# AS Relationships: Inference and Validation

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## ABSTRACT

Research on performance, robustness, and evolution of the global Internet is fundamentally handicapped without accurate and thorough knowledge of the nature and structure of the contractual relationships between Autonomous Systems (ASs). In this work we introduce novel heuristics for inferring AS relationships. Our heuristics improve upon previous works in several technical aspects, which we outline in detail and demonstrate with several examples. Seeking to increase the value and reliability of our inference results, we then focus on validation of inferred AS relationships. We perform a survey with ASs' network administrators to collect information on the actual connectivity and policies of the surveyed ASs. Based on the survey results, we find that our new AS relationship inference techniques achieve high levels of accuracy: we correctly infer 96.5% customer to provider (c2p), 82.8% peer to peer (p2p), and 90.3% sibling to sibling (s2s) relationships. We then cross-compare the reported AS connectivity with the AS connectivity data contained in BGP tables. We find that BGP tables miss up to 86.2% of the true adjacencies of the surveyed ASs. The majority of the missing links are of the p2p type, which highlights the limitations of present measuring techniques to capture links of this type. Finally, to make our results easily accessible and practically useful for the community, we open an AS relationship repository where we archive, on a weekly basis, and make publicly available the complete Internet AS-level topology annotated with AS relationship information for every pair of AS neighbors.

## Categories and Subject Descriptors

C.2.5 [**Local and Wide-Area Networks**]: Internet; C.2.1 [**Network Architecture and Design**]: Network topology

## General Terms

Measurement, Verification

## Keywords

AS relationships, inference, routing policies

## 1. INTRODUCTION

The global Internet routing system is composed of thousands of Autonomous Systems (ASs) that operate individual parts of the Internet infrastructure. ASs engage in a variety of relationships to collectively and ubiquitously route traffic in the Internet. These relationships are usually realized in the form of business agreements that, in turn, translate into engineering constraints on traffic flows within and across individual networks.

Understanding the underlying business AS relationships plays a critical role in many research and operational tasks ranging from realistic simulations of packets routed in the Internet to selection of peers or upstream providers based on connectivity and AS relationships of candidate ISPs. Further, statistical data on these relationships are useful for development of more advanced interdomain routing protocols and architectures that take into account the presence of AS relationships to improve their performance [27]. Moreover, business behavior patterns of Internet players influence directions of ISPs' collaboration and ultimately the evolution of the macroscopic infrastructure of the Internet.

In this study we follow previous works [15, 26, 4, 14] in considering the following three major categories of AS relationships: *customer-to-provider* (c2p), *peer-to-peer* (p2p), and *sibling-to-sibling* (s2s). In the c2p category, a customer AS pays a provider AS for any traffic sent between the two.[1] In the p2p category, two ASs freely exchange traffic between themselves and their customers, but do not exchange traffic from or to their providers or other peers. In the s2s category, two ASs administratively belong to the same organization and freely exchange traffic between their providers, customers, peers, or other siblings.

Our work makes the following contributions:

1. We introduce novel heuristics for inferring c2p, p2p, and s2s relationships. Our heuristics improve the state-of-the-art in several technical aspects, one of them being a more realistic problem formulation that accepts that AS paths do not always exhibit a hierarchical pattern. We demonstrate using several examples our enhancements that lead to more accurate inference results.
2. We conduct a survey with organizations operating ASs, from which we retrieve company-verified information about the actual types of relationships they have with other networks. We use this information to validate the AS relationships we infer and find that they are highly accurate. To our knowledge, this study is the most exhaustive AS relationship validation effort to date.

[1] We use acronym *c2p* to refer to customer to provider relationships in general, as well as to links $A$-$B$, where AS $A$ is a customer of AS $B$. In contrast, we use acronym *p2c* to refer only to links $A$-$B$, where AS $A$ is a provider of AS $B$.

3. Using company-verified data we confirm previous measurement results [9, 23] on the poor coverage of AS topologies. In addition, we verify the commonly held assumption that most of the missing links are of p2p type.
4. To promote further analysis and discussion of the macroscopic Internet topology, we introduce a publicly available AS relationships repository [7]. We automate our heuristics and archive datasets of annotated AS links on a weekly basis. We also compute and publish ranking of ASs based on inferred AS relationship hierarchies [8].

This paper follows our earlier work [13] on inferring c2p relationships. It addresses the issue left open of how to select the most realistic from the candidate solutions to our c2p problem formulation. It then extends our previous work by: 1) introducing new heuristics for the inference of p2p and s2s relationships, 2) validating our inferences, and 3) developing an open AS relationships repository.

We organize the paper as follows. In the next section we introduce and describe in detail our heuristics. We compare our approach to inferring AS relationships with previous ones and discuss our improvements. In section 3 we apply the developed heuristics to Internet data and fully annotate a snapshot of the AS topology with the computed types of relationships. We also briefly discuss our ranking of ASs based on inferred AS relationship hierarchies. In section 4, we describe the results of our AS survey, validate our heuristics, and analyze the true AS relationships that we learned from the participating ASs. Finally, we conclude in section 5.

# 2. INFERENCE HEURISTICS

## 2.1 Preliminaries

Gao's seminal work [15] was the first to formulate and systematically study the AS relationships inference problem. Gao assumed that every BGP path must comply with the following hierarchical pattern: an uphill segment of zero or more c2p or s2s links, followed by zero or one p2p links, followed by a downhill segment of zero or more p2c or s2s links. Paths with this hierarchical structure are *valley-free* or *valid*. Paths that do not follow this hierarchical structure are called *invalid* and may result from BGP misconfigurations or from BGP policies that are more complex and do not distinctly fall into the c2p/p2p/s2s classification. Following this definition of valid paths, Gao proposed an inference heuristic (which we denote as GAO) that identified top providers and peering links based on AS degrees and valid paths.

Following Gao's work, Subramanian *et al.* [26] developed a mathematical formulation of the inference problem. They cast the inference of AS relationships into the *Type of Relationship* (ToR) combinatorial optimization problem: given a graph $G(V, E)$ derived from a set of BGP paths $P$, assign the edge type (c2p or p2p; s2s relationships are ignored) to every edge $i \in E$ such that the total number of valid paths in $P$ is maximized. The authors speculated that ToR is NP-complete and developed a heuristic solution, which we refer to as SARK below.

The SARK approach takes as input the BGP tables collected at different vantage points and computes a *rank* for

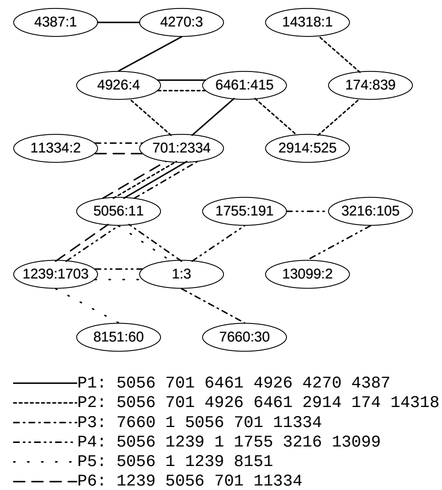



**Figure 1: An instance of the ToR problem that does not admit a solution. Each circle is marked with a tuple $X : Y$, where $X$ is the AS number and $Y$ is the AS degree seen in our AS topology. The paths at the bottom yield the ToR instance.**

every AS. This rank is a measure of how close to the graph core an AS lies and it is equivalent to node *coreness* [3, 2]. The heuristic then infers AS relationships by comparing ranks of adjacent ASs. If the ranks are similar, the algorithm classifies the link as p2p, otherwise it is c2p.

Di Battista *et al.* [4] and Erlebach *et al.* [14] independently showed that ToR is indeed NP-complete, developed mathematically rigorous approximate solutions to the problem and proved that it is impossible to infer p2p relationships under the ToR formulation framework. For this reason, their solutions (referred to as DPP and EHS) infer c2p relationships only and ignore p2p and s2s relationships.

Despite the ToR formulation being substantially studied, we find that it bears limitations that lead to incorrect inferences. We describe these limitations with the following examples.

***Example 1.*** **Ignoring s2s relationships causes proliferation of erroneous inferences.** Consider a path $p = \{ij\} \in P$ that includes an edge $i$ appearing in multiple paths, and an edge $j$, appearing only in this path $p$. Suppose that in reality $i$ is a sibling edge and $j$ is a c2p edge. It is convenient to represent a c2p edge annotation by making the edge directed *from* the customer AS *to* the provider AS. Depending on the structure of other paths containing the sibling edge $i$, a ToR solution can direct it as either c2p or p2c. If it gets directed as p2c, then to make the path $p$ valid, the algorithm has to erroneously direct the edge $j$ as p2c, too.

***Example 2.*** **A solution maximizing the number of inferred-to-be-valid paths is not necessarily correct.** Consider the real-world instance of the ToR problem in Figure 1, which was used in [4] to introduce ToR. In this setting there are four distinct combinations of edge orientations, each maximizing the number of valid paths, but rendering one of the paths P1, P2, P4, or P5 as invalid. In path P6, AS 5056 with degree 11 appears to transit traffic between large providers, AS 701 (UUNET) and AS 1239 (Sprint). A ToR solution will treat path P6 as valid. Thus, it must infer AS 5056 as a provider of either UUNET or Sprint, or a provider of both, all of which are incorrect. The key point of this example is that while it is reasonable to assume that most AS paths in the Internet have a valid hierarchical structure, it is still possible that some paths in real networks have a non-hierarchical invalid structure. An attempt to annotate such paths based on the valley-free model will result in unrealistic relationships.

***Example 3.*** **In cases when there are multiple solutions with the same number of valid paths, ToR has no means to deterministically select the most realistic solution.** Instead, it has to randomly attribute validity to one of the available solutions. Consider path $p \in P$ that is a sequence of edges $i_1, i_2, \ldots, i_{|p|-1}, j \in E$. Suppose that the last edge $j$ appears only in this one path $p$ and that it is from a large provider (such as UUNET) to a small customer. Suppose that other edges $i_1, i_2, \ldots, i_{|p|-1}$ appear in several other paths and are correctly inferred as c2p. In this scenario both orientations of the edge $j$ (i.e. correct p2c and incorrect c2p) render path $p$ valid. Thus, this edge cannot receive a deterministic direction from the ToR solution. This example explains why Rimondini [24] found several well-known large providers such as AT&T, Sprint, Level3 to be inferred as customers of smaller ASs such as AS2685 (degree 2), AS8043 (1) and AS13649 (7), respectively. We also observed incorrect inferences of this type in our experiments.

The above examples illustrate that: 1) it is necessary to account for s2s relationships; 2) trying to simply maximize the number of valid AS paths may result in incorrect AS relationship inferences; and 3) without additional information, the ToR framework by itself is insufficient to ensure a deterministic inference of AS relationships.

In the following subsections we address these shortcomings and present heuristics to determine AS relationships more accurately.

## 2.2 Inferring s2s relationships

Sibling links connect ASs belonging to the same organization, and thus communication between s2s ASs is not subject to the export restrictions found in c2p and p2p relationships. For example, rules such as "prefixes learned from a peer cannot be announced to other peers" do not apply for sibling ASs. Therefore, sibling ASs can implement much more flexible and diverse policies than non-affiliated ASs, making it very difficult to infer s2s relationships from BGP data. For this reason we utilize the IRR databases to annotate s2s links. We then remove s2s edges from both graph $G$ and path set $P$ to avoid proliferation of incorrect c2p inferences. In effect, we abrogate the limitations of Example 1 by independently inferring s2s relationships.

Specifically, we track the organization to which each AS is registered in the databases and create groups of sibling ASs registered to the same organization. In several cases sibling ASs are registered to syntactically different organizational names, which still represent the same organization by other measures. For example, ASs 7018 and 3339 are registered to "AT&T WorldNet Services" and "AT&T Israel", respectively. To find such cases, we examine the organization names and manually create a dictionary of organization name synonyms. Then, we infer as s2s the ASs that are registered to the same organization name or to synonymous organization names.

The strength of our approach is that it takes advantage of explicit information contained in the IRR databases. Although we realize these databases are not always up-to-date or perfectly accurate, the organization names change less frequently than BGP policies and other more dynamic attributes. We can therefore treat the IRR databases as a source of publicly available information, which is reasonably accurate for the purpose of inference of s2s relationships.

## 2.3 Improving the integrity of c2p inferences

In Example 2 we demonstrated that trying to maximize the number of inferred-to-be-valid paths can lead to incorrect inferences since in reality AS paths are not always hierarchical. To address this limitation we construct a c2p-inference heuristic that is based on the idea of relaxing the requirement for a maximal number of valid paths and using the AS degree information to detect paths that are invalid and that we should not try to direct as valid. We formalize this idea as follows.

For every edge $i \in E$ we introduce a weight $f(d_i^-, d_i^+)$ that is a function of the degrees $d_i^-$ and $d_i^+$ ($d_i^- \leq d_i^+$) of the ASs adjacent to the edge $i$. The weight $f$ is large when there is a significant degree difference ($d_i^- \ll d_i^+$) between these neighboring ASs, and small otherwise. In directing the edges of the graph, we use $f$ in the following way: when an edge $i$ is directed from a small-degree AS to a large-degree AS, it earns a *bonus* $b_i$ equal to $f(d_i^-, d_i^+)$, otherwise $b_i = 0$. We then formulate the inference problem as the following multiobjective optimization problem:

$O_1$ Maximize the number of valid paths in $P$;
$O_2$ Maximize the sum $\sum_{i \in E} b_i$.

These two methodological objectives can be conflicting. Consider again the Example 3 using Figure 1. According to the objective $O_1$, at least one of the edges 1239-5056 or 5056-701 in P6 must be directed against the node degree gradient in order to render P6 valid. By introducing the second objective $O_2$, we relax the first objective's requirement for the maximal number of valid paths. We can thus accept an "invalid" orientation for P6 based on the strong degree-gradient indication ($O_2$) that neither 1239 nor 701 are customers of 5056.

This formulation combines the strengths of previous works. First, it is similar to SARK, DPP and EHS, in that it respects the valley-free model and tries to maximize the number of valid paths in the input path set $P$. Secondly, it is similar to GAO, in that it uses the implicit knowledge embedded in AS degree information to assign directions to edges along the node degree gradient by giving certain weighted preference to edge orientations collinear with this gradient.

To solve the newly formulated optimization problem, we map the c2p or p2c relationship of edge $i$ to boolean variable $x_i$ as follows: assuming an arbitrary initial direction of $i$, an assignment of *true* to $x_i$ means that edge $i$ keeps its original direction, while an assignment of *false* to $x_i$ reverses the direction of $i$. We find assignments to variables $x_i$ by reducing the multiobjective optimization problem to the well-known MAX2SAT problem.

MAX2SAT is a boolean algebra problem: given a set of clauses with two boolean variables per clause $l_i \vee l_j$, find an assignment of values to variables maximizing the number of simultaneously satisfied clauses [16]. If the clauses are weighted, the problem is to maximize the sum of weights of the simultaneously satisfied clauses. MAX2SAT is NP-complete, however, the semidefinite programming (SDP) approach [17] delivers an approximate answer that differs from the exact answer by not more than a factor of 0.94.

To reduce the objective $O_1$ (ToR) to MAX2SAT we use the approach of DPP and EHS [4, 14]. This gives a set of $x_i \vee x_j$ clauses, where $i, j \in E$.

To reduce the objective $O_2$ to MAX2SAT, we introduce a clause $x_i \vee x_i$ for every edge $i \in E$ that has an initial direction along the node degree gradient, and a clause $\bar{x_i} \vee \bar{x_i}$ for every edge with an initial direction against the node degree gradient. We thus ensure that if an edge is directed along the node degree gradient, then the corresponding clause is satisfied. To make our MAX2SAT instance equivalent to $O_2$, we weight every clause by $b_i = f(d_i^-, d_i^+)$.

We then reduce the resulting multiobjective optimization problem to MAX2SAT by refining the weights of the clauses. We introduce a parameter $\alpha$ and weight the objective $O_1$ by $\alpha$ and the objective $O_2$ by $1 - \alpha$:

$$w_{ij}(\alpha) = \begin{cases} c_1 \alpha & \text{for } O_1 \text{ clauses,} \\ c_2(1-\alpha) f(d_i^+, d_i^-) & \text{for } O_2 \text{ clauses.} \end{cases} \tag{1}$$

The normalization coefficient $c_1$ is determined from the condition $\sum_{i \neq j} w_{ij}(\alpha) = \alpha \Rightarrow c_1 = 1/m_1$, where $m_1$ is the number of $O_1$ clauses. The normalization coefficient $c_2$ is determined from the condition $\sum_i w_{ii}(\alpha) = 1 - \alpha$. Varying $\alpha$ in the region between 0 and 1 controls the relative preference of the two objectives.[2] We explore the tradeoff between the objectives $O_1$ and $O_2$ and adjust $\alpha$ to the region or the point that results in the most accurate AS relationship inferences (cf. discussion of the optimal value of $\alpha$ in section 3.2).

Function $f$ encodes dependence on AS degrees into our inference process. This function should take large values when its two degree arguments differ significantly, otherwise its values should be small, because neighboring ASs with significant size difference typically have a customer to provider relationship and AS size is strongly correlated to AS degree [28]. We note that a given absolute difference in AS degrees is of different importance for small ASs and for large ASs. For example, a degree difference of 50 says more about the relative size of two ASs of degrees 1 and 51, than of 3000 and 3050. To account for this relative importance, we normalize the degree difference in $f$ to the *relative* node degree gradient $(d_i^+ - d_i^-)/(d_i^+ + d_i^-)$. In addition, topology graphs derived from BGP data provide only approximations of the true AS degrees. They tend to underestimate degrees of small ASs but yield more accurate degree approximations for larger ASs [9]. To model this effect, we introduce a logarithmic factor reflecting our stronger confidence in accuracy of large AS degrees, compared to small ones. We thus construct $f$ as:

$$f(d_i^+, d_i^-) = \frac{d_i^+ - d_i^-}{d_i^+ + d_i^-} \log(d_i^+ + d_i^-). \tag{2}$$

In summary, our formulation of the c2p relationship inference problem exploits the structure of the AS paths to address the limitations that we illustrated in Examples 2 and 3 of section 2.1.

[2] In the terminology of multiobjective optimization [11], we consider the simplest scalar method of weighted sums.

## 2.4 Inferring p2p relationships

The inference of p2p relationships is more challenging than the inference of c2p relationships. As both DPP and EHS show, it is impossible to infer p2p relationships within the ToR formulation framework. Indeed, a valid path can have only one p2p link adjacent to the top provider in the path. If we replace this p2p link with a c2p or p2c link, the path remains valid, as it still has a valley-free, hierarchical structure. Therefore, maximizing the number of valid paths as is done by ToR, one cannot deterministically infer any p2p relationships at all. Confirming the difficulty of inferring p2p relationships comes a work by Xia and Gao [30], who find that GAO and SARK's p2p inference heuristics yield a low accuracy of, respectively, 49.08% and 24.63% of correct p2p inferences.

To improve the inference of p2p relationships, we develop a heuristic that combines GAO and DPP strengths. We start from a set of BGP paths $P$ and extract a graph $G$ from it. Then we preprocess $P$ to identify links that are not of p2p type (non-p2p).

According to the valley-free model, a path can have at most one p2p link and this link must be adjacent to the top provider of the path. We thus parse all paths in $P$ and denote all links that are not adjacent to the highest degree AS in a path as non-p2p. This approach is similar but not identical to GAO. GAO assumed that 1) a p2p link can lie only between the highest degree AS in a path and its highest degree neighbor and 2) that the degree ratio between the two edge ASs of a p2p link is smaller than an external parameter (discussed below). This method is aggressive in excluding non-p2p links. To illustrate, consider an AS path A-B-C-D, where AS degrees are $d_A = 10$, $d_B = 500$, $d_C = 1000$, and $d_D = 501$. GAO allows only link C-D to be of p2p type and denotes the others as non-p2p. However, the degree difference between B and D is too small to make this judgment reliably. Our heuristic addresses this shortcoming by including both B-C and C-D as candidate p2p links. We denote by $R$ the set of possible p2p edges constructed this way.

We then introduce a weight $g(d_i^-, d_i^+)$ for every edge $i \in R$. Weight $g$ is large when the ASs adjacent to the edge $i$ have similar degrees, and small otherwise. Such weighting expresses our higher confidence that a pair of neighboring ASs are peers when their degrees are similar. Our selected weight $g$ complements the weight $f$ used for the inference of c2p links:

$$g(d_i^-, d_i^+) = 1 - c_3 f(d_i^-, d_i^+), \tag{3}$$

where $c_3 = 1/\max_{i \in E} f(d_i^-, d_i^+)$ is a normalization coefficient.

Next, we remove from $R$ any links that connect ASs with large degree differences $d_i^- \ll d_i^+$. More specifically, we introduce a threshold $w_e \in [0, 1]$ and remove every edge $i$ with $g(d_i^-, d_i^+) < w_e$. The GAO heuristic used an empirically selected value of 60 or $\infty$ for a similar threshold. We improve upon this approach by using information learned from our survey (see section 4) to choose a proper value for $w_e$. Namely, for each true p2p and c2p link present both in our survey results and in $R$, we examine what selection of $w_e$ leads to: 1) erroneously excluding a true p2p link from the set of possible p2p links $R$, meaning that $g(d_i^-, d_i^+) < w_e$ for a true p2p link; and 2) erroneously not excluding a true c2p link from the set of possible p2p links $R$, meaning that

$g(d_i^-, d_i^+) > w_e$ for a true c2p link. We find that the value of $w_e$ that minimizes errors is $g(3, 545)$. The need for external threshold $w_e$ is unfortunate, but the large degree difference between $d^- = 3$ and $d^+ = 545$ indicates that this threshold simply cleans $R$ of links that are unlikely to be of p2p type.

At the last step of our p2p inference process, we examine those paths in $P$ that contain more than one edge from $R$. Such paths violate the valley-free model, and we need to classify some links from $R$ as non-p2p in order to resolve this violation. DPP showed that the problem of finding a maximal set of p2p links that do not introduce invalid paths in $P$ is equivalent to the Maximum Independent Set (MIS) problem. In the MIS formulation, we are given a graph with nodes in $N$ and arcs in $A$ and we need to find the maximum subset of $N$ such that no two nodes of the subset are joined by an arc in $A$. To increase the reliability of the p2p link determination, we utilize our assigned link weights $g$ and turn the MIS problem into the Maximum Weight Independent Set (MWIS) problem. In the MWIS formulation, we give preference to edges with large weights because we know that these edges are more likely to be of p2p type. We solve the NP-complete MWIS problem by means of a polynomial time approximation [6] and find a maximal weight subset of $R$ that does not create invalid paths in $P$. We denote this subset as $F$ and admit it as our final set of p2p links.

## 2.5 Summary of inference heuristics

In summary, our inference heuristics take as input a set of BGP paths $P$ and a corresponding graph $G(V, E)$ and perform the following three consecutive steps:

1. Use IRRs to infer s2s relationships and create set $S \subset E$ of s2s links;
2. Remove the subset $S$ from consideration and apply our heuristic assigning c2p/p2c relationships to the links remaining in $E \setminus S$;
3. Use $P$ and $G$ to infer p2p relationships and to create set $F \subset E$ of p2p links.

The final result is set $S$ of s2s links, set $F$ of p2p links, and set $E \setminus F \setminus S$ of c2p links.

## 2.6 Related work

In comparison with other approaches to AS relationship inference, our heuristics offer a number of improvements. In contrast to DPP [4] and EHS [14], we identify not only c2p, but p2p and s2s relationships as well. Moreover, our c2p heuristic addresses the limitations we discussed in section 2.1 with ToR solutions.

The work by Subramanian *et al.* [26] introduced the ToR problem and the SARK heuristic [26] for solving ToR. SARK used node coreness [3, 2], which reflects ASs' topological positions in AS graphs, as a metric for inferring c2p and p2p relationships. In contrast, our heuristics use AS degrees and policies encoded in AS paths to infer c2p and p2p relationships.

The work by Gao [15] used AS degrees and the valley-free model to infer c2p, p2p, and s2s relationships. GAO algorithm treats every AS path as a hint of true types of links in the path. It takes a set of AS paths as input, directs every link toward the highest degree AS in the path, and after parsing all paths, counts the directions each link has accumulated. If a link has received consistent directions throughout the process, it is marked as c2p with the provider being at the top of the directed link. Otherwise, the link is marked as s2s. Similarly to this work, our heuristics employ the valley-free model and AS degrees to infer c2p and p2p relationships, but we make a number of technical enhancements, which we outline in detail in sections 2.3 and 2.4. We use the IRR databases to infer s2s relationships, since it is hard to reliably infer them from BGP data.

Xia and Gao [30] used the IRR databases to extract relationships among a subset of ASs and proposed a variation of the GAO heuristic that takes this subset as an input to infer other AS relationships. They demonstrated how accurate and current IRR databases provide explicit information on AS relationships. On the other hand, dealing with IRR data has its own intrinsic methodological problems: 1) it is much harder to automate; 2) the data is not always accurate and its accuracy level is hard to estimate; 3) not all ASs are registered. In our work we also use the IRR data but only for s2s relationship inference. For this task, we process the organization description records, which are relatively stable over time, compared to policy-related records.

Mao *et al.* [20] proposed an AS relationship inference technique that employs the valley-free model to infer c2p and p2p relationships. The technique introduces a set of new interesting ideas based on the assumption that ASs prefer shorter AS paths over longer AS paths. This assumption does not however hold when ASs use routing policies to select the next-hop AS on the basis of its policy ranking, regardless of AS path lengths.

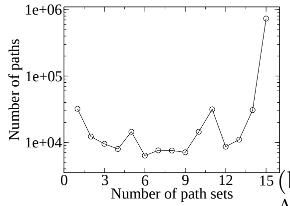
Recent work by Muehlbauer *et al.* [22] introduced a shift from inferring AS relationships to inferring AS paths using a model with *agnostic* AS relationships and multiple routers per AS. They found that their model leads to more accurate results, as far as accuracy of capturing path diversity is concerned, than a model using inferred AS relationships and a single router per AS. By definition, agnostic approaches cannot however capture precise characteristics of individual ASs. Therefore, agnostic approaches are not appropriate for tasks such as constructing realistic economy-based evolution models of ASs. In addition, [22, 29] assumed that models with c2p/p2p/s2s relationships are equivalent to models with a single router per AS. The former models can however be extended to use multiple routers per AS, and such extensions may result in significantly higher path diversity than [22] reported.

# 3. APPLYING HEURISTICS TO THE DATA

## 3.1 Collecting and sanitizing the data

We first construct the input BGP path set $P$ and the corresponding graph $G(V, E)$. We collected BGP tables from RouteViews [21], at 8-hour intervals, over the period from 03/01/2005 to 03/05/2005, for a total of 15 BGP table instances. After cleaning out AS prepending and AS sets, each BGP table yields a path set $P_k$.

Invalid paths caused by BGP misconfigurations occur quite often and affect 200-1200 BGP table entries each day [19]. To mitigate the impact of these misconfigurations, we sanitize the input data as follows. We define the *persistency* of a path $p \in \cup_{k=1}^{15} P_k$ as the number of sets $P_k$ containing $p$. The persistency distribution (Figure 2(a)) shows that although the majority of the paths appear in most of the 15 sets, a significant number of paths appear only in a few of the sets.




(a) Persistency distribution of paths in $\cup_{k=1}^{15} P_k$.

| | Stable paths ($P$) (decrease) | Average over $P_k$ |
|---|---|---|
| Paths | 733,931 (-12.49%) | 838,734 |
| Links | 38,282 (-4.34%) | 40,023 |
| ASs | 18,943 (-1.87%) | 19,305 |

(b) Numbers of paths, links, and ASs in the stable path set vs. the same numbers averaged over $P_k$, $k = 1 \ldots 15$.

**Figure 2: Stable paths set $P$ vs. unprocessed path sets $P_k$.**

**Table 1: Number of unique degree-valleys and total number of degree-valleys found in stable and unstable path sets.**

| window $w$ | Unique degree-valleys | | Total degree-valleys | |
|---|---|---|---|---|
| | Stable paths | Unstable paths | Stable paths | Unstable paths |
| 5 | 206 | 241 (+17%) | 1290 | 2368 (+83.6%) |
| 10 | 167 | 208 (+24.6%) | 1178 | 2290 (+94.4%) |
| 15 | 150 | 190 (+26.6%) | 1135 | 2135 (+88.1%) |
| 20 | 141 | 171 (+21.2%) | 1119 | 1609 (+43.8%) |

Since BGP misconfigurations are temporal events, we select as input $P$ to our algorithm the paths that appear in all of the 15 sets $P_k$. We call $P$ the *stable path* set and the paths that are not selected the *unstable path* set.

In Figure 2(b) we compare the number of paths, links and ASs in the stable paths set with the corresponding averages over $P_k$. Even though $P$ is 12.49% smaller than the average size of $P_k$, our filtering of unstable paths does not entail significant information loss in terms of number of links (4.34% reduction) and ASs (1.87% reduction).

We also verify that the unstable paths include more non-hierarchical degree sequences, which is often an indication of a misconfiguration [19], than the stable paths. We call a *degree-valley* any AS sequence A-B-C with degrees $d_A$, $d_B$, and $d_C$, such that both $d_A$ and $d_C$ are larger than $d_B$ plus a small margin constant $w$: $d_A, d_C > d_B + w$. (The small margin $w$ is added to filter out trivial differences between $d_B$ and $d_A$, $d_C$.) Then, for both the stable and unstable path sets, we 100 times randomly select 10,000 paths and count the number of degree-valleys for different $w$. Table 1 shows the average number of unique degree-valleys and the average of the total number of degree-valleys in the selected paths. The number of degree-valleys in the unstable paths is between 17% and 94.4% larger than in the stable paths.

## 3.2 Inferring AS relationships

***s2s relationships.*** To infer s2s relationships in our graph we use the RIPE, ARIN, and APNIC databases, collected on 06/10/2004.[3] We analyze the databases according to the methodology outlined in section 2.2 and find 1,943 organizations that own multiple AS numbers. We then examine the input graph $G$ and discover 177 edges between ASs that belong to the same organization ($|S| = 177$).

***c2p relationships.*** We remove edges inferred as s2s from $E$ and apply our methodology detailed in section 2.3 to the remaining links $E \setminus S$. Our implementation uses parts of the code from EHS [14], the LEDA v4.5 software library [1], and a publicly available SDP solver DSDP v4.7 [5]. We compute orientations of the edges in $E \setminus S$ for different values of $\alpha$, sampling densely the interval between 0 and 1. Recall that when $\alpha = 1$, our problem formulation is equivalent to the original ToR formulation, whereas $\alpha = 0$ corresponds to entirely degree-based relationship inference.

To evaluate the computed orientations, we introduce a metric called *reachability*. We define reachability of an AS $X$ as the number of ASs one can reach from this AS traversing only p2c edges. The reachability of an AS has the following two properties: 1) it is determined entirely from the inferred c2p relationships; and 2) it induces a natural hierarchy of ASs based on the size of their customer trees. These two properties enable us to perform an initial validation of the inferred c2p relationships by matching the top ASs in the calculated hierarchy against the empirically known largest ISPs in the Internet.

We sort all ASs by their reachability, and group ASs with the same reachability into *levels*. ASs at the highest level have the largest trees of customer ASs. ASs at the lowest level have the smallest reachability. We then define the position *depth* of an AS $X$ as the number of ASs at the reachability levels above the level of the AS $X$. We define the position *width* of an AS $X$ as the number of ASs at the same level as the AS X.

In Table 2 we examine the top five ASs in the hierarchies calculated for the two extreme cases, $\alpha = 0$ and $\alpha = 1$. When $\alpha = 0$, the well-known ISPs: UUNET, AT&T, Sprint, Level 3, and Qwest occupy the top five positions in the hierarchy. On the other hand, when $\alpha = 1$, these positions are taken by ASs of very small degrees, e.g., AS13987 of degree 3. The columns in the table track the position of these ASs in the hierarchies induced for $\alpha$ equal to 0, 0.01, 0.05, 0.1, 0.5, and 1. We observe that as $\alpha$ gets closer to 1, the well-known ASs drift away from the top of the hierarchies, thus highlighting an increasingly stronger deviation from reality. This deviation is maximized when $\alpha = 1$ (the original ToR formulation), demonstrating the limitations of AS relationship inference based solely on maximization of the number of valid paths.

The induced hierarchies suggest that solutions with values of $\alpha$ close to 1 are incorrect since they propel small ASs to the top of the hierarchy, while well-known ISPs sink to lower positions. On the other hand, the percentage of invalid paths, listed at the top of Table 2, attains its maximum of 12.75% when $\alpha = 0$. The latter observation suggests that the solution with $\alpha = 0$ is also incorrect since a large number of paths violates the valley-free routing model. Taken together, these two observations indicate that intermediate values of $\alpha$ yield best solutions to our multiobjective optimization resulting both in realistic hierarchies and in small numbers of invalid paths. We emphasize that there is no oracle, intrinsic to the multiobjective optimization problem formulation, that would reveal the proper balance between the two objectives and the corresponding "right" value of $\alpha$. As is typically the case with multiobjective optimization [11], we must exercise our external expert knowledge of data specifics to sift out the most realistic relative weight of the objectives. For our problem, we formalize this expert insight as follows: we search for the value of $\alpha$ corresponding to the smallest percentage of invalid paths among all the solutions that have only well-known ISPs at the top of the

[3]Since we extract from these databases the information that changes slowly with time, the date of the database dump is not critically important.

**Table 2: The reachability-based hierarchy of ASs and percentage of invalid paths as functions of $\alpha$. For different values of $\alpha$, we show the position *depth* (the number of AS at the levels above) and *width* (the number of ASs at the same level) for the ten ASs that occupy the top five positions when $\alpha$ takes its two extreme values: $\alpha = 0$ and $\alpha = 1$. The AS numbers are matched to AS names using the WHOIS databases.**

| | | | | $\alpha = 0.00$ | | $\alpha = 0.01$ | | $\alpha = 0.05$ | | $\alpha = 0.10$ | | $\alpha = 0.50$ | | $\alpha = 1.00$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Percentage of invalid paths | | | | | | | | | | | | | | | |
| | | | | 12.75% | | 1.79% | | 0.69% | | 0.46% | | 0.36% | | 0.33% | |
| Top of reachability based hierarchy | | | | | | | | | | | | | | | |
| | AS # | name | degree | dep. | wid. | dep. | wid. | dep. | wid. | dep. | wid. | dep. | wid. | dep. | wid. |
| $\alpha = 0$ | 701 | UUNET | 2334 | 0 | 1 | 1 | 1 | 0 | 105 | 0 | 120 | 2 | 201 | 11 | 319 |
| | 7018 | AT&T | 1911 | 1 | 1 | 2 | 1 | 0 | 105 | 0 | 120 | 2 | 201 | 11 | 319 |
| | 1239 | Sprint | 1703 | 2 | 1 | 0 | 1 | 0 | 105 | 0 | 120 | 2 | 201 | 11 | 319 |
| | 3356 | Level 3 | 1228 | 3 | 1 | 3 | 1 | 0 | 105 | 0 | 120 | 2 | 201 | 11 | 319 |
| | 209 | Qwest | 1105 | 4 | 1 | 4 | 1 | 0 | 105 | 0 | 120 | 2 | 201 | 11 | 319 |
| $\alpha = 1$ | 14551 | UUNET | 35 | 128 | 1 | 137 | 2 | 138 | 1 | 151 | 1 | 260 | 2 | 0 | 1 |
| | 13987 | IBASIS Inc. | 3 | 1792 | 955 | 1802 | 963 | 1830 | 976 | 1847 | 971 | 1885 | 966 | 1 | 2 |
| | 8631 | Routing Arbiter | 48 | 108 | 1 | 123 | 1 | 122 | 2 | 0 | 120 | 0 | 1 | 1 | 2 |
| | 23649 | Hong Kong Teleport | 4 | 1792 | 955 | 1802 | 963 | 899 | 121 | 916 | 121 | 967 | 119 | 3 | 8 |
| | 4474 | Village Communications | 2 | 2747 | 16136 | 2765 | 16118 | 2806 | 16077 | 2818 | 16065 | 2 | 201 | 3 | 8 |

**Table 3: Summary statistics of the inferred relationships.**

| | Total $\lvert E\rvert$ | c2p links $\lvert E \setminus F \setminus S\rvert$ | p2p links $\lvert F \setminus S\rvert$ | s2s links $\lvert S\rvert$ |
|---|---|---|---|---|
| number of links | $38,282$ | $34,552$ | $3,553$ | $177$ |
| percentage | 100% | 90.26% | 9.28% | 0.46% |

hierarchy. In our experiments, this most realistic value of $\alpha$ is 0.01 (cf. Table 2).

***p2p relationships.*** We implement our p2p heuristic detailed in section 2.4, using the QUALEX [6] solver to approximate the MWIS problem. We then infer p2p relationships in the AS topology $G$ and construct the set $F$ of p2p links. After removing from $F$ the set $S$ of s2s links, we obtain our final answer that contains $3,553$ p2p links ($|F \setminus S| = 3,553$).

Table 3 summarizes our results for the whole graph $G(V, E)$.

## 3.3 Repository of AS Relationships and AS Rank

To make our results easily accessible and practically useful for the community, we automated our inference heuristics. We archive the inferred AS relationships on a weekly basis and make them available for download at the AS relationship data repository [7].

We also created an interactive web site [8] where we apply our automated relationship inferences to rank ASs based on their *customer cones*. We define the customer cone of an AS A as the AS A itself plus all the ASs that it can reach "for free", that is, following only p2c and s2s links. In other words, AS A's customer cone is A, plus A's customers, plus its customers' customers, and so on. We use the following three metrics to measure the size of customer cones: the number of ASs in the cone, the number of unique prefixes advertised by these ASs, and the number of /24 blocks in the union of these prefixes.

AS ranking is valuable not only for conceptual understanding of relative importance of Internet players, but also for network vendors and operators in prioritizing their customer lists and in solving other practical tasks. Users of our AS ranking have an option to group multiple sibling ASs into one organizational entity by specifying sibling groups either from the IRRs data, or as user-provided sibling lists.

# 4. SURVEY AND VALIDATION

Measuring, understanding, and modeling AS relationships in the Internet are challenging tasks hampered by the fact that these relationships are sensitive business information and generally considered private by ISPs. Nevertheless, without validation against truth, we have no way of evaluating the integrity of our heuristics.

Most of the previous works relied on implicit validation. However, indirect approaches are not always reliable. For example, the authors of [26, 4, 14, 20] used the number of valid paths as an indicator of the accuracy of the inferred relationships. As we discussed in section 2.1, a large number of valid paths does not necessarily result in a large number of correctly inferred AS relationships.

In contrast with previous works, we augment our validation based on implicit metrics (e.g., reachability, section 3.2) with the explicit data that we collected via private communications with engineers from the ASs under observation.

We contacted several ASs ranging from large continental or national ISPs, to content providers, and university networks. We sent the list of AS relationships that we inferred for a given AS to this AS's network administrator, peering negotiator, informed engineer, or researcher. We included three questions in our email inquiry:[4]

**Q1:** For the listed inferred AS relationships, specify how many are incorrect, and what are the correct types of the relationships that we mis-inferred?

**Q2:** What fraction of the total number of your AS neighbors is included in our list?

**Q3:** Can you describe any AS relationships, more complex than c2p, p2p, or s2s, that are used in your networks?

We performed the survey in the period between 06/07/05 and 06/30/05 and received answers from 38 out of the 78

[4]We also offered to sign a non-disclosure agreement (NDA) that protected peering information from being released to the public and regulated our data analysis to anonymizing the participating ASs. Only one organization (a government agency) required an NDA and two commercial ISPs did not have a policy in place (or the policy was not) to deal with such requests. They still helpfully provided us with general answers regarding what percent of peers we inferred incorrectly.

Table 4: **Validation of the inference results using the survey data. Each row shows the total number, number of correct, and percentage of correct inferred AS relationships.**

| | links | inferred c2p links | inferred p2p links | inferred s2s links |
|---|---|---|---|---|
| total number of | 3,724 | 3,070 | 623 | 31 |
| number of correct | 3,508 | 2,964 | 516 | 28 |
| percentage of correct | 94.2% | 96.5% | 82.8% | 90.3% |

ASs we contacted. Among these, 5 were tier-1 ISPs, 13 were smaller ISPs, 19 were universities, and 1 was a content provider. These ASs reported to us the true relationship types for 3,724 of our inferred AS relationships. The universities reported only 54 of those 3,724 relationships, whereas all the remaining relationships came from the ISPs and the content provider. The BGP-derived AS degrees for the universities ranged from 1 to 8, while for the remaining ASs it ranged from 1 to almost 2000.

## 4.1 Validation of inferred AS relationships

We validate our heuristics by counting the number of correctly inferred AS relationships. Among the 3,724 verified AS relationships, 82.6% were c2p, 16.1% were p2p, and 1.2% were s2s. Table 4 demonstrates that our heuristics correctly infer 96.5% of c2p, 82.8% of p2p, and 90.3% of s2s relationships. The total percentage of correctly inferred AS relationships is 94.2%. This accuracy level demonstrates that our heuristics produce reliable and veracious inferences of the true types of AS relationships in the Internet.

The data in our survey bear certain limitations and our results should be interpreted accordingly. First, the self-selection aspect of the sampling of ASs may induce biases into the resulting statistics. Second, the obtained 3,724 links with confirmed relationships represent 9.7% of the total number of links in our data. While acknowledging these limitations, we note that providing rigorous validation of inferred AS relationships is an extremely challenging task because of the difficulty in collecting ground-truth data against which one can check the inferences.

## 4.2 Missing AS links

In this section we analyze the relationships of the full set of adjacencies of the participating ASs, including the links that we do not see in BGP tables and, consequently, in our graph. The second question in our survey asks ASs for the ratio of the number of their neighbors in our AS topology data to the total number of AS neighbors they actually have. Out of the 38 ASs, 27 (3 of which were tier-1 ISPs) provided us not only with this ratio, but also with the types of relationships their ASs have with the missing neighbors. Out of the total of 1,114 true reported adjacencies, the BGP tables observe only 552. This finding agrees with the conclusion of previous works [9, 23] that a significant number of existing AS connections remain hidden from most BGP routing tables.

To improve our understanding of the missing AS links, we analyze the true relationships of these links. Figure 3 illustrates the per-relationship breakdown of the true and observed adjacencies of the ASs. It shows that we only see 38.7% out of the 865 p2p links, whereas we see 86.7% out of the 218 c2p links, and 93.3% of the 30 s2s links. This gap

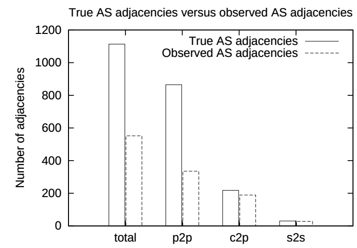



Figure 3: **Numbers of true and observed AS links for different types of AS relationships in the survey.**

demonstrates that BGP-derived AS topologies miss predominantly p2p links.

The reasons for this bias stem from the intrinsic nature of p2p relationships. In a p2p relationship, prefixes learned from a peer AS are not advertised to any providers. Consequently, a link between two p2p ASs is not seen (as a part of some AS path) at any upstream ASs. It follows that we can only observe a p2p link in the BGP tables of the customers or siblings of the p2p ASs. The periphery of the Internet has many small interconnecting ASs. Thus, in order to observe p2p links in the periphery, we should have a significant number and variety of BGP tables from these small ASs. BGP tables with small number of data feeds alone do not provide representative statistics of p2p links.

Figure 3 also shows that the majority of the 1,114 true adjacencies are in reality p2p: 865 (77.6%) are p2p, while only 218 (19.6%) and 30 (2.7%) are c2p and s2s, respectively. We thus face a large number of p2p relationships which appear to be very popular among small and medium size ASs. Interestingly, some tier-1 ISPs have several dozens or even hundreds of p2p relationships, frequently with ASs of smaller size.

Next, we seek to evaluate how representative the BGP-derived AS degrees are of the true AS degrees. In Figure 4 we plot the number of true AS adjacencies of the surveyed ASs versus the number of AS adjacencies derived from our BGP data. At the bottom-left corner of the diagram, 20 ASs[5] that are mainly university networks, have their true numbers of adjacencies close or identical to the measured numbers of adjacencies. We find that most of the adjacencies of these small ASs are c2p links. As we have seen above, our AS topology captures c2p links relatively well. Examining the rest of the diagram, we first observe that the percentage of missed adjacencies can be as large as 86.2%. The degrees for most of the highly connected ASs are under-sampled, half of them missing more than 70.5% links. Further examination of the missed AS links reveals that most of them are of p2p type, which is consistent with Figure 3.

Our results confirm the common assumption that p2p relationships, while widespread in the Internet, are not amenable to observation from few BGP feeds and can render BGP-derived AS degrees significantly smaller than the true AS degrees. On one hand, the identified deficiencies should inspire further pursuit of representative statistics on the number of p2p links, for example via the deployment of distributed measurement infrastructures [25]. On the other hand, we emphasize that these missing links do not qualita-

[5]Note that points (1,1) and (2,2) in the figure correspond to more than one AS.

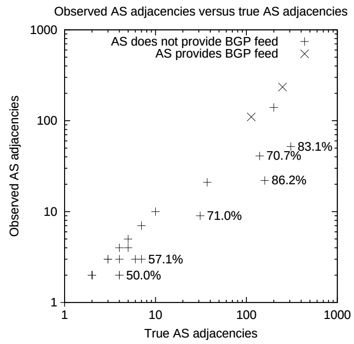



**Figure 4: True vs. observed degrees of the surveyed ASs. We mark 1) the percentage of missed links for a few ASs with the highest values of this percentage; and 2) the ASs that did (not) provide feeds in our BGP data.**

tively change a set of frequently-used *statistical* characteristics of the BGP-derived AS topologies [12, 18, 10].

## 4.3 Complex AS relationships

The last question of our survey asks about more complex configuration scenarios the AS may be using. From the responses we learn that although the majority of AS relationships are simply c2p or p2p, in few cases their configurations are either more specialized versions of the basic c2p or p2p types, or a hybrid of c2p and p2p (c2p/p2p).

For example, the backup provider relationship is a specialized variant of the basic c2p relationship. In this case, a customer AS has a c2p relationship with a provider AS, but this relationship only allows traffic to flow during an emergency situation such as a disruption of connectivity to the main upstream provider of the customer AS. Hence, the backup provider relationship is a temporally conditioned version of the c2p relationship.

A hybrid c2p/p2p relationship occurs when two ISPs interconnect at multiple peering points and have different types of relationships at these points. For instance, two ISPs can have a p2p relationship at a peering point in Europe and a c2p relationship at a peering point in the U.S. Another flavor of a hybrid c2p/p2p relationship is when two ASs have different types of relationships for different IP prefixes. In this case the ISPs may have a p2p relationship for one set of IP prefixes and a c2p relationship for another set of IP prefixes. These examples of hybrid c2p/p2p relationships illustrate that AS relationships may involve also spatial and/or prefix-based aspects.

In other words, based on the configuration descriptions we collected in our survey, we conclude that AS relationships vary across the following three dimensions: space, time, and prefix. Therefore, to fully characterize a relationship between a pair of ASs, including more complex relationship scenarios, one has to gain access to information identifying the ASs' policy configurations per peering location(s), per time, and per prefix. Although limited per-prefix and per-time data are presently available, identifying more complex relationships for the complete Internet AS topology is a formidable task as it likely requires significantly more sources of more detailed data than currently available.

A natural question that arises is how a c2p, p2p, or s2s inference for an AS link, which in fact is a more complex relationship, distorts reality and how prominent this artifact is. For a backup relationship, a c2p, p2p, or s2s inference misses the temporal component of the relationship. For a hybrid c2p/p2p relationship, a c2p or p2p inference misses one part of the hybrid relationship. Such artifacts, however, do not occur often. Indeed, more complex relationships are likely to exist only between large ASs. However, most AS links in the Internet connect small ASs to large ASs or connect small ASs to each other [18]. Such AS pairs are known to employ consistent routing policies over their usually single or sometimes multiple peering points.

# 5. CONCLUSION

The relationships among ASs in the Internet represent the outcome of policy decisions governed by technical and business factors of the global Internet economy. Precise knowledge of these relationships is therefore an essential building block needed for any reliable and effective analysis of technical and economic aspects of the global Internet, its structure, and its growth.

In this work we introduced novel heuristics that significantly improve the state-of-the-art in inferring c2p relationships and carefully address the particularly difficult problems of inferring p2p and s2s relationships from currently available data.

In comparison with previous studies that primarily used implicit validation of inferred AS relationships, we go a step further. In addition to implicit validation, we make an effort to collect explicit ground-truth data via direct communication with ASs. Using the true relationships of 3,724 links we confirmed that our heuristics achieve very high accuracy of 96.5% (c2p), 82.8% (p2p), and 90.3% (s2s) of correctly inferred relationships, with the overall accuracy being 94.2%. Given the overall difficulty of validating inference results and that surveys like the one in this paper tend to be extremely involved procedures in practice, we hope that our work will serve to cast ponderable confidence on such inference studies.

Using the data of our survey we followed previous studies [9, 23] in finding that measured AS topologies miss a significant number of AS links. We take this result further by verifying the commonly held assumption that most of the missing links are of p2p type.

Easy access to accurate AS relationship data is essential to a variety of studies dealing with aspects of Internet architecture and policy. To support the research community with as objective data as possible, we have automated our heuristics and calculate and archive AS relationships on a weekly basis [7]. As an example of using the inferred relationships we provide a ranking of ASs [8].

From the perspective of empirical research, the global Internet compares to an economy or an ecosystem. As such, cross-disciplinary approaches that combine knowledge of the Internet macroscopic structure with insights into its economics and policy are required to advance our understanding of its technical and economical viability. We believe our work significantly benefits Internet research that strives to build more encompassing models validated against reliable and accurate data.

## Acknowledgment

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