Aspects of Axion $F(R)$ Gravity

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We provide a compact review on recent developments on axion $F(R)$ gravity. The axion field is a string theory originating theoretical particle that is a perfect candidate for low-mass particle dark matter. In this review we present how a viable inflationary phenomenology and a viable late-time evolution can be described by an axion $F(R)$ gravity theory, in which the $F(R)$ gravity part can drive in a geometric way the inflationary and the late-time era, and the axion field behaves as dark matter, with its energy density ρ_a behaving as a function of the scale factor as $\rho_a \sim a^{-3}$. We also briefly discuss the effect of a non-trivial axion Chern-Simons coupling on the inflationary phenomenology of the R^2 model. Finally, we briefly discuss the effects of a non-minimal coupling of the axion field with the curvature on neutron stars, and also the propagation of gravity waves in Chern-Simons axion gravity.

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I. INTRODUCTION

Dark matter and dark energy are two of the most persisting problems in theoretical physics and theoretical cosmology. Dark energy is a name that characterizes the still unknown mechanism that drives the acceleration of the Universe, firstly observed in the late 90's [1]. This observational result was one of the most surprising phenomena ever observed, and the whole problem becomes even more difficult at present time, since observational data indicate that the expansion rate of the Universe based on low redshift sources is different in comparison to the data obtained by the Cosmic Microwave Background radiation [2]. This problem is nowadays known as the H_0 tension problem [3, 4], and there exist a plethora of data coming from different sources that verify this tension in the exact value of the Hubble rate. With regard to dark matter, the emergence of the dark matter concept appeared after the first galactic rotation curves were studied, and its existence is a theoretical proposal even up to date, since no dark matter particle has ever been found. The experiments for nearly three decades focused on finding a Weakly Interacting Massive Particle (WIMP) with mass MeV or even hundreds GeV with no results. For reviews and some characteristic articles related to the heavy mass dark matter particle searches can be found in Refs. [5–10]. The last five years, experiments and observations are focused on finding small mass WIMP, in the mass range of eV or even much smaller than the eV scale. The most important small mass WIMP candidate is the axion [11–14], which is a string theory and quantum chromodynamics (QCD) originating particle. The QCD axion is accompanied by many theoretical problems, since the primordial $U(1)$ Peccei-Quinn symmetry is not broken during inflation, and thus the most plausible axion scenario for viable model building is the misalignment axion scenario [11]. In the literature there exist quite many articles focusing on the axion dark matter perspective, see for example [11, 15–59]. In addition to the theoretical works, there exist additional experimental and theoretical proposals based on the assumption of the existence of axion particle dark matter [60–68]. In the majority of the experimental proposals, the axion-photon conversion in a magnetic field feature is utilized [69–71]. A particularly interesting approach to low mass scalar dark matter particles is the effect of the latter on gravity waves coming from astrophysical sources [38–40] or even from primordial gravitational waves [72], where the possibility of finding non-trivial polarizations is mainly studied or stressed [73, 74]. The non-trivial polarization of gravitational waves in theories involving Chern-Simons axionic couplings is also discussed in the literature [72]. For an important stream of paper on Chern-Simons coupling terms see [18, 72, 75–87]. Moreover, the axions can be related with the H_0-tension problem [88].

Modified gravity [89–94] can successfully describe in a viable way in many cases, the dark energy and dark matter problems. For an important stream of articles on the modified gravity descriptions of the dark sector of our Universe, see for example [95–105]. Although certain dynamical aspects of dark matter can be explained by modified gravity [106, 107], observational data coming from the Bullet cluster indicate that dark matter might have eventually a particle nature. To our opinion, unless supersymmetry is found in the Large Hadron Collider, which may offer new insights in the particle dark matter problem, the axion is particle dark matter's last hope.

In this short review we shall present the latest developments on axion $F(R)$ gravity phenomenology. Particularly, we shall present several models of $F(R)$ gravity in the presence of a misalignment axion scalar field, and we shall demonstrate how a viable inflationary phenomenology can be achieved. In the simplest case, the axion scalar and the $F(R)$ gravity are decoupled, and due to the fact that at early times the misalignment axion is frozen in its primordial vacuum expectation value, the inflationary era is controlled by the R^2 part of the $F(R)$ gravity, and as the Hubble rate drops and becomes approximately equal to the axion scalar mass, the axion begins to oscillate, in a way so that its energy density ρ_a scales as $\rho_a \sim a^{-3}$. Thus, the axion field behaves perfectly as dark matter, and this behavior continues until late times. The $F(R)$ gravity contains terms which also affect the late-time era, and thus we will show that a viable phenomenology, compatible with the ΛCDM model in most of the cases, is obtained. Qualitatively similar results are obtained for a model with a non-minimal coupling between the axion and the one of the $F(R)$ gravity components. Moreover, we discuss how a Chern-Simons coupling can reduce the tensor-to-scalar ratio of the Starobinsky model [108] to a great extent. Finally, we demonstrate how a non-minimal coupling between the scalar axion and the curvature can affect the maximum mass of a neutron star, while leaving the radius unaffected.

II. MISALIGNMENT AXION-$F(R)$ GRAVITY MODEL WITH NO AND WITH NON-MINIMAL COUPLING

A. Model I

In this subsection we analyze the first axion $F(R)$ model, the details on this model can be found in Ref. [59]. The first axion-$F(R)$ gravity model we shall present, has the following action [20, 59]

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}F(R) - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) + \mathcal{L}_m\right] , \tag{1}$$

with $\kappa^2 = \frac{1}{8\pi G} = \frac{1}{M_p^2}$, and G is Newton's gravitational constant while M_p is the reduced Planck mass. The Lagrangian $\mathcal{L}_m$ represents all the perfect matter fluids present. The functional form of the $F(R)$ gravity has the following form,

$$F(R) = R + \frac{1}{M^2}R^2 - \gamma\Lambda\Big(\frac{R}{3m_s^2}\Big)^\delta , \tag{2}$$

In addition m_s in Eq. (2) stands for $m_s^2 = \frac{\kappa^2\rho_m^{(0)}}{3}$, and we choose the parameter δ to be in the interval $0 < \delta < 1$. For phenomenological reasons, the parameter M is $M = 1.5 \times 10^{-5}\left(\frac{N}{50}\right)^{-1} M_p$, with N standing for the e-foldings number. For a flat Friedmann-Robertson-Walker (FRW) metric,

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3}\left(dx^i\right)^2 , \tag{3}$$

the equations of motion are,

$$3H^2F_R = \frac{RF_R - F}{2} - 3H\dot{F}_R + \kappa^2\left(\rho_r + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right) , \tag{4}$$

$$-2\dot{H}F_R = \kappa^2\dot{\phi}^2 + \ddot{F}_R - H\dot{F}_R + \frac{4\kappa^2}{3}\rho_r ,$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{5}$$

with $F_R = \frac{\partial F}{\partial R}$. Moreover the "dot" and the "prime" denote differentiation with respect to the cosmic time and the scalar field respectively, and also the only matter fluid present is that of radiation $p_r = \frac{1}{3}\rho_r$. As we show in the next section, the axion scalar field will act as the dark matter perfect fluid.

The dynamical evolution of the misalignment axion field is crucial for all the axion $F(R)$ gravity models we shall develop. The scalar field potential of the misalignment axion after the primordial $U(1)$ Peccei-Quinn symmetry was broken, has the form $V(\phi) \simeq \frac{1}{2}m_a^2\phi_i^2$, where ϕ_i is the vacuum expectation value of the axion field after the $U(1)$ symmetry is broken. During the early times when inflation occurs, the dynamical evolution of the axion is overdamped, and satisfies the initial conditions $\ddot{\phi}(t_i) \simeq 0$, $\dot{\phi}(t_i) \simeq 0$ and $\phi(t_i) \equiv \phi_i = f_a\theta_a$ [11], where t_i is a characteristic time scale during inflation, and f_a is the axion decay constant, while θ_a is the misalignment angle. The axion remains frozen as long as $H \gg m_a$, however, when $H \sim m_a$, axion oscillations occur. Assuming a slowly varying evolution, it can be shown that the axion energy density scales as $\rho_a \sim a^{-3}$ [11, 59].

1. The Inflationary Era: R^2 Gravity Domination

Now let us show that indeed during the inflationary era, the axion field has no effect on the dynamics. The equations of motion (27) during inflation are,

$$3H^2\left(1+\frac{2}{M^2}R-\delta\gamma\Big(\frac{R}{3m_s^2}\Big)^{\delta-1}\right)=\frac{R^2}{2M}+(\gamma-\gamma\delta)\frac{\left(\frac{R}{3m_s^2}\right)^{\delta}}{2}-3H\dot{R}\Big(\frac{2}{M^2}-\gamma\delta(\delta-1)\Big(\frac{R}{3m_s^2}\Big)^{\delta-2}\Big)+\kappa^2\left(\rho_r+\frac{1}{2}\kappa^2\dot{\phi_i}^2+\frac{1}{2}m_a^2\phi_i^2\right). \tag{6}$$

For late-time phenomenological reasons, we choose,

$$\gamma=\frac{1}{0.5},\quad \delta=\frac{1}{100}, \tag{7}$$

and $\Lambda \simeq 11.895\times 10^{-67}\mathrm{eV}^2$. Also for $N\sim 60$, M is $M\simeq 3.04375\times 10^{22}\mathrm{eV}$. In addition, during inflation, $H=H_I\sim 10^{13}\mathrm{GeV}$ so $R\sim 1.2\times 10^{45}\mathrm{eV}^2$. The radiation fluid term is $\kappa^2\rho_r\sim e^{-N}$ so it can safely be ignored, and also since $\phi_i=\mathcal{O}(10^{15})\mathrm{GeV}$ we have $m_a\simeq\mathcal{O}(10^{-14})\mathrm{eV}$. In effect, the potential term is $\kappa^2V(\phi_i)\sim\mathcal{O}(8.41897\times 10^{-36})\mathrm{eV}^2$. The curvature scalar related terms are of the order$R\sim 1.2\times\mathcal{O}(10^{45})\mathrm{eV}^2$, and $R^2/M^2\sim\mathcal{O}(1.55\times 10^{45})\mathrm{eV}^2$, and in addition, $\sim\left(\frac{R}{3m_s^2}\right)^{\delta}\sim\mathcal{O}(10)$, and also $\sim\left(\frac{R}{3m_s^2}\right)^{\delta-1}\sim\mathcal{O}(10^{-111})$ and finally $\sim\left(\frac{R}{3m_s^2}\right)^{\delta-2}\sim\mathcal{O}(10^{-223})$. From the above it is evident that the Ricci scalar related terms dominate the inflationary evolution, and specifically the R^2 term, hence the model is reduced to the R^2 model, which yields a viable inflationary phenomenology compatible with the observational data coming from Planck 2018 [2].

2. Dark Energy Era

Let us now consider the late-time era, and the equations of motion can be cast in the Einstein field equations form for a FRW metric as follows,

$$\begin{aligned} 3H^2&=\kappa^2\rho_{tot}\,, \\ -2\dot{H}&=\kappa^2(\rho_{tot}+P_{tot})\,, \end{aligned} \tag{8}$$

with $\rho_{tot}=\rho_\phi+\rho_G+\rho_r$ being the total energy density and $P_{tot}=P_r+P_a+P_G$ being the total pressure of the cosmological fluid consisting of the axion scalar field, the radiation perfect fluid and the geometric contribution coming from the $F(R)$ gravity. In our case ρ_G and P_G are equal to,

$$\kappa^2\rho_G=\frac{F_RR-F}{2}+3H^2(1-F_R)-3H\dot{F}_R\,, \tag{9}$$

$$\kappa^2P_G=\ddot{F}_R-H\dot{F}_R+2\dot{H}(F_R-1)-\rho_G\,. \tag{10}$$

Using the redshift z as a dynamical variable,and by introducing the statefinder quantity $y_H(z)$ [109, 110],

$$y_H(z)=\frac{\rho_G}{\rho_m^{(0)}}\,, \tag{11}$$

with $\rho_m^{(0)}$ being the cold dark matter energy density at present time. We can write $y_H(z)$ in terms of the Friedmann equation (8),

$$y_H(z)=\frac{3H^2}{\kappa^2\rho_m^{(0)}}-\frac{\dot{\phi}^2}{2\rho_m^{(0)}}-\frac{V(\phi)}{\rho_m^{(0)}}-\frac{\rho_r}{\rho_m^{(0)}}\,. \tag{12}$$

With regard to the radiation energy density this scales as $\rho_r=\rho_r^{(0)}a^{-4}$, with $\rho_r^{(0)}$ being the value of the radiation energy density at present time , hence $\frac{\rho_r}{\rho_m^{(0)}}=\chi(1+z)^4$, with $\chi=\frac{\rho_r^{(0)}}{\rho_m^{(0)}}\simeq 3.1\times 10^{-4}$. We can write Eq. (12) as follows,

$$y_H(z)=\frac{H^2}{m_s^2}-(1+z)^3-\chi(1+z)^4\,. \tag{13}$$

with $m_s^2 = \frac{\kappa^2 \rho_m^{(0)}}{3} = H_0 \Omega_c = 1.37201 \times 10^{-67} \mathrm{eV}^2$. The Friedmann equation of motion can be cast in terms of the statefinder function $y_H(z)$ as follows [110],

$$\frac{d^2 y_H(z)}{dz^2} + J_1 \frac{d y_H(z)}{dz} + J_2 y_H(z) + J_3 = 0 \,, \tag{14}$$

with the functions J_1, J_2 and J_3 being defined in the following way,

$$\begin{aligned} J_1 &= \frac{1}{z+1}\left(-3 - \frac{1-F_R}{\left(y_H(z) + (z+1)^3 + \chi(1+z)^4\right) 6 m_s^2 F_{RR}}\right) \,, \\ J_2 &= \frac{1}{(z+1)^2}\left(\frac{2-F_R}{\left(y_H(z) + (z+1)^3 + \chi(1+z)^4\right) 3 m_s^2 F_{RR}}\right) \,, \\ J_3 &= -3(z+1) - \frac{(1-F_R)\Big((z+1)^3 + 2\chi(1+z)^4\Big) + \frac{R-F}{3m_s^2}}{(1+z)^2\Big(y_H(z) + (1+z)^3 + \chi(1+z)^4\Big) 6 m_s^2 F_{RR}} \,, \end{aligned} \tag{15}$$

with $F_{RR} = \frac{\partial^2 F}{\partial R^2}$. We shall solve numerically the above equation, focusing on the redshift interval $z = [z_i, z_f]$ with $z_i = 0$ and $z_f = 10$. In addition we shall use the following initial conditions at redshift $z_f = 10$,

$$y_H(z_f) = \frac{\Lambda}{3m_s^2}\left(1 + \tilde{\gamma}(1+z_f)\right) \,, \quad \frac{d y_H(z)}{dz}\Big|_{z=z_f} = \tilde{\gamma}\frac{\Lambda}{3m_s^2} \,, \tag{16}$$

where $\tilde{\gamma}$ is $\tilde{\gamma} = \frac{1}{10^3}$. Using the numerical solution for $y_H(z)$ we can compare several statefinder quantities of the axion $F(R)$ model with the ΛCDM model. Moreover $\Omega_r/\Omega_M \simeq \chi$, where χ is defined below Eq. (12). In the literature many statefinder quantities are studied [111], however here we shall consider only the deceleration parameter $q = -1 - \frac{\dot{H}}{H^2}$. In Fig. 1 we present the comparison of the deceleration parameter q as a function of the redshift for the axion $F(R)$ gravity model (blue curve) and for the ΛCDM model (red curve), for $z = [0, 9]$. As it can be seen the models are indistinguishable, and only for large redshifts $z > 6$ the deceleration parameter of the axion $F(R)$ gravity models shows some oscillating behavior.

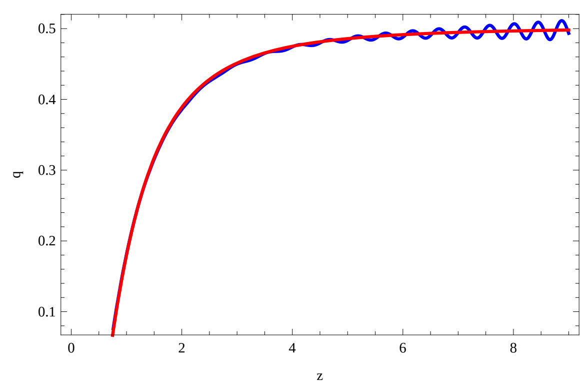

FIG. 1: The deceleration parameter q of the axion $F(R)$ gravity model (blue curve) and of the ΛCDM model for $\tilde{\gamma} = 1/10^3$ for $z = [0, 9]$.

B. Model II

The second axion $F(R)$ gravity model which we shall consider has the following gravitational action [20],

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2} f(R) + \frac{1}{2\kappa^2} h(\phi) G(R) - \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - V(\phi)\right] \,, \tag{17}$$

We introduce the notation for simplicity,

$$\mathcal{F}(R, \phi) = \frac{1}{\kappa^2} f(R) + \frac{1}{\kappa^2} h(\phi) G(R) \,. \tag{18}$$

Varying the gravitational action with respect to the metric we get,

$$3H^2F=\frac{1}{2}\dot{\phi}^2+\frac{RF-\mathcal{F}+2V}{2}-3H\dot{F}\,, \tag{19}$$
$$-3FH^2-2\dot{H}F=\frac{1}{2}\dot{\phi}^2-\frac{RF-\mathcal{F}+2V}{2}+\ddot{F}+2H\dot{F}\,,$$

with $F=\frac{\partial\mathcal{F}}{\partial R}$, and the variation with respect to the scalar field is,

$$\ddot{\phi}+3H\dot{\phi}+\frac{1}{2}\left(-\mathcal{F}'(R,\phi)+2V'(\phi)\right)=0\,. \tag{20}$$

We choose the $f(R)$ gravity to be the same as in the previous section, and for late-time phenomenological reasoning, we choose $G(R)$ to be,

$$G(R)\sim R^{\gamma}\,, \tag{21}$$

with γ taking values $0<\gamma<0.75$. The early time era behavior of the model is similar to the model presented in the previous section, since the R^2 model dominates the evolution of the Universe [20]. At late-times the $\sim R^2$ terms are subdominant, thus the Friedmann equation becomes,

$$3H^2\left(1+h(\phi)G'(R)\right)\simeq\frac{1}{2}\kappa^2\dot{\phi}^2+2V\kappa^2-3H\dot{R}G''(R)h(\phi)\,. \tag{22}$$

Thus at late times, the dominant terms in the left hand side of Eq. (22) are $\sim h(\phi)G'(R)$ since $G'(R)\sim R^{\gamma-1}$ due to the fact that $0<\gamma<0.75$. On the other hand, in the right hand side of Eq. (22), the term $-3H\dot{R}G''(R)h(\phi)$ dominates, since it contains powers of the curvature of the form $G''(R)\sim R^{\gamma-2}$. In effect, the leading order form of the Friedmann equation reads [20],

$$RH\simeq(1-\gamma)\,\dot{R}\,. \tag{23}$$

By using $R=12H^2+6\dot{H}$ the solution of the resulting differential equation at leading order is approximately,

$$H(t_0)\simeq\frac{\sqrt{2}\sqrt{1-\gamma}\sqrt{\Lambda}}{\sqrt{3-4\gamma}}\,, \tag{24}$$

so we obtain a de Sitter late-time evolution. Hence it is obvious that the axion-$f(R)$ gravity non-minimal coupling controls the late-time era.

C. Effect of Chern-Simons Axion Terms on the R^2 Model

In this section we shall demonstrate how a Chern-Simons coupling can reduce the tensor-to-scalar ratio of the standard R^2 model. The action is [17],

$$\mathcal{S}=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}f(R)-\frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi-V(\phi)+\frac{1}{8}\nu(\phi)R\tilde{R}\right]\,, \tag{25}$$

with $R\tilde{R}=\epsilon^{abcd}R_{ab}^{ef}R_{cdef}$, and with ϵ^{abcd} being the totally antisymmetric Levi-Civita tensor. We shall assume that $f(R)$ will have the form,

$$f(R)=R+\frac{1}{M^2}R^2\,. \tag{26}$$

The equations of motion for a FRW metric are,

$$3H^2F=\kappa^2\frac{1}{2}\dot{\phi}^2+\frac{RF-f+2V\kappa^2}{2}-3H\dot{F}\,, \tag{27}$$
$$-3FH^2+2\dot{H}F=\kappa^2\frac{1}{2}\dot{\phi}^2-\frac{RF-f+2V}{2}+\ddot{F}+2H\dot{F}\,.$$

The Chern-Simons term does not affect the background field equations, and issue stressed also in the literature [112, 113]. As it was shown in [112, 113] only the tensor perturbations are affected by the Chern-Simons coupling, while the scalar perturbations remain unaffected. It can be shown that the tensor-to-scalar ratio and the spectral index are, [17],

$$n_s = 1 - \frac{2}{N}, \;\; r \simeq \frac{r_s^v}{2} \left(\frac{1}{|1 - \frac{\kappa^2 x}{F}|} + \frac{1}{|1 + \frac{\kappa^2 x}{F}|} \right) . \tag{28}$$

with $r_s^v = 48\epsilon_1^2$ being the tensor-to-scalar ratio of the vacuum $f(R)$ gravity. The effect of the Chern-Simons term is quantified by the presence of the term $\sim \frac{\kappa^2 x}{F}$ in the tensor-to-scalar ratio. The value of the tensor-to-scalar ratio for the vacuum $f(R)$ gravity case is $r_s^v = 0.0033$, however the Chern-Simons induced term can further reduce the value of the tensor-to-scalar ratio, for example if $\frac{\kappa^2 x}{F} = \mathcal{O}(3 \times 10^8)$, the tensor-to-scalar ratio becomes $r = \mathcal{O}(10^{-11})$. Thus we demonstrated that the tensor-to-scalar ratio of the R^2 model can be further reduced by the Chern-Simons term.

III. BRIEF DISCUSSION ON FUTURE PERSPECTIVES: NEUTRON STARS AND GRAVITATIONAL WAVES

In this compact review we studied the phenomenology of axion $F(R)$ gravity models. What remains to be done in this type of theories is to appropriately study the reheating era. During that era we expect the $F(R)$ gravity to dominate this era, since the axion scalar scales as a dark matter particle. In addition, another non-trivial study is related to the matter curvature perturbations. Both the axion field and the $F(R)$ gravity have a non-trivial effect on the matter curvature perturbations, and this study is very relevant and necessary since the growth index is strongly related to matter curvature perturbations. A vital study and very timely is related to calculating the modified gravity effects on neutron stars [114]. As was shown in [114], the axion field can affect the maximum allowed mass for the neutron star. In addition, in [18], it was shown that the axion field causes inequivalent propagation in the circularly polarized gravitational wave modes. Both the neutron star and gravity waves issues are timely, and perhaps should be further be studied in the axion perspective.

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