

ARRAY ELEMENT LOCALIZATION OF THE HARO STRAIT EXPERIMENT USING A TWO-STEP INVERSION METHOD

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OVERVIEW OF THE PROBLEM

The Haro Strait experiment [1] was designed for ocean and geoacoustic tomography inversion. Three vertical line arrays (VLAs) were deployed at sea to record acoustic pressure fields generated by light bulb explosions over a period of several days. Tomography inversion requires an accurate knowledge of the receiver and source locations. Such knowledge is usually not available and this is particularly true in the case of the Haro Strait experiment. GPS measurements of the ship position were done during the deployment of the VLAs and sources. The uncertainty of these measurements varies from 5 to 20m. The source depth was estimated by the length of the immersed cable carrying the light bulb (uncertainty $\pm 3m$). In addition, due to strong tidal currents, the arrays were expected to be tilted and a GPS measurement at one VLA while being recovered revealed a drift of about 100m from the original position. It is thus necessary to localize the VLAs as continuously as possible (ideally at each explosion time). Localization is usually done by inverting the measured travel times of the direct paths between one source and the receivers. For the Haro Strait data, the large number of unknowns in the problem (x,y and z coordinates of each receiver and each source, and the absolute time of the explosions) requires additional information (data) in order to obtain an accurate solution. In this paper, we present a method for array element localization that takes advantage of additional travel times provided by the use of multiple sources and multiple paths. To overcome the lack of exact knowledge of the source positions, this method includes 2 inversion steps : a relative localization of the receivers which depends on the depth of the sources, and then the absolute localization of both sources and receivers.

INVERSIONS

Relative localization Inversion of travel times for localization is a nonlinear problem. However it is not highly nonlinear and local linearization using Newton's method has been applied successfully. Since the source and receiver x and y coordinates are not known, we first concentrate on their relative positions and define \mathbf{m} , the model of parameters to be determined, as follow:

$$\mathbf{m} = \{Z_j^r, D_{ij}, t_i^s; \quad j = 1, N_{rec}, i = 1, N_{src}\},$$

where N_{src} and N_{rec} are the number of sources and receivers respectively, Z_j^r is the depth of the j^{th} receiver, D_{ij} is the horizontal distance between the i^{th} source and the j^{th} receiver and t_i^s is the time delay for the i^{th} source. The total number of parameters is $M = N_{rec}(1 + N_{src}) + N_{src}$.

The source depths Z_i^s are assumed to be known. Given a constant sound speed c in the water, the relative onset travel times of the direct (d) and surface (s) reflected paths between the i^{th} source and the j^{th} receiver are given by eqs. 1 and 2 respectively:

$$t_{ij}^d = (\sqrt{d_{ij}^2 + (Z_i^s - Z_j^r)^2} - ct_i^s)/c, \quad (1)$$

$$t_{ij}^s = (\sqrt{d_{ij}^2 + (Z_i^s + Z_j^r)^2} - ct_i^s)/c. \quad (2)$$

The measured travel times of both paths define the vector of data: $\mathbf{t} = \{\mathbf{t}^d, \mathbf{t}^s\}$ ($N = 2N_{src} \times N_{rec}$ elements). Eqs 1 and 2 can be written in the more general form $\mathbf{t} = \mathbf{T}(\mathbf{m})$, where \mathbf{T} is a nonlinear function. The expansion of $\mathbf{T}(\mathbf{m})$ in a Taylor series (linearization) to first order about an arbitrary starting model \mathbf{m}_0 gives:

$$\mathbf{T}(\mathbf{m}) = \mathbf{T}(\mathbf{m}_0 + \Delta\mathbf{m}) \simeq \mathbf{T}(\mathbf{m}_0) + \mathbf{J}\Delta\mathbf{m}$$

where $J_{kl} = \delta T_k(\mathbf{m}_0)/\delta m_l$ is a Jacobian matrix. Applying the jumping method [2]($\Delta\mathbf{m} = \mathbf{m} - \mathbf{m}_0$) leads to:

$$\mathbf{J}\mathbf{m} = [\mathbf{t} - \mathbf{T}(\mathbf{m}_0)] + \mathbf{J}\mathbf{m}_0 \equiv \mathbf{t}_o. \quad (3)$$

This equation defines a linear problem (the right side of the equation is known). At this stage, it is possible to invert for \mathbf{m} . However, the solution is nonunique and the inversion can be unstable. One way to address this problem is regularization i.e. including *a priori* information about the model to stabilize the inversion. Regularized inversion consists in minimizing the objective function Φ that combines a least square data misfit term and a regularizing term :

$$\Phi = |\mathbf{G}(\mathbf{J}\mathbf{m} - \mathbf{t}_o)|^2 + \mu |\mathbf{H}(\mathbf{m} - \hat{\mathbf{m}})|^2. \quad (4)$$

In this equation, \mathbf{G} is a diagonal matrix whose diagonal elements are the inverse of the estimated data standard deviations (the noise is assumed Gaussian with zero mean), \mathbf{H} is the regularization matrix, $\hat{\mathbf{m}}$ is the *a priori* estimate of \mathbf{m} , and μ is a trade-off parameter controlling the relative importance of the 2 terms in the minimization. Minimizing Φ with respect to \mathbf{m} leads to the solution:

$$\mathbf{m} = \hat{\mathbf{m}} + [\mathbf{J}^t \mathbf{G}^t \mathbf{G} \mathbf{J} + \mu \mathbf{H}^t \mathbf{H}]^{-1} \mathbf{J}^t \mathbf{G}^t \mathbf{G} (\mathbf{t}_o - \mathbf{J} \hat{\mathbf{m}}). \quad (5)$$

Since nonlinear terms are neglected during the linearization, the solution \mathbf{m} may not be satisfactory (large misfit $\chi^2 = |\mathbf{G}(\mathbf{T}(\mathbf{m}) - \mathbf{t})|^2$). The inversion is then repeated by updating the starting model ($\mathbf{m} \rightarrow \mathbf{m}_0$) until $\chi^2 = N$.

Absolute localization The result of the inversion described in the previous part is the estimate of receiver depths and horizontal distances between sources and receivers. The next step is to estimate the absolute position (x and y coordinates) of the acoustic elements. However, the new model \mathbf{m}' of parameters to be determined is reduced to the $2N_{src}$ source coordinates (X_i^s, Y_i^s) since analytical expressions of the receiver coordinates (X_j^r, Y_j^r) can be derived from (X_i^s, Y_i^s) and the distances D_{ij} . The candidate models are sampled on a grid covering the uncertainty surface of the source locations. For each model, the receiver positions are determined and a coefficient β is calculated to characterize the shape of the array:

$$\beta = \sum_{j=1}^{N_{rec}-1} \sqrt{|X_{j+1}^r - X_j^r|^2 + |Y_{j+1}^r - Y_j^r|^2 + |Z_{j+1}^r - Z_j^r|^2}. \quad (6)$$

The model \mathbf{m}' corresponding to the minimum β (minimum structure in the array shape) is the estimate of the source x and y coordinates.

If the depths of the source are known, it is thus possible to determine the absolute positions of the sources and receivers ($\mathbf{m}'' = \{X_i^s, Y_i^s, X_j^r, Y_j^r, Z_j^r\}$). However, this is not true for the Haro Strait data and using different sets of source depths in the inversion would result in different estimates of \mathbf{m}'' that could still fit the data. In order to reduce the number of these solutions, a second inversion is done. This inversion consists in 1) repeating the estimation of \mathbf{m}'' for different sets of source depths and 2) for each estimate, predicting the onset travel times of the bottom (b), surface-bottom(sb) and bottom-surface (bs) reflected paths. Similar equations to eqs 1 and 2 do not exist to calculate the travel times of these additional paths. Instead, this 2nd inversion is a model-based inversion: the modeled travel times $\tau = \{\tau^d, \tau^s, \tau^b, \tau^{bs}, \tau^{sb}\}$ are computed using a ray code. The travel times of the bottom interacting paths ($\tau^b, \tau^{bs}, \tau^{sb}$) are then compared to the corresponding measured travel times (t^b, t^{bs}, t^{sb}):

$$\chi_b^2 = \sum_{i=1}^{N_{src}} \sum_{j=1}^{N_{rec}} 3/\sigma_{ij} (|\tau_{ij}^b - t_{ij}^b|^2 + |\tau_{ij}^{bs} - t_{ij}^{bs}|^2 + |\tau_{ij}^{sb} - t_{ij}^{sb}|^2), \quad (7)$$

where the modeled travel times τ have been preliminary calibrated such that $\tau_{i0}^d = t_{i0}^d$, and σ_{ij} is the uncertainty of the travel time between the i^{th} source and the j^{th} . The estimate of the source depths (and by extension \mathbf{m}'') is given by the model that minimizes $|\chi_b^2 - N_b|$ where N_b is the number of data ($N_b = 3N_{src} \times N_{rec}$).

RESULTS

For each VLA localization, 2 sources were selected such that 1) they were close enough in time, 2) their positions relative to the VLA would allow good x and y resolution (ie not in line with VLA) and 3) the 5 paths were easily identifiable. Picking the measured onset travel times $\mathbf{t} = \{t^d, t^s, t^b, t^{bs}, t^{sb}\}$ of the different paths from the time series was done using an adaptative matched filter calculating correlation between the time series and a reference waveform (direct path). For the regularization, $\hat{\mathbf{m}}$

was set to the preliminary estimates of the parameters at the deployment and \mathbf{H} was a diagonal matrix whose diagonal elements are the inverse of the uncertainties of these preliminary estimates. Table 1 shows the result of localization for 2 of the VLAs (noted NW and SW) as well as the *a priori* model $\hat{\mathbf{m}}$. X and Y coordinates are minutes of longitude 123° and latitude 48° respectively. Results for the top, middle and bottom receivers are given. In figure 1, the travel times τ calculated for the various paths from the final estimate of source and receiver locations are compared to the measured travel times. A good agreement can be observed.

param.	NW		SW	
	a priori	estimate	a priori	estimate
X_1^r (min)	12.880	13.140	12.880	12.778
X_2^r (min)	12.880	13.137	12.880	12.778
X_{15}^r (min)	12.880	13.131	12.880	12.778
Y_1^r (min)	38.700	39.479	38.700	38.741
Y_2^r (min)	38.700	39.478	38.700	38.735
Y_{15}^r (min)	38.700	39.470	38.700	38.717
Z_1^r (m)	30.00	40.94	50.00	66.98
Z_2^r (m)	73.75	84.32	93.75	110.10
Z_{15}^r (m)	123.75	132.54	143.75	150.02
X_1^s (min)	13.195	12.680	13.195	12.681
Y_1^s (min)	38.985	39.017	38.985	39.020
Z_1^s (m)	70.00	71.00	70.00	71.00
X_2^s (min)	13.195	13.184	13.195	13.186
Y_2^s (min)	38.985	38.982	38.985	38.986
Z_2^s (m)	70.00	71.00	70.00	71.00

Table 1: Result of localization for 2 arrays.

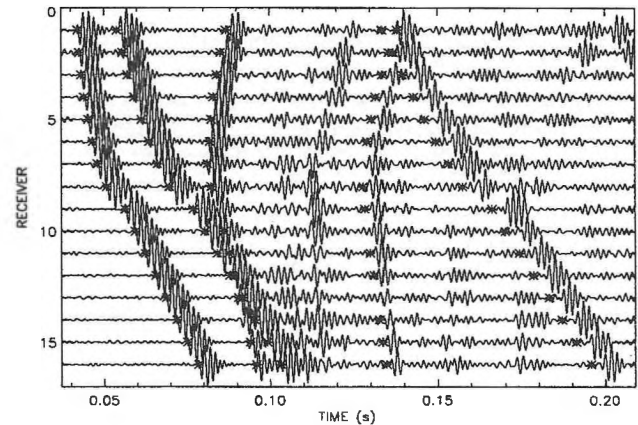


Figure 1: Pressure field recorded at the SW array for source 1. The modeled onset travel times τ are indicated by asterisk.

REFERENCES

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