

# Array Gain and Capacity for Known Random Channels with Multiple Element Arrays at Both Ends

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**Abstract**—Two arrays with  $M$  and  $N$  elements are connected via a scattering medium giving uncorrelated antenna signals. The link array gain relative to the case of one element at each end is treated for the situation where the channels are known at the transmitter and receiver. It is shown that the maximum mean gain achieved through adaptive processing at both the transmitter and the receiver is less than the free space gain, and cannot be expressed as a product of separate gains. First, by finding the singular values of the transmission matrix, fundamental limitations concerning the maximum gain and the diversity orders are given, indicating that the gain is upper bounded by  $(\sqrt{M} + \sqrt{N})^2$  and the diversity order is  $MN$ . Next an iterative technique for reciprocal channels which maximizes power at each stage transmitting back and forth is described. The capacity or spectral efficiency of the random channel is described, and it is indicated how the capacity is upper bounded by  $N$  parallel channels of gain  $M(N < M)$  for large values of  $N$  and  $M$ .

**Index Terms**—Antenna gain, array capacity, random channel.

## I. INTRODUCTION

THE INCREASING demand for bandwidth in wireless networks leads to a corresponding demand for signal power in order to achieve a reasonable range. For fixed wireless access systems operating above rooftops, high gain antennas at millimeter wave frequencies may be used to achieve a high antenna gain and thus circumvent the pathloss problem. For the mobile case the random and angularly widespread signals make it difficult to use traditional high gain antennas with fixed patterns, instead adaptive multielement antennas are needed. It is the main purpose of this paper to investigate the antenna gain situation, when adaptive arrays are present at both ends of the link. It should be noted that there are essentially two types of array gain when combining signals. One is the average power of the combined signal relative to the individual average powers from element to element, which we shall call the array gain for obvious reasons. The other gain is the diversity gain related to a certain outage probability level, e.g., 1%. This gain is of course highly dependent on the spatial correlation coefficients between the antenna signals. In this paper it will be assumed that all the antenna signals are spatially uncorrelated, which is a limiting case approached in indoor situations.

A *known* channel is a channel where information about the channel is available both at the transmitter and receiver. A *reciprocal* channel is one where the channel is identical in both

directions as in a TDD (time-division-duplex) situation; and a *nonreciprocal* channel is one where the channels are different in the two directions as in an FDD (frequency-division-duplex) situation. In the latter case, we may have two known channels, although this requires some form of feedback. In practice, this means that we have two optimizations instead of one.

The novel problem addressed in this paper is primarily the array gain problem of having  $M$  transmit antennas and  $N$  receive antennas jointly illuminating a random medium, which gives rise to spatially uncorrelated antenna signals. For a link budget, we would like to know the composite antenna array gain and diversity gain of joint transmit-receive diversity. The gain is maximized by adjusting the antenna weights at both ends. For the case of  $M$  or  $N$  equal to one, the results are well known; it is known that the antenna gain is  $N$  or  $M$ , respectively [1], when the array can be optimized in isolation. But what is it in the general  $(M, N)$  case? (We have here assumed negligible interaction between the antenna elements.) The main contribution of the paper is finding the mean value and the distribution of the received power under theoretically optimum conditions, where all  $MN$  transmission coefficients between all antenna elements are known. This is based on some recent results concerning the distribution of eigenvalues of complex matrices.

The antenna gain is relevant when the same information is sent over all antennas, but antenna gain is not the only variable of interest. Winters [2] showed that the theoretical capacity of an  $(M, M)$  array is much larger than for a single channel, since  $M$  independent channels may be established with each channel having about the same maximum data rate as a single channel. There has recently been a number of publications on the use of multiple antennas in a scattering environment [3]–[6]. The emphasis has in all cases been for the case of unknown channels at the transmitter, where different signals are sent over the antenna elements and coding is applied to take advantage of the diversity gain on the transmit side, the so-called space-time coding. For those situations, the same power is sent over the  $M$  antennas, and the array transmit gain is not realized, only the diversity gain. In this paper the capacity of the  $(M, N)$  array is perfectly realized by exciting the array with the correct weights for each parallel channel and distributing the total power among the channels by “water filling.” Thus, the antenna gain is also realized for each channel in contrast to the previous work, with achievement of the corresponding additional capacity. The knowledge of the channel makes it possible to derive some simple expressions for the capacity of the system in limiting situations.

In Section II, the channel model is introduced; and in Section III, a matrix solution lends itself to an analytical approach

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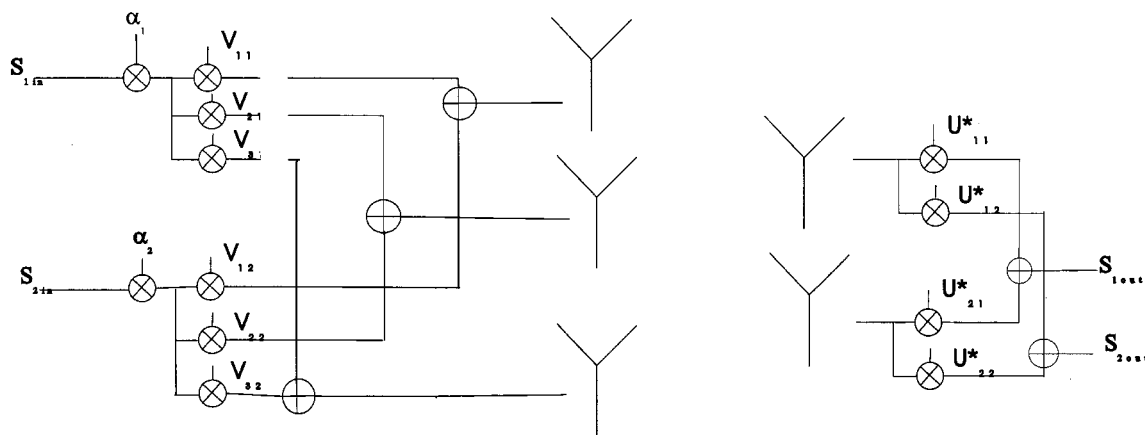


Fig. 1. Transmission from three ( $M$ ) transmit antennas to two ( $N$ ) receive antennas. There are two independent channels (the minimum of  $M$  and  $N$ ), which are excited by the  $\mathbf{V}$  vectors on the transmit side and weighted by the  $\mathbf{U}$  vectors on the receive side. The power is divided between the two channels according to the “water filling” principle. This is a maximum capacity excitation of the medium. In case only maximum array gain is wanted, only the maximum eigenvalue is chosen (one channel).

where the eigenvalues of the transmission system lead to a definition of the maximum gain as the largest eigenvalue. The singular value decomposition, SVD, is an elegant tool for this, since it also gives the proper weights at the transmit side and receive side. In this way, both the antenna gain and the diversity gain can be conveniently described. In Section IV, a simple iterative solution for achieving maximum gain for a reciprocal channel is presented. In Section V, the eigenvalues are used for an evaluation of the theoretical spectral efficiency of ( $M$ ,  $N$ ) arrays.

## II. THE CHANNEL MODEL

The model considered consists of  $M$  transmit elements in an array illuminating a scattering medium, which rescatters the energy to  $N$  receive elements. In the following it is assumed for simplicity that  $M > N$ . The antenna weights on the receive side are described as a column vector  $\mathbf{U}$  with elements  $U_1, U_2, \dots$ , and the vector is normalized so the norm is unity ( $\mathbf{U}'\mathbf{U} = 1$ ). Similarly,  $\mathbf{V}$  denotes the transmit weight vector. For notation, \* signifies complex conjugation, and ' transpose and conjugate. ( $M$ ,  $N$ ) means  $M$  antenna elements at one end and  $N$  antenna elements at the other end.

The transfer matrix from the transmit antennas to the receive antennas is described by transmission matrix  $\mathbf{H}$  with elements  $H_{ik}$ . They are random complex Gaussian quantities. A normalization such that

$$E\langle |H_{ik}|^2 \rangle = 1 \quad (1)$$

is applied.

The model represents an extreme case with angular spreads seen from both sides being so large, that the antenna signals are spatially uncorrelated.

## III. GAIN OPTIMIZATION BY SINGULAR VALUE DECOMPOSITION (SVD)

### A. Singular Value and Eigenvalue Decompositions

The SVD is an appropriate way of diagonalizing the matrix and finding the eigenvalues [7]. The matrix  $\mathbf{H}$  will in general be

rectangular with  $N$  rows and  $M$  columns. An SVD expansion is a description of  $\mathbf{H}$  itself as

$$\mathbf{H} = \mathbf{U}_\lambda \cdot \mathbf{D} \cdot \mathbf{V}'_\lambda \quad (2)$$

where  $\mathbf{D}$  is a diagonal matrix of real, nonnegative singular values, the square roots of the eigenvalues of  $\mathbf{G}$ , where  $\mathbf{G} = \mathbf{H}' \cdot \mathbf{H}$  is an  $M \times M$  Hermitian matrix consisting of the inner product of  $\mathbf{H}'$  and  $\mathbf{H}$ . The columns of the unitary matrices  $\mathbf{U}_\lambda$  and  $\mathbf{V}_\lambda$  are the corresponding singular vectors mentioned above. Thus, (2) is just a compact way of writing the set of independent channels

$$\begin{aligned} \mathbf{H}\mathbf{V}_1 &= \sqrt{\lambda_1}\mathbf{U}_1 \\ \mathbf{H}\mathbf{V}_2 &= \sqrt{\lambda_2}\mathbf{U}_2 \\ &\vdots \\ \mathbf{H}\mathbf{V}_N &= \sqrt{\lambda_N}\mathbf{U}_N. \end{aligned} \quad (3)$$

The SVD is particularly useful for interpretation in the antenna context. Choosing one particular eigenvalue, it is noted that  $\mathbf{V}_i$  is the transmit weight factor for excitation of the singular value  $\sqrt{\lambda_i}$ . A receive weight factor of  $\mathbf{U}'_i$ , a conjugate match, gives the receive voltage  $S_r$  and the square of that the received power

$$\begin{aligned} S_r &= \mathbf{U}'_i \mathbf{U}_i \sqrt{\lambda_i} = \sqrt{\lambda_i} \\ P_r &= |S_r|^2 = \lambda_i. \end{aligned} \quad (4)$$

This clearly shows that the matrix  $\mathbf{H}$  of transmission coefficients may be diagonalized leading to a number of independent orthogonal modes of excitation, where the power gains of the  $i$ th mode or channel is  $\lambda_i$ . The weights applied to the arrays are given directly from the columns of the  $\mathbf{U}_\lambda$  and  $\mathbf{V}_\lambda$  matrices. Thus, the eigenvalues and their distributions are important properties of the arrays and the medium, and the maximum gain is of course given by the maximum eigenvalue. The number of nonzero eigenvalues may be shown to be the minimum value of  $M$  and  $N$ . The situation is illustrated in Fig. 1, where the total power is distributed among the  $N$  parallel channels by weight

factors  $\alpha$  (see later). An important parameter is the trace of  $G$ , i.e., the sum of the eigenvalues

$$\text{Trace} = \sum_i \lambda_i \quad (5)$$

which may be shown to have a mean value of  $MN$  [8].

### B. (2, 2) Example

The  $(M, N) = (2, 2)$  example may illustrate the case. The  $G$  matrix is given by

$$\begin{aligned} G &= \begin{pmatrix} |H_{11}|^2 + |H_{12}|^2 & H_{11}H_{21}^* + H_{12}H_{22}^* \\ H_{11}^*H_{21} + H_{12}^*H_{22} & |H_{22}|^2 + |H_{21}|^2 \end{pmatrix} \\ &= \begin{pmatrix} a & c \\ c^* & b \end{pmatrix} \end{aligned} \quad (6)$$

and the two eigenvalues are

$$\lambda_{\max} = \frac{1}{2} \left( a + b + \sqrt{(a-b)^2 + 4|c|^2} \right) \quad (7)$$

and

$$\lambda_{\min} = \frac{1}{2} \left( a + b - \sqrt{(a-b)^2 + 4|c|^2} \right). \quad (8)$$

Note that

$$\begin{aligned} \text{Trace} &= \sum_i \lambda_i = a + b \\ &= |H_{11}|^2 + |H_{12}|^2 + |H_{21}|^2 + |H_{22}|^2 \end{aligned} \quad (9)$$

so the sum of the eigenvalues displays the full fourth-order diversity. In this particular case, it may be shown that the mean values are

$$E\langle \lambda_{\max} \rangle = 3.5, \quad E\langle \lambda_{\min} \rangle = 0.5. \quad (10)$$

The distribution of ordered eigenvalues may be found in [9] from which the distributions for  $\lambda_{\min}$  and  $\lambda_{\max}$  may be derived

$$p_{\lambda_{\min}} = 2e^{-2\lambda} \quad (11)$$

$$p_{\lambda_{\max}} = e^{-\lambda}(\lambda^2 - 2\lambda + 2) - 2e^{-2\lambda}. \quad (12)$$

The minimum eigenvalue is Rayleigh distributed with mean power 0.5. The cumulative probability distributions are shown in Fig. 2, where the maximum eigenvalue (the array gain) follows the fourth-order maximum ratio diversity distribution quite closely. The cumulative probability distribution for  $\lambda_{\max}$  is

$$\begin{aligned} \text{Prob}(\lambda_{\max} < x) &= 1 - e^{-x}(x^2 + 2) + e^{-2x} \\ &\approx x^4/12 \quad x \ll 1. \end{aligned} \quad (13)$$

It is interesting to compare to the case of standard diversity  $(M, N) = (1, 4)$

$$\begin{aligned} \text{Prob}(P < x) &= 1 - e^{-x}(1 + x + x^2/2 + x^3/6) \\ &\approx x^4/24 \quad x \ll 1 \end{aligned} \quad (14)$$

so the  $(2, 2)$  case displays full fourth-order diversity but with twice the cumulative probability for the same power level, or a shift of  $2^{1/4} = 0.75$  dB in power for a given small cumulative probability. The mean values are shifted 0.58 dB  $(3.5/4)$ .

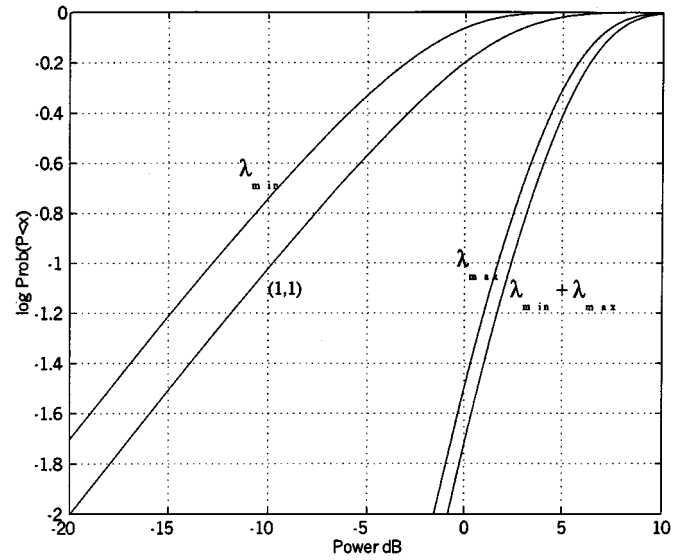


Fig. 2. Cumulative probability distribution of eigenvalues for a  $(T, R) = (2, 2)$  array with four uncorrelated paths. The maximum eigenvalue follows closely the fourth-order diversity with a shift of 0.75 dB.

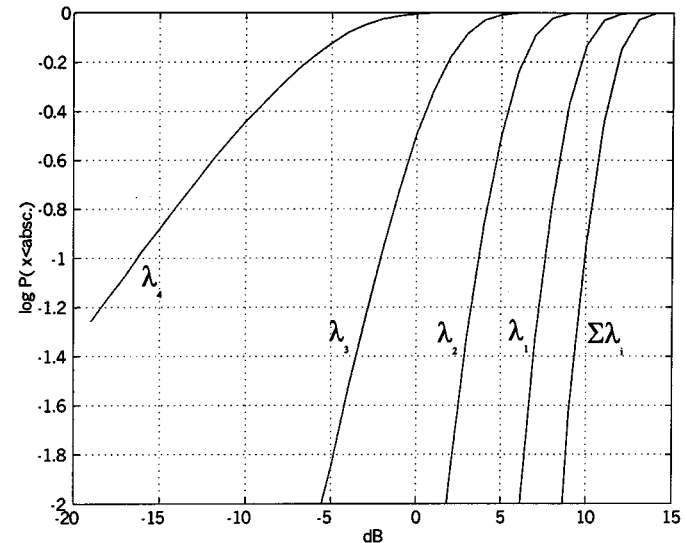


Fig. 3. Cumulative probability distribution of eigenvalues (power) for two arrays of each 4 elements including the sum of eigenvalues corresponding to a  $(1, 16)$  case.

In order to make full benefit of the maximum eigenvalue, it is of course a necessity to know the channel at the transmitter, otherwise the eigenvectors cannot be found.

### C. The General $(M, N)$ Case

The  $(2, 2)$  case above may be generalized to more elements, and as an example consider the  $(4, 4)$  case in Fig. 3. The two arrays have 16 different uncorrelated transmission coefficients, so the diversity order is 16. As before the mean antenna gain  $(E[\lambda_1])$  is reduced, now from 12 dB to 10 dB, and one intuitive explanation for this diminishment is that the available weight vectors on both sides only allow a total of eight complex weights, not enough for the combining of 16 transmission coefficients.

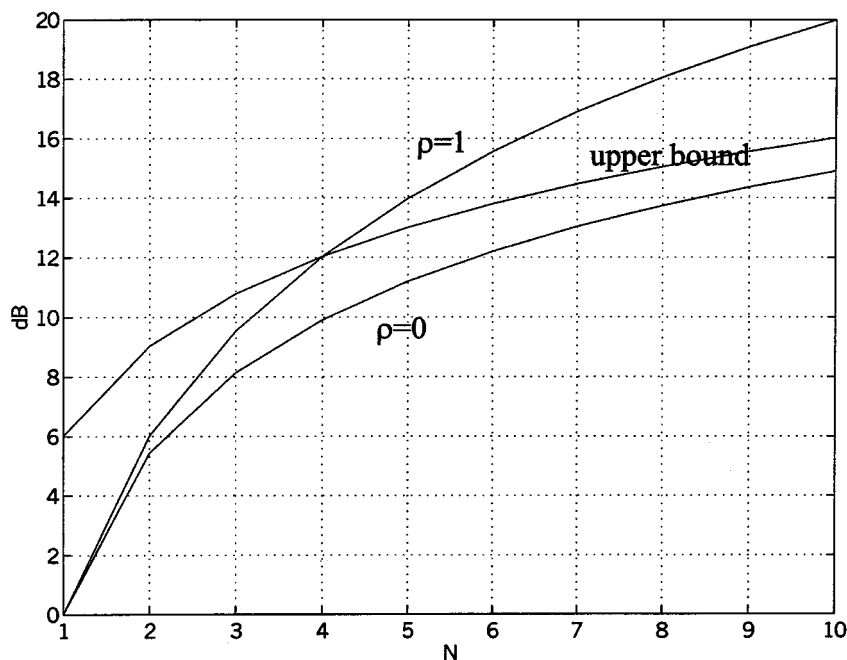


Fig. 4. The gain relative to one element of  $(N, N)$  arrays in a correlated situation ( $\rho = 1$ ), and in an uncorrelated case ( $\rho = 0$ ). The upper bound equals  $4N$ , and is the asymptotic upper bound for the maximum eigenvalue for  $N$  tending to infinity.

Recent results concerning the distribution of the eigenvalues of a random Hermitian matrix can give some insight into the maximum gain and how it varies with  $M$  and  $N$ . In the asymptotic limit when  $N$  is large, it may be shown [8], [10], [11] that the largest eigenvalue is bounded above by

$$\lambda_{\max} < (\sqrt{c} + 1)^2 N \quad c \geq 1 \quad (15)$$

where  $c$  equals  $M/N$ , and the smallest eigenvalue is bounded below by

$$\lambda_{\min} > (\sqrt{c} - 1)^2 N \quad c > 1. \quad (16)$$

In the previous examples,  $c = 1$ , and the upper asymptotic bound for this case is  $4N$ . These bounds should not be understood as absolute bounds, but rather as a limit approached as  $N$  tends to infinity for a fixed  $c$ .

The mean array gains (mean of the maximum eigenvalues) are shown in Fig. 4 together with the upper bound and the gain for the correlated, free space case,  $N^2$ . For  $N = 10$ , the true mean gain is just 1 dB below the upper bound. Thus, the price to pay for the random scattering is a diminishment of the gain from  $N^2$  to  $4N$  for  $N$  large. For a partly correlated case, we can expect the gain to lie between the  $\rho = 0$  and the  $\rho = 1$  cases, where  $\rho$  is the spatial correlation coefficient between the elements.

In some situations, it might be advantageous to have more antennas on one side than on the other, especially for asymmetric situations with heavy downloading of data from a base station. Again, the asymptotic, upper bound for the largest eigenvalue is useful (15). Introducing  $M$  directly we find

$$G_{\text{upper bound}} = (\sqrt{M} + \sqrt{N})^2 \quad (17)$$

which asymptotically will approach  $M$  for large values of  $M$  and fixed  $N$ . This clearly illustrates that the composite gain of

the link cannot be factored into one belonging to the transmitter and one belonging to the receiver.

#### IV. GAIN OPTIMIZATION BY ITERATION FOR A RECIPROCAL CHANNEL

Since the channel is reciprocal, exactly the same weights may be used for transmission as for reception. Consider now the situation with transmission from  $M$  transmit antennas to  $N$  receive antennas (or vice versa). The iteration starts with an arbitrary  $\mathbf{V}_1$ , in the numerical calculations chosen as unit vector with equal elements. At the receive side, the weights are adjusted for maximum gain, and the same weights are then used for transmit since the channel is reciprocal. This may then be repeated a number of times. In principle, the process might converge to an eigenvalue different from the maximum one, but experience shows excellent performance.

An example of the convergence in the mean is shown for a  $(3, 3)$  case in Fig. 5. After a few iterations, the gain has converged to the steady state. Note that the mean gain starts with a gain of 3 (4.77 dB), since the left array has a mean gain of 1 before adaptation. This is actually an easily proven result; for an unknown channel, the transmit antenna has a mean gain of 1 independent of the number of elements. What has been done is just an iterative way of finding the eigenvector belonging to the largest eigenvalue. The motivation for this section is that it might actually be a computationally efficient way of finding the maximum gain solution in practice without going through the trouble of finding the eigenvectors.

#### V. SPECTRAL EFFICIENCY OF PARALLEL CHANNELS

A looking at Fig. 3 with four independent channels makes it clear that there are other uses of the eigenvalues than using the largest for maximum gain. Another use is to keep them as

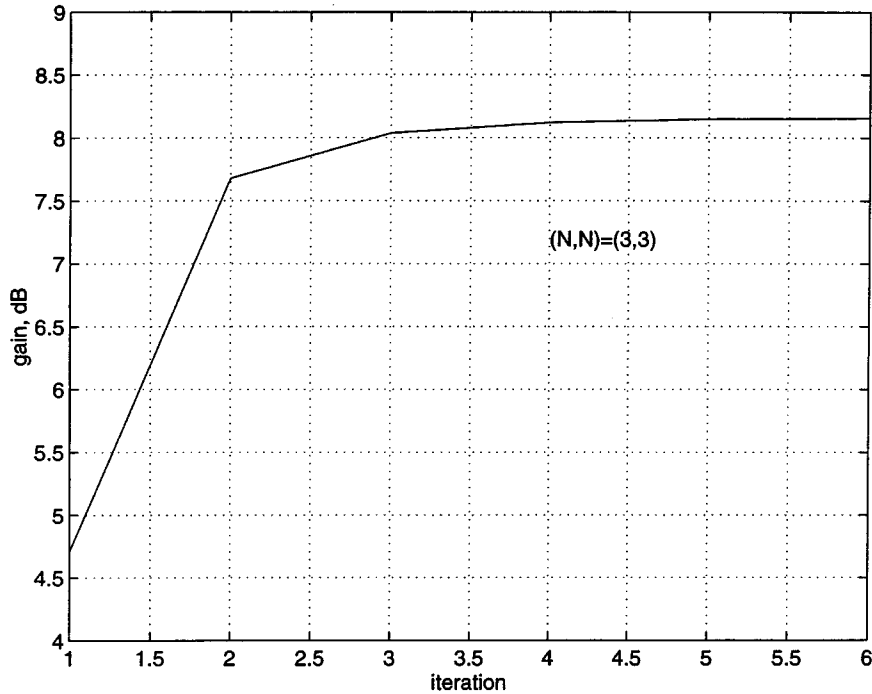


Fig. 5. Convergence of gain by iterative transmissions between receiver and transmitter for a (3, 3) case. The gain values are mean values.

parallel channels with independent information [2]. The knowledge about the distribution of the eigenvalues and the upper and lower bounds may now be used for evaluating bounds on the theoretical capacity of the link. Shannon's capacity measure gives an upper bound on the realizable information rates through parallel channels, and how the power should be distributed over the channels to achieve maximum capacity through "water filling" [12], [6]. The basic expression for the spectral efficiency measured in bits/s/Hz for one Gaussian channel is given by

$$C = \log_2(1 + P) \quad \text{bits/s/Hz} \quad (18)$$

where  $P$  is the signal-to-noise ratio,  $SNR$ , for one channel.

Assuming all noise powers to be the same, the "water filling" concept is the solution to the maximum capacity, where each channel is filled up to a common level  $D$

$$\frac{1}{\lambda_1} + P_1 = \frac{1}{\lambda_2} + P_2 = \frac{1}{\lambda_3} + P_3 = \dots = D. \quad (19)$$

Thus, the channel with the highest gain receives the largest share of the power. The constraint on the powers is that

$$\sum P_i = P. \quad (20)$$

The weight factors  $\alpha_i$  in Fig. 1 equal  $P_i/P$ . In case the level  $D$  drops below a certain  $1/\lambda_i$  then that power is set to zero. In the limit where the  $SNR$  is small ( $P < 1/\lambda_2 - 1/\lambda_1$ ), only one eigenvalue, the largest, is left and we are back to the maximum gain solution of the previous section. For the case of  $(M, N) = (2, 2)$  (Fig. 2) for  $P < 2 - 2/7 = 2.34$  dB, only the largest eigenvalue is active, using the mean values from (10).

The capacity equals

$$C = \sum_{N'} \log_2(1 + \lambda_i P_i) = \sum_{N'} \log_2(\lambda_i D) \quad (21)$$

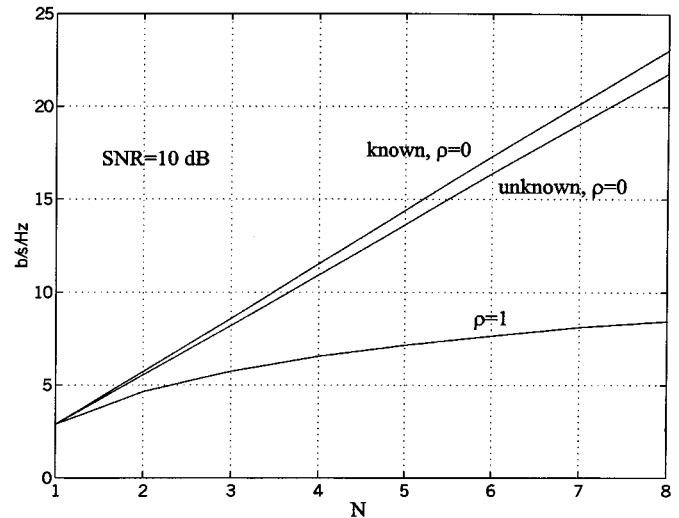


Fig. 6. Mean capacity for two arrays of each  $N$  elements. The capacity grows linearly with the number of elements and is approximately the same for the known and the unknown channel. The total transmitted power is constant.

where the summation is over all channels with nonzero powers. The water filling is of course dependent on the knowledge of the channels on the transmit side, which is the case of the known channel in the terminology of the previous sections.

In the case where the channel is unknown at the transmitter, the only reasonable division of power is a uniform distribution over the antennas, i.e.,

$$P_i = \frac{P}{M} \quad (22)$$

$M$  being the number of transmit antennas [4]. It may also be argued that the transmit antenna "sees"  $M$  eigenvalues, not taking into account that there are only  $N$  nonzero eigenvalues. Thus, power is lost by allocating power to the zero-valued eigenvalues.

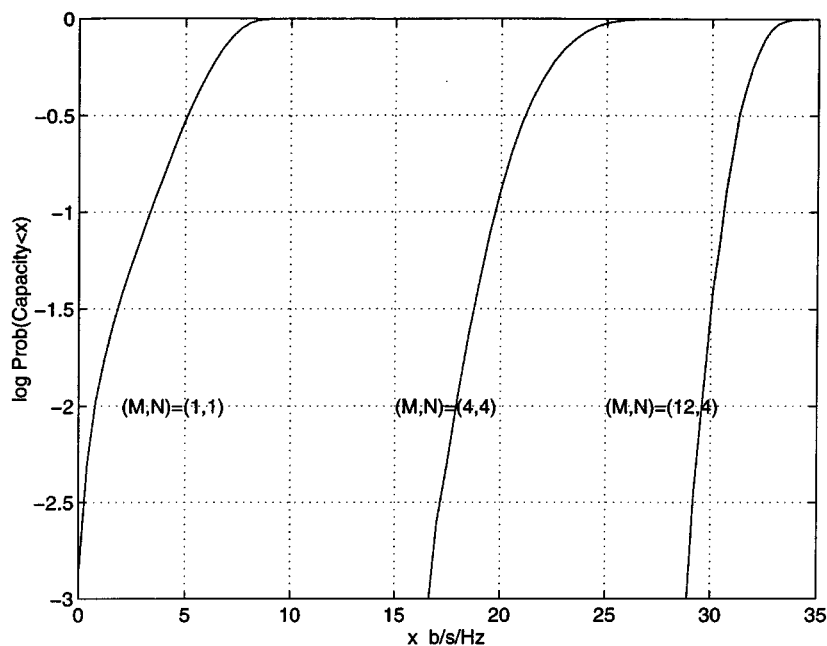


Fig. 7. The cumulative probability distribution of capacity on a log scale for the  $(M, N) = (1, 1)$ ,  $(4, 4)$ , and  $(12, 4)$  case. The basic signal-to-noise ratio is 20 dB. The total radiated power is the same in all cases.

It also follows that when  $M = N$ , the difference between the capacity for known and unknown channels is small for large  $P$ .

#### A. Capacity of the $(M, N)$ Array

It follows from (15) and (16) that the instantaneous eigenvalues are limited by

$$\left(\sqrt{M} - \sqrt{N}\right)^2 < \lambda_i < \left(\sqrt{M} + \sqrt{N}\right)^2. \quad (23)$$

So for  $M$  much larger than  $N$ , all the eigenvalues tend to cluster around  $M$ . Furthermore, each of them will be nonfading due to the high  $MN$ th order diversity. Thus, the uncorrelated asymmetric channel with many antennas has a very large theoretical capacity of  $N$  equal, constant channels with high gains of  $M$ . The above illustrates in a mathematical sense the observation of Winters [2] that  $M$  should be of the order  $2N$ . In the limit of large  $M$  and  $N$ , with  $M$  much larger than  $N$ , the capacity is easily found to be

$$C = N \log_2 \left(1 + \frac{P}{N} M\right) \quad (24)$$

with the result that the theoretical capacity grows linearly with the number of elements  $N$  [2], [4], [6] for  $M/N$  fixed. The result may conveniently be interpreted as  $N$  parallel channels, each with one  $N$ th of the power and each having a gain of  $M$ . Note that this capacity is higher than the one used in [2], [3], and [6], where the power is divided between the  $M$  antennas instead of the  $N$  channels (25)

$$C_{\text{unknown}} = N \log_2(1 + P). \quad (25)$$

The numerical results shown in Fig. 6 support this approximate analysis for the mean values. It should be remembered that the potential gains are higher when a certain outage probability is studied due to the high order diversity effects. This

is illustrated in Fig. 7, which shows the cumulative probability distribution (on a log scale) of the capacity for the case of 4 receiving elements, and 4 and 12 transmitting elements. The signal-to-noise ratio is 20 dB for the (1,1) case, and it is worth emphasizing that the total power radiated remains constant. The improvement going from 4 to 12 transmitting antennas is mainly due to the improved gain of the smallest eigenvalues as indicated by (23). Applying (24) to the (12, 4) case gives 32.9 b/s/Hz.

## VI. DISCUSSION

The benefits of having arrays with multiple elements at each end of a communication link are large. In this paper it is generally assumed that the channel is known, and it is shown that a simple iterative technique with optimizing of the power on each side at each iteration rapidly converges to the maximum gain for reciprocal channels. Reciprocal channels may be found in some modern TDD systems, and if the terminals are quasi-stationary, it is a good approximation to assume a reciprocal channel. A more rigorous matrix method defines the maximum gain as the maximum eigenvalue of the correlation matrix of the system; and by applying some new results from statistics of complex matrices, some upper bounds for the gain are given. The result is that two  $N$  element arrays (which are connected by  $N^2$  paths) have an  $N^2$ -order diversity, so the diversity gain rapidly becomes large with growing  $N$ . The mean gain, also denoted the array gain, grows at a slower pace, approximately bounded by  $4N$ . In case the system is asymmetric with  $M(M > N)$  transmit elements, the gain is even further increased, asymptotically approaching  $M$ . This asymmetry also has an effect on the spectral efficiency, which in this case is bounded by  $N$  parallel channels with a constant gain of  $M$ .

In a real environment, there will not be complete decorrelation among the antenna elements, which means that the capacity will decrease, since the number of useful independent channels

will decrease. From a gain point of view, it is actually an advantage with high correlation since the mean gain (the mean of the largest eigenvalue) will increase toward  $MN$ . In practice, high correlation between elements is indicative of a few paths, adaptively locking on to those will lead to a gain approaching the free space gains. In order to assess the gain and capacity as a function of carrier frequency, number of elements, and environment, more experimental propagation data are needed.

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