsted | \$ 3,992,810 | \$ 150,000 | 1,700 | \$ 576.11 | \$ 47.66 |
| Colebrook | 4,290,281 | 10,000 | 830 | 524,32 | 45.60 |
| Hartland | 4,206,385 | 160,000 | 1,100 | 529.65 | 68.36 |
| New Hartfard | 13,269,250 | 205,000 | 3,300 | 529.65 | 32.75 |
| Norfoik | 7,919,907 | 180,000 | 1,900 | 578.31 | 33.51 |
| Winchester <br> City of Winsted | $\begin{aligned} & 39,37.5,850 \\ & 28,992,720 \end{aligned}$ | $\begin{aligned} & 1,519,000 \\ & 1,793,000 \end{aligned}$ | 11,000 | 460.17 | 15.64 |

Data for this chart from Reqister and Manual, Connecticut, 1965.

## DISTRIBUTION OF STAFF AND PLPPIL POPULATION

IN THE SIX TOWNS COVERED

| TOTAL <br> STAFF | ELEMENTARY <br> PUPILS |  | SECONDARY <br> PUPILS | TOTAL <br> PUPILS |
| :--- | :---: | :---: | :---: | :---: |
| Public <br> School | 152 | 3,132 | 1,171 | 4,303 |
| Nonpublic <br> School | 54 | 479 | 603 | 1,082 |
| Total | 206 | 3,611 | 1,774 | 5,385 |

